Categories for Cryptographic Composability

Riley Shahar
Advised by Angélica Osorno (Math) and Adam Groce (CS)

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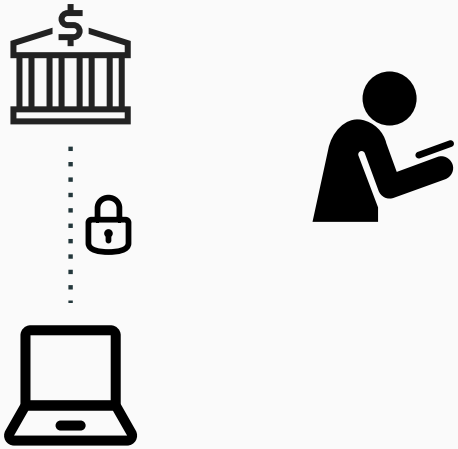

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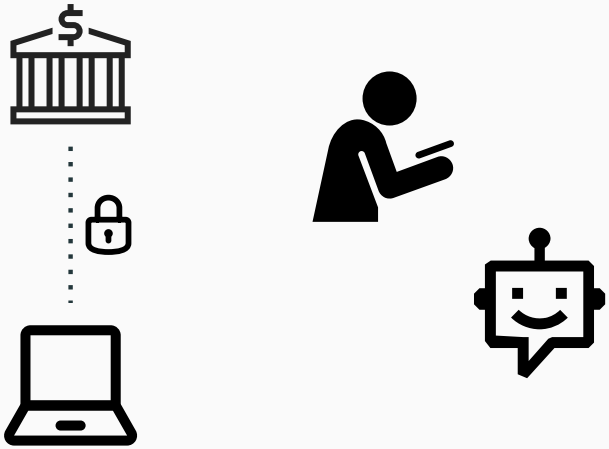

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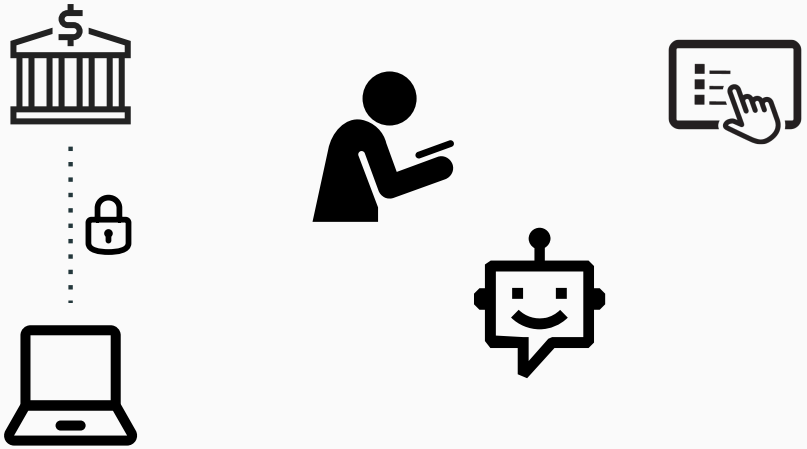

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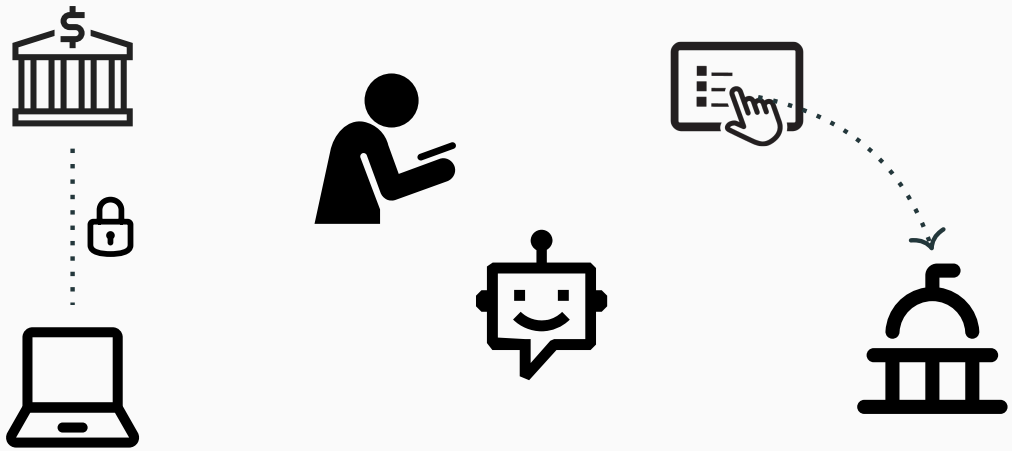

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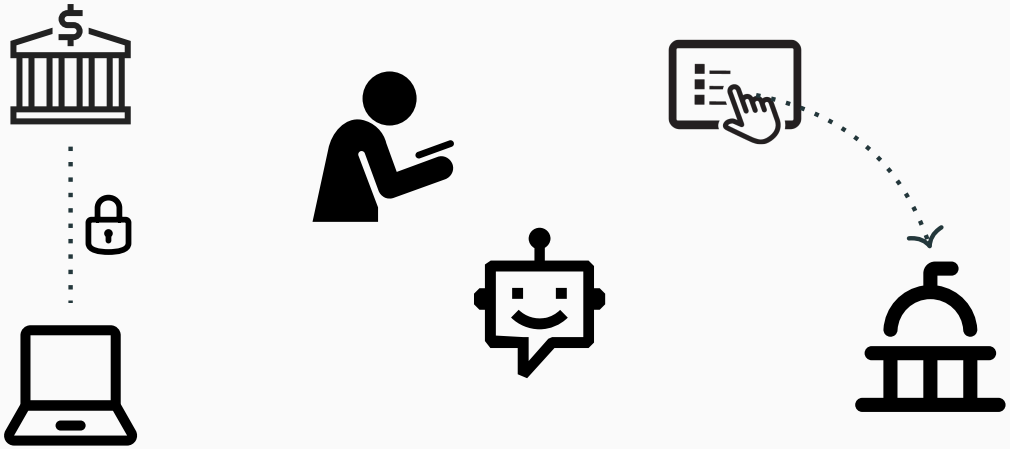

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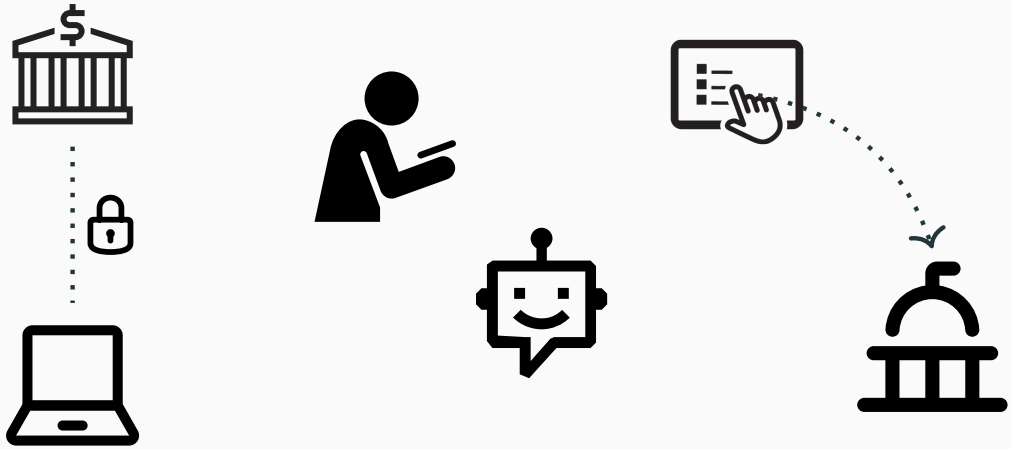

We want *proofs* that these things are secure.

Cryptographic Composability

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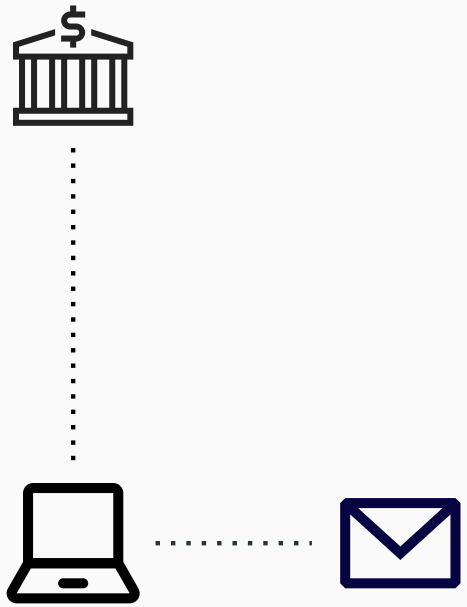

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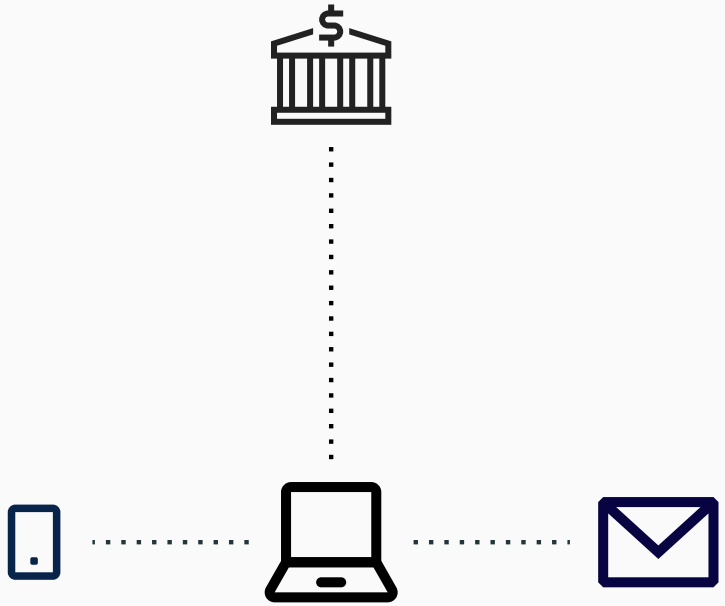

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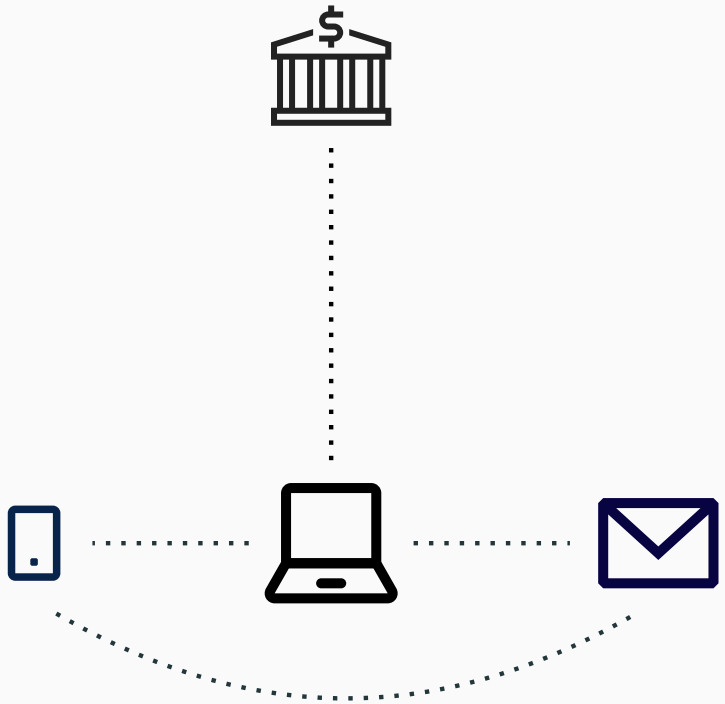

