Lab 5 - Joseph Riley Guest - MAT 275 Lab

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The Mass-Spring System

EX 1

A)

```
omega = 3.4; period = 2*pi/omega; % declaring omega and period
omega0  = 4; c = 2; % declaring omega not and c
alpha = ((c*omega)/(omega0.^2-omega.^2)); % calculating alpha
alpha_deg = rad2deg(alpha); % converting alpha to degrees
period                          % display period
alpha                           % display alpha in radians
alpha_deg                       % display alpha in degrees
```

```
period =

    1.8480


alpha =
```

```
    1.5315


alpha_deg =

   87.7503
```

B)

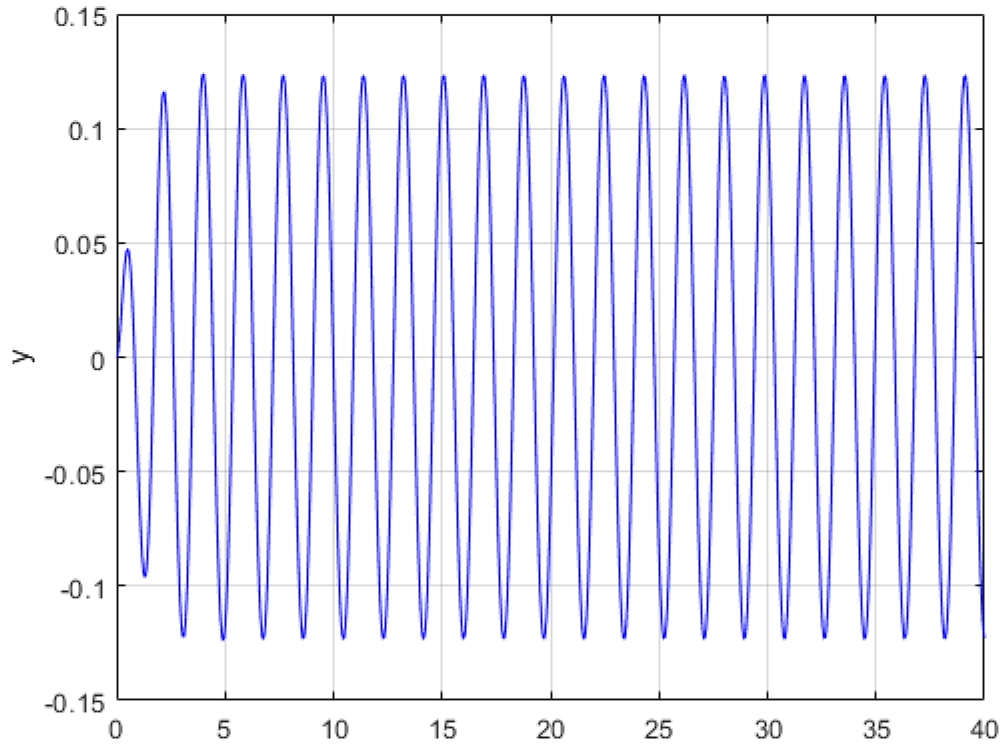
Looking at the graph below it is obvious that it is an exponentially decreasing oscillation.

