No.

Date

1) $m(a+bX) =$

$\frac{1}{N}\sum_{i=1}^{N}(a+bx_i) = \frac{1}{N}\left[\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i\right]$

$= \frac{1}{N}[Na] + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$

$= a + bm(X)$

$\therefore \boxed{m(a+bX) = a + b \times m(X)}$

$m(X) = \frac{1}{N}\sum_{i=1}^{N} x_i$

2) $cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - (a+bm(Y)))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(b(y_i - m(Y)))$

$= b\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))\right]$

$= \boxed{b\,cov(X,Y)}$

3) $cov(a+bX, a+bX) = b\,cov(a+bX, X)$

$m(a+bX) = a + bm(X)$

$cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}((a+bx_i) - (a+bm(X)))^2$

$= \frac{1}{N}\sum_{i=1}^{N}(b(x_i - m(X)))^2$

$= b^2\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\right]^2$

$= b^2 s^2$

$cov(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X)) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2$

$= s^2$

$\therefore cov(a+bX, a+bX) = b^2 cov(X,X)$ and $cov(X,X) = s^2$

4) A non-decreasing transformation keeps the order of the values the same, like $g(x) = 2+5x$ and $g(x) = arcsinh(x)$. Thus, the non-decreasing transformation of the median is the median of the transformed variable. This applies to the quantiles, because the 75th percentile is still the same after transformation. Measures based on differences (IQR or range) stay the same unless g is linear because nonlinear transforms change the spacing between values.

5) For non-decreasing $g$, $m(g(X))$ does not always equal $g(m(X))$.

e.g. $X = \{0, 2\}$ $g(x) = x^2$

$m(X) = \frac{0+2}{2} = 1$ $g(m(X)) = 1^2 = 1$

$m(g(X)) = \frac{0^2+2^2}{2} = \frac{4}{2} = 2$

$1 \neq 2$