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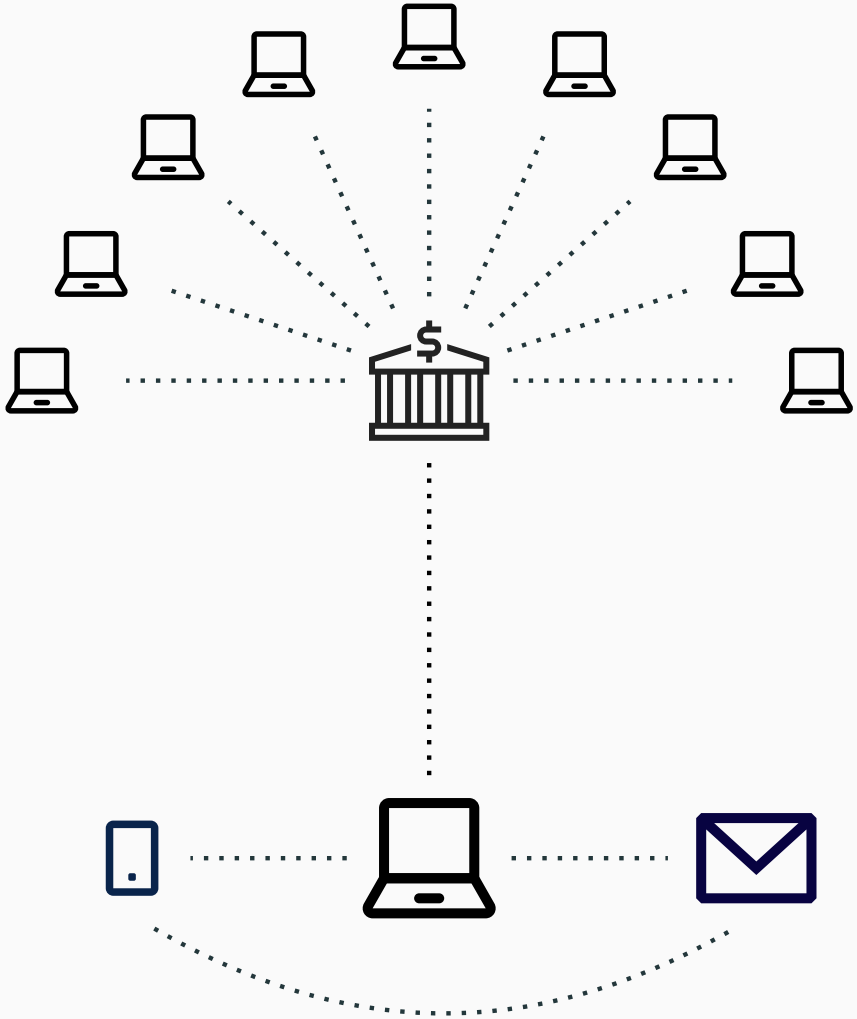

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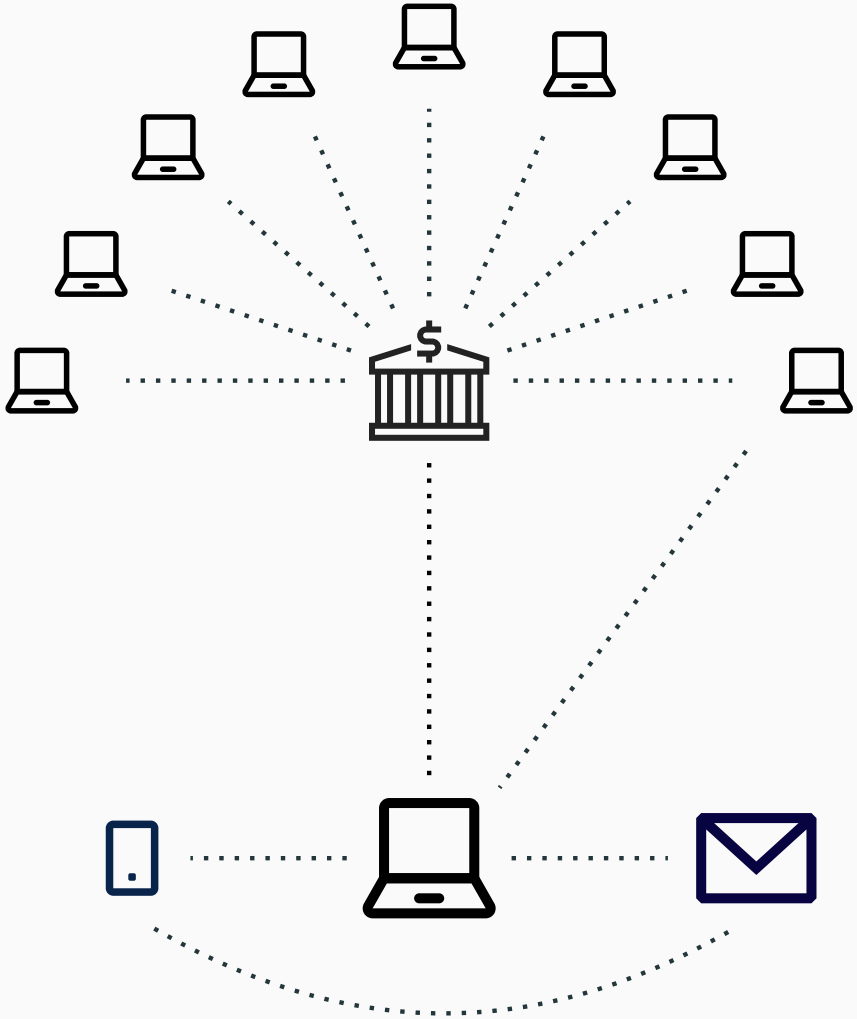

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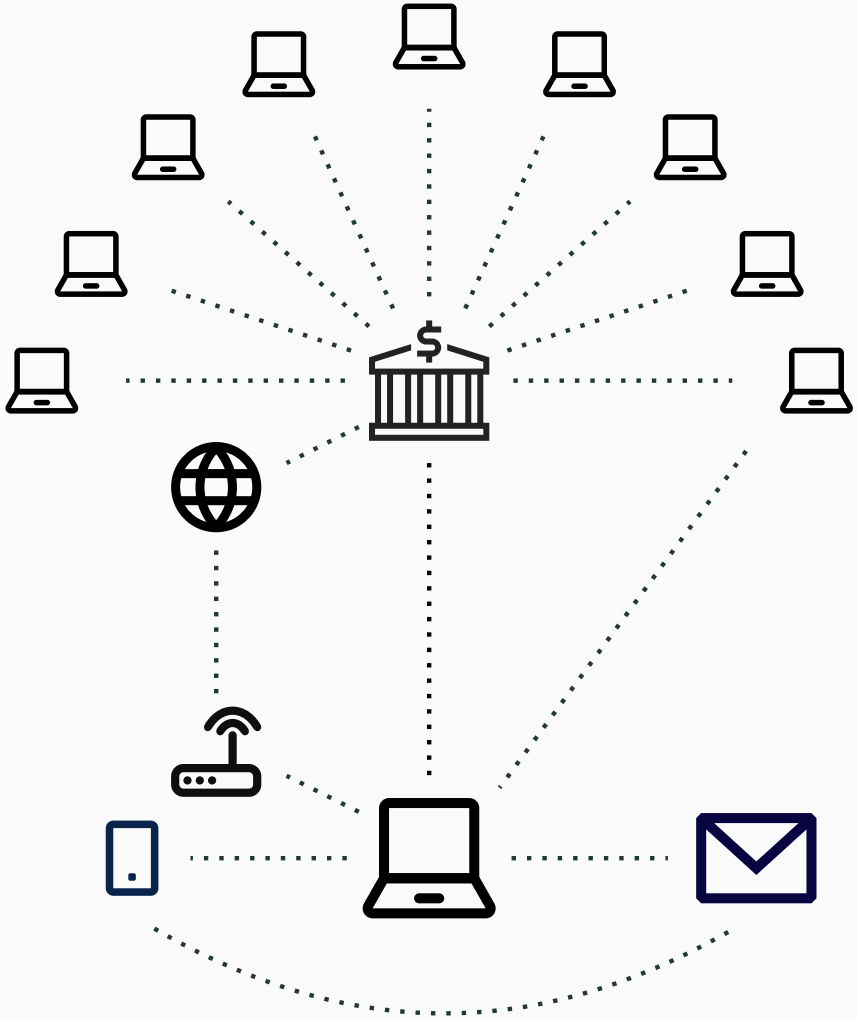

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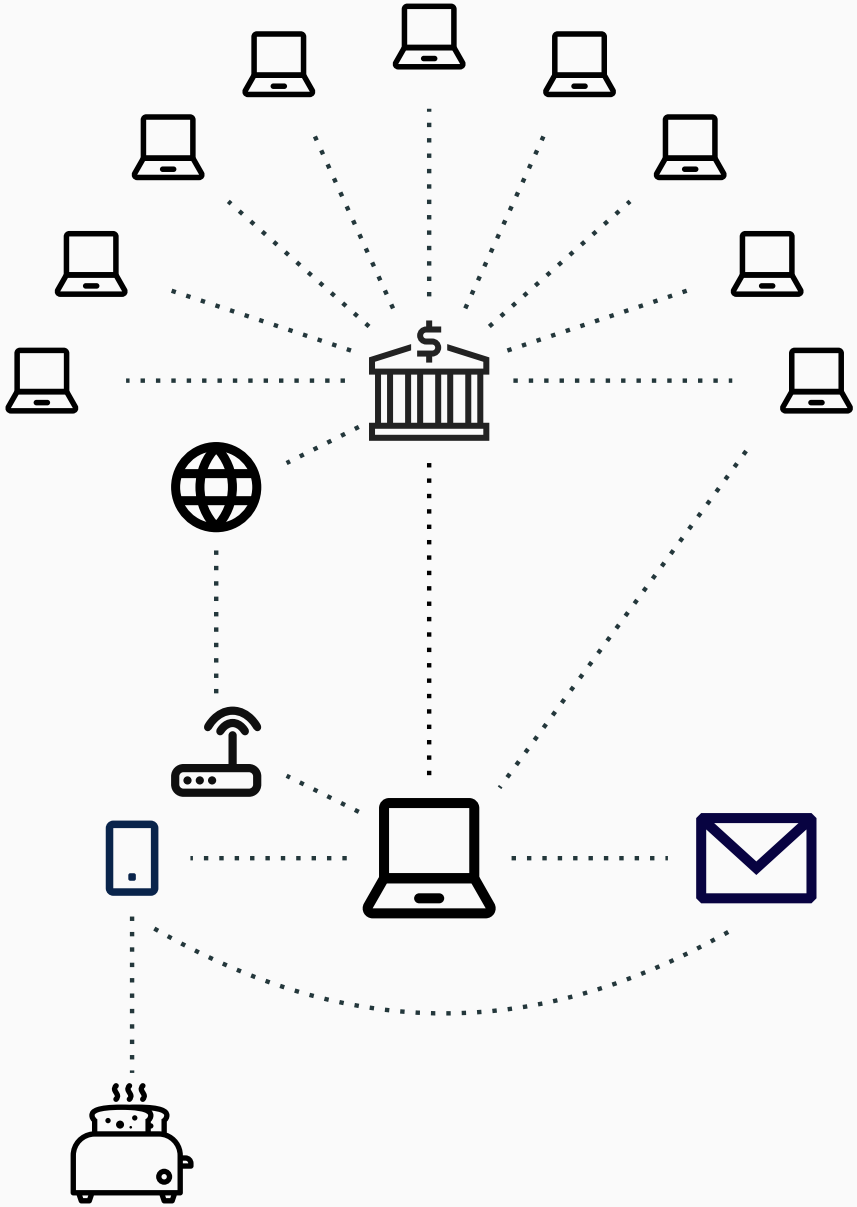

