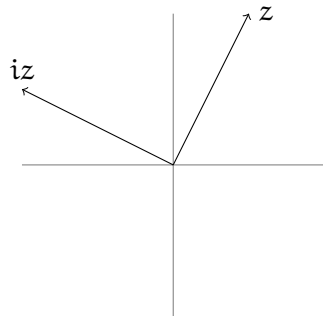


Complex

$$i^2 := -1$$

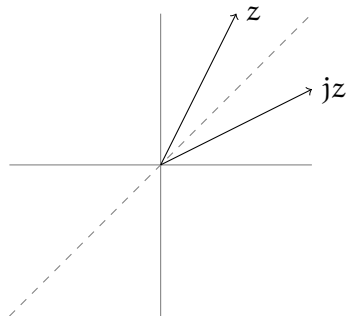
$$i(x + iy) = -y + ix$$



Hyperbolic

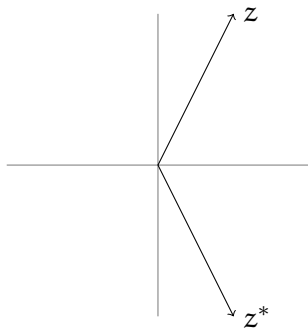
$$j^2 := +1$$

$$j(x + jy) = y + jx$$

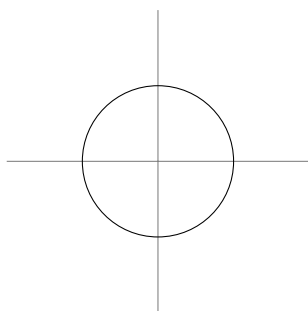


Conjugation and norm

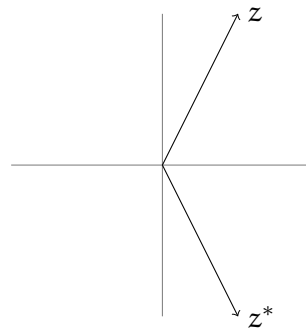
$$(x + iy)^* = x - iy$$



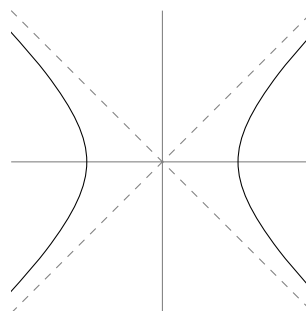
$$|z|^2 := zz^* = (x + iy)(x - iy) \\ = x^2 + y^2$$



$$(x + jy)^* = x - jy$$



$$|z|^2 := zz^* = (x + jy)(x - jy) \\ = x^2 - y^2$$



## Exponentiation

$$i^n = \begin{cases} 1 \\ i \\ -1 \\ -i \end{cases}$$

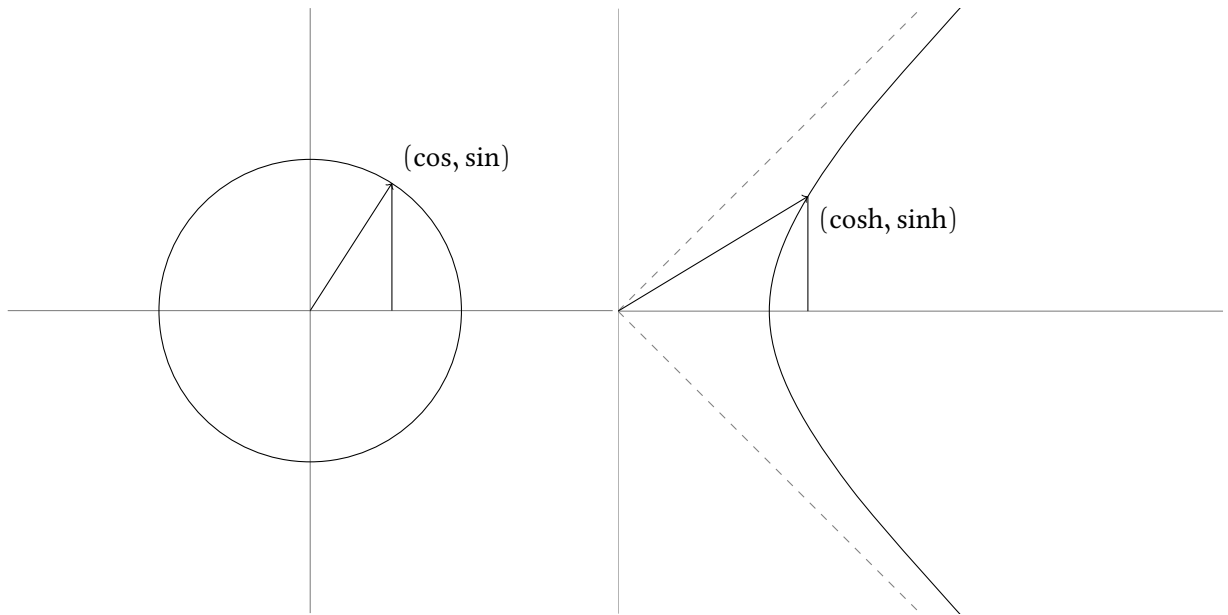
$$e^{it} = 1(\dots) + i(\dots) \\ -1(\dots) - i(\dots)$$

