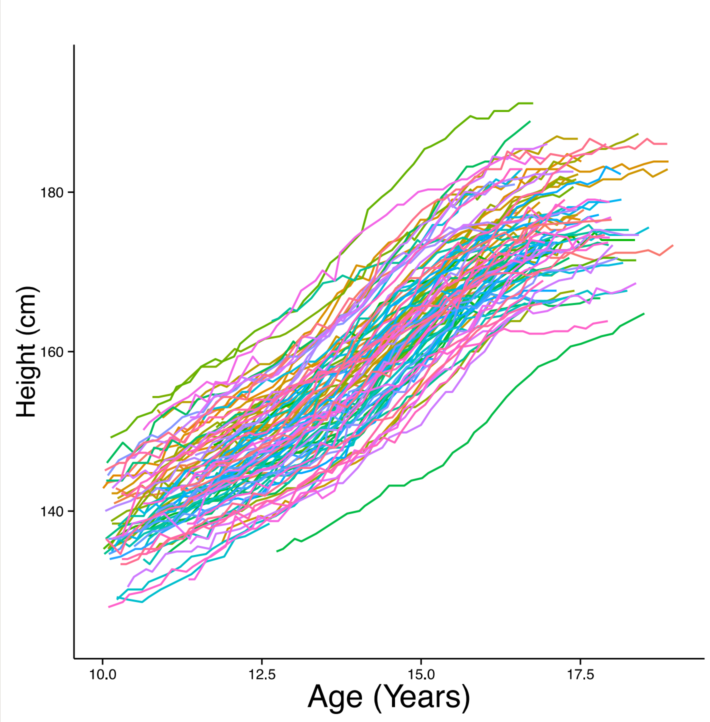
DR. ANDREW SIMPKIN – LECTURER IN BIOSTATISTICS

Longitudinal data analysis: modelling data which are collected over time in the same individuals/companies/countries/regions

- This topic would involve applying linear mixed models (LMMs) to publicly available data
- LMMs are an extension of the linear regression model
- Could extend to nonlinear methods (e.g. splines) if suitable
- Examples
  - Monthly Irish homeless figures by region, age, sex from 2016 – 2019
  - Yearly Contagious disease data for US states from 1928 – 2011



Final Year Projects 2020/2021

Possible topics

Tobias Rossmann

Areas

- I enjoy **algebra**, **number theory**, and related **geometry**. If there's a group, prime, or ring in it, then I'm probably interested!
- Recommended background knowledge is indicated for each of the projects that I propose.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Magic squares

A **magic square** is an $n \times n$ matrix of non-negative integers such that all row and column sums ("line sums") are equal.

This project is devoted to the following.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Theorem (Stanley)

Let $H_n(r)$ denote the number of $n \times n$ magic squares with line sum r.

Then $H_n(r)$ is a polynomial in r of degree $(n-1)^2$.

Background: some ring theory

Algebra of programming

This project explores how **category theory** provides an abstract algebraic framework for core topics of theoretical computer science.

Possible directions include

- abstract data types (e.g. lists, trees, ...),
- recursion,
- λ-calculus, and
- various optimisation problems.

Background: basic computer science, interest in algebra

Quaternion algebras

In 1843, Hamilton discovered the quaternions with basis $1, \mathrm{i}, \mathrm{j}, \mathrm{k}$ and

$$\mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = \mathrm{ijk} = -1.$$

This project will explore the more general class of **quaternion algebras** over fields and their connection to **quadratic forms**. Through the latter, quaternion algebras have important applications in **number theory**.

You will e.g. see that Hamilton's construction is the only interesting quaternion algebra over the real numbers but that there are infinitely many examples over the rationals.

Background: some field and ring theory

Derivative valuations

(A derivative is a financial instrument whose value depends on more basic underlying variables.)

- Description of different derivatives e.g. forward, futures, swaps & options
- Mathematical prerequisites e.g. Ito calculus
- Mathematical modelling of the underlying e.g. stock price, interest rate, exchange rate, price of beef
- Method 1: Black Scholes Merton model
- Method 2: Monte Carlo simulation
- Method 3: Binomial model
- Method 4: Trinomial model
- Method 5: Finite difference methods
- Method 6: Control variate technique
- Use the Greeks to hedge positions

M. Hayes

Projects could include valuing derivatives based on some of the following i.e. with real life applications:

- Stocks with no dividends
- Stocks with dividends
- Inflation e.g. in the UK ALL pensions increase in line with Consumer Price Inflation but capped at 2.5% p.a. and have a zero floor i.e. both a put option and call option i.e. a collar.

There is a shortage of these types of stocks to match hundreds of billions £ of liabilities

- Longevity swaps
- Interest rate (Black Scholes model & Vasicek model)
- Currency
- Derivative on a derivative e.g. option on a future

M. Hayes

Statistics/Data Science Project

John Newell

- Theory and applications of Statistical Learning Methods (e.g. penalised regression, regression trees, random forests) applied to high dimensional data.

- A topic of mutual interest.

- In each case you will be expected to source relevant data from an online repository (e.g. gov.ie, kaggle) and build a shiny app to communicate your findings.

The travelling salesman meets the travelling archaeologist: Incidence matrices and seriation in archaeology

Angela Carnevale
angela.carnevale@nuigalway.ie

Petrie matrices are special matrices. They lie at the core of mathematical models of **seriation** in archaeology.

