Chapter Review Sheets for
Elementary Differential Equations and Boundary Value Problems, 8e

Chapter 9: Nonlinear Differential Equations and Stability

Definitions:

- Equilibrium Solutions
- Critical Points
- Trajectory
- Phase P1ane,
- Phase Portrait
- Node; Nodal Sink, Nodal Source, Saddle Point, Proper Node, (Star Point)
- Improper Node, (Degenerate Node), Spiral Sink, Spiral Source, Autonomous
- Stable, Unstable Isolated Critical Point, Almost Linear System,
- Basis of attraction
- Globally asymptotically stable
- Region of Asymptotic Stability, Nullclines
- Separatrix
- Liapunov's Method
- Positive Definite, Negative Definite, Positive Semidefinite, Negative Semidefinite
- Limit Cycle
- Asymptotically Stable
- Strange attractors
- Chaotic system/equation

Theorems:

- Theorem 9.3.1: Stability of critical points of linear systems dependence on eigenvalues.
- Theorem 9.3.2: Stability of critical points of almost linear systems.
- Theorem 9.6.1: Stability of critical points dependence on negative definite and negative semidefinite nature of the Liapunov function, and its derivative.
- Theorem 9.6.2: Conditions on definiteness for an unstable critical point.
- Theorem 9.6.3: Conditions for positive definiteness.
- Theorem 9.6.4: Conditions for Y(x, (,,y)= ax ~ + 6 x y + c y` to be positive or negative definite
- Theorem 9.7.1: Existence of closed trajectories
- Theorem 9.7.2: Nonexistence of closed trajectories.
- Theorem 9.7.3: Poincare-Bendixon Theorem.

Important Skills:

- Be able to determine the phase plane and phase portraits of a 2 by 2 linear system . The solutions will depend on cigenvalues. Pages 484-491 cover the five important cases. Table 9.1.1 on p. 492 summarizes the eigenvalue results.
- Determine the trajectories for a system of ODE's. (Examples 1 & 2, p.500)
- Know how to determine whether a system of ODE's is almost linear. (Examples 1 & 2, p.505)
- Be able to determine the linear system associated with the almost linear system, (Example 3, p. 507)
- Relating the ODE system to the possible motions of a pendulum (Example 4, p. 510)
- Sketch phase portraits for competing species. (Example 1 or 2, p.539 and p.541)
- Sketch phase portraits for predator-prey. (Example 1, p.539)
- Use Liapunov's method to determine the stability of a critical point. (Example 2, p.541; Example l, p.539)

- Determine periodic solutions of systems of QDE's. (F;xample 1, p.548)
- Study the solution of van der Pol's equation (Example 2, p. 552)

Relevant Applications:

- Population Modeling, Competing Species, Predator-Prey Modeling