Categories for Cryptographic Composability

Riley Shahar
Advised by Angélica Osorno (Math) and Adam Groce (CS)

Cryptography

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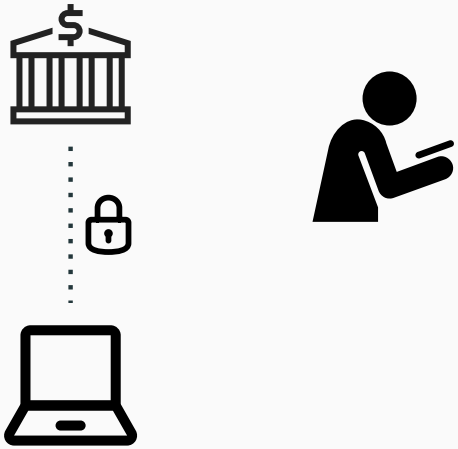

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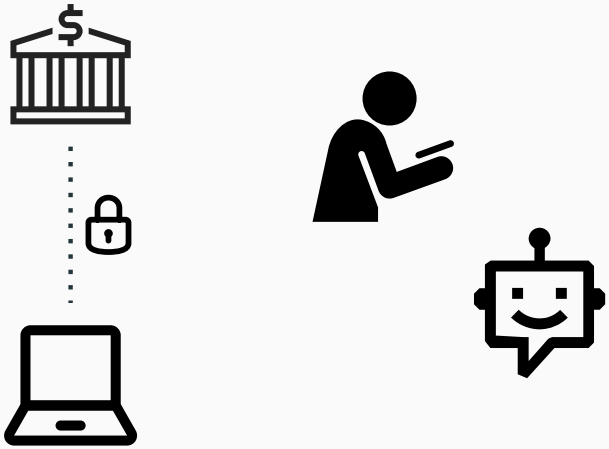

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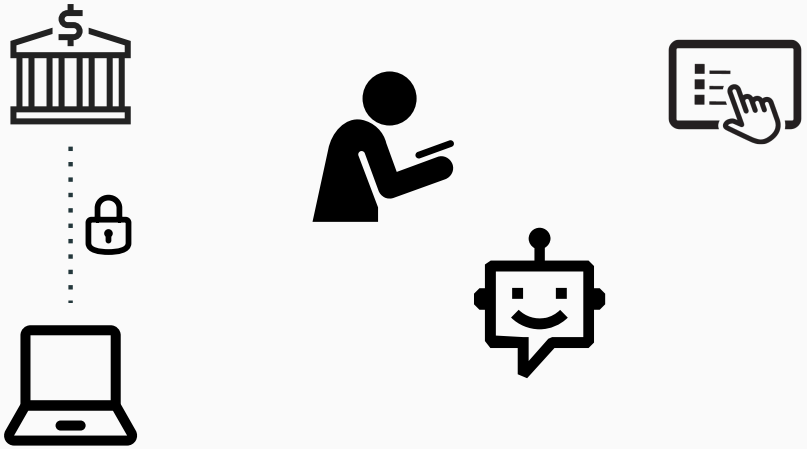

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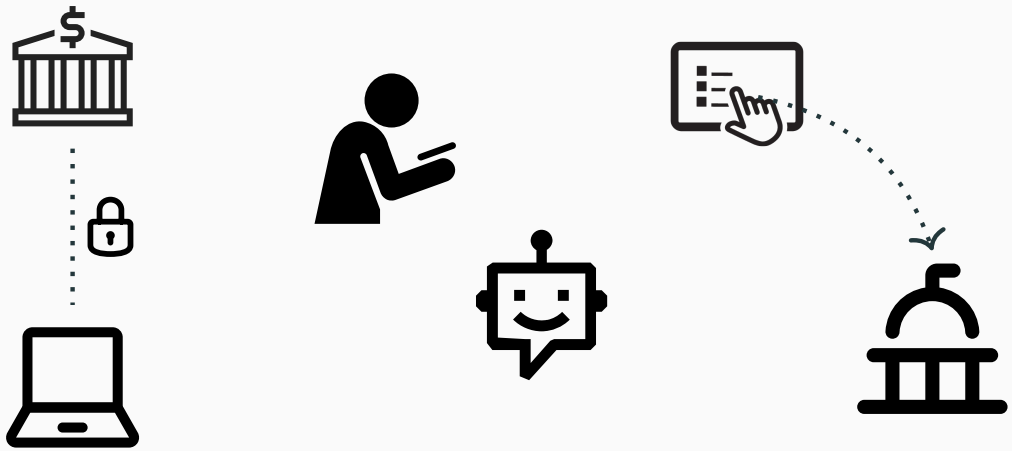

