

Laboratory Exercise #3 – Perform Surveying Calculations

Background

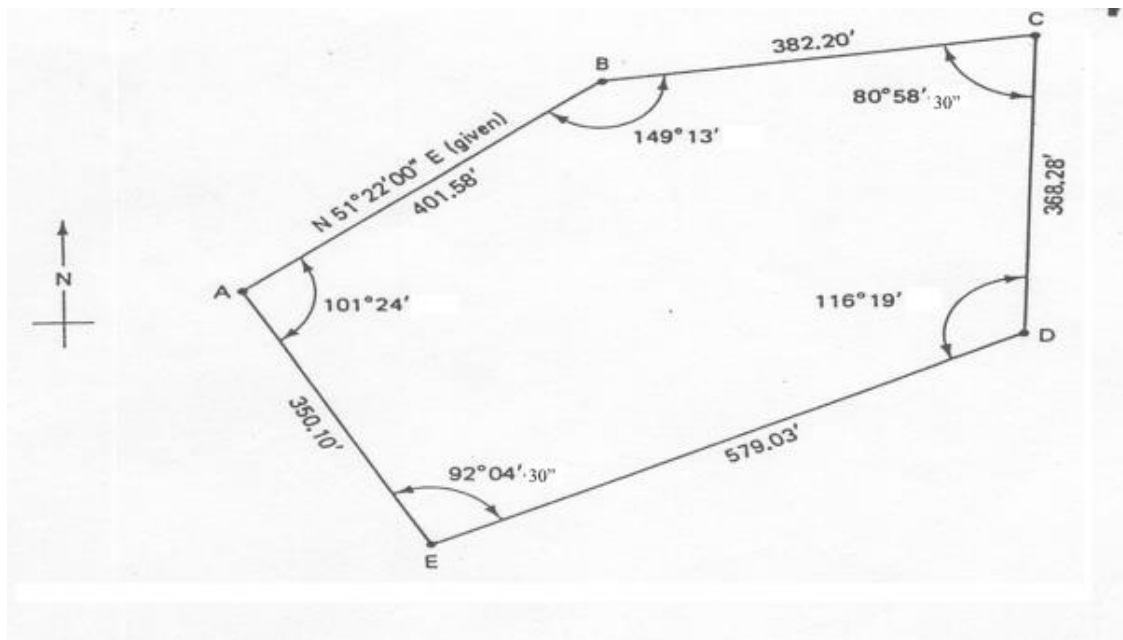
A traverse is a series of continuous connected lines of known lengths and angles beginning at a known point. Traverse surveys are used to determine the positions of existing boundary markers with unknown coordinates using the known location and the lengths and angles. Other important numbers about the traverse can be calculated including area enclosed and precision.

Objectives:

1. Learn how to properly adjust the measured values of a closed traverse to achieve mathematical closure.
2. Determine the error of closure and compute the accuracy of the work.
3. Calculate the area of a closed traverse.

Practice Question:

Given Northing, Easting (y, x) coordinates at A of (1000.00, 500.00) and measured interior angles and distances for a closed traverse:



1. Balance the interior angles
2. Calculate the azimuths and bearings of each side
3. Calculate the latitudes and departures
4. Calculate the errors in latitudes and departures
5. Compute the precision of the survey
6. Adjust the latitudes and departures by the Compass Rule
7. Determine the Northing (y) and Easting (x) coordinates of each point
8. Calculate the area of the closed traverse from the coordinate method

Balancing Horizontal Angles

To balance the horizontal angles, add the measured angles and compare them to the theoretical value. The geometric sum of the interior angles of any closed traverse is:

$$\sum \text{Interior Angles} = (n - 2) * 180^\circ \quad n = \text{number of } \frac{\text{angles}}{\text{sides}}$$

If the sum of the measured angles does not add up to the geometric value, the difference between them is the error. If $\text{True Value} + \text{Error} = \text{Measured Value}$, to get the correct value you need to subtract the error. $\text{True Value} = \text{Measured Value} - \text{Error}$. Following this method, the signs will cancel appropriately. Since we want to distribute the error across all the angles, you will subtract $1/5$ of the error from each side; or error/5.

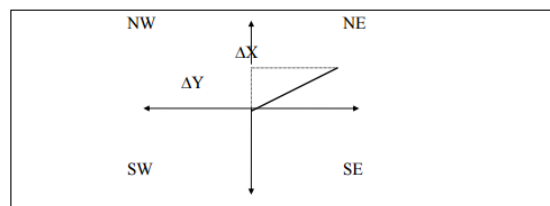
Azimuth and Bearing Angles

You measured an azimuth angle using the total station. That angle indicates the orientation of the line you were measuring. Through geometry, it can be determined that

$$AZ_{AB} + 180^\circ + \angle B = AZ_{BC}$$

If you are trying to find azimuth angles moving counter-clockwise around the traverse. If you labeled your points clockwise, you can use the same equation except you would subtract angle B. Most people make fewer mistakes when adding rather than subtracting DMS since it's a base 60 system.

