Categories for Cryptographic Composability

Riley Shahar
Advised by Angélica Osorno and Adam Groce

Outline

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- Why categories?

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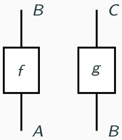

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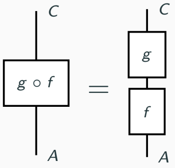

Sequential (Vertical) Composition

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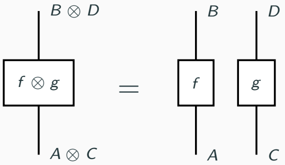

Parallel (Horizontal) Composition

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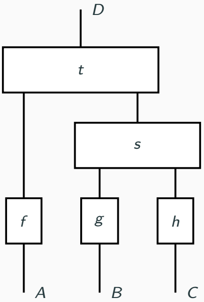

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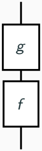
Is 

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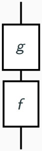
Is 

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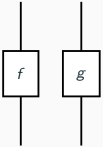
Is 

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4. Bob reveals b

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Game-based composability

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1. Bob picks two messages
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Many natural commitment protocols suffer from *malleability.*

It's taken *decades* for a missing property to be noticed [PW91; BHL05]!

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However, [GK96] gave a protocol for zero-knowledge proof that's simulation secure, but doesn't compose in parallel.

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- are wildly dependent on small technical details;
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- are very hard to trust.

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. . . **category theory** is an excellent candidate for such a theory.

Categories and Programming Languages

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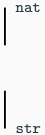
- objects are types;



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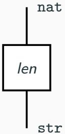

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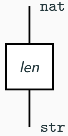

(We've already been doing category theory!)

Composing Programs

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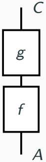
We can *always* make a program



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Composition "works" in PL theory.

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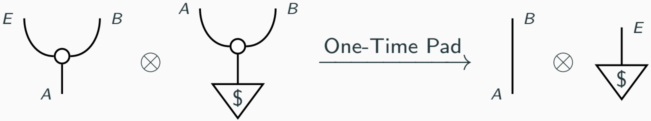

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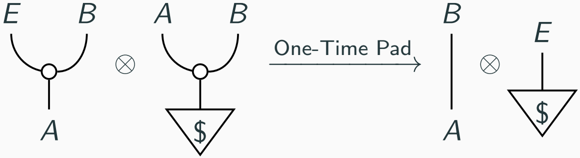

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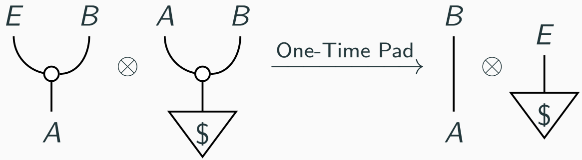

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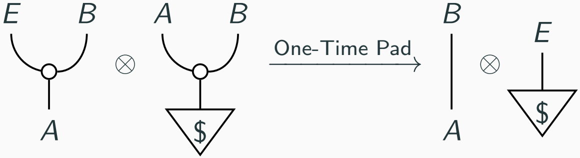

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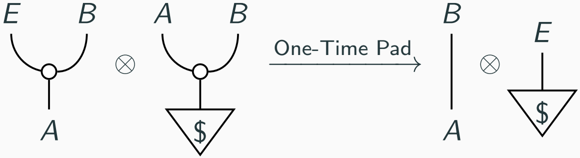

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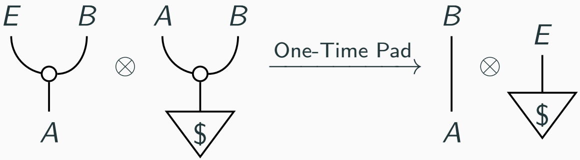

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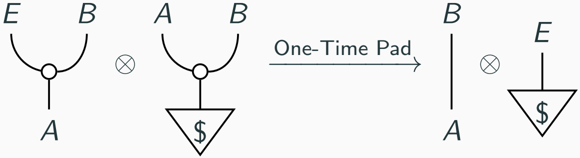

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This is the standard paradigm in programming language theory, transported to cryptographic language.

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Open Question 1: *Can the axioms be formulated as functoriality plus some conditions? If $\mathcal{A}$ is a functor, what should its codomain be?*

Open Question 2: *How broad is the definition of an attack model? Does it capture enough of modern cryptography?*

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We need some way to deal with asymptotic behavior. Our current idea is to value the metric in $\mathbb{R}^{\mathbb{N}}$.

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- Can we incorporate categorical notions from game theory, programming languages, etc. into cryptography?
- What does the presence of various categorical structure ((co)limits, monads, etc.) say cryptographically?