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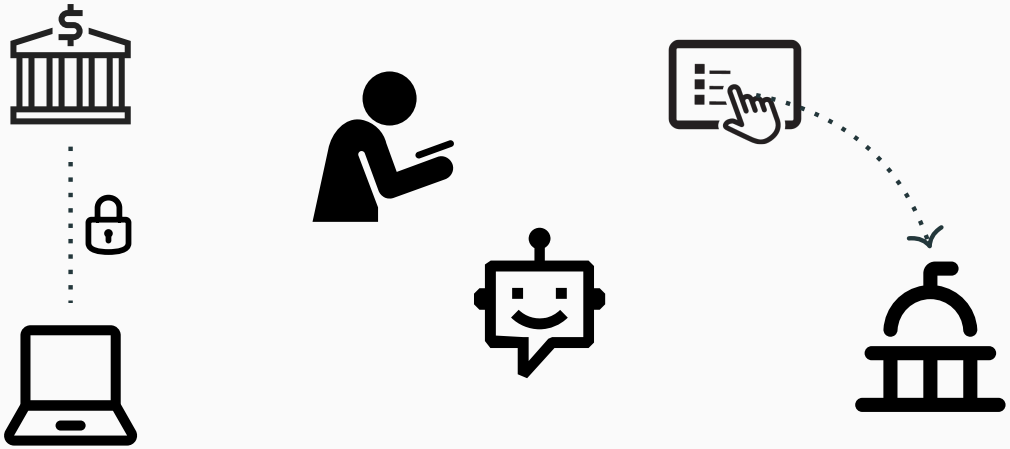

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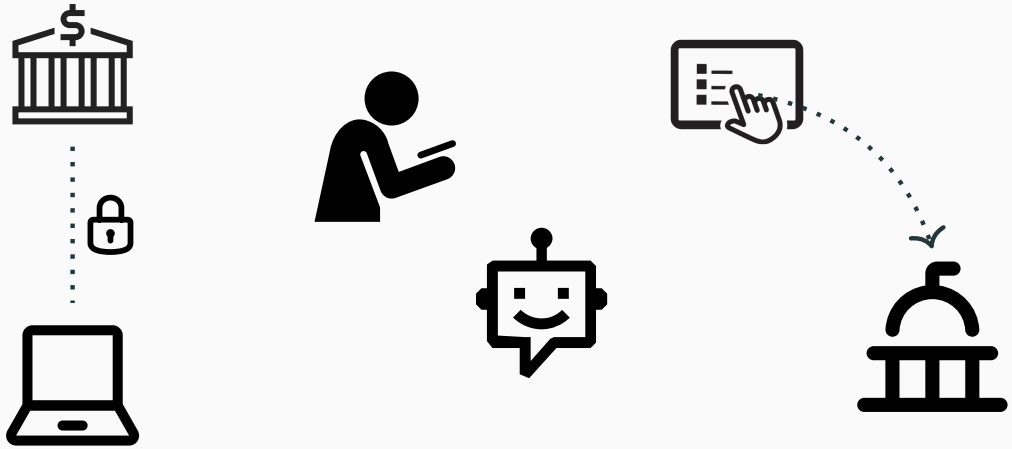

We want *proofs* that these things are secure.