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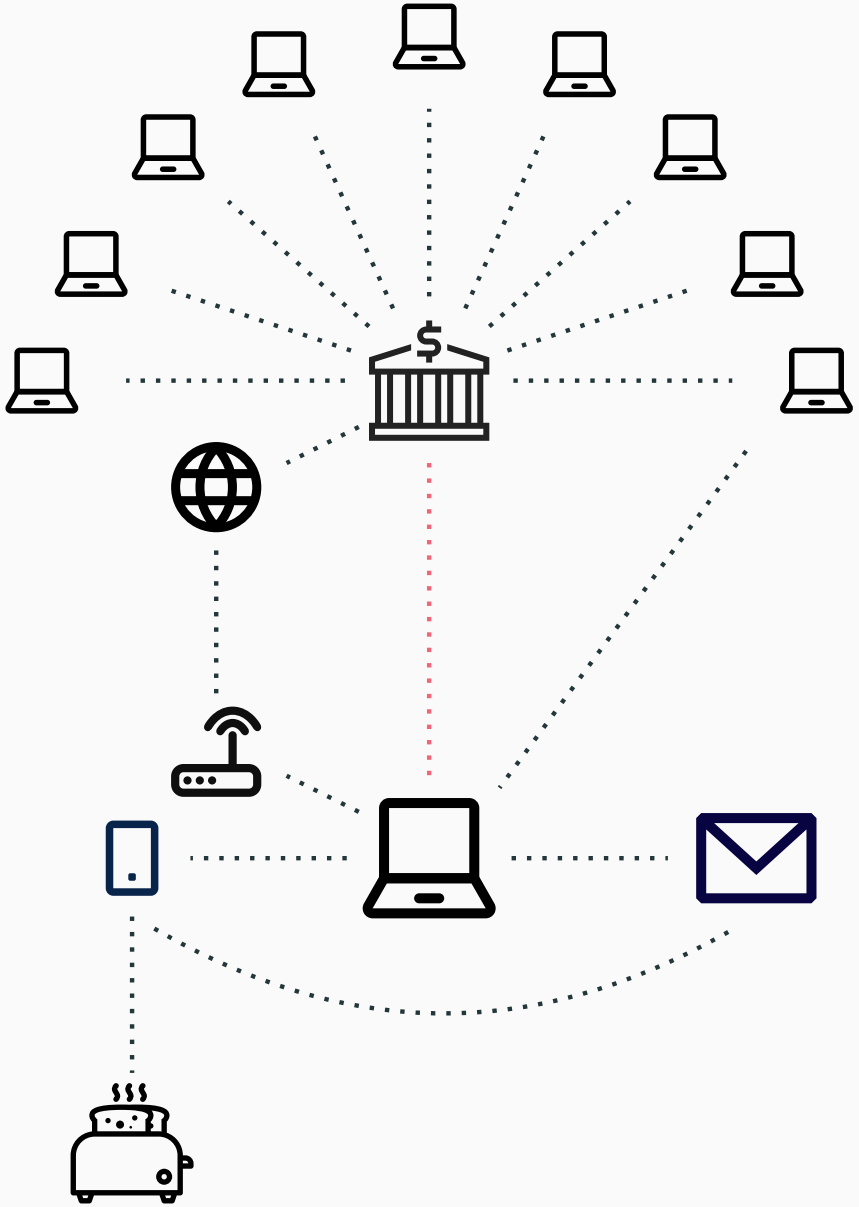

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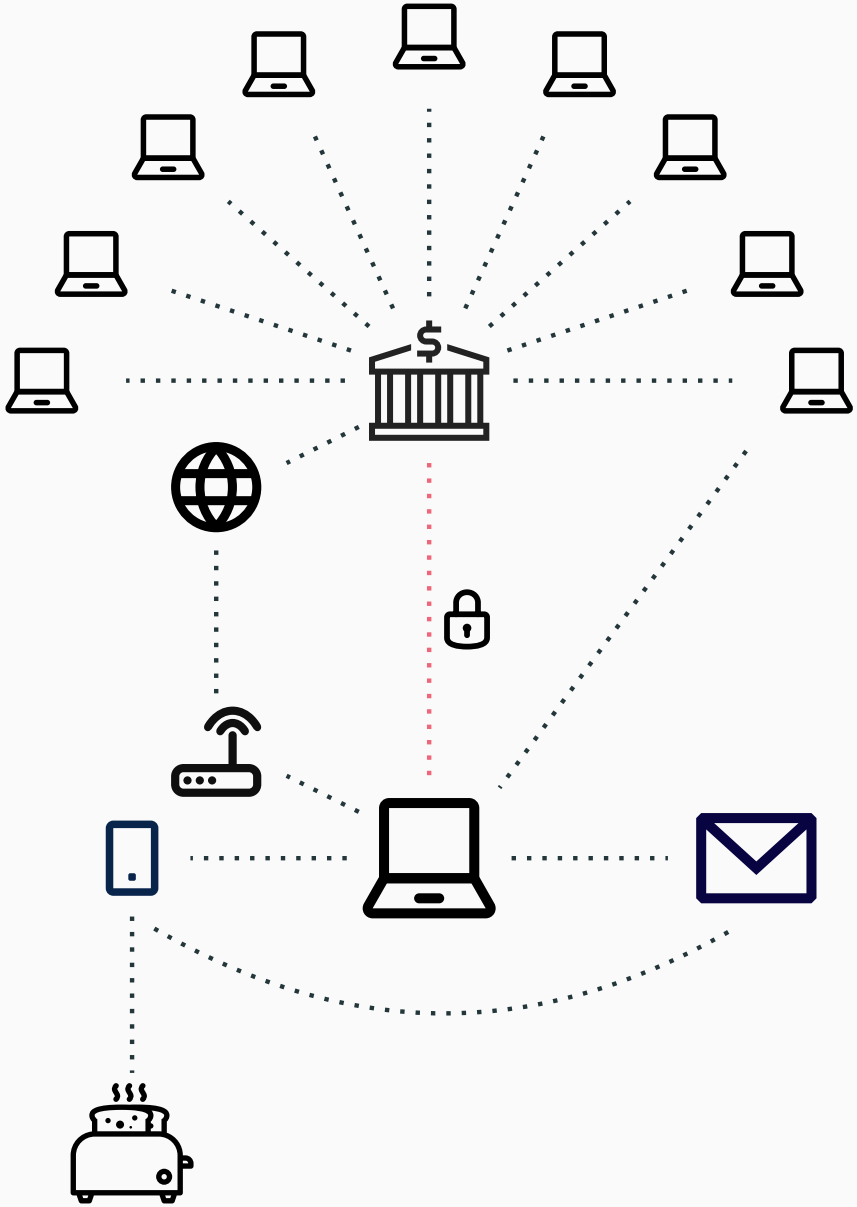

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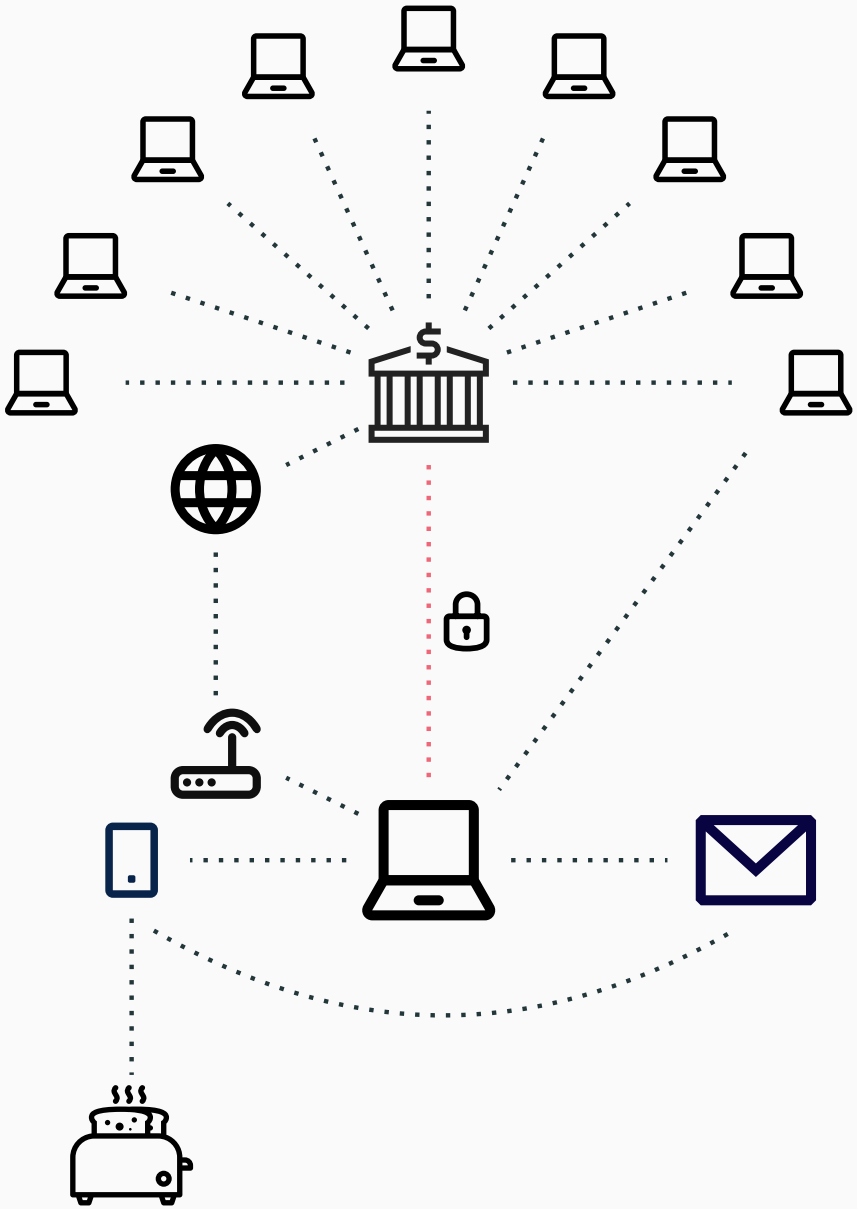

What do we need to prove about a computation in a vacuum so that it's still secure no matter what else is going on?

Universal Composability

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Due to Canetti 2000.