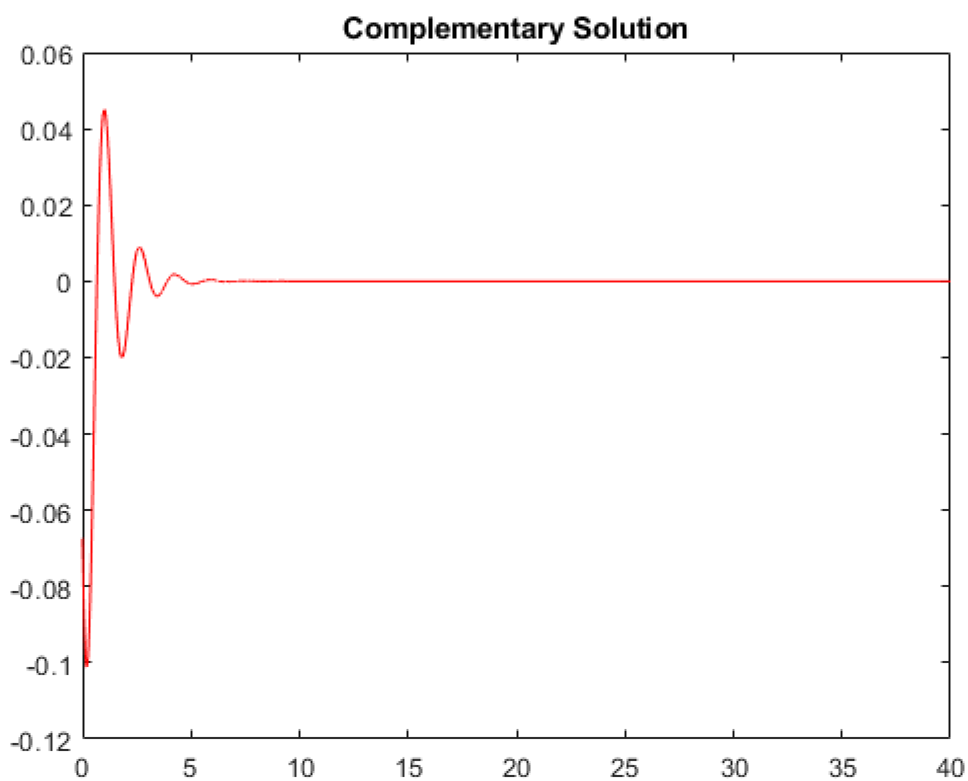
```
LAB06ex1;
type LAB06ex1;

computed amplitude of forced oscillation = 0.12313
theoretical amplitude = 0.12313

clear all; %this deletes all variables
omega0 = 4; c = 2; omega = 3.4;
param = [omega0,c,omega];
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 40;
options = odeset('AbsTol',1e-10,'relTol',1e-10);
[t,Y] = ode45(@f,[t0,tf],Y0,options,param);
y = Y(:,1); v = Y(:,2);
figure(1)
plot(t,y,'b-'); ylabel('y'); grid on;
t1 = 13; i = find(t>t1);
C = (max(Y(i,1))-min(Y(i,1)))/2;
disp(['computed amplitude of forced oscillation = ', num2str(C)]);
Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);
disp(['theoretical amplitude = ', num2str(Ctheory)]);
%---------------- added code to lab06ex1.m-----------------------
alpha_rad = atan((c*omega)/(omega0.^2-omega.^2)); % calculating alpha
 in radians
yc = y-Ctheory*cos(omega*t-alpha_rad); % declaring complementary
 solution yc
figure(2)
plot(t, yc, 'r-');
title('Complementary Solution');

%----------------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
end
```


0.15
0.1
0.05
0
-0.05
-0.1
-0.15
y
0
5
10
15
20
25
30
35
40

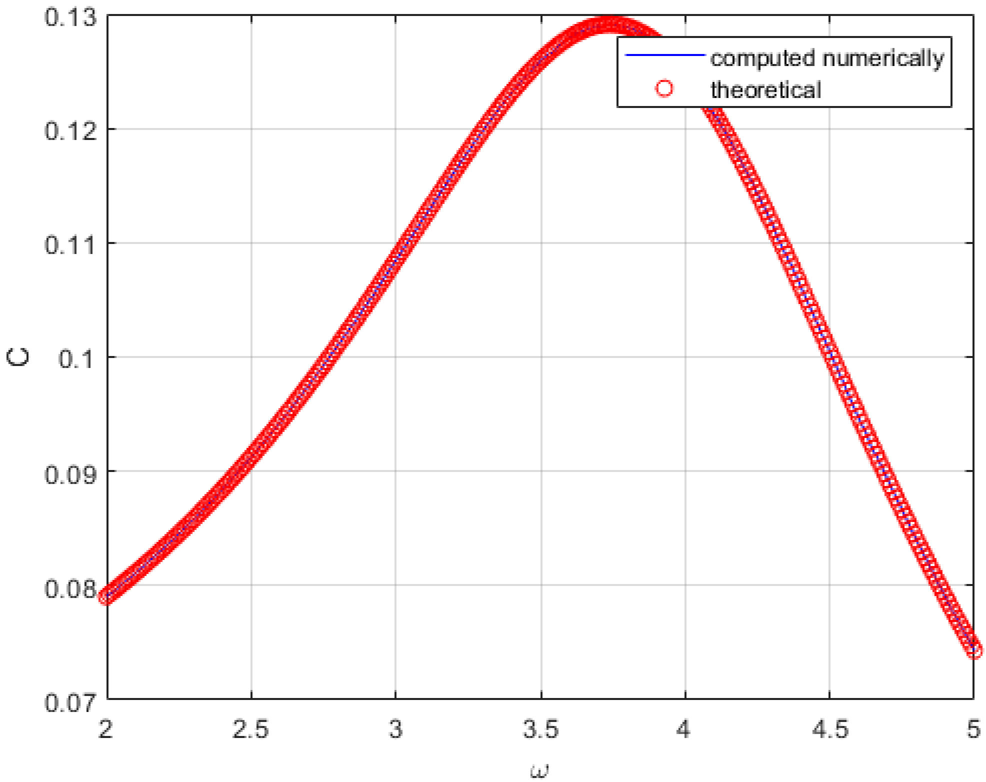

Complementary Solution
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
0
5
10
15
20
25
30
35
40