Cryptographic Composability

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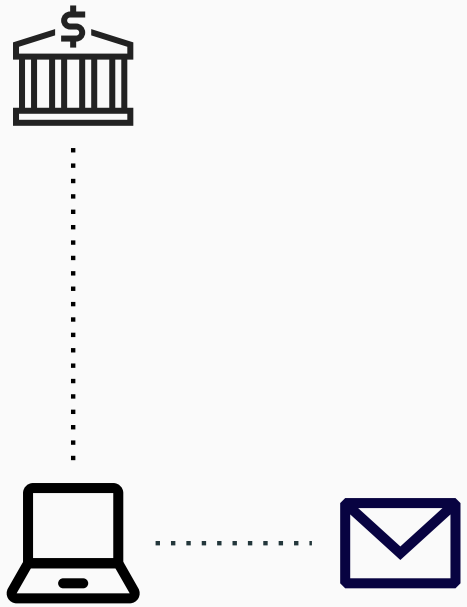

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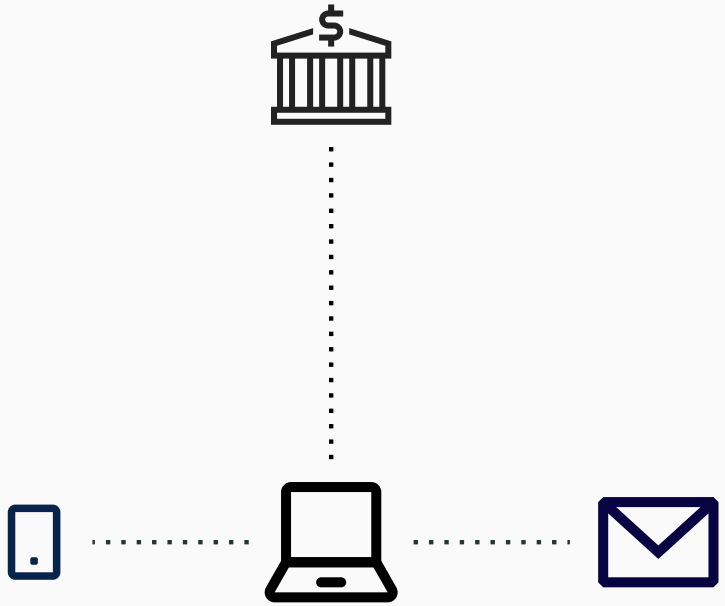

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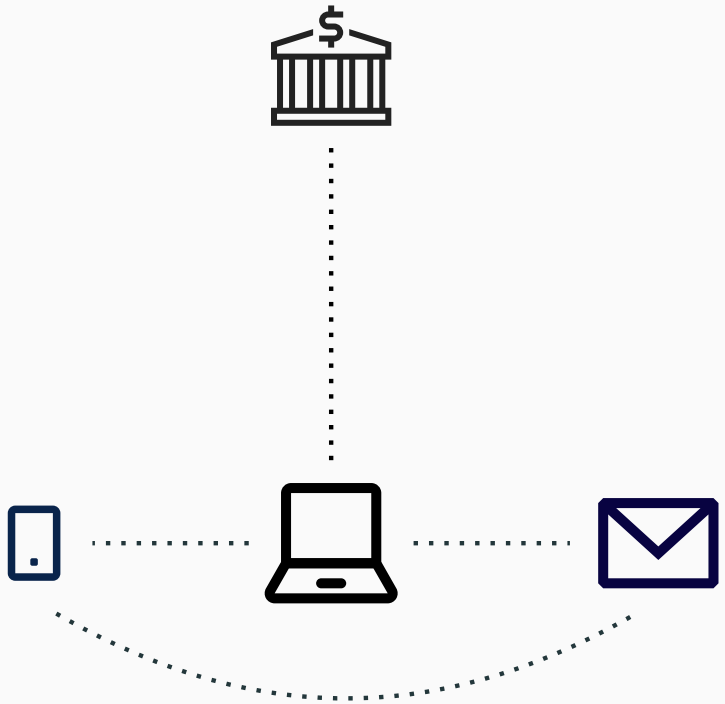

