Complex

$$i^2 \coloneqq -1$$
$$i(x+iy) = -y + ix$$

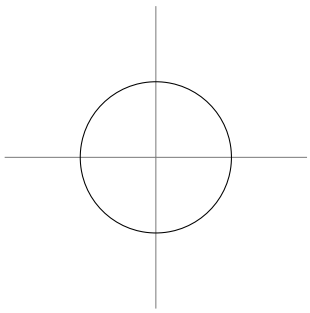
Hyperbolic

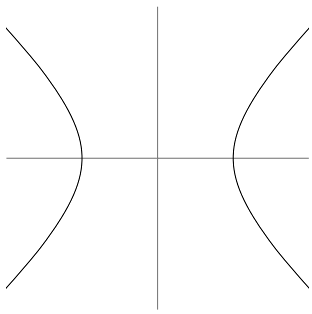
$$j^2 \coloneqq +1$$
$$j(x+jy) = y + jx$$

Conjugation and norm

$$\begin{aligned}(x+iy)^* &= x - iy \\ |z|^2 &\coloneqq zz^* = (x+iy)(x-iy) \\ &= x^2 + y^2\end{aligned}$$

$$\begin{aligned}(x+jy)^* &= x - jy \\ |z|^2 &\coloneqq zz^* = (x+jy)(x-jy) \\ &= x^2 - y^2\end{aligned}$$





Exponentiation

$$i^n = \begin{cases} 1 \\ i \\ -1 \\ -i \end{cases}$$
$$\begin{aligned}e^{it} &= 1(\dots) + i(\dots) \\ &\quad - 1(\dots) - i(\dots) \\ e^{it} &= \cos t + i \sin t\end{aligned}$$

$$j^n = \begin{cases} 1 \\ j \\ 1 \\ j \end{cases}$$
$$\begin{aligned}e^{jt} &= 1(\dots) + j(\dots) \\ &\quad + 1(\dots) + j(\dots) \\ e^{jt} &= \cosh t + j \sinh t\end{aligned}$$

Pythagorean identity

$$\begin{aligned}e^{-it} &= \cos(-t) + i\sin(-t) \\ &= \cos t - i \sin t \\ e^{-it} &= \left(e^{it}\right)^* \\ \left|e^{it}\right|^2 &= e^{it-it} \\ &= 1 \\ 1 &= \cos^2 t + \sin^2 t\end{aligned}$$

$$\begin{aligned}e^{-jt} &= \cosh(-t) + j\sinh(-t) \\ &= \cosh t - j \sinh t \\ e^{-jt} &= \left(e^{jt}\right)^* \\ \left|e^{jt}\right|^2 &= e^{jt-jt} \\ &= 1 \\ 1 &= \cosh^2 t - \sinh^2 t\end{aligned}$$

Pythagorean corollaries

$$\begin{aligned} \cos^2 t &= 1 - \sin^2 t \\ \sin^2 t &= 1 - \cos^2 t \\ \sec^2 t &= 1 + \tan^2 t \\ \csc^2 t &= \cot^2 t + 1 \end{aligned}$$

$$\begin{aligned} \cosh^2 t &= 1 + \sinh^2 t \\ \sinh^2 t &= \cosh^2 t - 1 \\ \operatorname{sech}^2 t &= 1 - \tanh^2 t \\ \operatorname{csch}^2 t &= \coth^2 t - 1 \end{aligned}$$

Angle sum formulae

$$\begin{aligned} e^{i(A+B)} &= e^{iA} e^{iB} \\ &= (c_A + i s_A)(c_B + i s_B) \end{aligned}$$

	c_B	$i s_B$
c_A	$c_A c_B$	$i c_A s_B$
$i s_A$	$i c_B s_A$	$-s_A s_B$

$$\begin{array}{r} \cos(A+B) \\ +i\sin(A+B) \end{array} = \begin{array}{c} c_A c_B - s_A s_B \\ +i(c_B s_A + c_A s_B) \end{array}$$

$$\begin{aligned} e^{j(A+B)} &= e^{jA} e^{jB} \\ &= (c_A + j s_A)(c_B + j s_B) \end{aligned}$$

	c_B	$j s_B$
c_A	$c_A c_B$	$j c_A s_B$
$j s_A$	$j c_B s_A$	$+s_A s_B$

$$\begin{array}{r} \cosh(A+B) \\ +j\sinh(A+B) \end{array} = \begin{array}{c} c_A c_B + s_A s_B \\ +j(c_B s_A + c_A s_B) \end{array}$$

Double angle

$$\begin{aligned} e^{2it} &= \left(e^{it}\right)^2 \\ \cos 2t &= \cos^2 t - \sin^2 t \\ i \sin 2t &= i\, 2 \cos t \sin t \end{aligned}$$

$$\begin{aligned} e^{2jt} &= \left(e^{jt}\right)^2 \\ \cosh 2t &= \cosh^2 t + \sinh^2 t \\ j \sinh 2t &= j\, 2 \cosh t \sinh t \end{aligned}$$

