Categories for Cryptographic Composability

Riley Shahar
Advised by Angélica Osorno and Adam Groce

Outline

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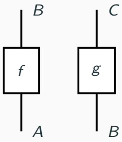

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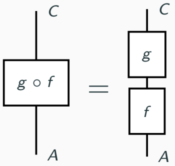

Sequential (Vertical) Composition

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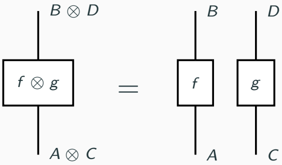

Parallel (Horizontal) Composition

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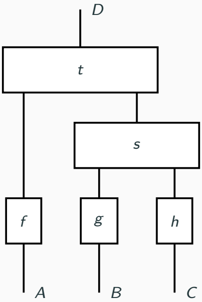

Cryptography

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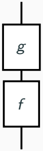
Is  secure?

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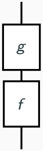
Is 

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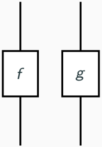
Is 

secure?

Simulation-based security

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A protocol is *secure* if it is computationally indistinguishable from the ideal world.

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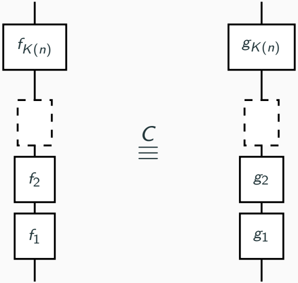

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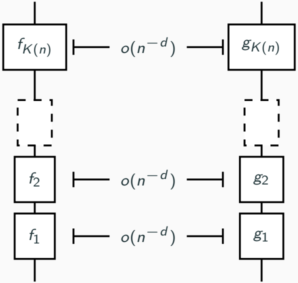

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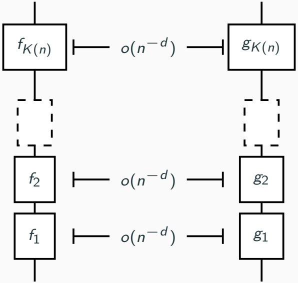

hold?   Only if $K(n) = O(n^d)$.

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However, [GK96] gave a protocol that's simulation secure, but doesn't compose in parallel.

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...**category theory** is an excellent candidate for such a theory.

Categories and Programming Languages

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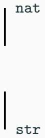
- objects are types;



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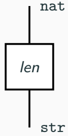

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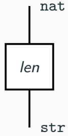

(We've already been doing category theory!)

Category Theory and Computer Science

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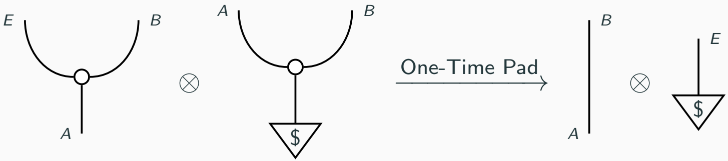
- cryptography [BK22]!

Categorical Composable Cryptography: The Idea



[Broadbent & Karvonen, 2022]

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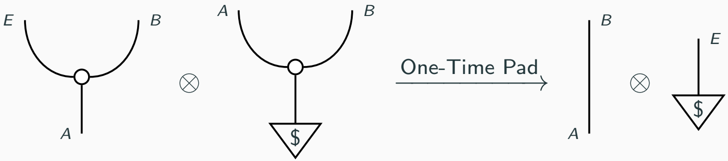

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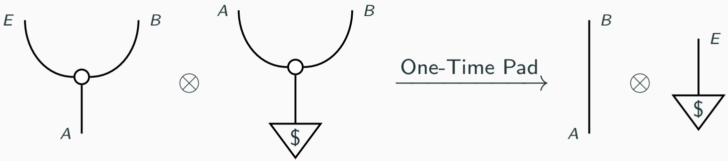

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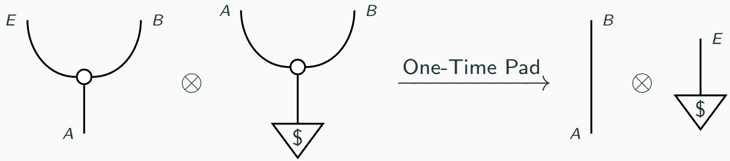

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- objects are resources, like channels or keys;
- morphisms are "protocols with holes";
- composition "plugs in the holes".

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There’s always a tradeoff: it relies on dense abstract machinery.

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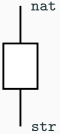

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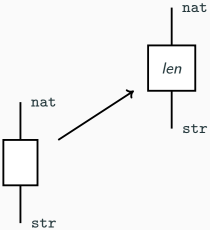

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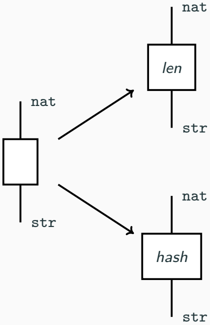

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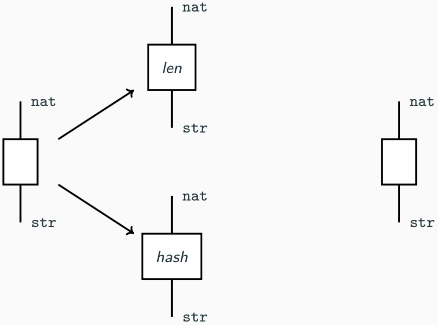

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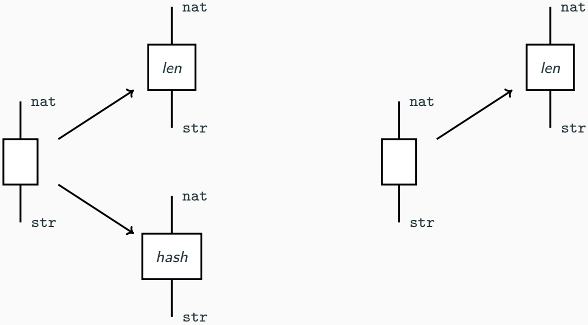

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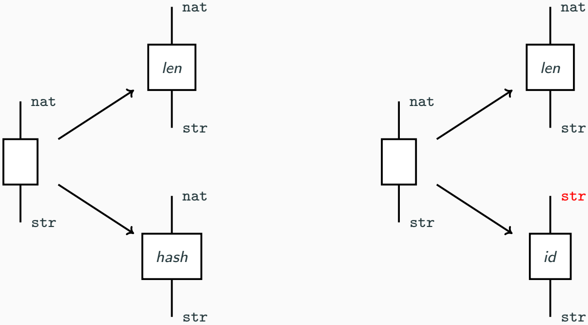

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~~Open~~ Question 1: *Can the axioms be formulated as functoriality plus some conditions?* Answer: **Yes.**

Open Question 2: *How broad is the definition of an attack model? Does it capture enough of modern cryptography?*

Computational Indistinguishability

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Open Question 3: *Is there a natural categorical model of computational indistinguishability?*

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- Can we incorporate categorical notions from game theory, programming languages, etc. into cryptography?

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- What does the presence of various categorical structure ((co)limits, monads, etc.) say cryptographically?

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