$$\mathrm{SL} = \det^{-1}(1) \implies \mathfrak{sl} = \mathrm{tr}^{-1}(0)$$

A basis for $\mathfrak{sl}(\mathbb{R}^N)$:

$$\begin{cases} u^\alpha \otimes u_\beta & \alpha \neq \beta \\ u^\alpha \otimes u_\alpha - u^N \otimes u_N & \alpha \in [0, N) \end{cases}$$

$$\begin{aligned} [u^\alpha|u_\beta, u^\gamma|u_\zeta] &= u^\alpha|u_\beta u^\gamma|u_\zeta - u^\gamma|u_\zeta u^\alpha|u_\beta \\ &= u^\alpha|u_\zeta \delta^\gamma_\beta - u^\gamma|u_\beta \delta^\alpha_\zeta \end{aligned}$$

In particular

$$\begin{aligned} [u^\alpha|u_\beta, u^\beta|u_\gamma] &= u^\alpha|u_\gamma \\ [u^\alpha|u_\beta, u^\beta|u_\alpha] &= u^\alpha|u_\alpha - u^\beta|u_\beta \end{aligned}$$

$$\begin{aligned} [u^\alpha|u_\beta, u^\gamma|u_\gamma - u^N|u_N] &= [u^\alpha|u_\beta, u^\gamma|u_\gamma] - [u^\alpha|u_\beta, u^N|u_N] \\ &= \left(u^\alpha|u_\gamma \delta^\gamma_\beta - u^\gamma|u_\beta \delta^\alpha_\gamma\right) - \left(u^\alpha|u_N \delta^N_\beta - u^N|u_\beta \delta^\alpha_N\right) \\ &= u^\alpha|u_\beta \left(\delta^\gamma_\beta - \delta^N_\beta - \delta^\alpha_\gamma + \delta^\alpha_N\right) \end{aligned}$$

$$\begin{aligned} [u^\alpha|u_\alpha - u^N|u_N, u^\gamma|u_\gamma - u^N|u_N] &= [u^\alpha|u_\alpha, u^\gamma|u_\gamma - u^N|u_N] - \cancel{[u^N|u_N, u^\gamma|u_\gamma]} \\ &= u^\alpha|u_\alpha \cancel{(\delta^\gamma_\alpha - \delta^N_\alpha - \delta^\alpha_\gamma + \delta^\alpha_N)} \\ &= 0 \end{aligned}$$