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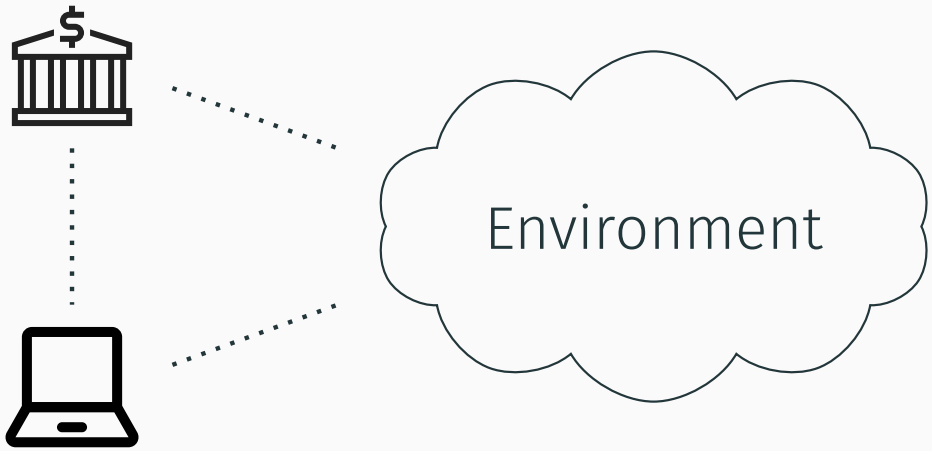

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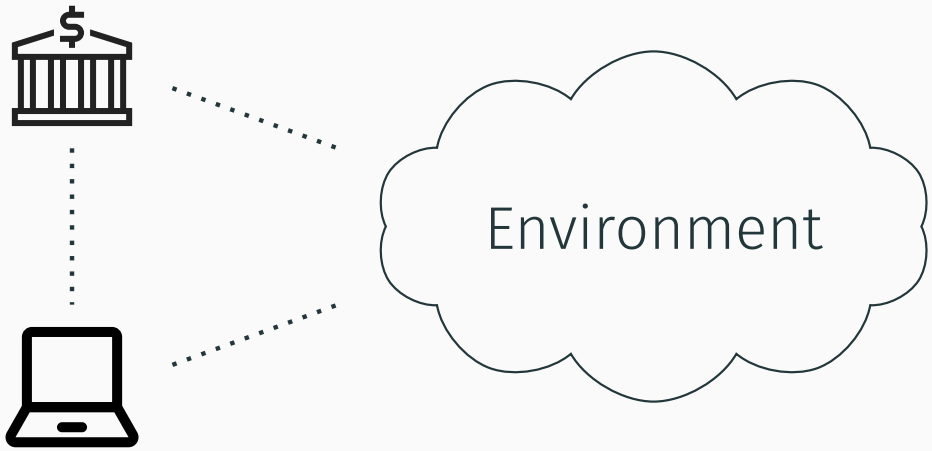

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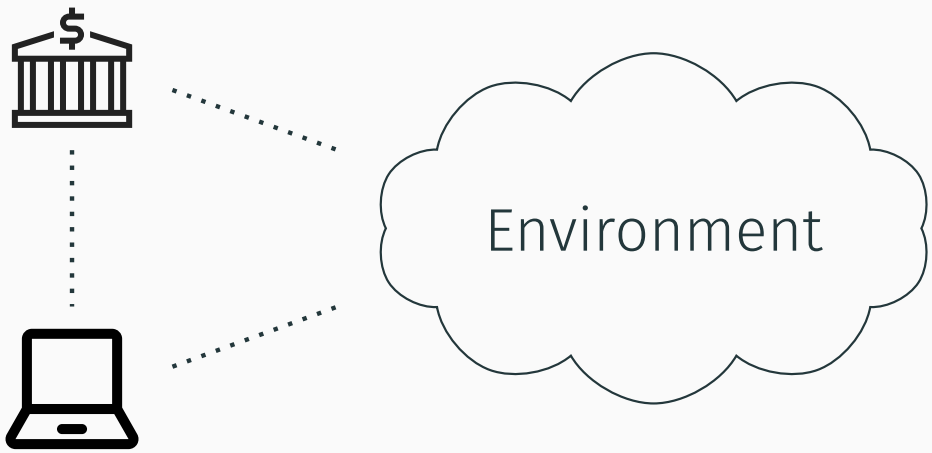

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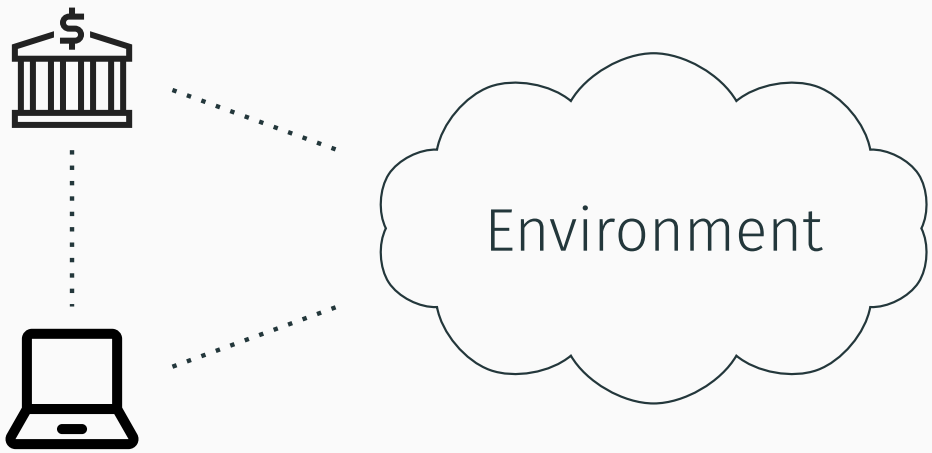

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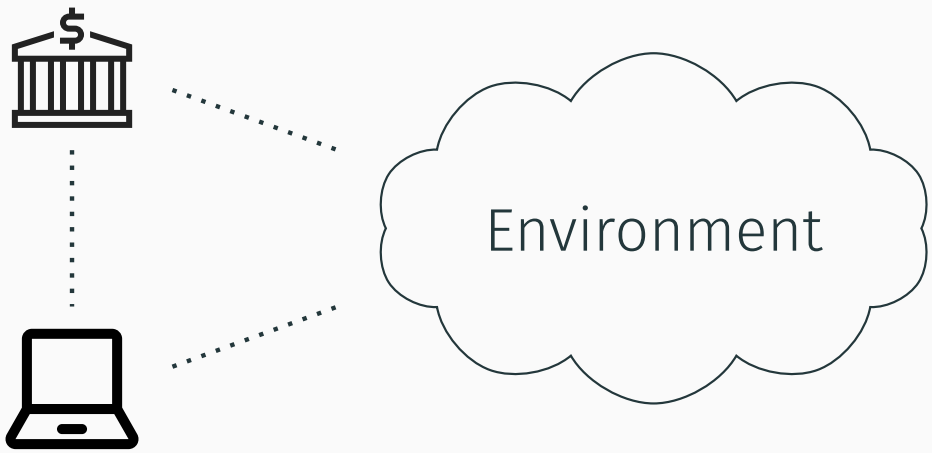

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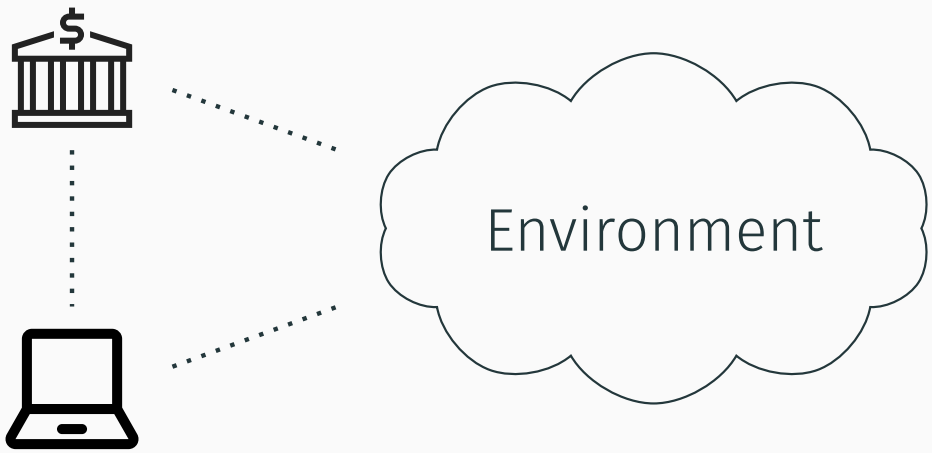
- 1 uses UC (Davies et al. 2023)



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Universal Composability

It is necessary that execution preserve security guarantees under composition. We refrain from proving UC security ... since such an analysis will be cumbersome. Instead, we prove the security of our protocols by constructing simulators.

–David et al. 2023

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...**category theory** is an excellent candidate for such a theory.

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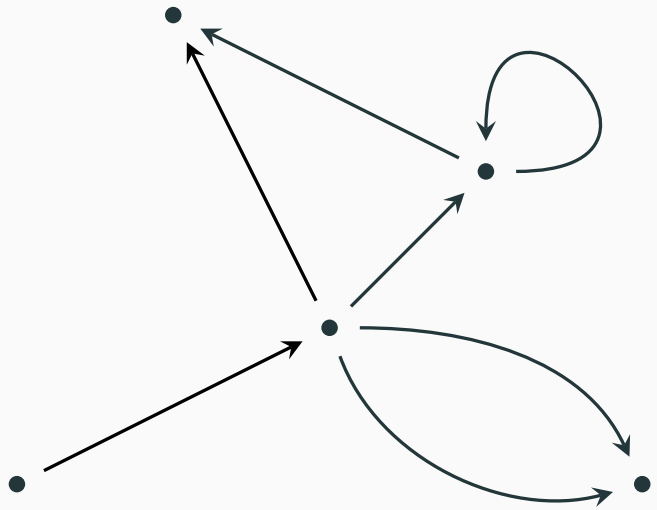

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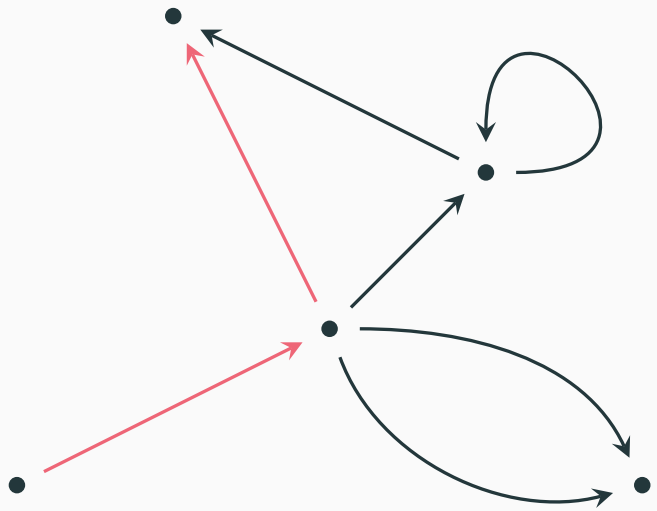

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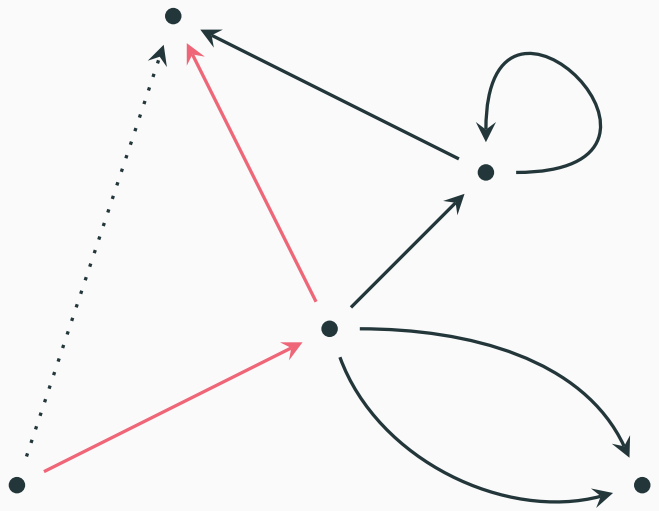

Categories and Cryptography

[Largely following Broadbent and Karvonen (2022)]