As part of this project you will explore this class of matrices and related graphs.

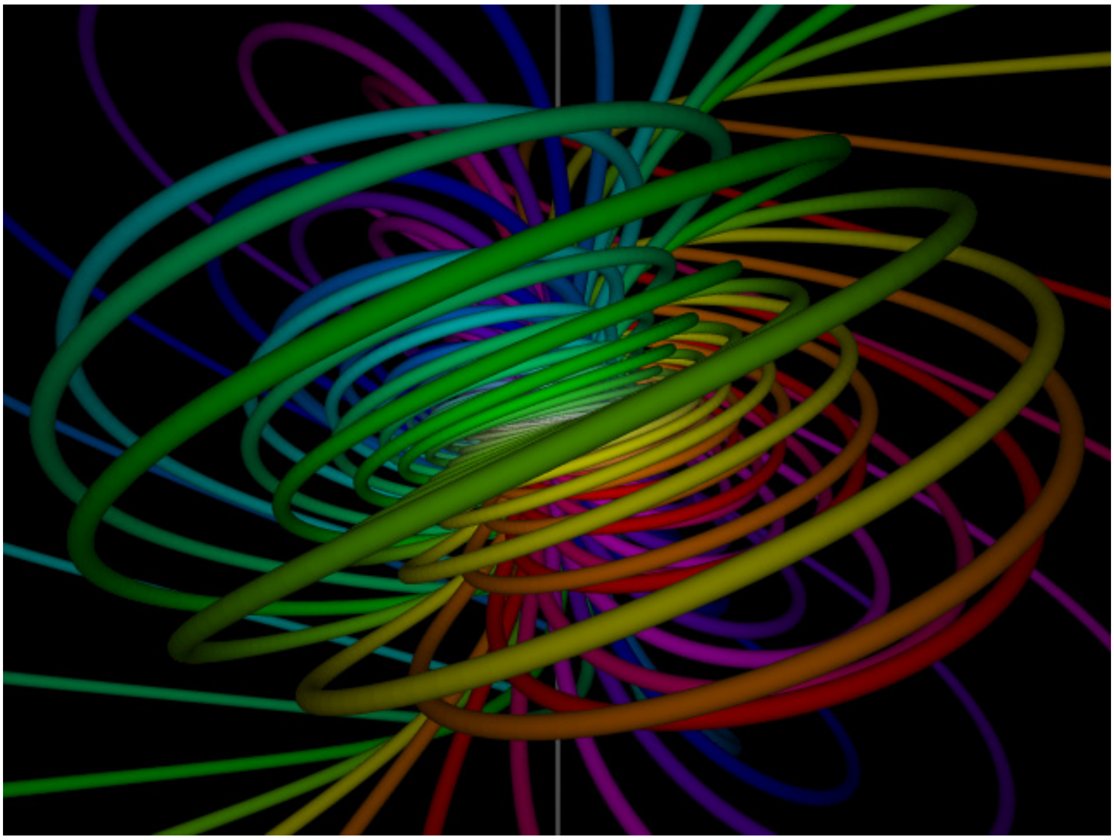
Milestones

- A characterisation of **non-singular** Petrie matrices.
- The study of **networks** associated with the **travelling salesman** & travelling archaeologist problems.

Feel free to contact me to discuss this further!

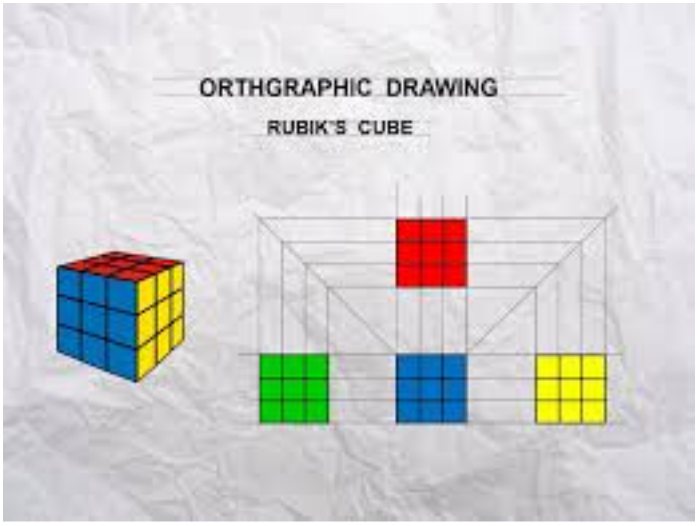
angela.carnevale@nuigalway.ie

PROJECT SUGGESTIONS 2020 (J.B.)