$$e^{it} = \cos t + i \sin t$$

$$j^n = \begin{cases} 1 \\ j \\ 1 \\ j \end{cases}$$

$$e^{jt} = 1(\dots) + j(\dots) \\ +1(\dots) + j(\dots)$$

$$e^{jt} = \cosh t + j \sinh t$$



## Pythagorean identity

$$e^{-it} = \cos(-t) + i \sin(-t) \\ = \cos t - i \sin t$$

$$e^{-it} = (e^{it})^*$$

$$|e^{it}|^2 = e^{it-it} \\ = 1$$

$$1 = \cos^2 t + \sin^2 t$$

$$e^{-jt} = \cosh(-t) + j \sinh(-t) \\ = \cosh t - j \sinh t$$

$$e^{-jt} = (e^{jt})^*$$

$$|e^{jt}|^2 = e^{jt-jt} \\ = 1$$

$$1 = \cosh^2 t - \sinh^2 t$$

Pythagorean corollaries

$$\cos^2 t = 1 - \sin^2 t$$

$$\sin^2 t = 1 - \cos^2 t$$

$$\sec^2 t = 1 + \tan^2 t$$

$$\csc^2 t = \cot^2 t + 1$$

$$\cosh^2 t = 1 + \sinh^2 t$$

$$\sinh^2 t = \cosh^2 t - 1$$

$$\operatorname{sech}^2 t = 1 - \tanh^2 t$$

$$\operatorname{csch}^2 t = \coth^2 t - 1$$

Angle sum formulae

$$e^{i(A+B)} = e^{iA} e^{iB}$$

$$= (c_A + i s_A)(c_B + i s_B)$$

	$c_B$	$i s_B$
$c_A$	$c_A c_B$	$i c_A s_B$
$i s_A$	$i c_B s_A$	$-s_A s_B$

$$\begin{aligned} \cos(A+B) &= c_A c_B - s_A s_B \\ +i \sin(A+B) &= +i(c_B s_A + c_A s_B) \end{aligned}$$

$$e^{j(A+B)} = e^{jA} e^{jB}$$

$$= (c_A + j s_A)(c_B + j s_B)$$

	$c_B$	$j s_B$
$c_A$	$c_A c_B$	$j c_A s_B$
$j s_A$	$j c_B s_A$	$-s_A s_B$

$$\begin{aligned} \cosh(A+B) &= c_A c_B + s_A s_B \\ +j \sinh(A+B) &= +j(c_B s_A + c_A s_B) \end{aligned}$$