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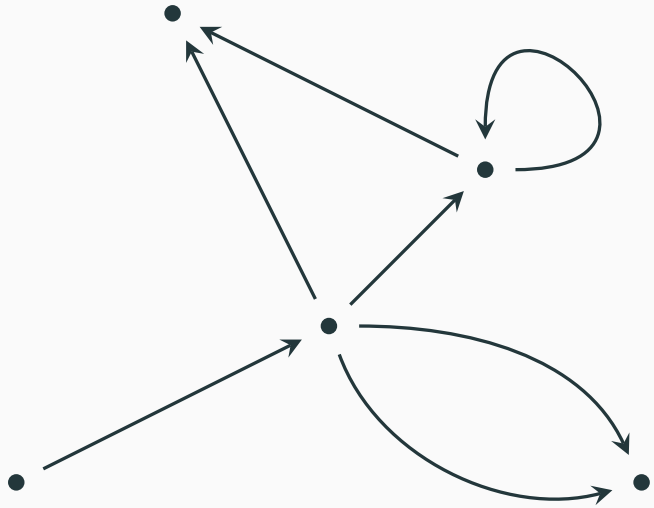

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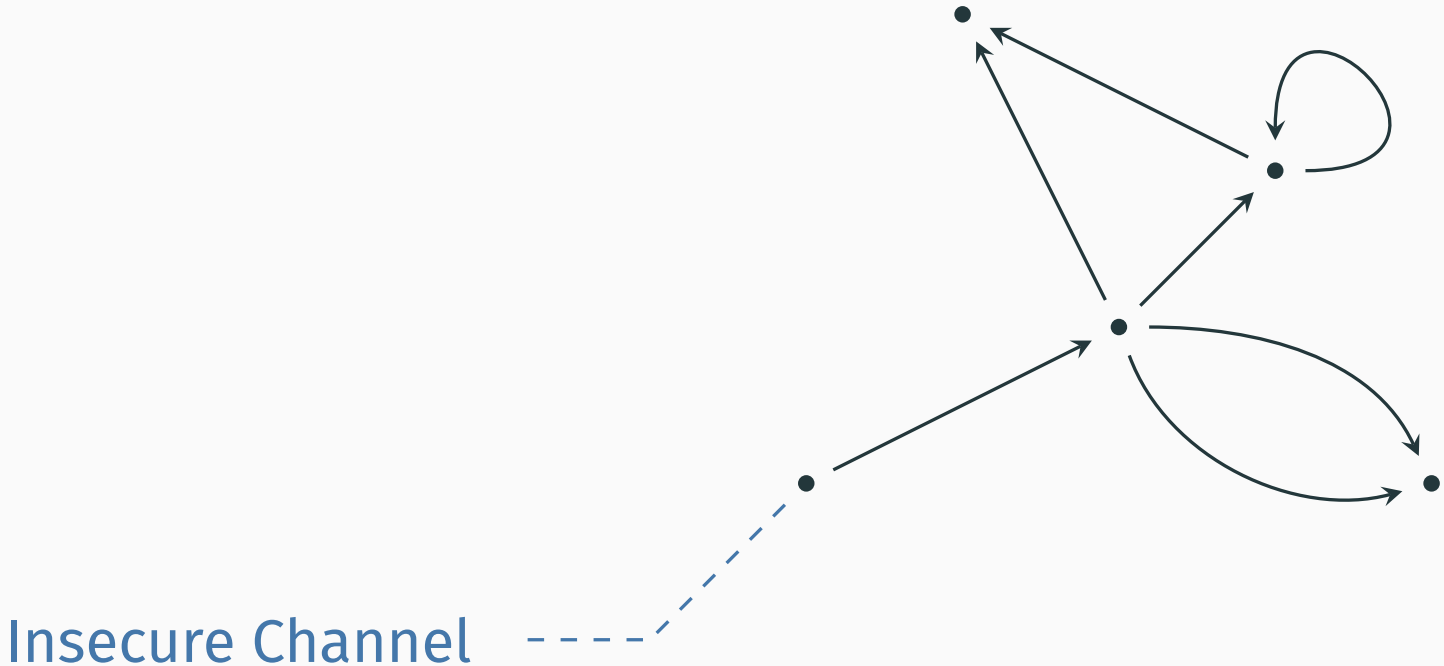

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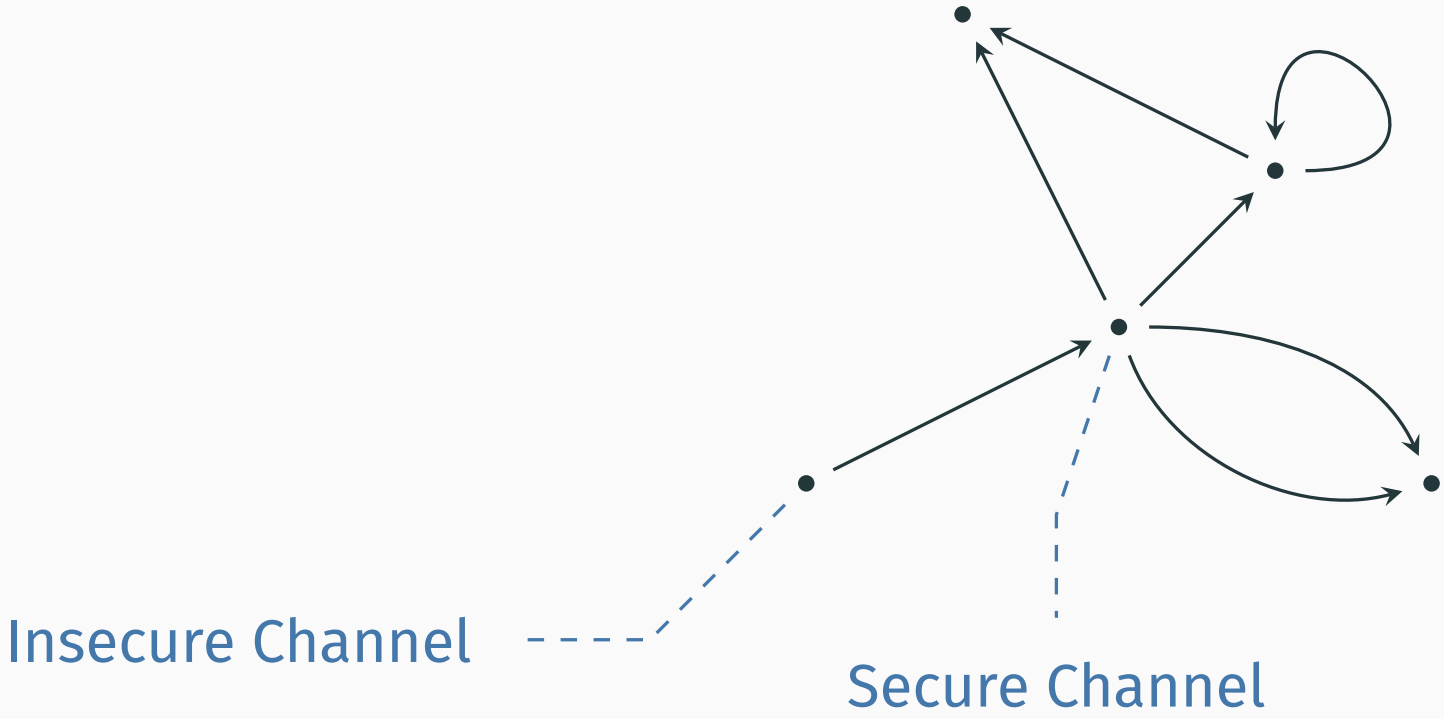

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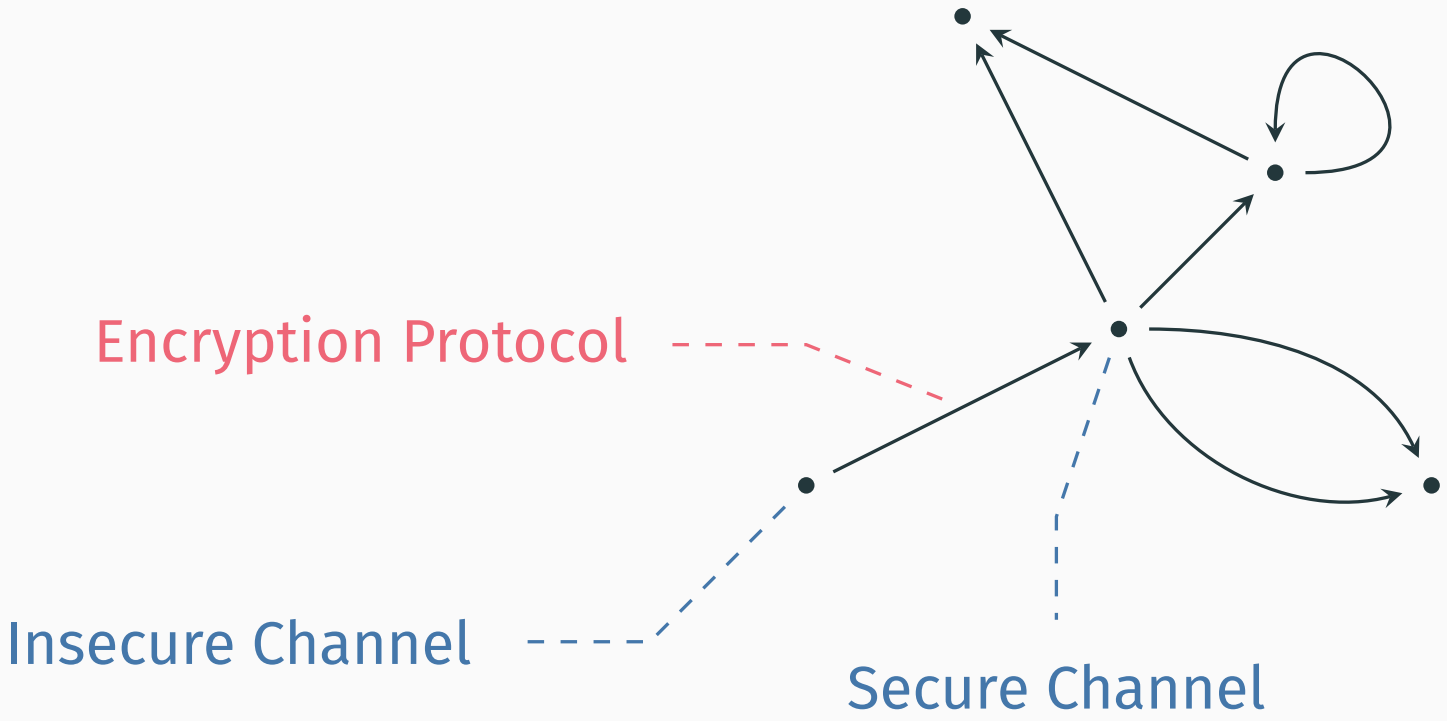