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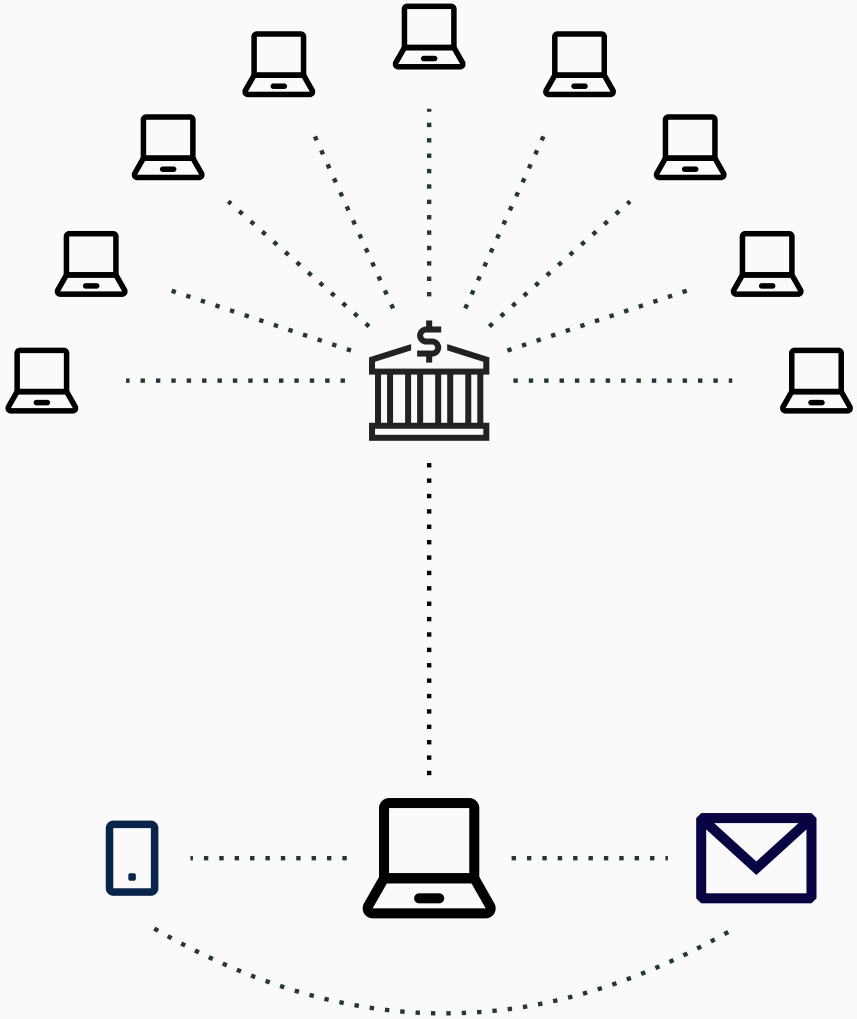