1. FOLIATIONS IN ECONOMICS AND THE SOCIAL SCIENCES

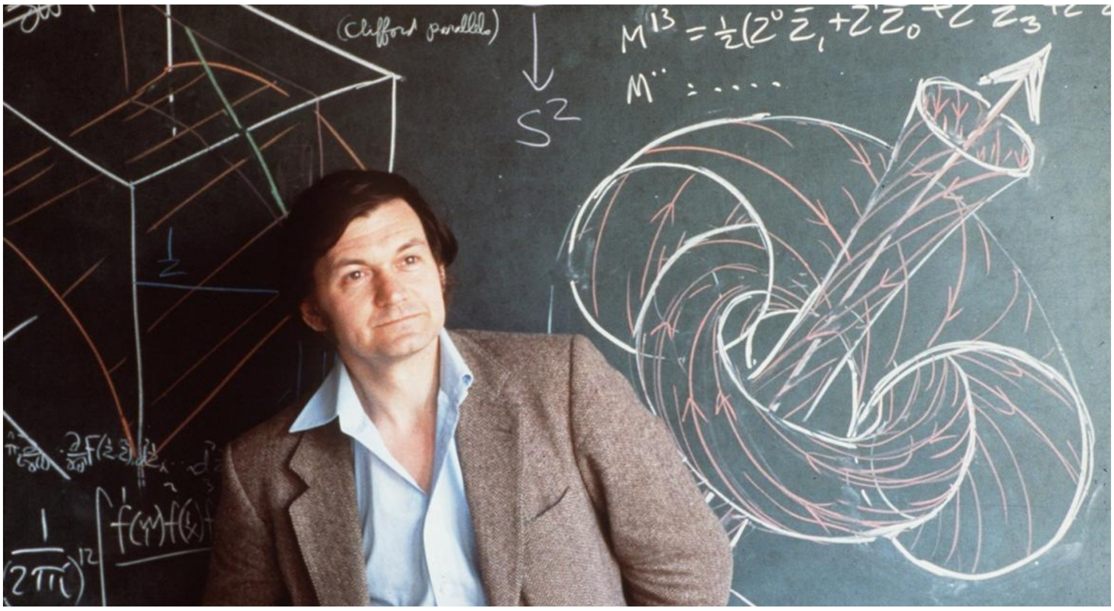
The level sets of a production function or more generally the indifference sets of preferences are examples of foliations. They suggest that one use of foliations is to conveniently catalogue information. Each leaf serves as an equivalence set of information or data that is of equal value relative to a specified objective or goal; for example, this goal could be captured by an agent's preferences or by a specified set of economic indicators. Different leafs then describe different equivalence classes of information. The project will be based on reading a research on this topic.



2. Drawing with complex numbers

Suppose a cube is orthographically projected and normalised so that a particular vertex is mapped to the origin. If a, b, c are the images of the three neighbouring vertices (considered as complex numbers), then the sum of the squares of a, b and c is zero. Conversely, if this equation is satisfied, then one can find a cube whose orthographic image is given in this way. The project will consider proofs of this fact and its generalisations, using Lie groups and projective geometry. Alternatively, the project could look at how solutions to the null equation above (satisfied by a, b and c) can be used to find minimal surfaces in R^3.

3. Twistor Theory



Twistor theory began with the work of Roger Penrose (above) who introduced the powerful techniques of complex algebraic geometry into general relativity. Loosely speaking it is the use of complex analytic methods to solve problems in real differential geometry or physics, with the emphasis on the geometry rather than the analysis. We will start with the so-called mini-twistor space of R^3 (the tangent bundle of the 2-sphere) and how it can be used to find harmonic functions on R^3, by an integral transform (our 1st Penrose transform).

Improving Golub's method on molecular classification prediction of Cancer

TR Golub et al. Molecular classification of cancer: class discovery and class prediction by gene expression monitoring. Science. 1999.

- **P**rimary samples: 38 bone marrow samples (27 ALL, 11 AML)
- **I**ndependent samples: 34 leukemia samples

4 years ago, a FYP by Robbie achieved 100% accuracy on **P** (LOO), **better** than Golub's. However, on **I**, it was **worse**.

In this project, you continue Robbie's effort.

- Have to understand Golub's work and Robbie's work first.
- Have to gain some insights by learning and thinking statistically and biologically.
- Using Matlab is preferred.
- This project can accommodate 2 or 3 pairs. In this case, each pair can develop its own private method independently. I can compare the performance and writing to evaluate the quality of projects.

Supervisor: Haixuan Yang

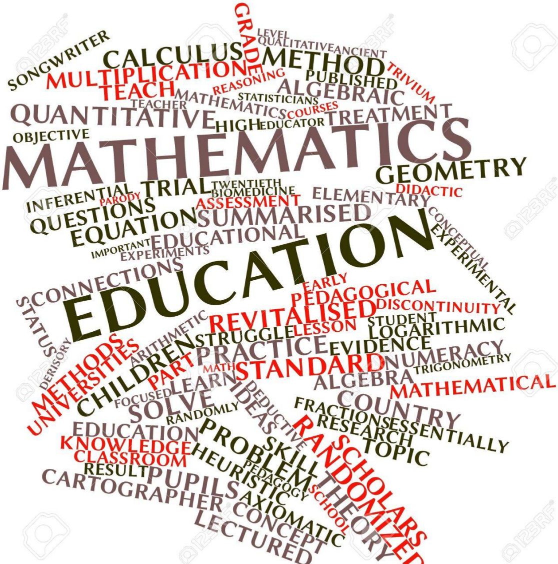
Mathematics Education

Ideas for Final Year Projects

Dr Kirsten Pfeiffer

School of Mathematics, Statistics and Applied Mathematics
NUI, Galway

Research in Mathematics Education



Research in Mathematics Education

Two main goals:

- To enhance our **knowledge** of mathematical thinking, learning, teaching, ...
  - How do people learn mathematics?
  - Study habits
  - Teaching styles
  - ...
- To enhance **practice** in the field to enable better learning and teaching
  - Curriculum design
  - Teacher development
  - Textbooks
  - Task design
  - ...