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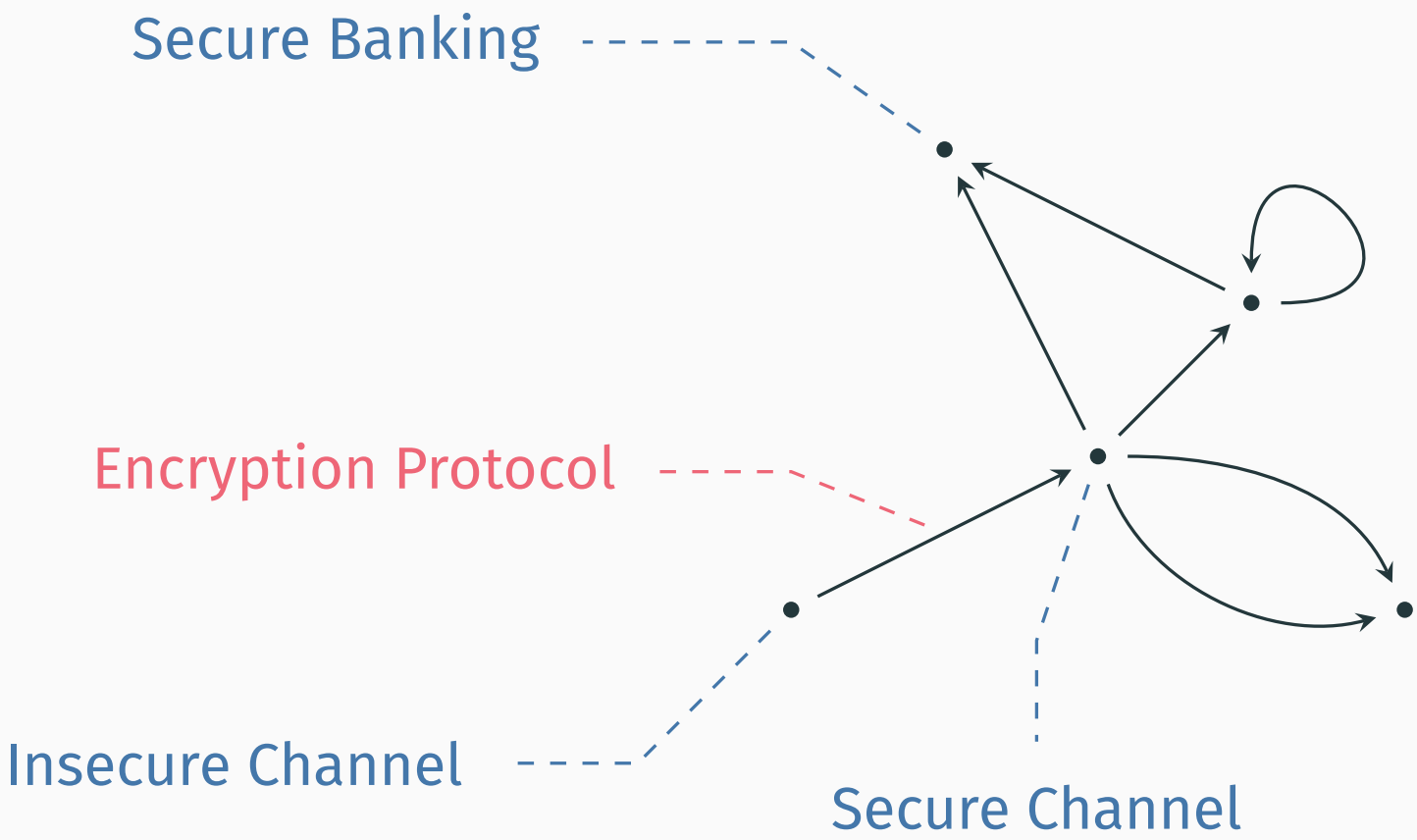

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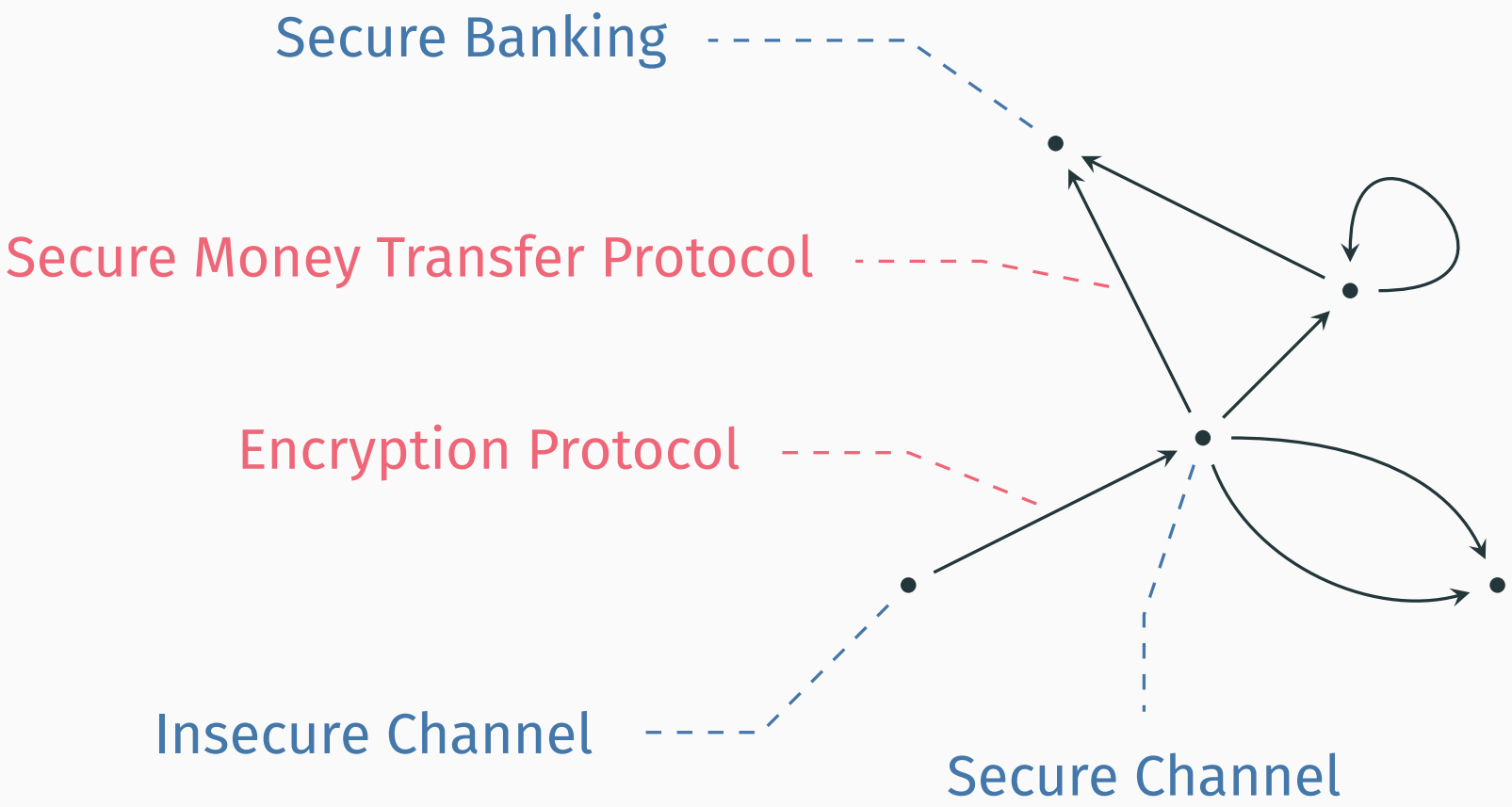

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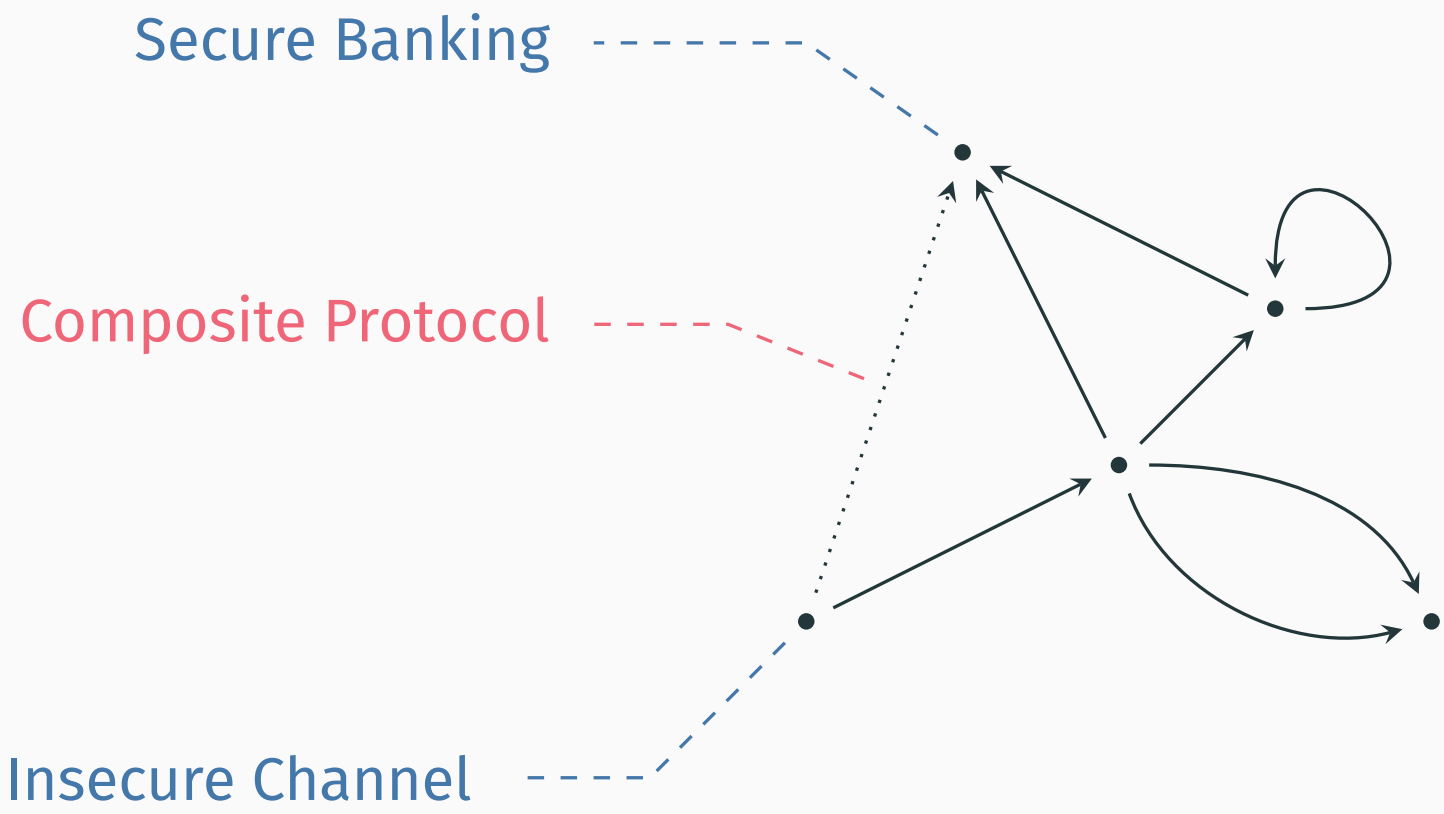