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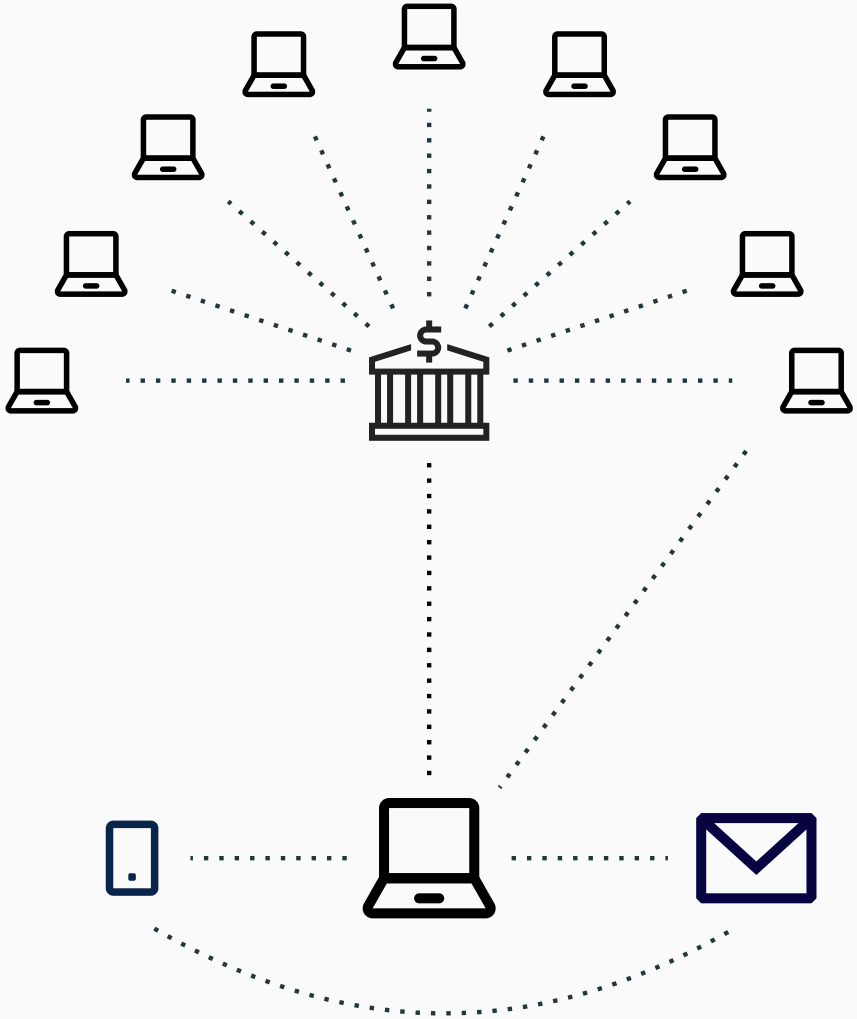

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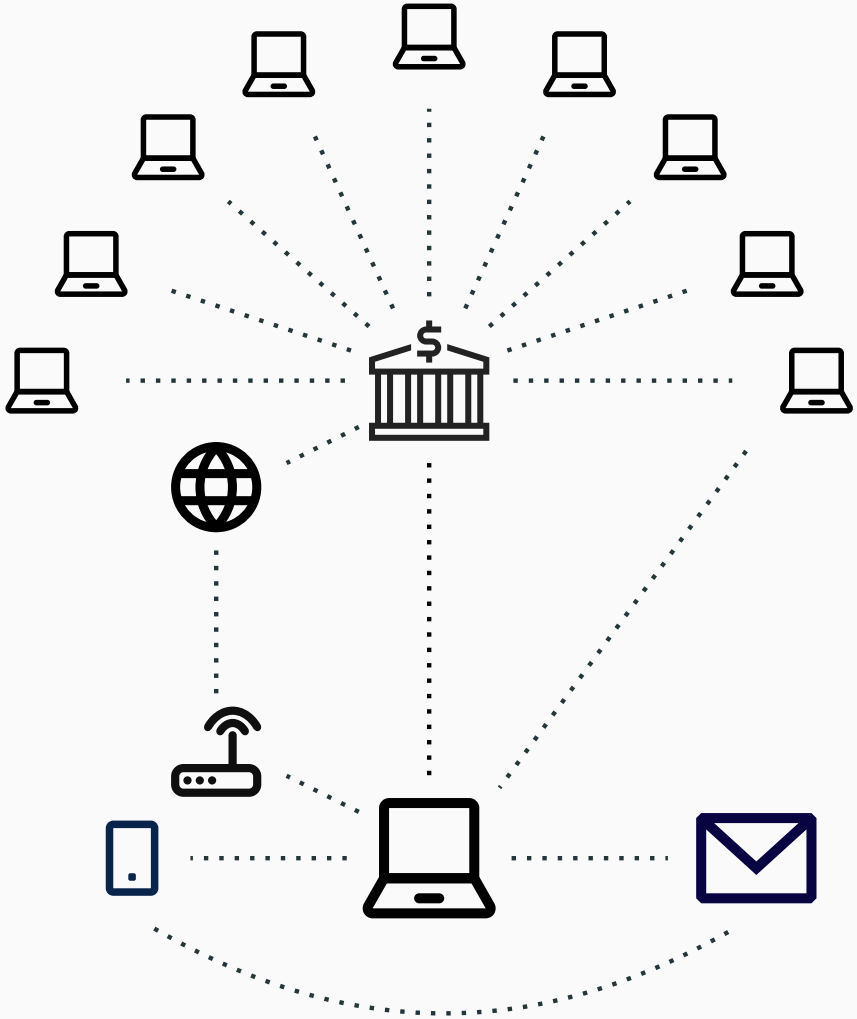

