

**Chapter Review Sheets for
Elementary Differential Equations and Boundary Value Problems, 8e**

Chapter 6: The Laplace Transformation

Definitions:

- Integral Transforms, Kernel
- Improper Integral Piecewise
- Continuous Exponential
- Order
- Unit Step Function (Heaviside Function)
- Unit Impulse Function, Delta Function
- Convolution
- Transfer Function, Impulse Response

Theorems:

- Theorem 6.1.1: Comparison Test for Improper Integrals
- Theorem 6.1.2: Existence of the Laplace Transform, $F(s)$
- Theorem 6.2.1: Laplace Transform of $f'(t)$
- Corollary 6.2.2: Laplace Transform of $f(t)$
- Theorem 6.3.1: Transform of the unit step function, $u(1)$, times a shifted function, $f(t)$
- Theorem 6.3.2: First Translation Theorem; Inverse Transforming $F(s - c)$
- Theorem 6.6.1: Second Translation Theorem; Convolution Result

Important Skills:

- The Laplace transformation is defined through an improper integral. You must be comfortable evaluating them. Hence you should review this topic in any calculus book.
- Be able to calculate the transform of all the basic functions, given in the table on page 319. (Examples 5, 6 & 7, p.311-312)
- Even more importantly, know how to compute inverse transform functions using manipulative translation methods. You may need to use partial fractions, but you should have already reviewed this for Chapter 2. (Examples 1 & 2, p.320)
- Know how to transform derivatives of functions and linear differential equations. (Theorem 6.2.1 and Corollary 6.2.2 p.315, Examples 1 & 2, p.320)
- Understand the unit function, $u(t)$, as well as, the unit impulse function, $\delta(t)$, and how to use them in transforming and inverse transforming functions. (Example 1, p.326; Example 1, p.343)
- The process of using the Laplace transform method is as follows; Given a differential equation, one transforms both sides of the equation. One will need to input the initial values when transforming derivatives. Derivatives with respect to t transform to polynomials in s . If the differential equation is linear, then the resulting equation is linear in $Y(s)$. You simply solve the equation for $Y(s)$, and then use all the methods available to recover $y(t)$. (Example 1, p.320 for continuous forcing; Example I, p.343 for discontinuous forcing.)

Relevant Applications:

- Mechanical and electrical problems with discontinuous forcing functions.