Complex

$$
\begin{aligned}
i^2 &\coloneqq -1 \\
i(x+iy) &= -y+ix
\end{aligned}
$$

Hyperbolic

$$
\begin{aligned}
j^2 &\coloneqq +1 \\
j(x+jy) &= y+jx
\end{aligned}
$$

Conjugation and norm

$$
\begin{aligned}
(x+iy)^* &= x-iy \\
|z|^2 \coloneqq zz^* &= (x+iy)(x-iy) \\
&= x^2+y^2
\end{aligned}
$$

$$
\begin{aligned}
(x+jy)^* &= x-jy \\
|z|^2 \coloneqq zz^* &= (x+jy)(x-jy) \\
&= x^2-y^2
\end{aligned}
$$

Exponentiation

$$
\begin{aligned}
i^n &= \begin{cases} 1 \\ i \\ -1 \\ -i \end{cases} \\
e^{it} &= 1(\dots)+i(\dots) \\
&\quad -1(\dots)-i(\dots) \\
e^{it} &= \cos t + i \sin t
\end{aligned}
$$

$$
\begin{aligned}
j^n &= \begin{cases} 1 \\ j \\ 1 \\ j \end{cases} \\
e^{jt} &= 1(\dots)+j(\dots) \\
&\quad +1(\dots)+j(\dots) \\
e^{jt} &= \cosh t + j \sinh t
\end{aligned}
$$

Pythagorean identity

$$
\begin{aligned}
e^{-it} &= \cos(-t) + i\sin(-t) \\
&= \cos t - i \sin t \\
e^{-it} &= \left(e^{it}\right)^* \\
\left|e^{it}\right|^2 &= e^{it-it} \\
&= 1 \\
1 &= \cos^2 t + \sin^2 t
\end{aligned}
$$

$$
\begin{aligned}
e^{-jt} &= \cosh(-t) + j\sinh(-t) \\
&= \cosh t - j \sinh t \\
e^{-jt} &= \left(e^{jt}\right)^* \\
\left|e^{jt}\right|^2 &= e^{jt-jt} \\
&= 1 \\
1 &= \cosh^2 t - \sinh^2 t
\end{aligned}
$$

Pythagorean corollaries

$$
\begin{aligned}
\cos^2 t &= 1 - \sin^2 t \\
\sin^2 t &= 1 - \cos^2 t \\
\sec^2 t &= 1 + \tan^2 t \\
\csc^2 t &= \cot^2 t + 1
\end{aligned}
$$

$$
\begin{aligned}
\cosh^2 t &= 1 + \sinh^2 t \\
\sinh^2 t &= \cosh^2 t - 1 \\
\operatorname{sech}^2 t &= 1 - \tanh^2 t \\
\operatorname{csch}^2 t &= \cot^2 t - 1
\end{aligned}
$$

Angle sum formulae

$$
\begin{aligned}
e^{i(A+B)} &= e^{iA}e^{iB} \\
&= (c_A + is_A)(c_B + is_B)
\end{aligned}
$$

$$
\begin{array}{c|cc}
 & c_B & is_B \\
\hline
c_A & c_A c_B & ic_A s_B \\
is_A & ic_B s_A & -s_A s_B
\end{array}
$$

$$
\begin{matrix} \cos(A+B) \\ +i\sin(A+B) \end{matrix} = \begin{matrix} c_A c_B - s_A s_B \\ +i(c_B s_A + c_A s_B) \end{matrix}
$$

$$
\begin{aligned}
e^{j(A+B)} &= e^{jA}e^{jB} \\
&= (c_A + js_A)(c_B + js_B)
\end{aligned}
$$

$$
\begin{array}{c|cc}
 & c_B & js_B \\
\hline
c_A & c_A c_B & jc_A s_B \\
js_A & jc_B s_A & +s_A s_B
\end{array}
$$

$$
\begin{matrix} \cosh(A+B) \\ +j\sinh(A+B) \end{matrix} = \begin{matrix} c_A c_B + s_A s_B \\ +j(c_B s_A + c_A s_B) \end{matrix}
$$