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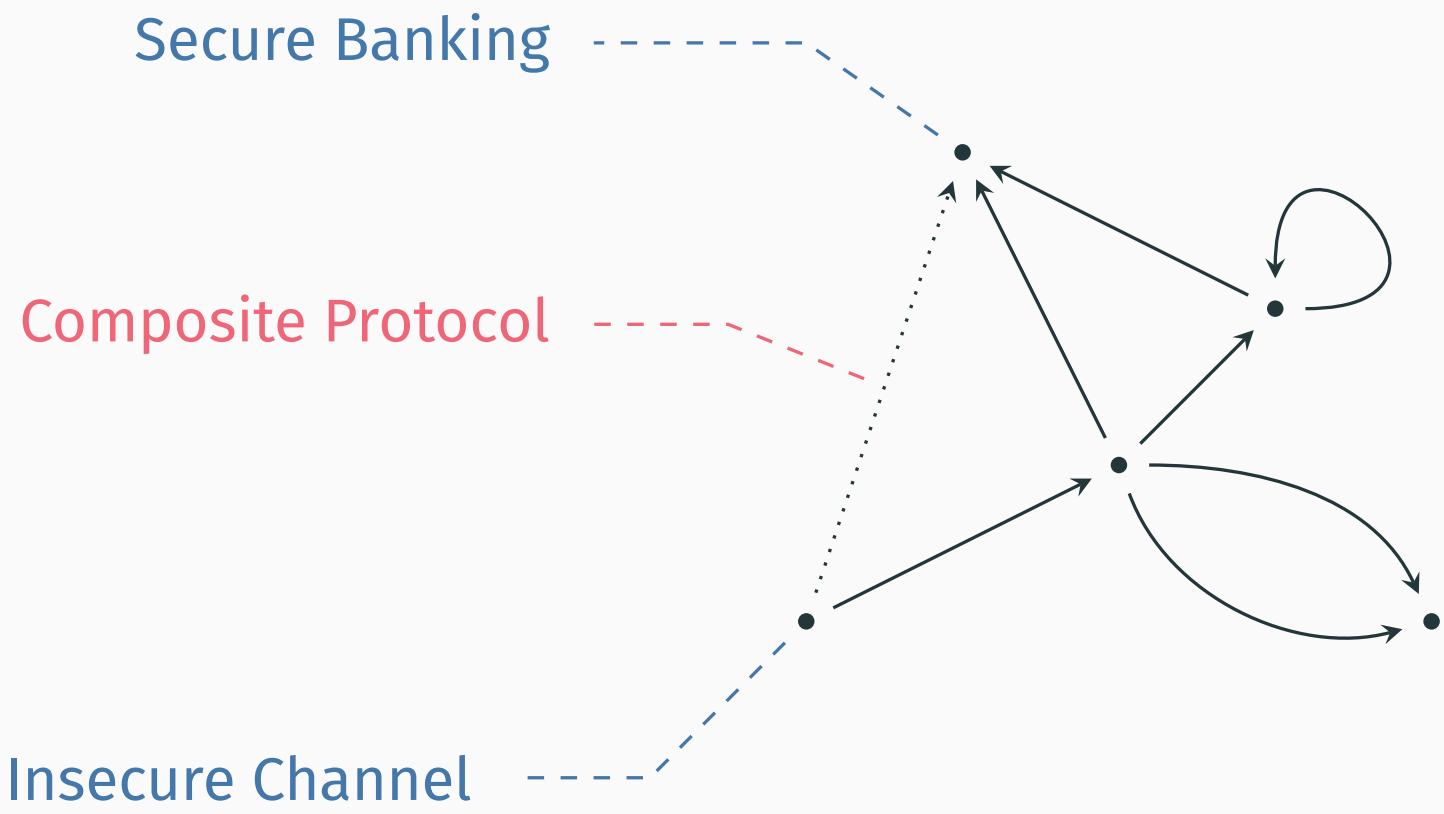

The challenge is to encode cryptographic objects as a category.

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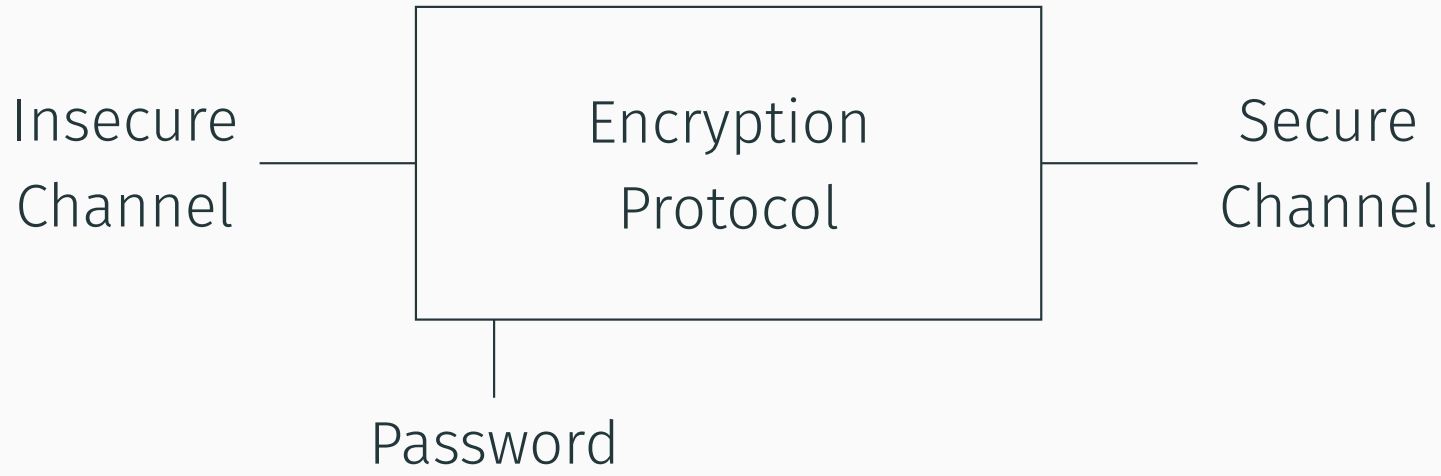

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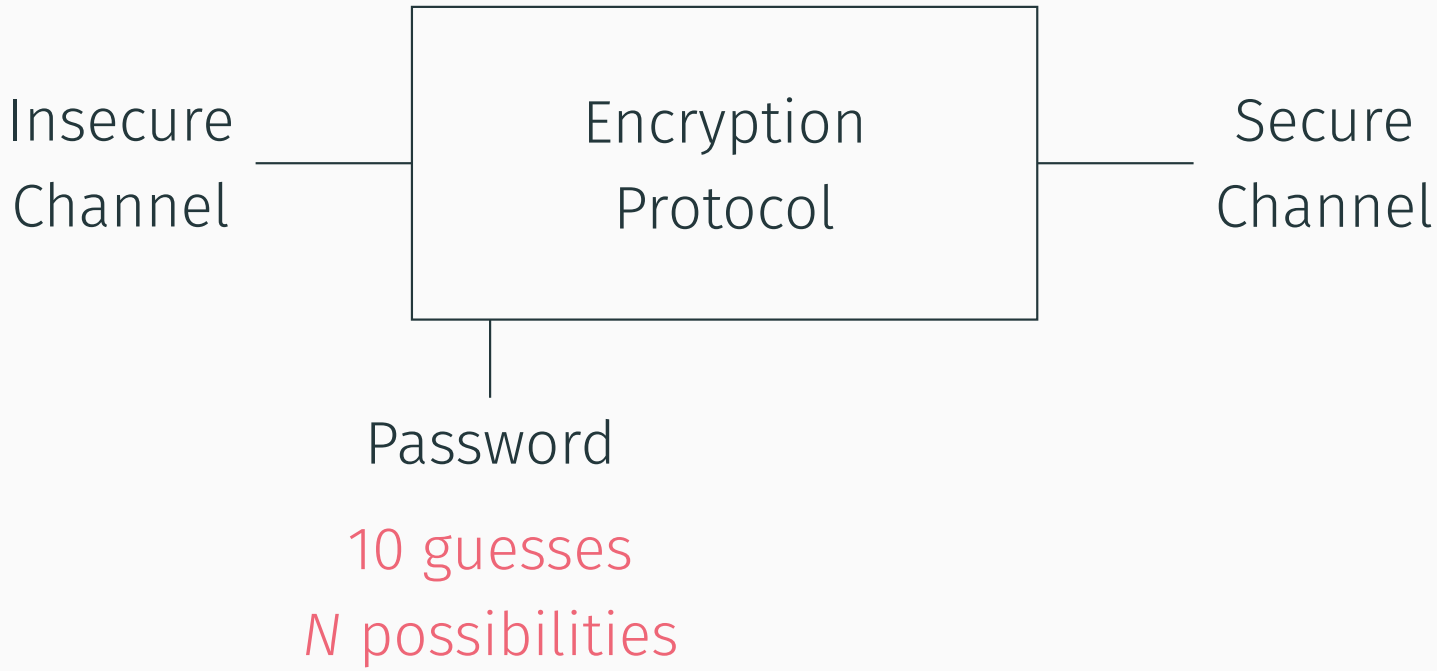

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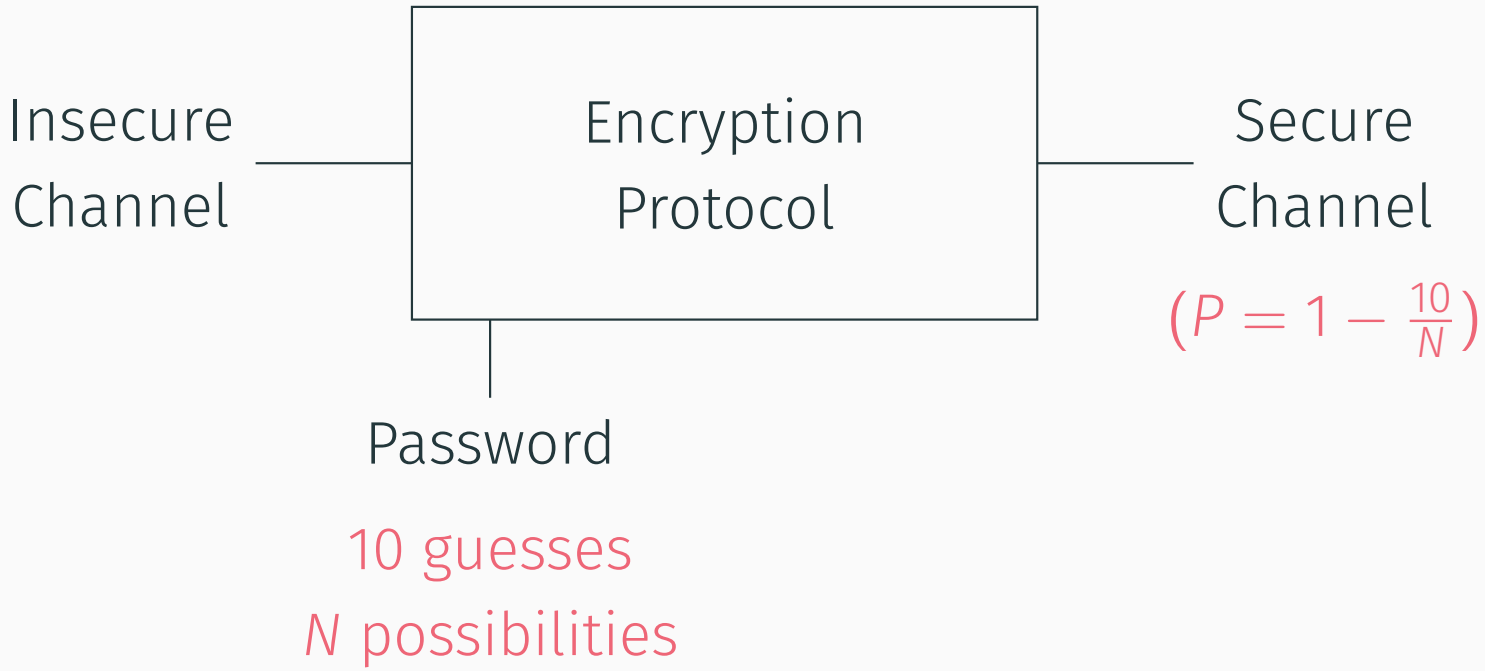