Research Questions

Just a few examples:

- How do students learn the concept of limit?
- Benefits or disadvantages of group work in the mathematics classroom
- In what ways can we design assessment to pay attention to real life contexts?
- Reasoning and proving in Irish school mathematics textbooks
- Exploring methods for using Geogebra in Geometry classes
- Exploring methods of teaching mathematics online

Any questions?

You may have come across with some oberservations or questions in your own teaching or learning experience which you want to explore further.

If you are interested in a project on Mathematics Education, please talk to me after this session or contact me per email:

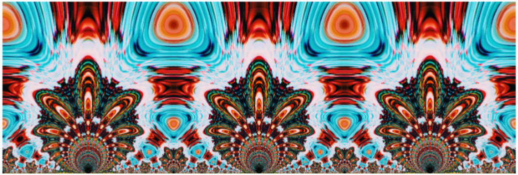
kirsten.pfeiffer@nuigalway.ie

Games hiding mathematics

- Playing pool with π. (Trigonometry and dynamics)
- Keeping track of two kids is easier than one. (Complex analysis)

Art hiding mathematics

- What can you draw? (Set/measure theory)
- Psychedelic art via geometry. (Planar/hyperbolic geometry)



Final year project - what to do?(Aisling McCluskey)

1. Look back: topics of an historical and foundational flavour - Euclid (geometry), Cantor (set theory/infinity), paradoxes; number theory (Fermat's Last Theorem...)
2. More specialised: an exploration of the notion of equidistant points ie given two points x, y in a space X. what do we mean by a point p being equidistant to $\{x, y\}$? *Reference: Proc. Amer. Math. Soc. 59, (1976), no. 1, 179 - 183 (S. Nadler, Jr)*

Final year project - what to do?

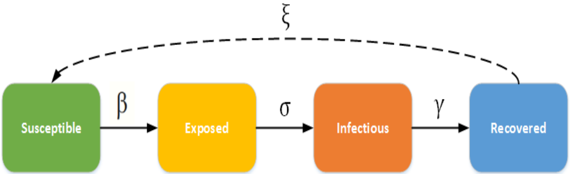
3. University mathematics education: the mathematical constructs of *definition, theorem, proof* - how do they arise? how do we find them? how do we teach them?
4. Tionscadal in ábhar atá luaite thuas nó ábhar matamaiticiúil eile atá oiriúnach dúinn/Project in Irish on any of the above topics or another mathematical topic of mutual interest
5. Any other topic in pure mathematics of mutual interest

Project Proposals

Martin Meere, NUI Galway

2020

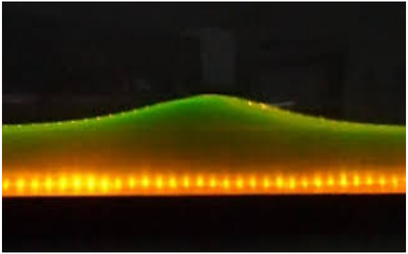
1. Modelling The Covid-19 Outbreak In Ireland Using Compartment Models



The project will cover the following topics:

- Compartmenmt models - what are they? ODE models.
- What is the SEIR model?
- Search the literature and the internet for parameter values.
- Dynamical systems analysis. Insight into R_0 and herd immunity.
- Take account of variations in population density across the country and different behaviours of different age cohorts.
- Use the model to gain insight into what has happened up to now. Optimal future public health measures?

