Z_1

Z_2

ω_o

V_{out}/V_{in}

ϕ

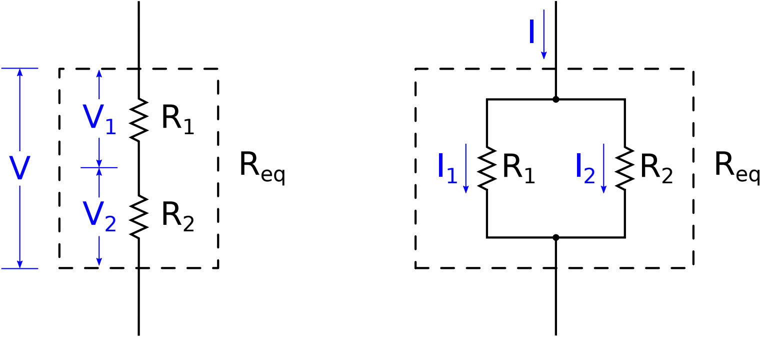
V

/

$$V = IR$$

R

$$P_{\text{diss}} = I^2 R = \frac{V^2}{R}$$



Series **(a)**

Parallel **(b)**

R_{eq}

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

$R_1||R_2||R_3\ldots$

V_1

$=$

$$\frac{R_1}{R_1 + R_2} V$$

V_2

$$\frac{R_2}{R_1 + R_2} V$$

I_1

$$\frac{R_2}{R_1 + R_2} I$$

I_2

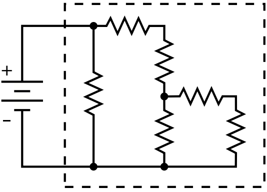
$$\frac{R_1}{R_1 + R_2} I$$

/1

R_1

l_2

R_2



Δ

Y

$$\Delta - Y$$

Resistor Network Analysis

1. Use the equivalent resistance expressions (Equations 3 and 4) to find the equivalent resistance of the entire network.

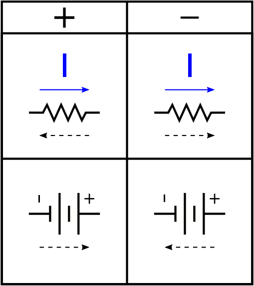
2. Use Ohm's law to calculate the current flowing through the voltage source.

3. Use the current division expressions (Equations 6) at each node to determine the currents in the network.

4. Use Ohm's law to determine the voltage drops across the resistors and from these, the voltages of all of the nodes.

$$\sum_{\text{loop}} V_i = 0$$

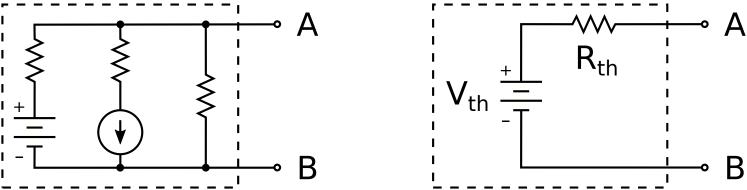
V_i


+
−
I
I
+
+

$$\sum_{\text{junction}} I_i = 0$$

Circuit Analysis using Kirchhoff's Rules

1. Draw the circuit with arrows and labels representing all of the unique currents.
2. Construct a set of loop-rule equations including every device in the circuit at least once.
3. Construct a set junction-rule equations including every current in the circuit at least once.
4. Given enough known quantities (usually the properties of the devices in the circuit — resistances, capacitances, source voltages, e.g.), these equations can be combined algebraically to determine the unknown quantities (usually currents and potential differences in the circuit).



(a) (b)

Finding the Thévenin Equivalent of a Circuit

1. The Thévenin equivalent voltage V_{th} is the voltage across the output terminals A and B of the circuit with no load attached.