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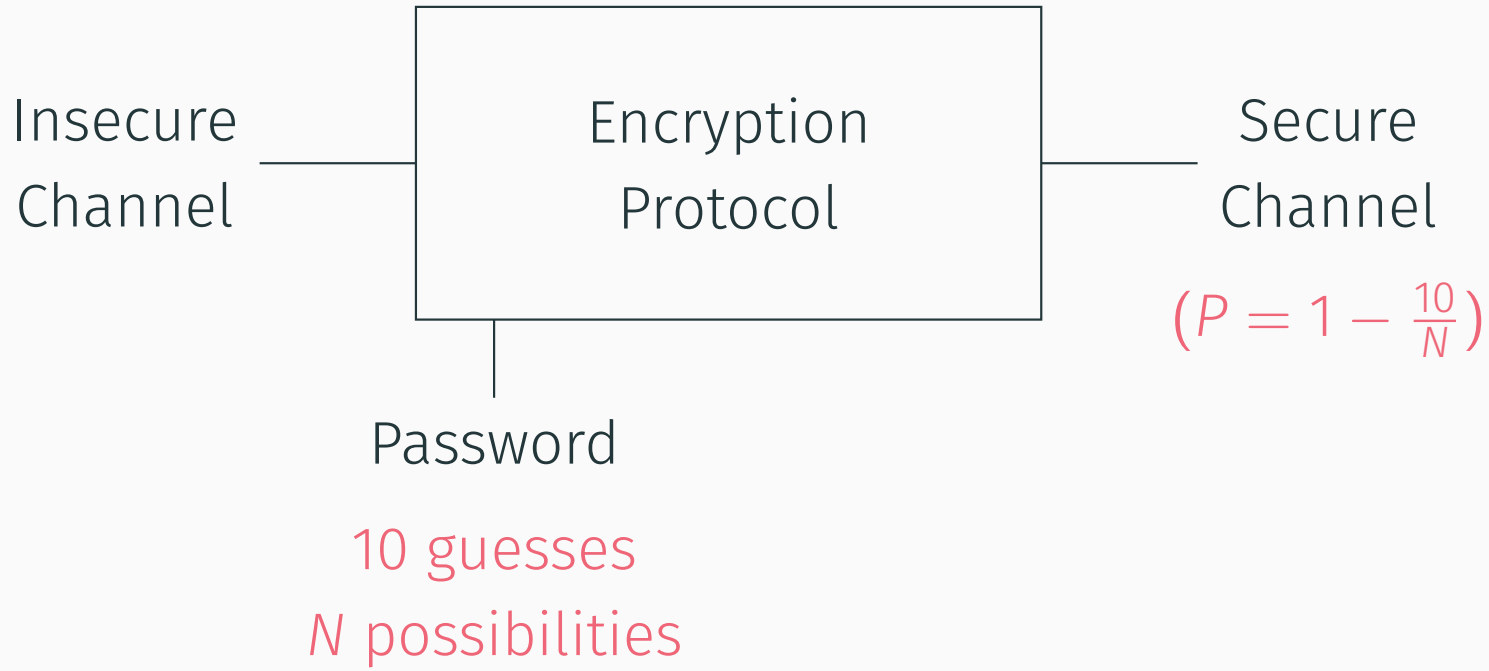
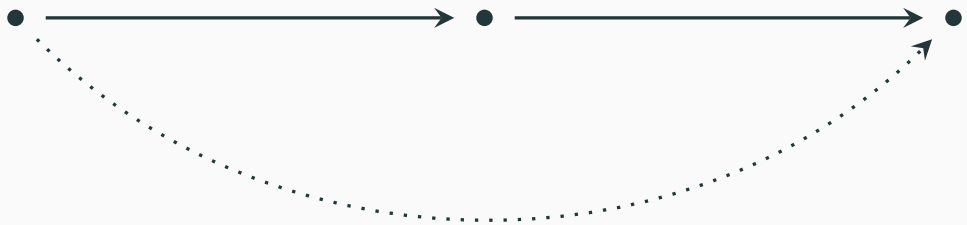


Cryptographers are very good at dealing with this.

Approximate Composition



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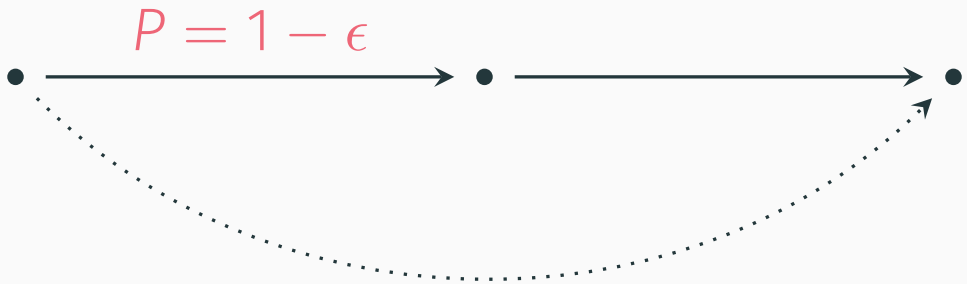

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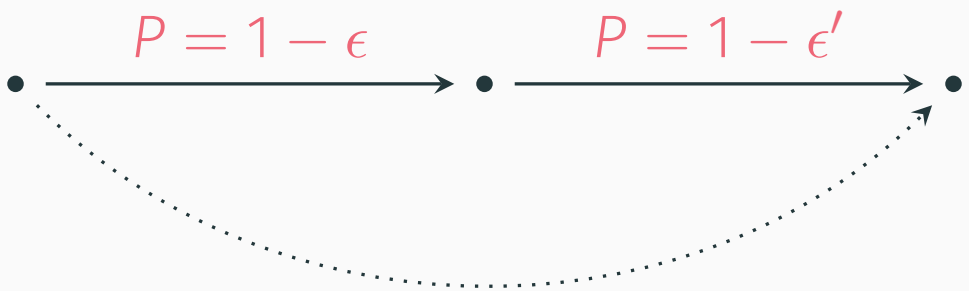

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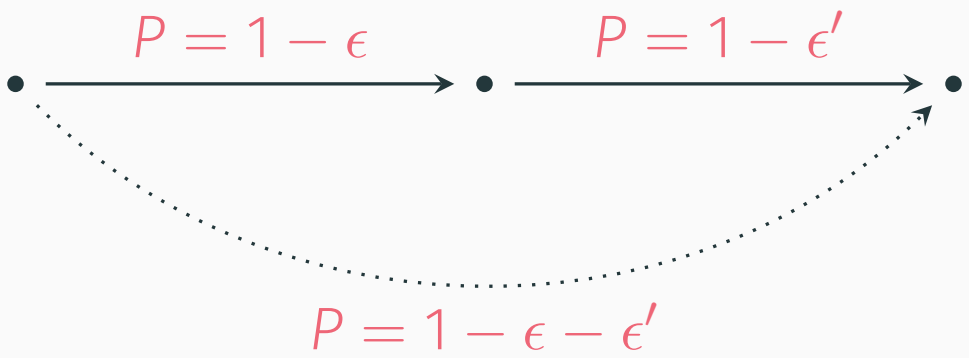

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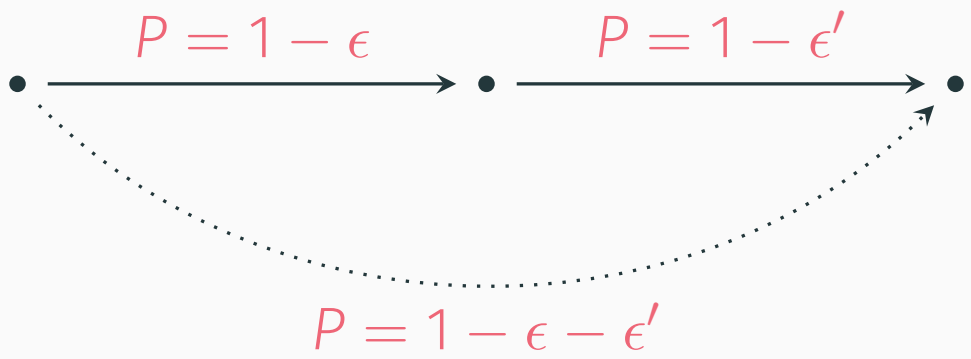

Small probabilities compound under composition.

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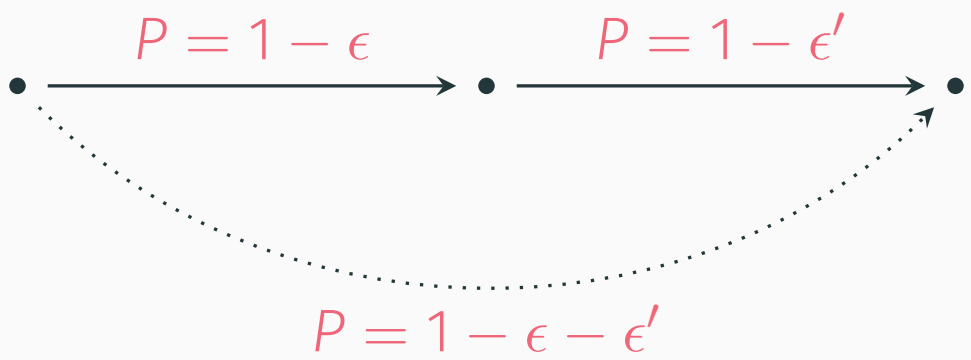

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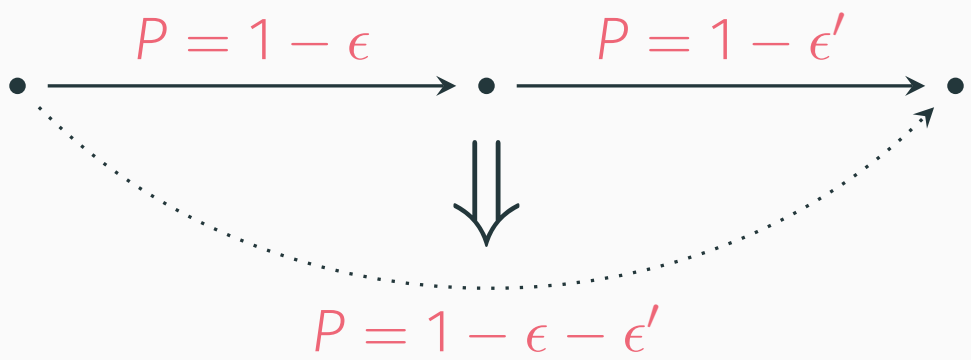

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We've made meaningful progress towards this goal.

There are major outstanding issues.

Thanks for your time!

References

Broadbent, Anne and Martti Karvonen (2022). **“Categorical composable cryptography”.** In: *Foundations of software science and computation structures*. Vol. 13242. Lecture Notes in Comput. Sci. Springer, Cham, pp. 161–183. ISBN: 9783030992538. DOI: 10.1007/978-3-030-99253-8_9.

Canetti, Ran (2000). ***Universally Composable Security: A New Paradigm for Cryptographic Protocols.*** Cryptology ePrint Archive, Paper 2000/067. URL: https://eprint.iacr.org/2000/067.

David, Bernardo et al. (2023). **“Perfect MPC over Layered Graphs”.** In: *Advances in Cryptology – CRYPTO 2023*. Ed. by Helena Handschuh and Anna Lysyanskaya. Cham: Springer Nature Switzerland, pp. 360–392. ISBN: 978-3-031-38557-5.

Davies, Gareth T. et al. (2023). ***Security Analysis of the WhatsApp End-to-End Encrypted Backup Protocol.*** Cryptology ePrint Archive, Paper 2023/843. URL: https://eprint.iacr.org/2023/843.