EX 2

A)

```
LAB06ex2;
type LAB06ex2;


clear all; %this deletes all variables
omega0 = 4; c = 2;
OMEGA =2:0.01:5;
C = zeros(size(OMEGA));
Ctheory = zeros(size(OMEGA));
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf =40; t1 = 13;
for k = 1:length(OMEGA)
   omega = OMEGA(k);
   param = [omega0,c,omega];
   [t,Y] = ode45(@f,[t0,tf],Y0,[],param);
   i = find(t>t1);
   C(k) = (max(Y(i,1))-min(Y(i,1)))/2;
   Ctheory(k) = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);; % FILL-IN
 the formula for Ctheory
end
figure(1)
plot(OMEGA, C, 'b-', OMEGA, Ctheory, 'ro'); grid on; % FILL-IN to plot
 C and Ctheory as a function of OMEGA
xlabel('\omega'); ylabel('C');
legend('computed numerically','theoretical')
%-----------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
end
```



B)

By looking at the graph and zooming in, you can see that the maximums of omega = 3.74 and C = 0.1291.

C)

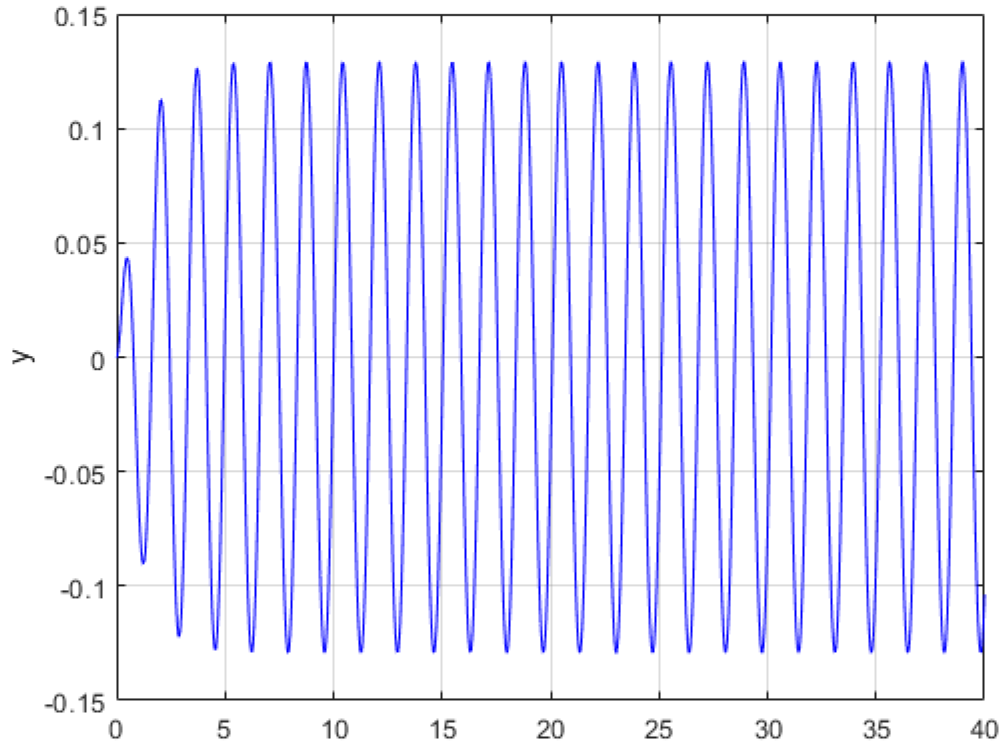
$\omega = 3.74$

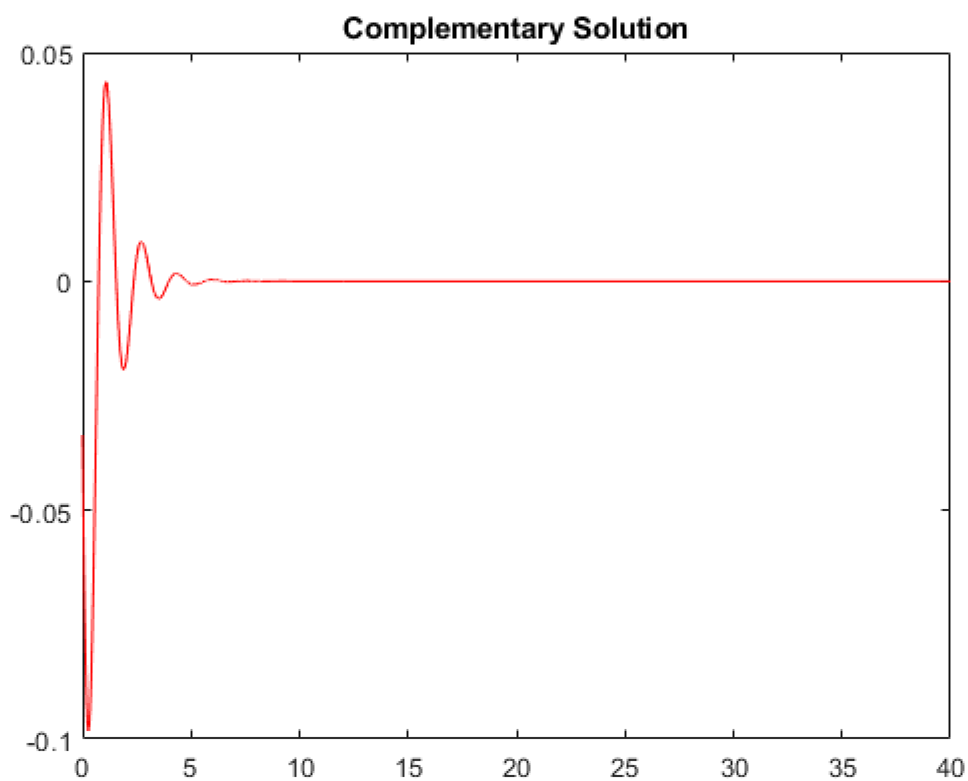
D)

The amplitude of the forced oscillation is 0.1291, which is larger than the amplitude of 0.12313. Running this file with any other value for omega, you can expect the amplitude of the solution to be smaller.

```
LAB06ex2d;
```

```
computed amplitude of forced oscillation = 0.1291
theoretical amplitude = 0.1291
```

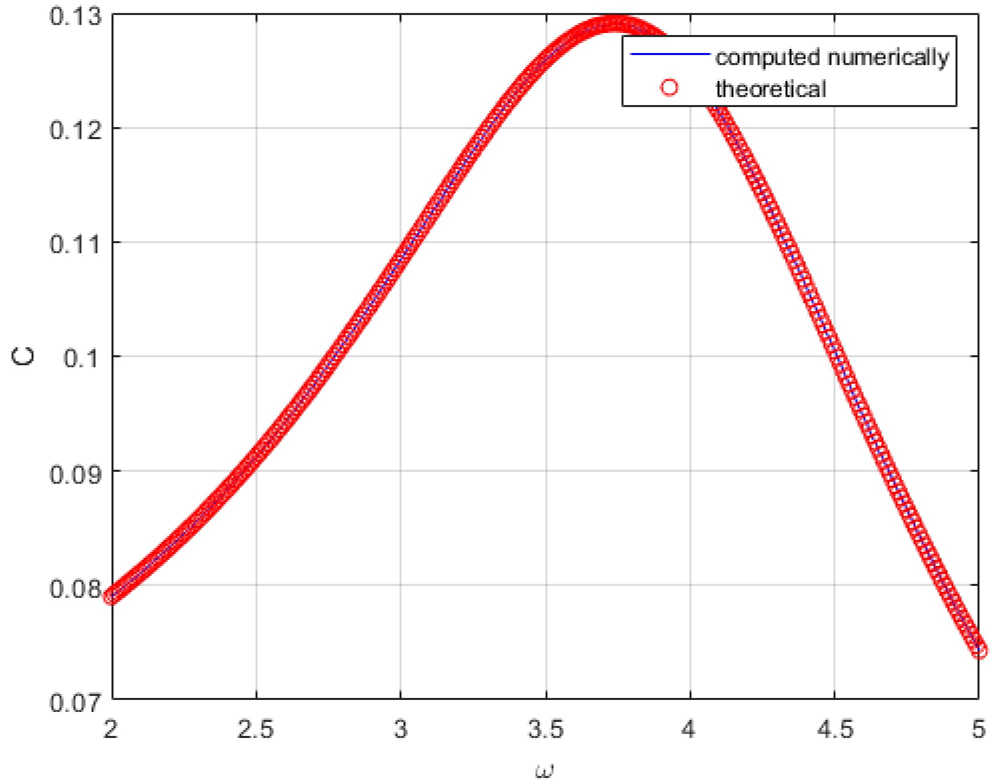

0.15
0.1
0.05
0
-0.05
-0.1
-0.15
y
0
5
10
15
20
25
30
35
40


Complementary Solution
0.05
0
-0.05
-0.1
0
5
10
15
20
25
30
35
40

E)