2. The Thévenin equivalent resistance R_{th} is the resistance of the circuit measured between the output terminals A and B, with all voltage and current sources replaced with their internal resistances. *Ideal voltage sources have zero resistance, and ideal current sources have infinite resistance.*

CIVIL

The mnemonic CIVIL can be used to remember that the voltage across a capacitor lags $\pi/2$ (half a period of oscillation) behind the current (C$\underline{\text{IV}}$), and the voltage across an inductor leads the current by $\pi/2$ ($\underline{\text{VI}}$L). The voltage across a resistor is always in phase with the current.

$$X_L = j\omega L$$

$$\omega = 2\pi f$$

L

j

$$\sqrt{-1}$$

i

$$X_C = -\frac{j}{\omega C}$$

C

Z

$v(t)$

$$Re(V\, e^{j\omega t+\phi_v}) = V\cos(\omega t + \phi_v)$$

$i(t)$

$$Re(I\, e^{j\omega t+\phi_i}) = I\cos(\omega t + \phi_i)$$

ϕ_i

ϕ_V

$$e^{j\omega t}$$

V

$$V\, e^{j\phi_v}$$

I

$$I\, e^{j\phi_i}$$

$$Re(\mathbf{V}\, e^{j\omega t})$$

$$Re(\mathbf{I}\, e^{j\omega t})$$

$$\mathbf{I} = \frac{\mathbf{V}}{Z}$$

$$\mathbf{P} = \mathbf{I}^*\mathbf{V} = IV\, e^{j(\phi_v - \phi_i)}$$

$$\langle P \rangle = \frac{IV}{2}\cos(\phi_v - \phi_i) = \frac{V^2}{2Z}\cos(\phi_v - \phi_i)$$

$$\cos^2(\omega t)$$

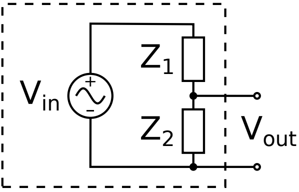
$$\phi_v - \phi_i$$

$\pi/2$

$$-\pi/2$$

Phasor Analysis

- Equivalent impedance calculations work the same way as equivalent resistance calculations (Equations 3 and 4).
- Kirchhoff's rules work for phasors $\mathbf{V}$ and $\mathbf{I}$.
- Thévenin's theorem applies to AC circuits containing only linear components. The Thévenin voltage and impedance are found via the process outlined in Section 1.5 with impedance in place of resistance.



Z1
Vin
+
−
Z2
Vout

I

$$\mathbf{V_{in}} - \mathbf{I}Z_1 - \mathbf{I}Z_2 = 0$$

$$\mathbf{V_{out}} = \mathbf{I} Z_2$$

$$\mathbf{V_{out}} = \frac{Z_2}{Z_1 + Z_2}\mathbf{V_{in}}$$

V_{out}

$$V_{out} = \sqrt{\mathbf{V}_{\mathbf{out}}^{*}\mathbf{V}_{\mathbf{out}}}$$

V_{in}

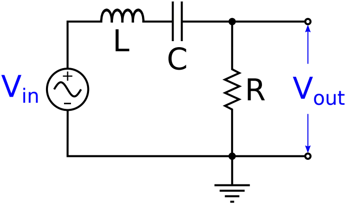
$$\tan \phi = \frac{Im\left(\frac{Z_2}{Z_1+Z_2}\right)}{Re\left(\frac{Z_2}{Z_1+Z_2}\right)}$$

A_V

$$A_V = \frac{V_{out}}{V_{in}} = \frac{|\mathbf{V_{out}}|}{|\mathbf{V_{in}}|} = \left|\frac{Z_2}{Z_1 + Z_2}\right| = \sqrt{\left(\frac{Z_2}{Z_1 + Z_2}\right)^* \left(\frac{Z_2}{Z_1 + Z_2}\right)}$$

$$\frac{1}{2}$$

$$A_v = \frac{1}{\sqrt{2}}$$


L
C
R
V_in
V_out
+
−

$$Z_2 = R$$

$$Z_1 = j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\frac{V_{out}}{V_{in}} = \sqrt{\frac{R^2}{R^2 + (\omega L - 1/\omega C)^2}}$$

$$a + jb$$

$$\frac{Z_2}{Z_1 + Z_2} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R^2 - jR\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan \phi = \frac{\frac{1}{\omega C} - \omega L}{R}$$