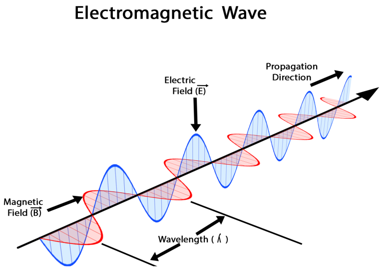
2. Solitons



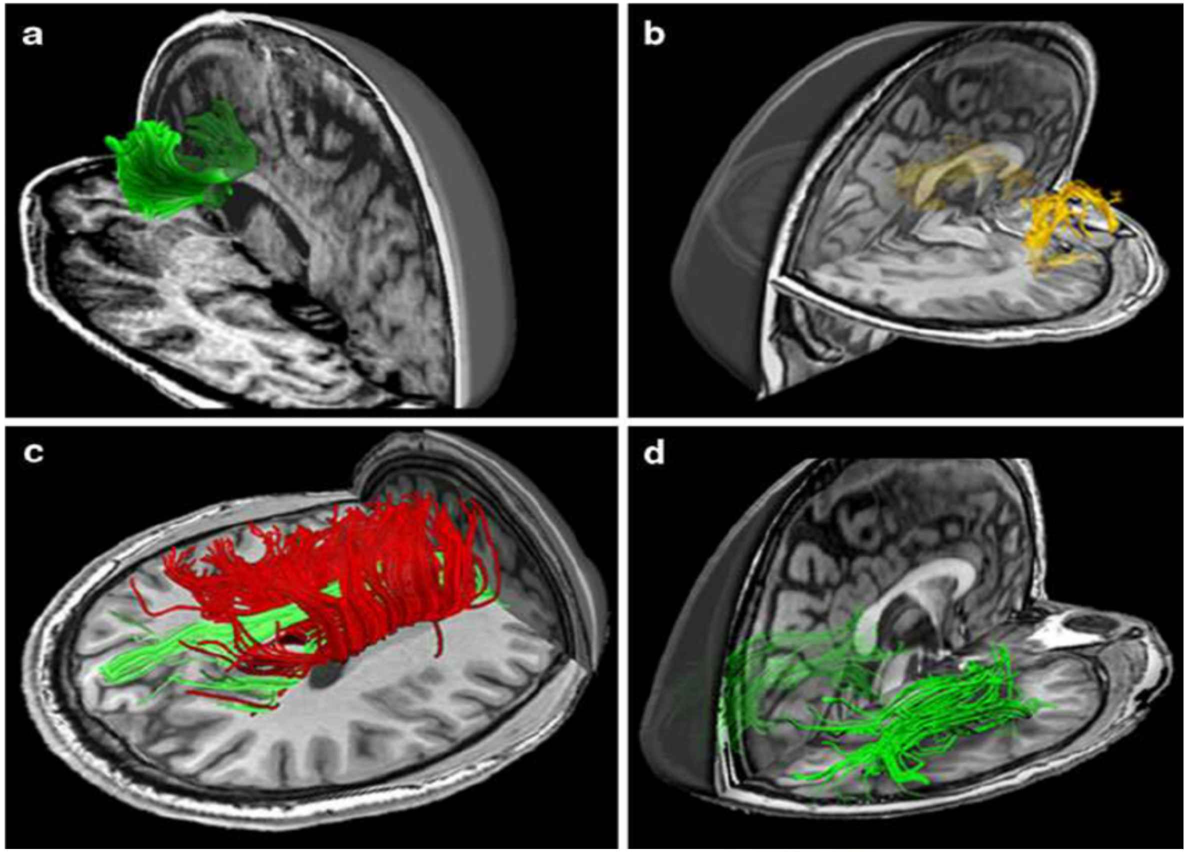
The project will consider the following:

- Solitons - what are they? Nonlinear waves of permanent form.
- Arise in many contexts - water wave theory, optics, magnetism, waveguides, ...
- The KdV equation.
- Travelling wave solutions of the KdV equation.
- More sophisticated solution techniques - the inverse scattering method.
- 2,3,4 ..... interacting solitons.

3. Electromagnetic Waves

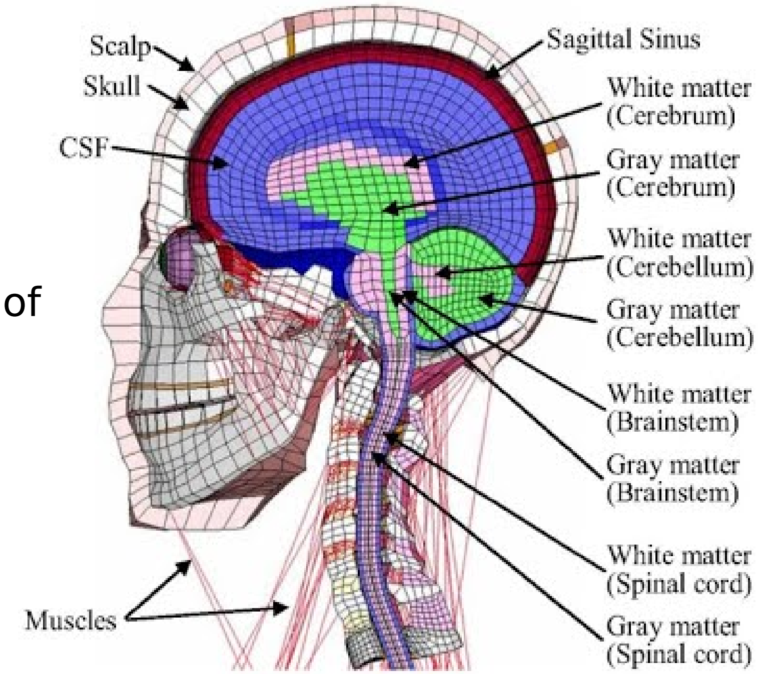


- Plane electromagnetic waves. Polarization.
- Incident, reflected, refracted waves at interface of two dielectric slabs
- Snell's law, Brewster's angle, total internal reflection.
- Designing an optical system - three layer systems.



MRI of Human head (Above)

Classical FEM mesh of head (Right)



AIM: Construction of a detailed 3D finite element mesh of human head ie skull + brain (including axons) + neck using GMSH.

Motivation: To simulate traumatic brain injury using this mesh.

Skills: Literature Survey; Open-source packages/OS: GMSH, Linux, Latex; collaborative skills, for instance, with School of Physics, NUIG for Human MRI data; prelimary understanding of human imaging

Deliverables: Potential publications in journals like Journal of computational science; open-source package for GMSH

Requirements: Motivation

Contact: bharat.tripathi@nuigalway.ie

2020/21 Project Suggestions

Dane Flannery, e-mail: dane.flannery@nuigalway.ie

1. **Expander graphs**. What is known (esp. constructions), what are applications (e.g., in computer science); what are the outstanding problems? Reference: *Expander families and Cayley graphs* by Krebs & Shaheen.