Modifying the initial conditions does not change the graphs or the outputs. Below is the output with initial conditions changed to y(0)=1 and v(0)=2.

```
LAB06ex2e;
```

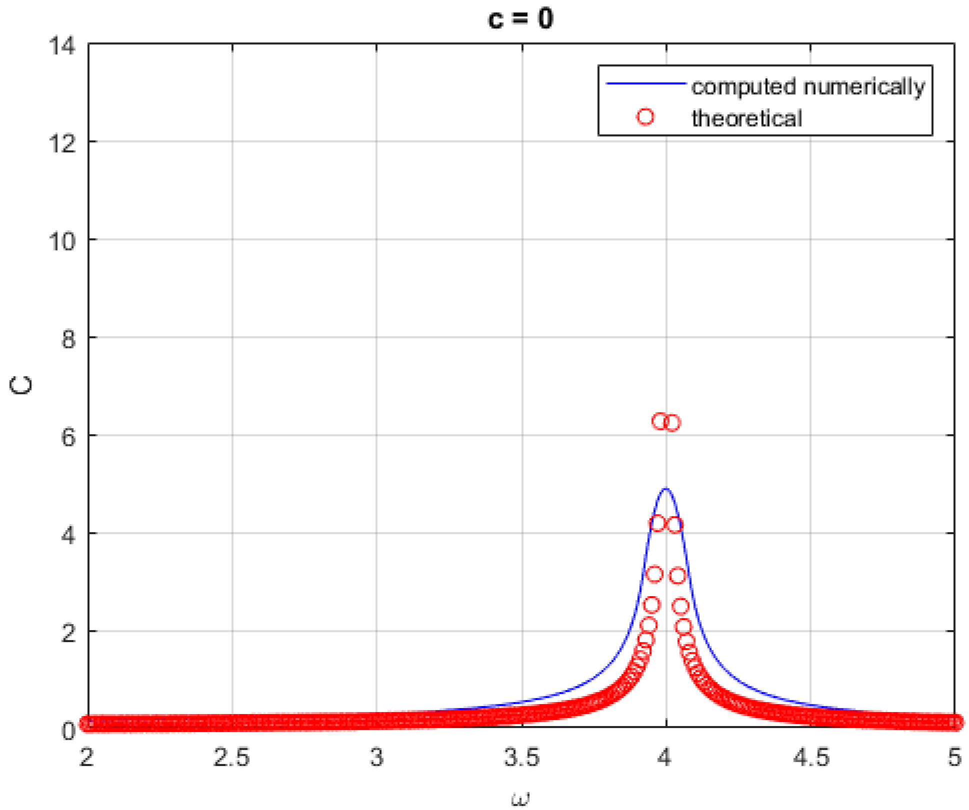


EX 3

A)

Omega = 3.99 at the maximal amplitude with corresponding C value of 12.52. Omega is the same as Omega0.

```
LAB06ex3a;
```