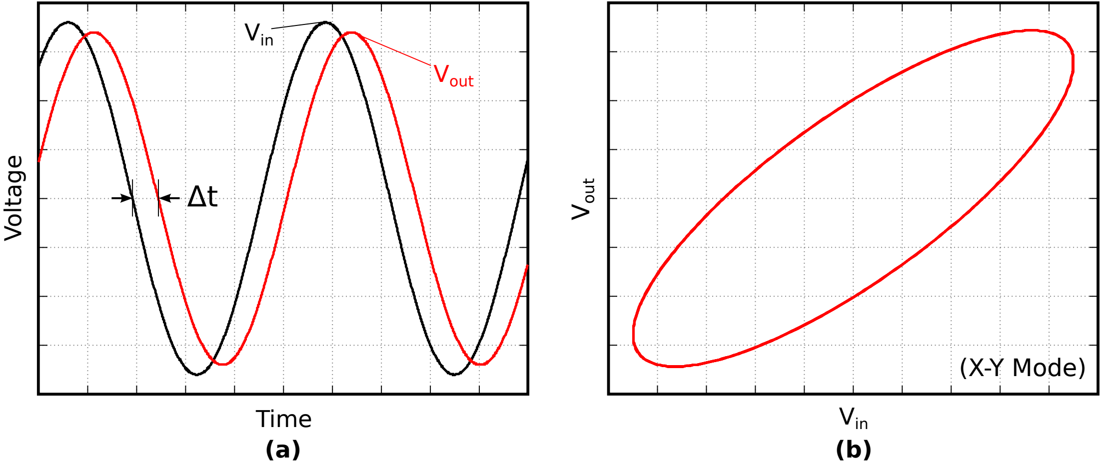
$v_{out}(t)$

$$v_{in}(t)$$

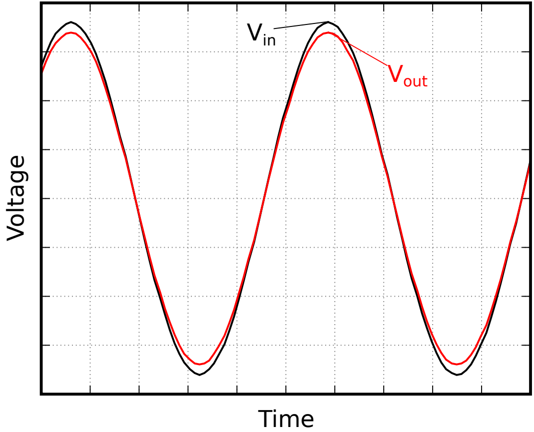
$$\Delta t = t_{V_{out}} - t_{V_{in}}$$

$$\Delta t = \frac{\phi}{\omega} = \frac{\phi}{2\pi f} = \frac{\phi}{2\pi} T$$

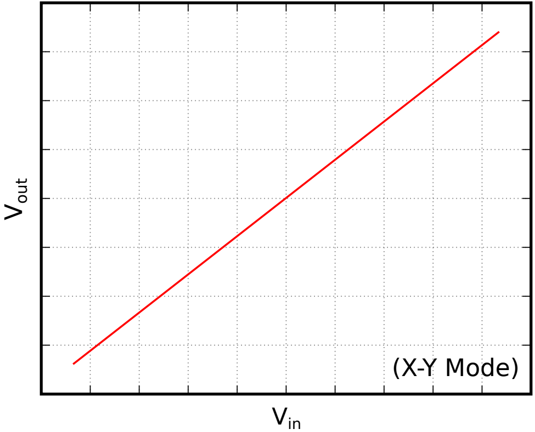
T



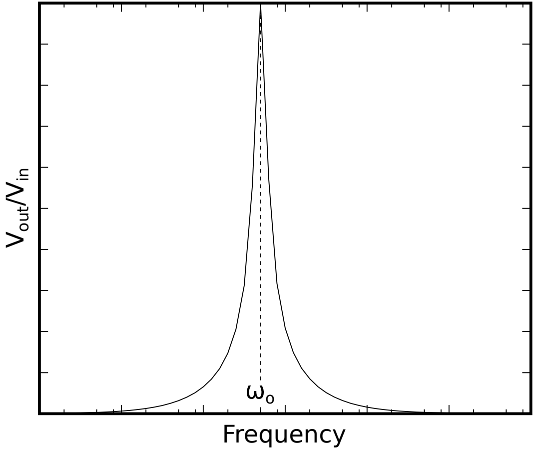
Voltage
Time
V_{in}
V_{out}
Δt
(a)
V_{out}
V_{in}
(X-Y Mode)
(b)



