

1 Lagrange's formula

$$X = \begin{bmatrix} X_0^0 & X_1^0 & X_2^0 \\ X_0^1 & X_1^1 & X_2^1 \\ X_0^2 & X_1^2 & X_2^2 \end{bmatrix}$$

$$\wedge^3 X = \begin{bmatrix} X_0^0 \\ X_0^1 \\ X_0^2 \end{bmatrix} \wedge \begin{bmatrix} X_1^0 \\ X_1^1 \\ X_1^2 \end{bmatrix} \wedge \begin{bmatrix} X_2^0 \\ X_2^1 \\ X_2^2 \end{bmatrix}$$

$$\begin{bmatrix} X_0^0 \\ 0 \\ 0 \end{bmatrix} \wedge \begin{bmatrix} X_1^0 \\ X_1^1 \\ X_1^2 \end{bmatrix} \wedge \begin{bmatrix} X_2^0 \\ X_2^1 \\ X_2^2 \end{bmatrix} = X_0^0 u_0 \wedge \begin{vmatrix} X_1^1 & X_2^1 \\ X_1^2 & X_2^2 \end{vmatrix} u_1 \wedge u_2 = +X_0^0 M_0^0 u_{012} = X_0^0 C_0^0 u_{012}$$

$$\begin{bmatrix} 0 \\ X_0^1 \\ 0 \end{bmatrix} \wedge \begin{bmatrix} X_1^0 \\ X_1^1 \\ X_1^2 \end{bmatrix} \wedge \begin{bmatrix} X_2^0 \\ X_2^1 \\ X_2^2 \end{bmatrix} = X_0^1 u_1 \wedge \begin{vmatrix} X_1^1 & X_2^1 \\ X_1^2 & X_2^2 \end{vmatrix} u_0 \wedge u_2 = -X_0^1 M_1^0 u_{012} = X_0^1 C_1^0 u_{012}$$

$$\wedge^3 X = \sum_{\alpha} X_{\alpha}^0 C_{\alpha}^0 u_{012}$$

2 Jacobi's formula

$$(A^T B)_{\beta}^{\alpha} = (A^T)_{\gamma}^{\alpha} B_{\beta}^{\gamma} = A_{\alpha}^{\gamma} B_{\beta}^{\gamma}$$

$$\text{tr}(A^T B) = (A^T B)_{\alpha}^{\alpha} = A_{\alpha}^{\gamma} B_{\alpha}^{\gamma}$$

$$\det A = \sum_k A_{ik} C_{ik}$$

$$\begin{aligned} \det(A + t u_i \otimes u_j) &= \sum_k A_{ik} C_{ik} + t C_{ij} \\ &= \det A + t C_{ij} \end{aligned}$$

$$\nabla_{u_i \otimes u_j} \det = C_{ij}$$

$$\begin{aligned} d\det_A(h) &= \sum_{ij} C_{ij} h_{ij} \\ &= \text{tr}(C^T h) \\ &= \text{tr}(\text{adj}(A)h) \end{aligned}$$