Double angle + Pythagorean

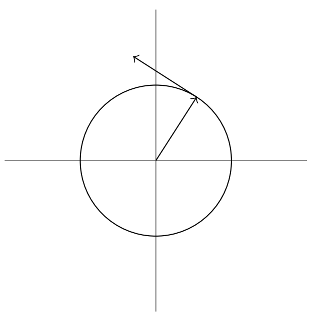
$$\begin{aligned} \cos 2t &= \cos^2 t - (1 - \sin^2 t) \\ &= 2\cos^2 t - 1 \\ &= (1 - \sin^2 t) - \sin^2 t \\ & 1 - 2\sin^2 t \end{aligned}$$

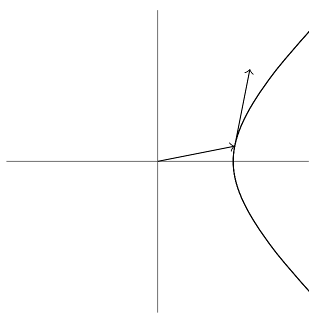
$$\begin{aligned} \cosh 2t &= \cosh 2t + (\cosh 2t - 1) \\ &= 2\cosh^2 t - 1 \\ &= (1 + \sinh^2 t) + \sinh^2 t \\ &= 1 + 2\sinh^2 t \end{aligned}$$

Derivatives

$$\begin{aligned} f(t) &\coloneqq e^{it} \\ f'(t) &= i e^{it} \\ &= i(\cos t + i \sin t) \\ \cos' t + i \sin' t &= -\sin t + i \cos t \end{aligned}$$

$$\begin{aligned} f(t) &\coloneqq e^{jt} \\ f'(t) &= j e^{jt} \\ &= j(\cosh t + j \sinh t) \\ \cosh' t + j \sinh' t &= \sinh t + j \cosh t \end{aligned}$$





Dot and cross product

$$\begin{aligned}
u^*v &= (\alpha - i\beta)(\gamma + i\delta) \\
&\begin{array}{c|cc} & \gamma & i\delta \\ \hline \alpha & \alpha\gamma & i\alpha\delta \\ -i\beta & -i\beta\gamma & +\beta\delta \end{array} \\
&= (\alpha\gamma + \beta\delta) + i(\alpha\delta - \beta\gamma) \\
u^*v &= (u \cdot v) + i(u \times v) \\
(iz) \cdot z &= \Re(iz^*z) = \Re(i|z|^2) = 0
\end{aligned}$$

$$\begin{aligned}
u^*v &= (\alpha - j\beta)(\gamma + j\delta) \\
&\begin{array}{c|cc} & \gamma & i\delta \\ \hline \alpha & \alpha\gamma & +j\alpha\delta \\ -j\beta & -j\beta\gamma & -\beta\delta \end{array} \\
&= (\alpha\gamma - \beta\delta) + j(\alpha\delta - \beta\gamma) \\
u^*v &= (u \cdot v) + j(u \times v) \\
(jz) \cdot z &= \Re(jz^*z) = \Re(j|z|^2) = 0
\end{aligned}$$

Arc length

$$\begin{aligned}
f(t) &\coloneqq e^{it} \\
S &= \int \sqrt{|f'(t)|^2}dt \\
&= \int \sqrt{|ie^{it}|^2}dt = \int \sqrt{1}dt \\
&= \Delta t
\end{aligned}$$

$$\begin{aligned}
f(t) &\coloneqq e^{jt} \\
S &= \int \sqrt{-|f'(t)|^2}dt \\
&= \int \sqrt{|je^{jt}|^2}dt = \int \sqrt{-(-1)}dt \\
&= \Delta t
\end{aligned}$$

Sector area

$$\begin{aligned}
A &= \frac{1}{2}\int f(t) \times f'(t)dt \\
&= \frac{1}{2}\int \Im\Big(f^*(t)f'(t)\Big)dt \\
&= \frac{1}{2}\int \Im\big(e^{-it}ie^{it}\big)dt \\
&= \frac{1}{2}\int 1\,dt \\
&= \frac{\Delta t}{2}
\end{aligned}$$

$$\begin{aligned}
A &= \frac{1}{2}\int f(t) \times f'(t)dt \\
&= = \frac{1}{2}\int \Im\big(f^*(t)f'(t)\big)dt \\
&= = \frac{1}{2}\int \Im\big(e^{-jt}je^{jt}\big)dt \\
&= \frac{1}{2}\int 1\,dt \\
&= \frac{\Delta t}{2}
\end{aligned}$$