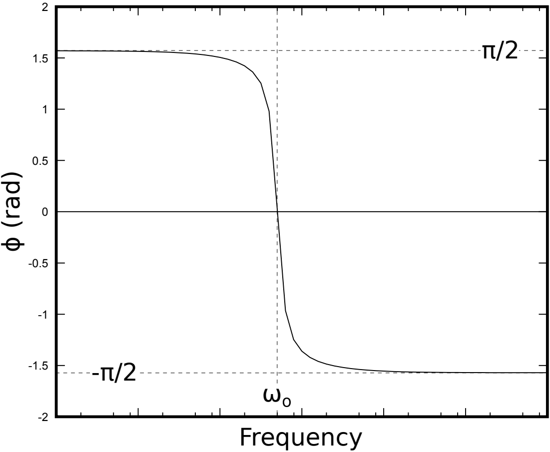
(c)



(d)



(a)



(b)

$$V_{out} = V_{in}$$

X_L

X_C

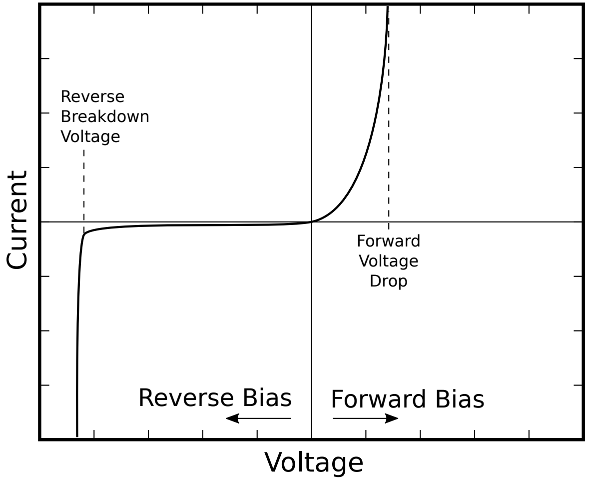
$$\omega_o = \frac{1}{\sqrt{LC}}$$

V_{out}

V_{in}

$$A_V = V_{out}/V_{in}$$

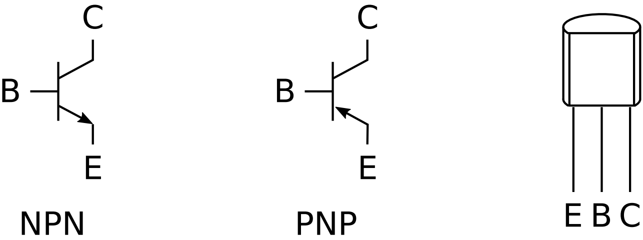
v_{out}



Reverse
Breakdown
Voltage
Forward
Voltage
Drop
Reverse Bias
Forward Bias
Current
Voltage

$V - I$

10^{-8}



(a) (b) (C)