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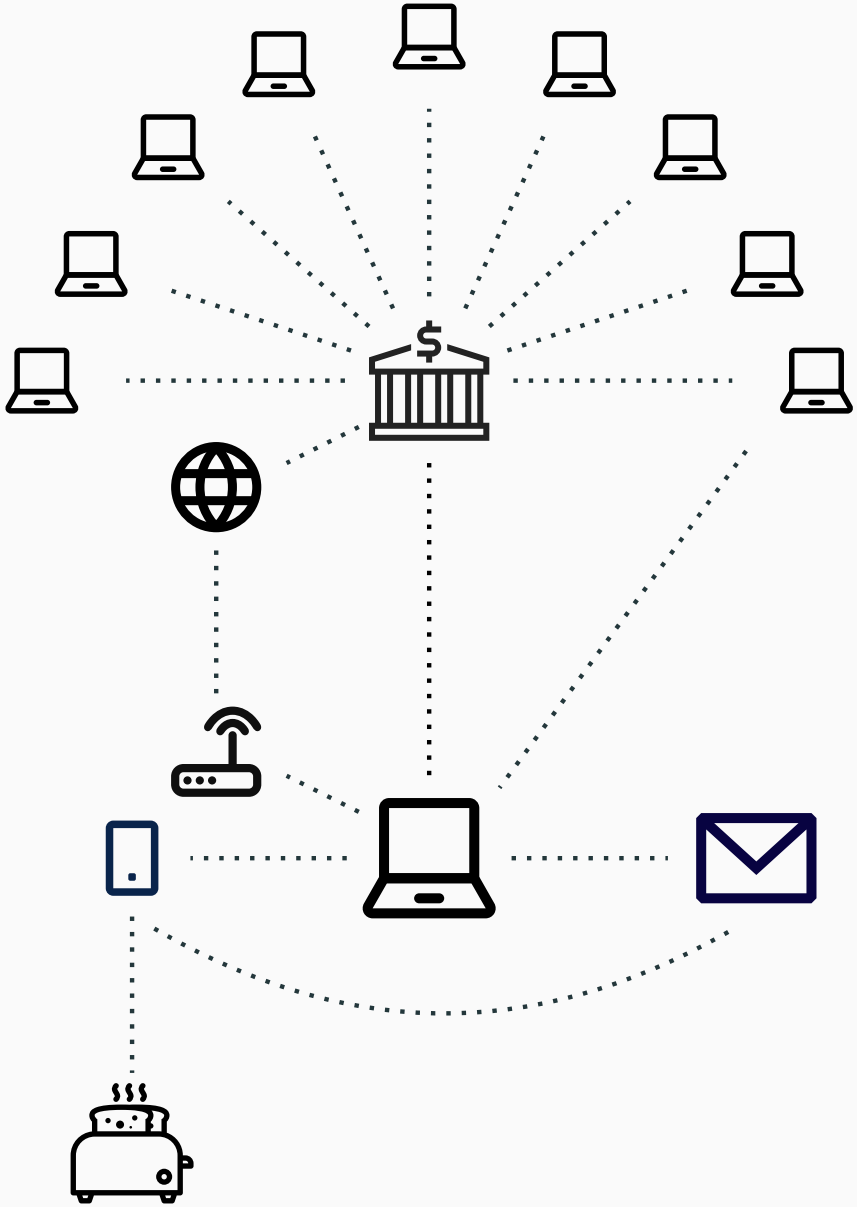