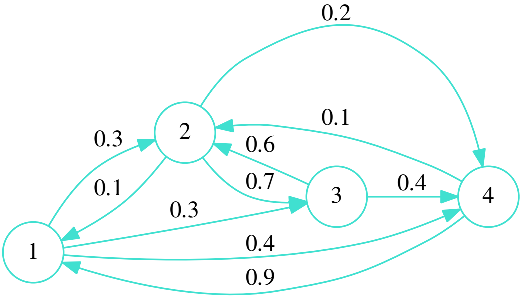
2. **Multipliers for difference sets**. Tools for finding difference sets in finite groups. Scope for computation. References: Chapter 6 of *Difference sets* by Moore & Pollatsek; Chapter 2 of *Applied abstract algebra* by Klima, Sigmon & Stitzinger.

3. A topic of mutual interest.

A 2.38 minutes video presentation of these is at https://youtu.be/oNb7EmelN84

rachel.quinlan@nuigalway.ie

Messages enter the network every day, each with one initial recipient and one intended recipient. Each spy can recognize messages intended for herself. Every day, messages are forwarded within the network.



$$\begin{pmatrix} 0 & 0.3 & 0.3 & 0.4 \\ 0.1 & 0 & 0.7 & 0.2 \\ 0 & 0.6 & 0 & 0.4 \\ 0.9 & 0.1 & 0 & 0 \end{pmatrix}$$

The number X_k of days taken for a message that starts with Spy k to reach its intended recipient is a random variable with expected value $E(X_k)$.

Theorem (Kemeny, 1960)

$$E(X_1) = E(X_2) = E(X_3) = E(X_4) = E(X).$$

This common value is called Kemeny's constant.
It has an expression in terms of the eigenvalues of the matrix.
It can be interpreted as a measure of efficiency of the flow in the network. It is related to the "time to mixing" of a Markov process.

A project on this topic could investigate

- Why is this value independent of the starting point?
- What is the effect of introducing (or deleting) a new arc?

Areas of mathematics involved:

- (Elementary) probability theory
- Non-negative matrices (Perron-Frobenius Theorem)
- (Spectral) Graph theory

Generating functions.

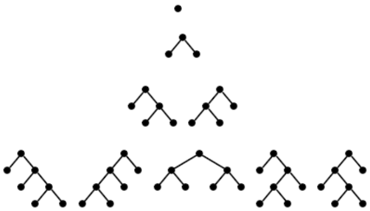
Generating functions are a widely used technique in mathematics for variety of counting problems.

Example The number of ways a_n of giving change for n Cent in terms of $1, 2, 5, 10, 20, 50$ Cent coins is described by the generating function

$$f(x) = \sum_{n \geq 0} a_n x^n = 1 + x + 2x^2 + 2x^3 + 3x^4 + \ldots + 4562x^{100} + \ldots$$

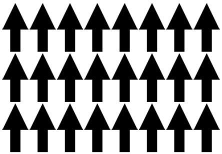
Example The number c_n of binary trees with n branches has generating function

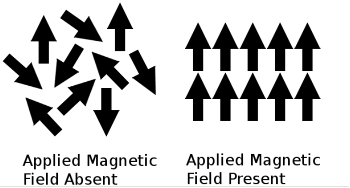
$$f(x) = \sum_{n \geq 0} c_n x^n = \frac{1 - \sqrt{1 - 4x}}{2x} = 1 + x + 2x^2 + 5x^3 + \ldots$$



Statistical mechanics and phase transitions.

Statistical mechanics describes the average behaviour of large systems of particles and used to understand such features as temperature, internal energy, entropy, phase transitions etc. This project will examine some background theory and consider both theoretical and computational models of spontaneous magnetization in ferromagnets.