I_C

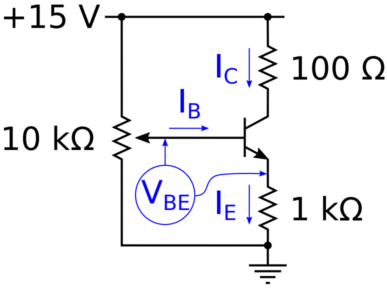
V_{BE}

$$I_C = I_o \left(e^{\frac{V_{BE}}{kT/e}} - 1 \right)$$

I_o

$$e = 1.6 \times 10^{-19}$$

$$k = 1.38 \times 10^{-23}$$


+15 V
I_C
100 Ω
I_B
10 kΩ
V_{BE}
I_E
1 kΩ

≈ 0.7

Ω

$$r_e = \frac{dV_{BE}}{dI_C} = \frac{kT/e}{I_C}$$

$(25\ \Omega)/I_C[\text{mA}]$

V_{CE}

$$V_{CE} \approx 0.25$$

I_B

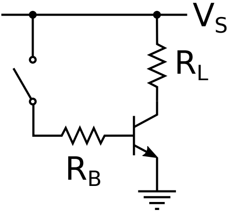
h_{FE}

$$I_C = h_{FE} I_B$$

I_E

Basic Transistor Behavior

- In order for a transistor to function, make $V_C > V_E$, and keep I_B, I_C, and V_{CE} below the rated maximum values of the transistor.
- **On:** In saturation, $V_{BE} \approx 0.7$ V, $V_{CE} \approx 0.25$ V, and $I_C = h_{FE} I_B$, where $h_{FE} \approx 100$.
- **Off:** If $V_{BE} < 0.7$ V (significantly), $I_C \approx 0$.
- **Active Region:** V_{BE} is between "off" and saturation. The collector current is governed by the Ebers-Moll equation (Equation 29). The emitter resistance at room temperature is $r_e = (25\ \Omega)/I_C[\text{mA}]$.



V_S
R_L
R_B

R_L

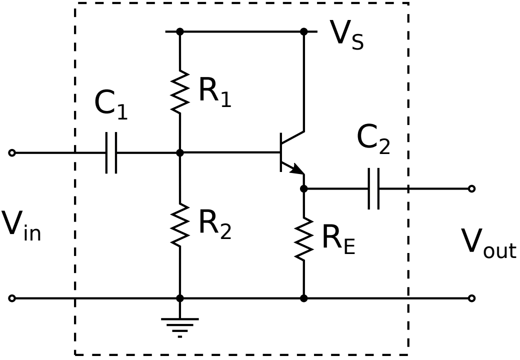
R_B

$$\approx V_S$$

≈ 0.25

V_S

$$\approx 10\ I_C/h_{fe}$$



V_S
R_1
C_1
C_2
R_2
R_E
V_in
V_out

$$A_v \approx 1$$

V_E

R_E

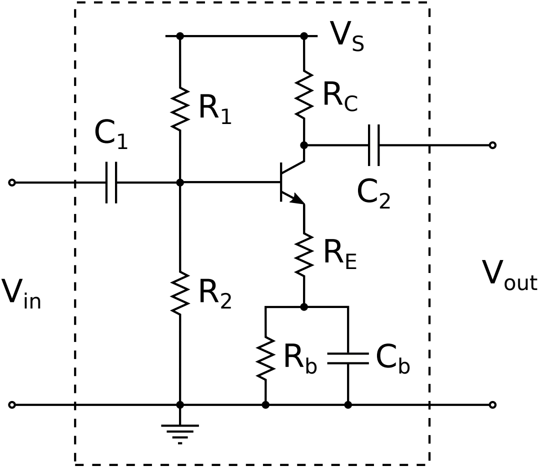
$$R_1/R_2$$

$$V_B \approx V_E + 0.6\ \mathrm{V}$$

$$R_1||R_2 \approx h_{FE}R_E/10$$

C_1

C_2



V_S
R_C
R_1
C_1
C_2
R_E
V_in
V_out
R_2
R_b
C_b

$$A_v = -\frac{R_C}{R_E}$$

R_C

r_e

R_b

C_b

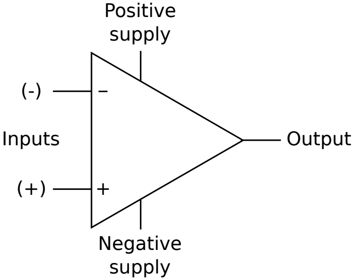
$$\approx \frac{R_b}{10}$$

$$R_E + r_e$$

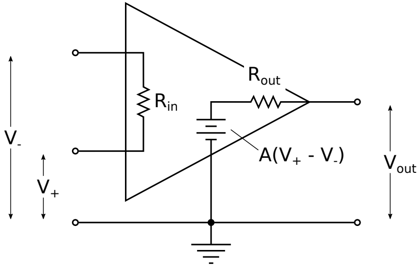
$$R_1||R_2 \approx h_{FE}(R_E + R_b)/10$$