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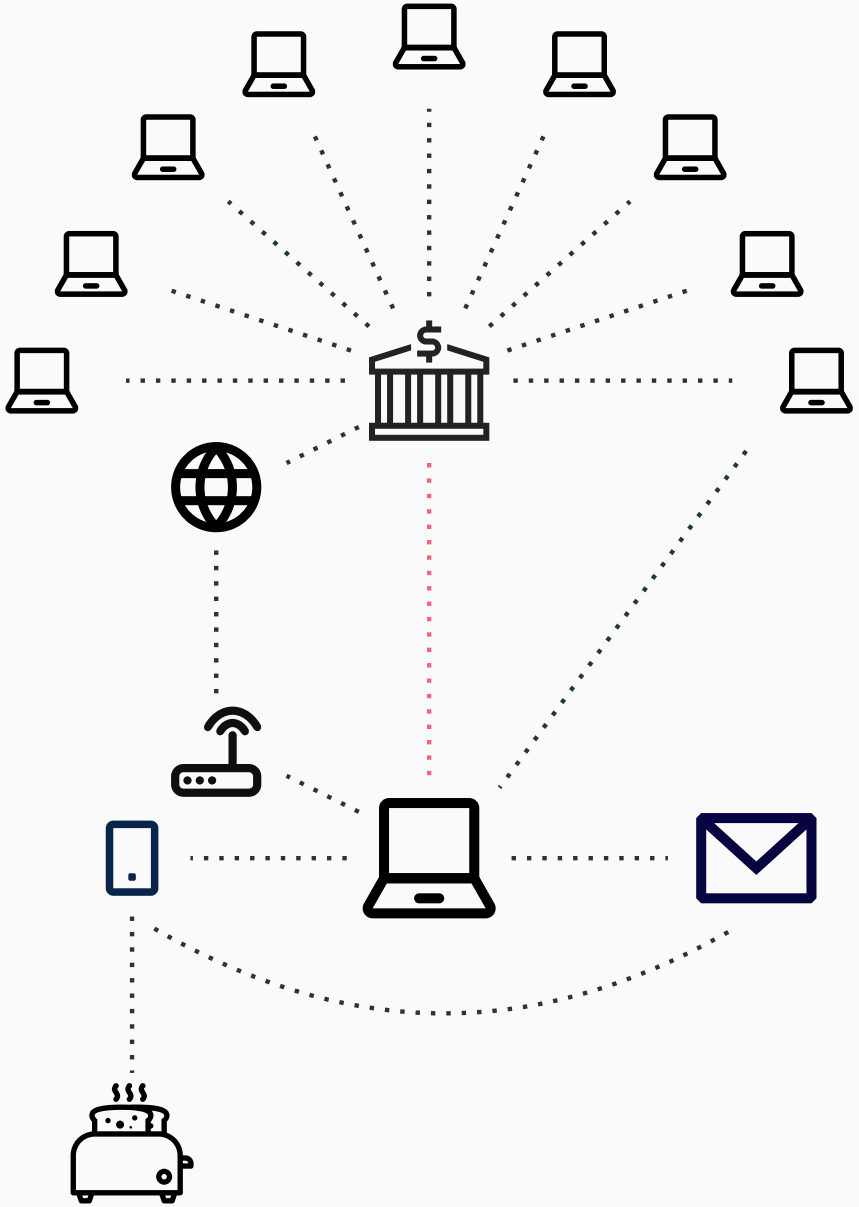