Double angle

$$
\begin{aligned}
e^{2it} &= \left(e^{it}\right)^2 \\
\cos 2t &= \cos^2 t - \sin^2 t \\
i\sin 2t &= i\,2\cos t \sin t
\end{aligned}
$$

$$
\begin{aligned}
e^{2jt} &= \left(e^{jt}\right)^2 \\
\cosh 2t &= \cosh^2 t + \sinh^2 t \\
j\sinh 2t &= j\,2\cosh t \sinh t
\end{aligned}
$$

Double angle + Pythagorean

$$
\begin{aligned}
\cos 2t &= \cos^2 t - (1 - \sin^2 t) \\
&= 2\cos^2 t - 1 \\
&= (1 - \sin^2 t) - \sin^2 t \\
& 1 - 2\sin^2 t
\end{aligned}
$$

$$
\begin{aligned}
\cosh 2t &= \cosh 2t + (\cosh 2t - 1) \\
&= 2\cosh^2 t - 1 \\
&= (1 + \sinh^2 t) + \sinh^2 t \\
&= 1 + 2\sinh^2 t
\end{aligned}
$$

Derivatives

$$
\begin{aligned}
f(t) &\coloneqq e^{it} \\
f'(t) &= ie^{it} \\
&= i(\cos t + i\sin t) \\
\cos' t + i\sin' t &= -\sin t + i\cos t
\end{aligned}
$$

$$
\begin{aligned}
f(t) &\coloneqq e^{jt} \\
f'(t) &= je^{jt} \\
&= j(\cosh t + j\sinh t) \\
\cosh' t + j\sinh' t &= \sinh t + j\cosh t
\end{aligned}
$$

Dot and cross product

$$
u^* v = (\alpha - i\beta)(\gamma + i\delta)
$$

$$
\begin{array}{c|cc}
 & \gamma & i\delta \\
\hline
\alpha & \alpha\gamma & i\alpha\delta \\
-i\beta & -i\beta\gamma & +\beta\delta
\end{array}
$$

$$
\begin{aligned}
&= (\alpha\gamma + \beta\delta) + i(\alpha\delta - \beta\gamma) \\
u^* v &= (u \cdot v) + i(u \times v) \\
(iz) \cdot z &= \Re(iz^* z) = \Re(i|z|^2) = 0
\end{aligned}
$$

$$
u^* v = (\alpha - j\beta)(\gamma + j\delta)
$$

$$
\begin{array}{c|cc}
 & \gamma & i\delta \\
\hline
\alpha & \alpha\gamma & +j\alpha\delta \\
-j\beta & -j\beta\gamma & -\beta\delta
\end{array}
$$

$$
\begin{aligned}
&= (\alpha\gamma - \beta\delta) + j(\alpha\delta - \beta\gamma) \\
u^* v &= (u \cdot v) + j(u \times v) \\
(jz) \cdot z &= \Re(jz^* z) = \Re(j|z|^2) = 0
\end{aligned}
$$

Arc length

$$\begin{aligned} f(t) &\coloneqq e^{it} \\ S &= \int \sqrt{|f'(t)|^2} dt \\ &= \int \sqrt{|ie^{it}|^2} dt = \int \sqrt{1} dt \\ &= \Delta t \end{aligned}$$

$$\begin{aligned} f(t) &\coloneqq e^{jt} \\ S &= \int \sqrt{-|f'(t)|^2} dt \\ &= \int \sqrt{|je^{jt}|^2} dt = \int \sqrt{-(-1)} dt \\ &= \Delta t \end{aligned}$$

Sector area

$$\begin{aligned} A &= \frac{1}{2} \int f(t) \times f'(t) dt \\ &= \frac{1}{2} \int \Im\Big(f^*(t) f'(t)\Big) dt \\ &= \frac{1}{2} \int \Im\big(e^{-it} i e^{it}\big) dt \\ &= \frac{1}{2} \int 1 \, dt \\ &= \frac{\Delta t}{2} \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2} \int \Big[\Big] f(t) \times f'(t) dt \\ &= = \frac{1}{2} \int \Im\big(f^*(t) f'(t)\big) dt \\ &= = \frac{1}{2} \int \Im\big(e^{-jt} j e^{jt}\big) dt \\ &= \frac{1}{2} \int 1 \, dt \\ &= \frac{\Delta t}{2} \end{aligned}$$