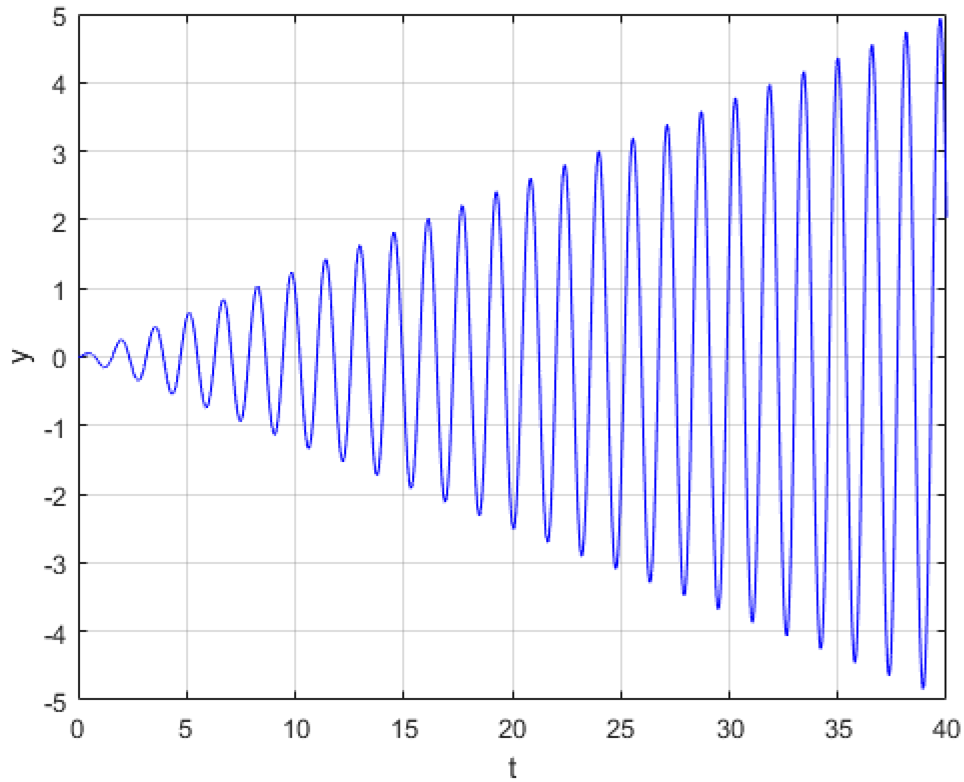


B)

Looking at the graph below, the amplitude is increasing. If you change the interval of time, you would see that the amplitude increases to infinity.

```
LAB06ex3b;

computed amplitude of forced oscillation = 4.8895
theoretical amplitude = 12.5156
```



