Chapter Review Sheets for
Elementary Differential Equations and Boundary Value Problems, 8e

Chapter 5: Series Solutions of Second Order Equations

Definitions:
- Radius of Convergence, Interval of Convergence
- Analytic
- Recurrence Relation
- Ordinary Point, Singular Point
- Regular and Irregular Singular Points
- Euler Equation, Indicial Equation Exponents of Singularity
- Chebyshev equation Hermite equation; Bessel Equation

Theorems:
- Theorem 5.3.1: Existence of series solutions to linear OBE's near ordinary points, and their convergence properties.
- Theorem 5.5.1: General solutions to Euler equations.
- Theorem 5.7.1: Series solutions near regular singular points.

Important Skills:
- Review power series, how to shift the index of summation, (Example 3, p.247) and tests for convergence. (Example 2, p.245)
- Know how to find the interval of convergence for a power series. (Example 2, p.245)
- Be able to determine all ordinary and singular points for a differential equation. (p. 250-51)
- For all singular points, be able to categorize as either regular or irregular; (Equations (7) and (8) page 271 give the criteria for a regular singular point.)
- For ordinary points, equation (3) on page 251 gives the form of the solution. Be able to derive recursion relation, as in example 1. If the recursion relation can be solved, one obtains the two linearly independent solutions of the homogenous problem. (Example 1, p.251)
- The method described in the second paragraph on page 244 can be used to find the first several terms in each of the linearly independent homogeneous solutions.
- Be able to determine lower bounds on the radius of convergence of the series solutions. (Example 4, p.264)
- Series solutions near regular singular points require the ability to solve Euler equations. Be able to recognize Euler equations, and know how to derive the characteristic equation. Know the general solutions for the three case of roots to the characteristic equation. (Theorem 5.5.1, Examples 2 and 3, p. 274-275)
- The assumption for the form of the series solution near regular points is given by (7) on page 280.
- Substitution into the differential equations will yield an indicial equation, as well as, a recursion relation. The solutions to the indicial equation are those to the associated Euler problem. (Example 1, p.281) In cases where the roots to the indicial equation are equal or differ by an integer, the method can be slightly modified to obtain solutions, or one can use reduction of order. (p. 289)
- Finally, Bessel equations give good examples of series solutions near regular singular points; several examples are given in section 5.8. Bessel functions are extremely important in applied mathematics, physics and engineering problems, and seem to arise when there are cylindrical symmetries.