Landau & Lifshitz thinks of dx as a small change in x. I try to think of it as a differential form. I believe they are equivalent in the presence of a metric, but without one, I'm not so sure. I suppose you can always use an arbitrary metric.

$$f(x + dx) = df(x)(dx)?$$

$$\begin{aligned}
S &= \int -m\, d\tau \\
S(x + \delta x) &= \int -m\sqrt{d(x + \delta x)_\alpha d(x + \delta x)^\alpha} \\
S(x) + \delta S(x)(\delta x) + o(\delta x) &= \int -m\sqrt{d(x + \delta x)_\alpha d(x + \delta x)^\alpha} \\
\delta S(x)(\delta x) &= \int -m \frac{dx_a d\delta x^\alpha}{d\tau} \\
&= \int -mu_\alpha\, d\delta x^\alpha \\
&= \int m\, du_\alpha \delta x^\alpha + \cancel{(-mu_\alpha \delta x^\alpha]_a^b} \\
\frac{\delta S}{\delta x} &= m\, du
\end{aligned}$$

$$\begin{aligned}
S &= -\int A_\alpha(x) dx^\alpha \\
S(x + \delta x) &= -\int A_\alpha(x + \delta x) d(x + \delta x)^\alpha \\
&= -\int A_\alpha(x) dx^\alpha - \int A_\alpha(x) d\delta x^\alpha + dA_\alpha(x)(\delta x) dx^\alpha + o(\delta x)
\end{aligned}$$