$R_1 || R_2$

$$R_C || (R_E + R_b)$$


Positive supply
(-)
–
Inputs
Output
(+)
+
Negative supply

(a)


R_out
R_in
V_-
A(V_+ - V_-)
V_out
V_+

(b)

$$V_+ - V_-$$

A

$$-V_S \leq V_{out} \leq V_S$$

$$-V_S < V_{out} < V_S$$

$-V_S$

$$A = \infty$$

$$R_{in} = \infty$$

$$R_{out} = 0$$

$$V_+ = V_-$$

10^3

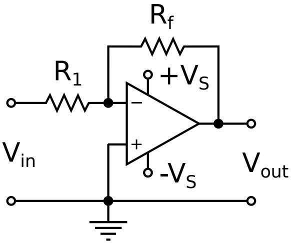
10^5

$$V_+ \approx V_-$$

Basic Op Amp Behavior

- $R_{in} = \infty, \; A = \infty, \; R_{out} = 0$
- **Linear region:** $V_+ = V_-, \; -V_S < V_{out} < V_S$
- **Saturation:** $V_+ > V_- \implies V_{out} = V_S$ or $V_+ < V_- \implies V_{out} = -V_S$

$10^{12}\ \Omega$



Rf
R1
+VS
−
+
Vin
Vout
-VS

R_f

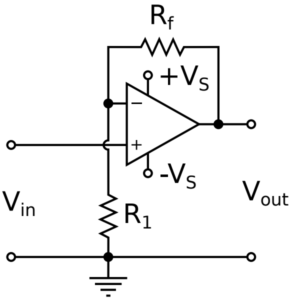
$$I = \frac{V_{in}}{R_1} = \frac{-V_{out}}{R_f}$$

$$A_v = -\frac{R_f}{R_1}$$

$R_{oa} || R_f$

R_{oa}

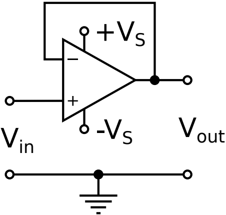
$$R_f / R_1$$


R_f
$+V_S$
$-V_S$
V_{in}
R_1
V_{out}

$$V_{in} = \frac{R_1}{R_1 + R_f} V_{out}$$

$$A_v = 1 + \frac{R_f}{R_1}$$

$$R_{oa}||(R_1 + R_f)$$

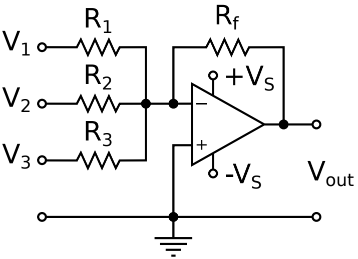


$+V_S$
$-V_S$
V_{in}
V_{out}

$$R_1 = \infty$$

$$R_f = 0$$

$$A_v = 1$$



R1
Rf
V1
+VS
R2
V2
−
R3
+
V3
-VS
Vout

$$\frac{V_1}{R_1}$$

$$\frac{V_2}{R_2}$$

I3

$$\frac{V_3}{R_3}$$

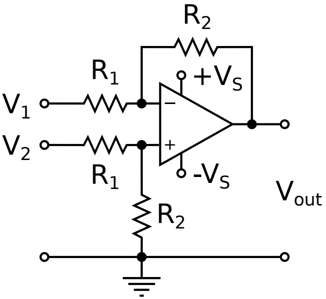
I_f

$$\frac{V_{out}}{R_f}$$

$$I_1 + I_2 + I_3 = I_f$$

$$V_{out} = -\left(\frac{R_1}{R_f}V_1 + \frac{R_2}{R_f}V_2 + \frac{R_3}{R_f}V_3\right)$$