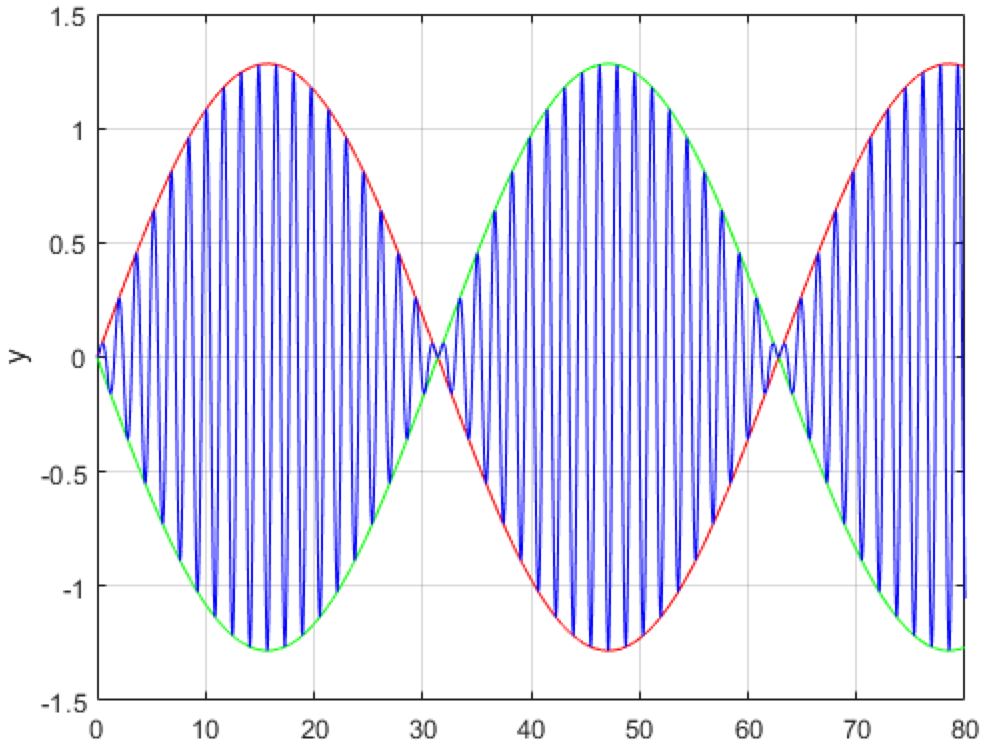
EX 4

A)

```
type 'LAB06ex4';
LAB06ex4;


clear all; %this deletes all variables
omega0 = 4; c = 0; omega = 3.8;
param = [omega0,c,omega];
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 80;
options = odeset('AbsTol',1e-10,'relTol',1e-10);
[t,Y] = ode45(@f,[t0,tf],Y0,options,param);
y = Y(:,1); v = Y(:,2);
figure(1)
C = 1/abs(omega0^2-omega^2); % declaring C
A = 2*C*sin(0.5*(omega0-omega)*t); % declaring A, envelope function
plot(t,A,'r-',t,-A,'g-',t, y,'b-'); ylabel('y'); grid on;
t1 = 13; i = find(t>t1);
C = (max(Y(i,1))-min(Y(i,1)))/2;
disp(['computed amplitude of forced oscillation = ', num2str(C)]);
Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);
disp(['theoretical amplitude = ', num2str(Ctheory)]);
%-----------------------------------------------------------------
```

```
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
end
computed amplitude of forced oscillation = 1.28
theoretical amplitude = 0.64103
```



B)

$T = \frac{4\pi}{\omega_0+\omega}$ The period of the fast oscillation is 1.61107 seconds.

C)

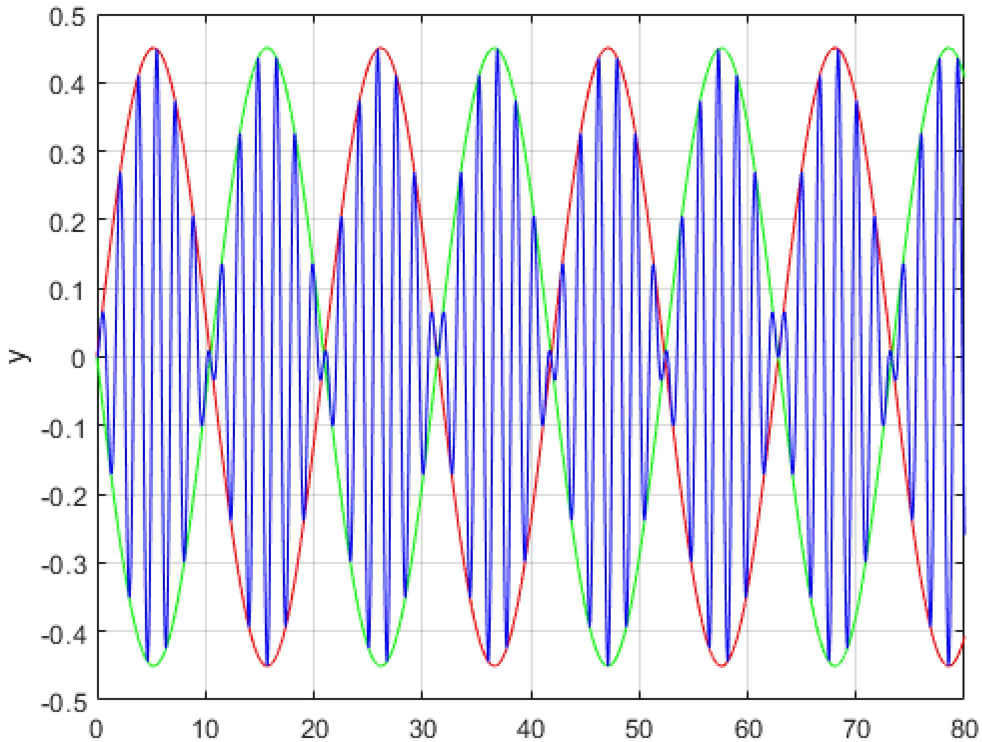
$L = 0.5\frac{4\pi}{|\omega_0 - omega|}$ The length of the beats is 31.41592 seconds.

D)

The length of the beats for omega = 3.9 is 62.83185 seconds. The length of the beats for omega = 3.4 is 10.47197 seconds. As omega increases, the period decreases.

```
LAB06ex4d;
```

computed amplitude of forced oscillation = 0.44964
theoretical amplitude = 0.22523

