$$
\begin{aligned}
f(x) &= f(0) + \int_0^x df(t)dt = f(0) + \int_0^x df(t)(x-t)^0 dt \\
&= f(0) - \int_x^0 \underbrace{df(t)}_{u}\underbrace{(x-t)^0 dt}_{dv} \\
&= f(0) - \Big(df(t)(-1)(x-t)\Big)_x^0 - (-1)\int_x^0 d^2 f(t)(-1)(x-t)dt \\
&= f(0) + df(0)x - \int_x^0 d^2 f(t)(x-t)dt \\
&= f(0) + df(0)x - \left(d^2 f(t)\frac{(-1)(x-t)^2}{2}\right)_x^0 - (-1)\int_x^0 d^3 f(t)\frac{(-1)(x-t)^2}{2}dt \\
&= f(0) + df(0)x + d^2 f(0)\frac{x^2}{2} - \int_x^0 d^3 f(t)\frac{(x-t)^2}{2}dt \\
&= f(0) + df(0)x + \ldots d^n f(0)\frac{x^n}{n!} + \underbrace{\int_0^x d^{n+1} f(t)\frac{(x-t)^n}{n!}dt}_{E}
\end{aligned}
$$

1 Holomorphic

Theorem 1 (Cauchy-Riemann)

$$df = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} z & iz \end{bmatrix}$$

Corollary 2

$$\oint_\gamma f dz = 0$$

Proof.

$$\int_{\partial A} f dz = \int_A df \wedge dz = \int_A (z dx + iz dy) \wedge (dx + i dy) = \int_A (iz - iz) dx \wedge dy = 0. \qquad \square$$

Theorem 3

$$f(a) = \frac{1}{i\tau}\oint \frac{f(z)}{a-z}dz$$

Proof. Set $z = re^{it} + a$.

$$\oint \frac{f(z)}{z-a}dz = \oint \frac{f(re^{it}+a)}{\cancel{re^{it}}}\left(i\cancel{re^{it}}dt\right) \to if(a)\oint dt = i\tau f(a) \qquad \square$$

Corollary 4 f *is complex-analytic.*

Proof.

$$f(u) = \frac{1}{i\tau}\oint \frac{f(z)}{z(1-u/z)}\,dz = \frac{1}{i\tau}\oint \frac{f(z)}{z}\left(1+\frac{u}{z}+\left(\frac{u}{z}\right)^2+\dots\right)dz$$

□