R_1/R_f



R2
R1
+VS
V1
V2
R1
-VS
Vout
R2

$$I_1 + I_2 = I_3 + I_4$$

$$\frac{V_1 - V_-}{R_1}$$

$$\frac{V_2 - V_+}{R_1}$$

$$\frac{V_{-} - V_{out}}{R_2}$$

/4

$$\frac{V_+}{R_2}$$

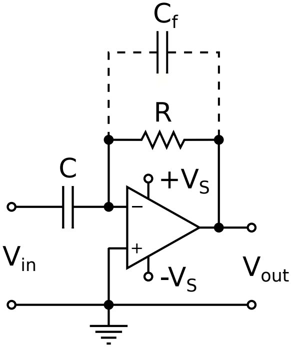
$$V_+ = \frac{R_2}{R_1 + R_2} V_2$$

$$V_{-} = V_{+}$$

$$V_{out} = -\frac{R_2}{R_1}\left(V_1 - V_2\right)$$

V_1

V_2



C_f
R
C
+V_S
-V_S
V_in
V_out

$$C_f < C$$

C_f

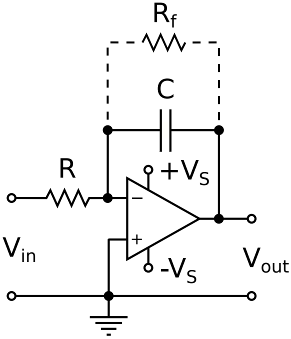
$$V_{out} = -IR$$

$$V_{in} = \frac{Q}{C}$$

$$I \equiv \frac{dQ}{dt}$$

$$V_{out} = -RC\frac{dV_{in}}{dt}$$

RC



Rf
C
R
+VS
-VS
Vin
Vout

$$I = \frac{V_{in}}{R}$$

$$V_{out} = -\frac{Q}{C}$$

$$I \equiv dQ/dt$$

$$Q = \int dQ = \frac{1}{R} \int V_{in}\, dt$$

$$V_{out} = -\frac{1}{RC}\int V_{in}\,dt$$

$Thing_1$

$Thing_2$

$$dB = 10 \log \left(\frac{\mathsf{Thing}_2}{\mathsf{Thing}_1} \right)$$

Thing_2

10^8

Thing_1

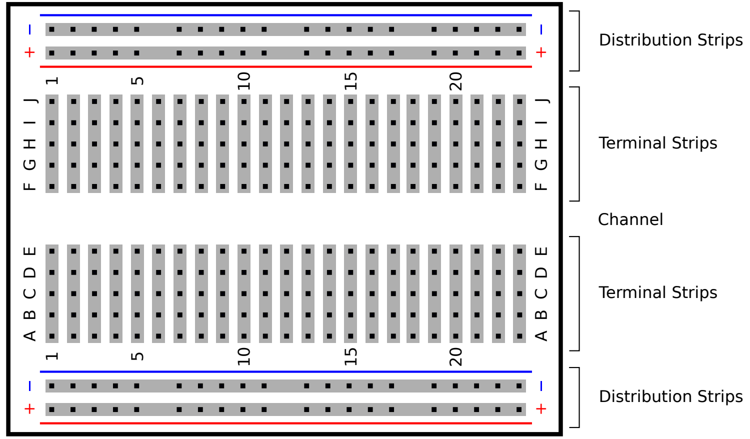
10^{-12}

2

$$10\ \log\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\frac{1}{2}\right) = 10(-0.3010) = -3.01$$

$$|V_{in}|/|V_{out}|$$

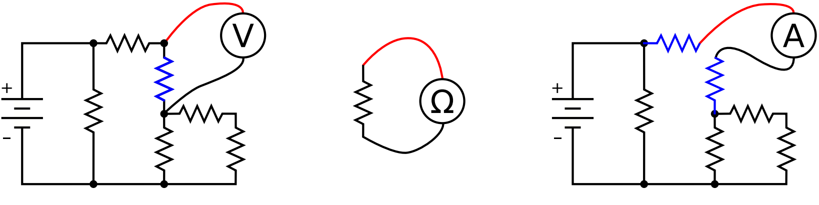
$$1/\sqrt{2}$$



Distribution Strips
Terminal Strips
Channel
Terminal Strips
Distribution Strips
1
5
10
15
20
J
I
H
G
F
E
D
C
B
A
−
+

$$R = 64 \times 10^2\ \Omega = 64 \times 100\ \Omega = 6400\ \Omega$$

$\pm 320\ \Omega$



(a) **(b)** **(c)**