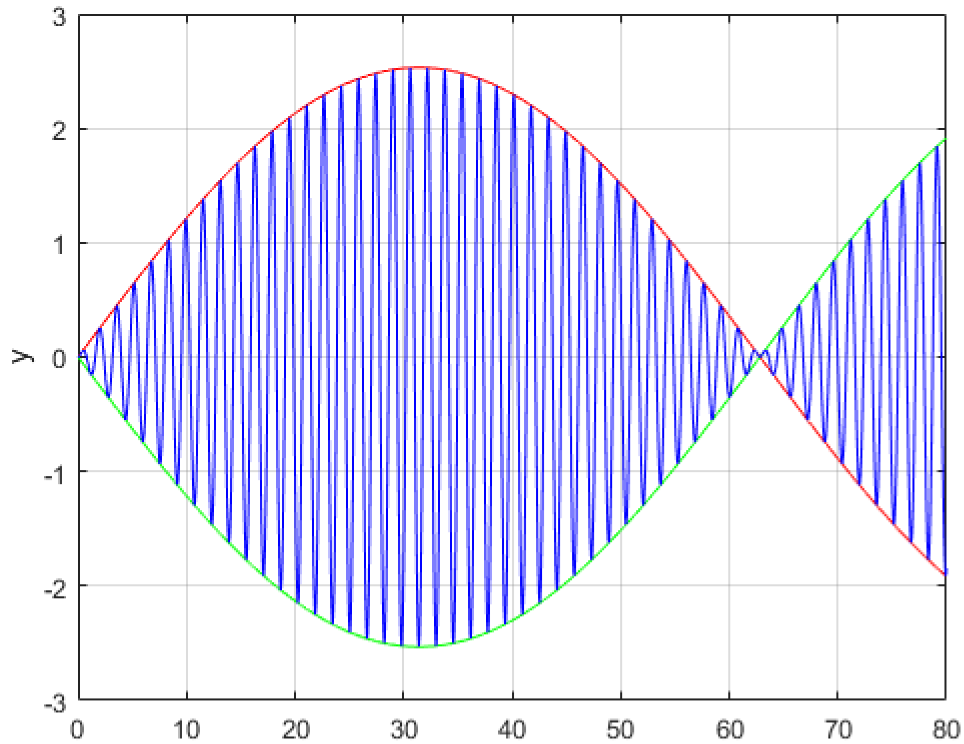


D)

Second Graph

```
LAB06ex4d2;
```

computed amplitude of forced oscillation = 2.5306
theoretical amplitude = 1.2658

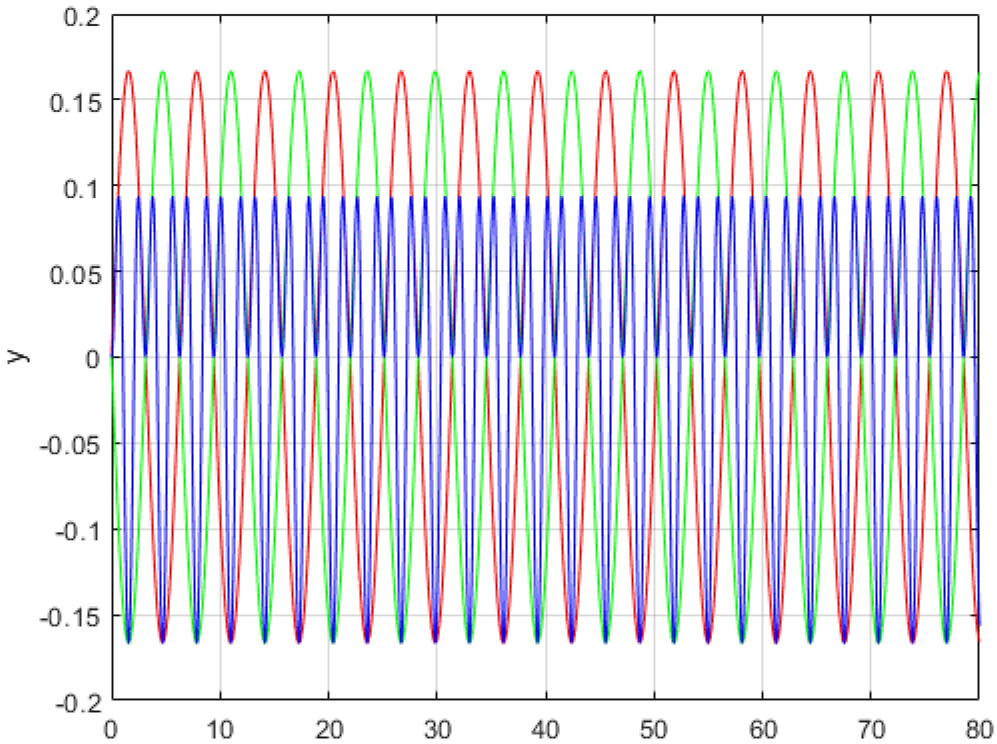


E)

The beats phenomenon is no longer present with omega = 2. This is because when omega is decreased, the length of the beats shortens.

```
LAB06ex4e;
```

computed amplitude of forced oscillation = 0.13021
theoretical amplitude = 0.083333



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