Bearing Angles can be thought of as a set of instructions. From the origin you first need to go north or south. From due north or south, you need to rotate a certain number of degrees to the west or the east. If my angle was 230 degrees, I would go south from the origin then 50 degrees toward the west. Thus S 50 W is the bearing angle.



Bearing grid showing four quadrants

Latitudes and Departures

There are both open and closed traverses; you performed a closed traverse for the last lab because you began and ended at the same location. A traverse is used to determine the exact location of an unknown point. By knowing a bearing angle and a distance from a known

point, the ΔX (also called easting, or departure) and ΔY (also called northing, or latitude) from the known point can be calculated. The rectangular coordinates of the new point can then be determined with respect to the known point. If the known point already has coordinates, the ΔX and ΔY are added algebraically to these coordinates. This procedure is followed around the traverse and the coordinates for each new point are determined.

The coordinate pairs that you were provided are x,y pairs measured from the origin in the NAD 1983 UTM Zone 12N projection. When you are finished with your computations, the coordinate pairs you are left with will be for the same projection and coordinate system. In this system, you measure, the coordinate pairs are measured in meters. The total station gives lengths in feet. Be sure to properly convert units before reporting your final answer. We will discuss the need for projections and coordinate systems as well as how to use these coordinate points later in class.

Either azimuth or bearing angles may be used, along with a horizontal distance to compute ΔX (departure) and ΔY (latitude):

Option 1 – Using the Azimuth Angles

$$\Delta X = D * \sin A$$

$$\Delta Y = D * \cos A$$

Option 2 – Using the Bearing Angles

Northeast Quadrant

$$\Delta X = D * \sin B$$

$$\Delta Y = D * \cos B$$

Northwest Quadrant

$$\Delta X = -D * \sin B$$

$$\Delta Y = D * \cos B$$

Southwest Quadrant

$$\Delta X = -D * \sin B$$

$$\Delta Y = -D * \cos B$$

Southeast Quadrant

$$\Delta X = D * \sin B$$

$$\Delta Y = -D * \cos B$$

Where:

ΔX = change in X

ΔY = change in Y

D = Horizontal Distance

A = Azimuth angle (measured clockwise from north)

B = Bearing angle

Bearings are measured to the east or west from an axis running north and south. The quadrants are set up like a rectangular coordinate system. The known point is the origin and the unknown point lies in one of the four quadrants. The unknown point is north ΔY and east ΔX of the known point in the figure below.

Error in Latitudes and Departures and Precision

If all measurements were perfect when you add up the ΔX 's and ΔY 's for a closed traverse they would each sum to 0 for perfect closure. Unfortunately, even with the best

equipment and practices this is impossible and so you will need to calculate the error of closure in the X (east or departure) direction, the Y (north or latitude) direction, and the total linear error. These are computed as follows:

$$E_x = \Sigma \Delta X \qquad E_y = \Sigma \Delta Y \qquad E_{linear} = \sqrt{E_x^2 + E_y^2}$$

The accuracy of your traverse = 1: (Perimeter/Linear Error). As an example, if you calculated the perimeter of a traverse to be 2934.23 feet, and you found error of closure (vector of closure) to be 0.37, the accuracy of the traverse would be 1: (P/E_{lin}) = (2934.23 feet)/ (0.37) = 1:7930, meaning there is one foot of error for every 7930 feet.

Adjusting Latitudes and Departures

The Compass Rule Adjustment is used in survey computations to distribute the error of closure proportionately between the different legs of the traverse. If done correctly the traverse will close precisely to the point of origin. The correction to each traverse leg is determined by the ratio of the distance between the two points and the total perimeter.

The correction to be applied to each of the latitudes and departures can be found by applying the following formulas:

Correction = $-\Sigma \Delta y / P * L_i$ or $-\Sigma \Delta x / P * L_i$ where L_i = the length of the particular segment you are correcting and P = the perimeter of the traverse.

As an example, let's say the perimeter of the traverse was measured to be 2124.34 feet. The length of section AB was measured to be 177.23 feet. The Latitude of section AB was found to be -132.45 feet, and the Departure was found to be 342.23 feet. The sum of the latitudes ($\Sigma \Delta y$) was found to be -0.25 and the sum of the departures ($\Sigma \Delta x$) was found to be 0.28. The corrected latitude of AB is equal to $-(-0.25/2124.34) * 177.23 = 0.02$. The corrected latitude is -132.45 feet + 0.02 = -132.43 feet. The corrected departure of AB is equal to $-(0.28/2124.34)*177.23 = -0.02$. The corrected departure is 342.23 feet - 0.02 = 342.21 feet.