Double angle

$$e^{2it} = (e^{it})^2$$

$$\cos 2t = \cos^2 t - \sin^2 t$$

$$i \sin 2t = i 2 \cos t \sin t$$

$$e^{2jt} = (e^{jt})^2$$

$$\cosh 2t = \cosh^2 t + \sinh^2 t$$

$$j \sinh 2t = j 2 \cosh t \sinh t$$

Double angle + Pythagorean

$$\cos 2t = \cos^2 t - (1 - \sin^2 t)$$

$$= 2 \cos^2 t - 1$$

$$= (1 - \sin^2 t) - \sin^2 t$$

$$1 - 2 \sin^2 t$$

$$\cosh 2t = \cosh^2 t + (\cosh^2 t - 1)$$

$$= 2 \cosh^2 t - 1$$

$$= (1 + \sinh^2 t) + \sinh^2 t$$

$$= 1 + 2 \sinh^2 t$$

Derivatives

$$f(t) := e^{it}$$

$$f'(t) = i e^{it}$$

$$= i(\cos t + i \sin t)$$

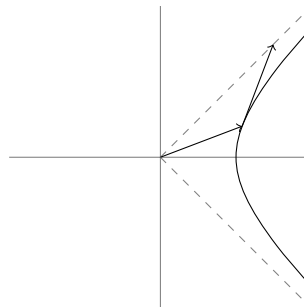
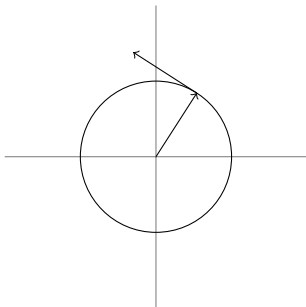
$$\cos' t + i \sin' t = -\sin t + i \cos t$$

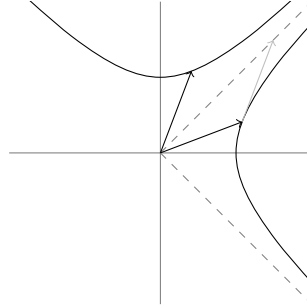
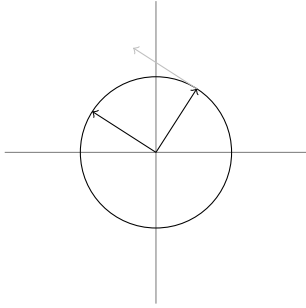
$$f(t) := e^{jt}$$

$$f'(t) = j e^{jt}$$

$$= j(\cosh t + j \sinh t)$$

$$\cosh' t + j \sinh' t = \sinh t + j \cosh t$$





Dot and cross product

$$\begin{aligned}
 \mathbf{u}^* \mathbf{v} &= (\alpha - i\beta)(\gamma + i\delta) \\
 &= \begin{vmatrix} \alpha & \gamma & i\delta \\ -i\beta & -i\beta\gamma & +\beta\delta \end{vmatrix} \\
 &= (\alpha\gamma + \beta\delta) + i(\alpha\delta - \beta\gamma) \\
 \mathbf{u}^* \mathbf{v} &= (\mathbf{u} \cdot \mathbf{v}) + i(\mathbf{u} \times \mathbf{v}) \\
 (iz) \cdot z &= \Re(iz^*z) = \Re(i|z|^2) = 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{u}^* \mathbf{v} &= (\alpha - j\beta)(\gamma + j\delta) \\
 &= \begin{vmatrix} \alpha & \gamma & i\delta \\ -j\beta & -j\beta\gamma & -\beta\delta \end{vmatrix} \\
 &= (\alpha\gamma - \beta\delta) + j(\alpha\delta - \beta\gamma) \\
 \mathbf{u}^* \mathbf{v} &= (\mathbf{u} \cdot \mathbf{v}) + j(\mathbf{u} \times \mathbf{v}) \\
 (jz) \cdot z &= \Re(jz^*z) = \Re(j|z|^2) = 0
 \end{aligned}$$

Arc length

$$\begin{aligned}
 f(t) &:= e^{it} \\
 S &= \int \sqrt{|f'(t)|^2} dt \\
 &= \int \sqrt{|ie^{it}|^2} dt = \int \sqrt{1} dt \\
 &= \Delta t
 \end{aligned}$$

$$\begin{aligned}
 f(t) &:= e^{jt} \\
 S &= \int \sqrt{|f'(t)|^2} dt \\
 &= \int \sqrt{|je^{jt}|^2} dt = \int \sqrt{-(-1)} dt \\
 &= \Delta t
 \end{aligned}$$

Sector area

$$\begin{aligned}
 A &= \frac{1}{2} \int f(t) \times f'(t) dt \\
 &= \frac{1}{2} \int \Im(f^*(t)f'(t)) dt \\
 &= \frac{1}{2} \int \Im(e^{-it}ie^{it}) dt \\
 &= \frac{1}{2} \int 1 dt \\
 &= \frac{\Delta t}{2}
 \end{aligned}$$

$$\begin{aligned}
 A &= \frac{1}{2} \int f(t) \times f'(t) dt \\
 &= \frac{1}{2} \int \Im(f^*(t)f'(t)) dt \\
 &= \frac{1}{2} \int \Im(e^{-jt}je^{jt}) dt \\
 &= \frac{1}{2} \int 1 dt \\
 &= \frac{\Delta t}{2}
 \end{aligned}$$