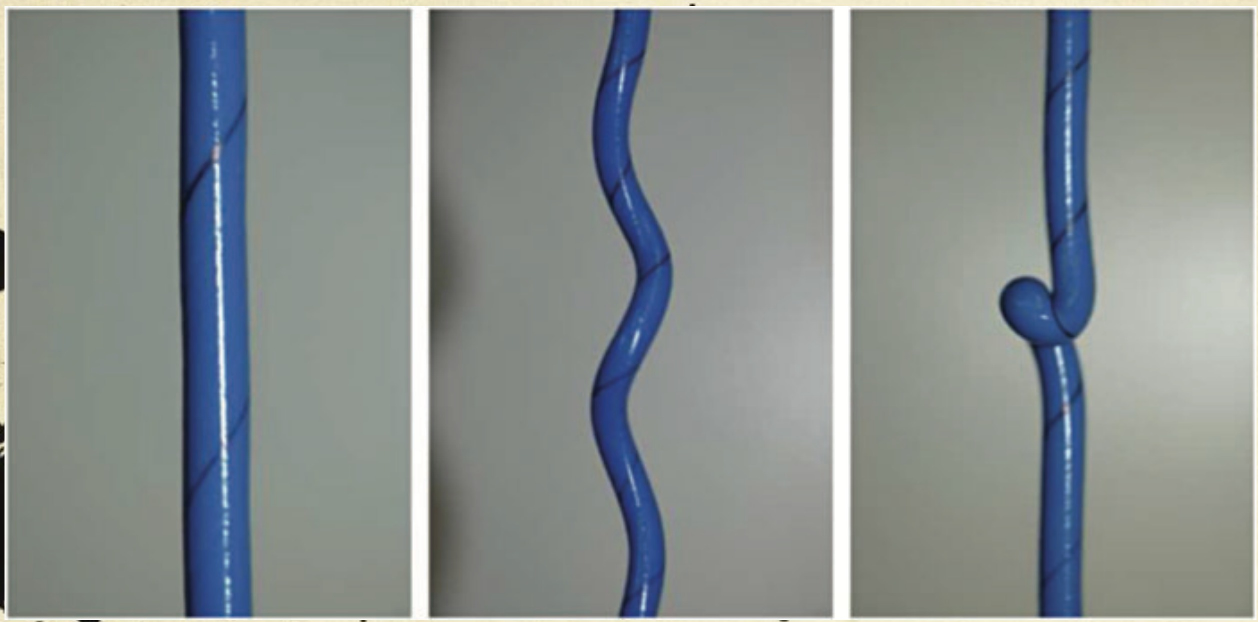




A topic of mutual interest.

Torsion of cylinders or tubes

with Michel Destrade (michel.destrade@nuigalway.ie)



Skills acquired:

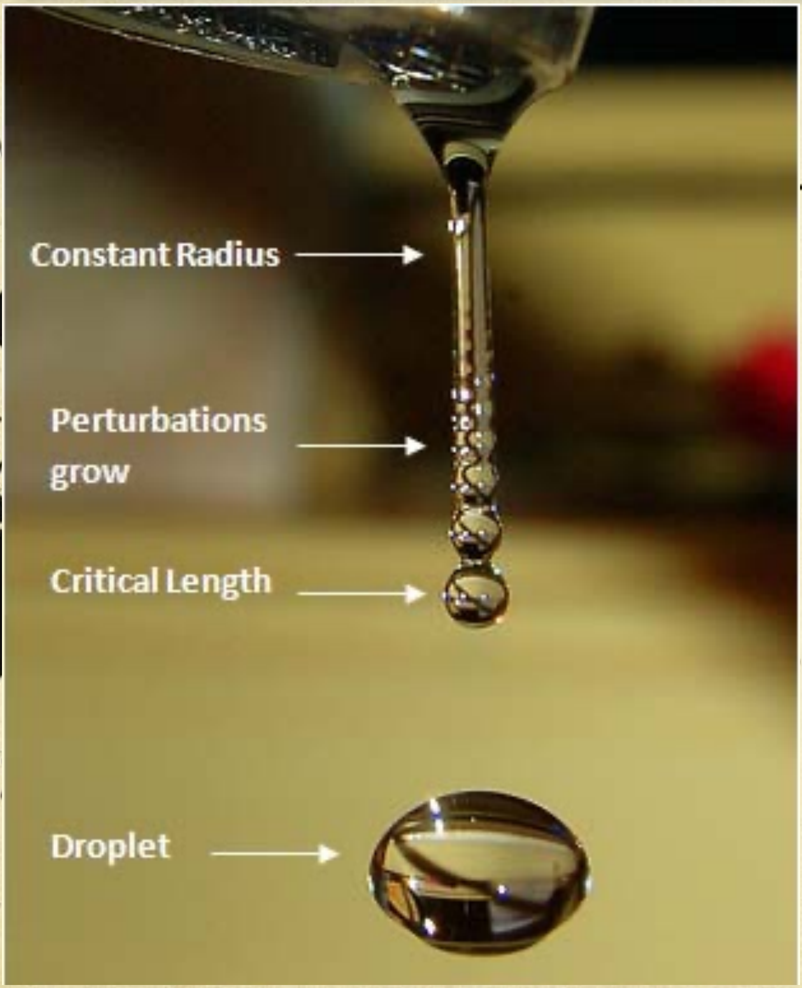
- literature search
- modelling
- scientific English
- LaTeX
- graphics, animations
- experimental data
- presentation

When a cylinder is twisted, it elongates: What about when a hollow tube is twisted? And can we study the kink formation?

Note: I'm open to other projects.
Search for keywords in the American Journal of Physics for ideas

Jet Instability

with Michel Destrade (michel.destrade@nuigalway.ie)



Rayleigh modelled jet instability in 1878.

- Can we reproduce his results?
- Can we relate them to inkjet printing or spray formation?
- Can very soft solids experience jet instability?