Calculating Coordinates for Each Point

Now that your latitudes and departures are adjusted, the sum of each should be zero. Beginning with the known coordinate points of the first position, add the adjusted latitudes and departures of each side to get the coordinate points of the next point.

Suppose I know the coordinates of point A and I want the coordinates of point B. I will add the adjusted latitudes and departures of line AB to get the coordinates of B. The latitudes and departures represent the change in x and y across that line; so, I will add the change in x to the x value for A and the change in y to the y value for A. If the coordinates of A are (500,1000), the departures of AB are 100, and the latitudes are -200, I would get the coordinates (600,800) for point B.

Computing the Area

When the coordinates of a traverse are known, the area can be calculated directly. The simplest method for computing the areas is the Area Coordinate method. This is done by arranging the coordinates in a clockwise direction as shown below. Cross multiplication of each (x, y) pair is performed and the result placed to the left or right side as indicated. The left and

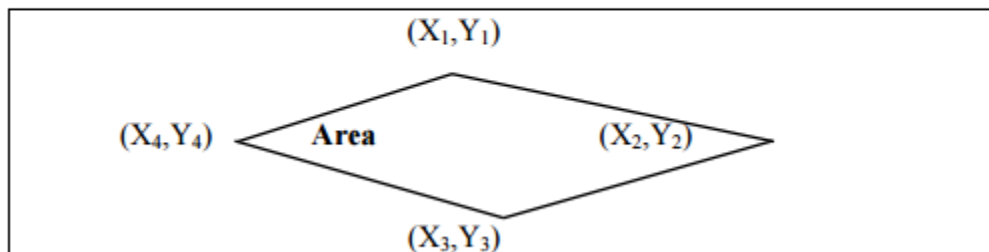
right columns are summed and the absolute value of the difference between Σ_1 and Σ_2 divided by two is the area of the polygon.

$$\begin{array}{rcccl}
 & X_1 & & Y_1 & \\
 Y_1 * X_2 & \leftarrow & X_2 & Y_2 & \rightarrow X_1 * Y_2 \\
 Y_2 * X_3 & \leftarrow & X_3 & Y_3 & \rightarrow X_2 * Y_3 \\
 Y_3 * X_4 & \leftarrow & X_4 & Y_4 & \rightarrow X_3 * Y_4 \\
 Y_4 * X_1 & \leftarrow & X_1 & Y_1 & \rightarrow X_4 * Y_1 \\
 \hline
 \Sigma_1 = & & & & \Sigma_2 = \\
 \text{Area} = |\Sigma_1 - \Sigma_2| / 2 \dots\dots\dots 5.17
 \end{array}$$

Where:

Σ_1 = sum of values in the left column

Σ_2 = sum of values in the right column



Area Computation

For bookkeeping purposes, the first point is repeated at the bottom of the column. If the coordinates are listed counter-clockwise the result will be negative but the magnitude will represent the correct area.

Deliverables-

- Complete parts 1-8 from the procedure for the example problem and your own traverse. Organize your answers in tables like the example answer sheet provided.
- Include scans of each page of computations done by hand. If you decide to use excel or another program instead of working by hand, you may use prints of the spreadsheet if you include the formulas you used in each cell. Otherwise, no partial credit can be earned.
- Include the initial measurements from the previous lab including: the lengths of each side of the traverse, the azimuth you measured for side AB, the number of the control point you started on.
- You will lose points for using Decimal Degrees and you will probably get the wrong answers.
- If you do not include the initial values, it will not be possible to check your work. You will either receive no credit or be penalized.

Grading Rubric

Item	Points
Practice problem- Results from steps 1-8	10
Your traverse- Results from steps 1-8. You must report the data you collected with total station to get any of the following points: <ul style="list-style-type: none">• 1 pt balancing the angles correctly• 5 pts Correct Azimuths• 5 pts Correct Latitudes and Departures• 2 pts Calculating error in Latitudes and Departures• 1 pt Correct Precision• 3 pts Adjust the Latitudes and Departures• 2 pts Calculating the Northings and Eastings of each point• 1 pt Calculate the area	20
Total	30

Lab 3 Answers

Measured
Azimuth _____

Balanced Angles	
A	
B	
C	
D	
E	

Measured Lengths	
AB	
BC	
CD	
DE	
EA	

#2		Azimuths	Bearings
	AB		
	BC		
	CD		
	DE		
	EA		

#3		Latitudes	Departures
	AB		
	BC		
	CD		
	DE		
	EA		

#4	Linear Error:	
	Error in Latitudes	
	Error in Departures	

#5	Precision:	
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#6		Adjusted Latitudes	Adjusted Departures
	AB		
	BC		
	CD		
	DE		
	EA		

#7		Northings	Eastings
	A		
	B		
	C		
	D		
	E		

Starting Control
Point

#8	Area:	
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