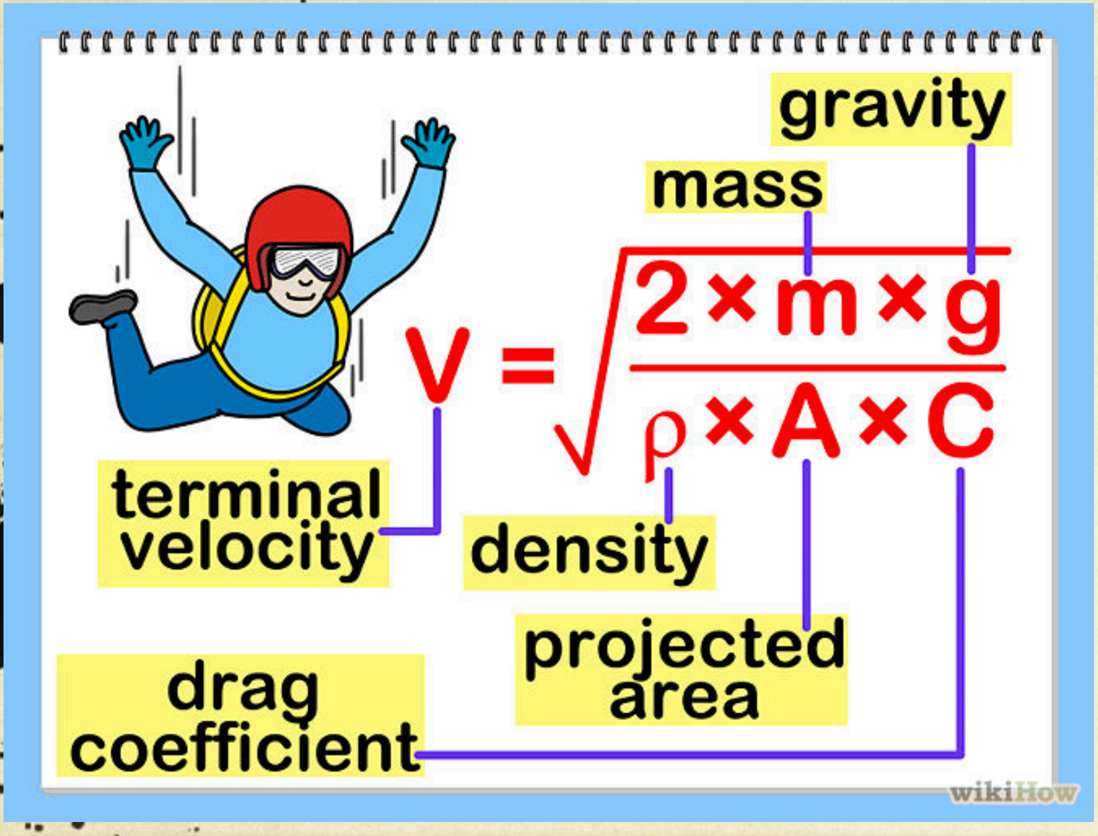
Skills acquired:

- literature search
- modelling
- scientific English
- LaTeX
- animations
- experimental data
- presentation

Note: I'm open to other projects!
Search for keywords in the American Journal of Physics for ideas

Aerodynamic Drag

with Michel Destrade (michel.destrade@nuigalway.ie)



Solve simple ODEs to find formulas for the **terminal velocity** of falling objects;

Move on to **Fluid Mechanics**, where viscosity might play a role

Skills acquired:

- literature search
- scientific English
- tabletop experiments
- LaTeX
- numerical integration
- graphics, animations
- presentation

Skydiving, Space Shuttle,
Bubbles , Cylinders or Spheres in viscous flows, etc.

Note: I'm open to other projects!
Search for keywords in the American Journal of Physics for ideas

$$6 = 2.3 = (1+\sqrt{-5})(1-\sqrt{-5})$$

Q1

$$e^{\pi\sqrt{163}/3} =$$

640320.0000000006...

- algebra: failure of unique factorization measured by a group
- geometry: algorithms for listing the group elements
- analysis: formula for the order of the group

Q2

Topology of viral evolution

- algebra: persistent homology captures vertical and horizontal evolution
- topology: Evolutionary data stored as high-dimensional topological spaces

---

Google:

"The Dirichlet class number formula for imaginary quadratic fields"
Jerry Shurman, Reed College

"Topology of viral evolution" by Chan, Carlsson, Rabadan
Proceedings National Academy of Science 2013

Götz Pfeiffer

Project Topics 2020/21.

- Algebra
- Computer Science
- Computational Algebra

Specifically:

- **Category Theory.**
  [Wikipedia]
- **Backtracking; Dancing Links.**
  [D. Knuth, The Art of Computer Programming, vol. 4B]

7.2.2.1. Dancing links. One of the chief characteristics of backtrack algorithms is the fact that they usually need to *undo* everything that they *do* to their data structures. In this section we'll study some extremely simple link-manipulation techniques that modify and unmodify the structures with ease. We'll also see that these ideas have many, many practical applications.

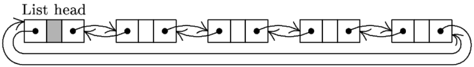
Suppose we have a doubly linked list, in which each node `X` has a predecessor and successor denoted respectively by `LLINK(X)` and `RLINK(X)`. Then we know that it's easy to delete `X` from the list, by setting

$$\texttt{RLINK(LLINK(X))} \leftarrow \texttt{RLINK(X)}, \qquad \texttt{LLINK(RLINK(X))} \leftarrow \texttt{LLINK(X)}. \tag{1}$$

At this point the conventional wisdom is to recycle node `X`, making it available for reuse in another list. We might also want to tidy things up by clearing `LLINK(X)` and `RLINK(X)` to Λ, so that stray pointers to nodes that are still active cannot lead to trouble. (See, for example, Eq. 2.2.5–(4), which is the same as (1) except that it also says '`AVAIL` $\Leftarrow$ `X`'.) By contrast, the dancing-links trick resists any urge to do garbage collection. *In a backtrack application, we're better off leaving* `LLINK(X)` *and* `RLINK(X)` *unchanged.* Then we can undo operation (1) by simply setting

$$\texttt{RLINK(LLINK(X))} \leftarrow \texttt{X}, \qquad \texttt{LLINK(RLINK(X))} \leftarrow \texttt{X}. \tag{2}$$

For example, we might have a 4-element list, as in 2.2.5–(2):



. (3)

If we use (1) to delete the third element, (3) becomes