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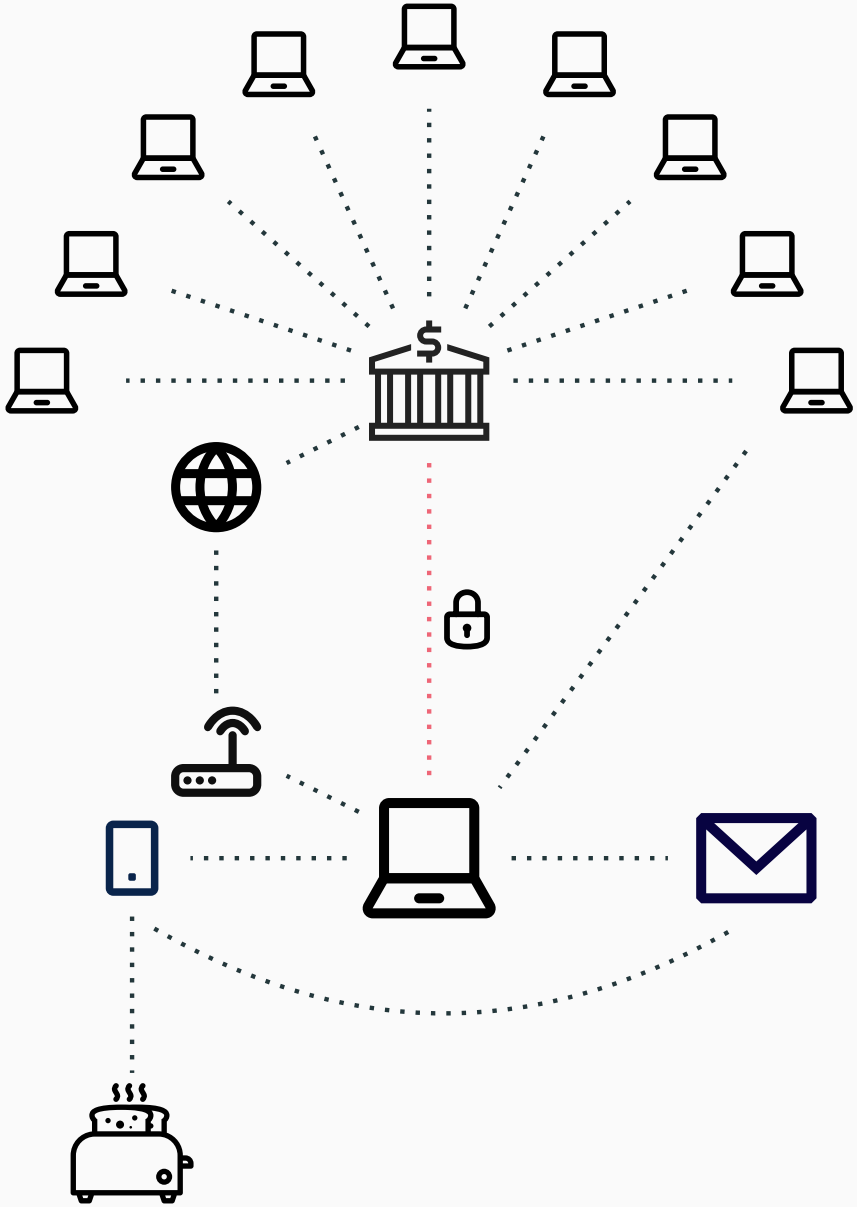

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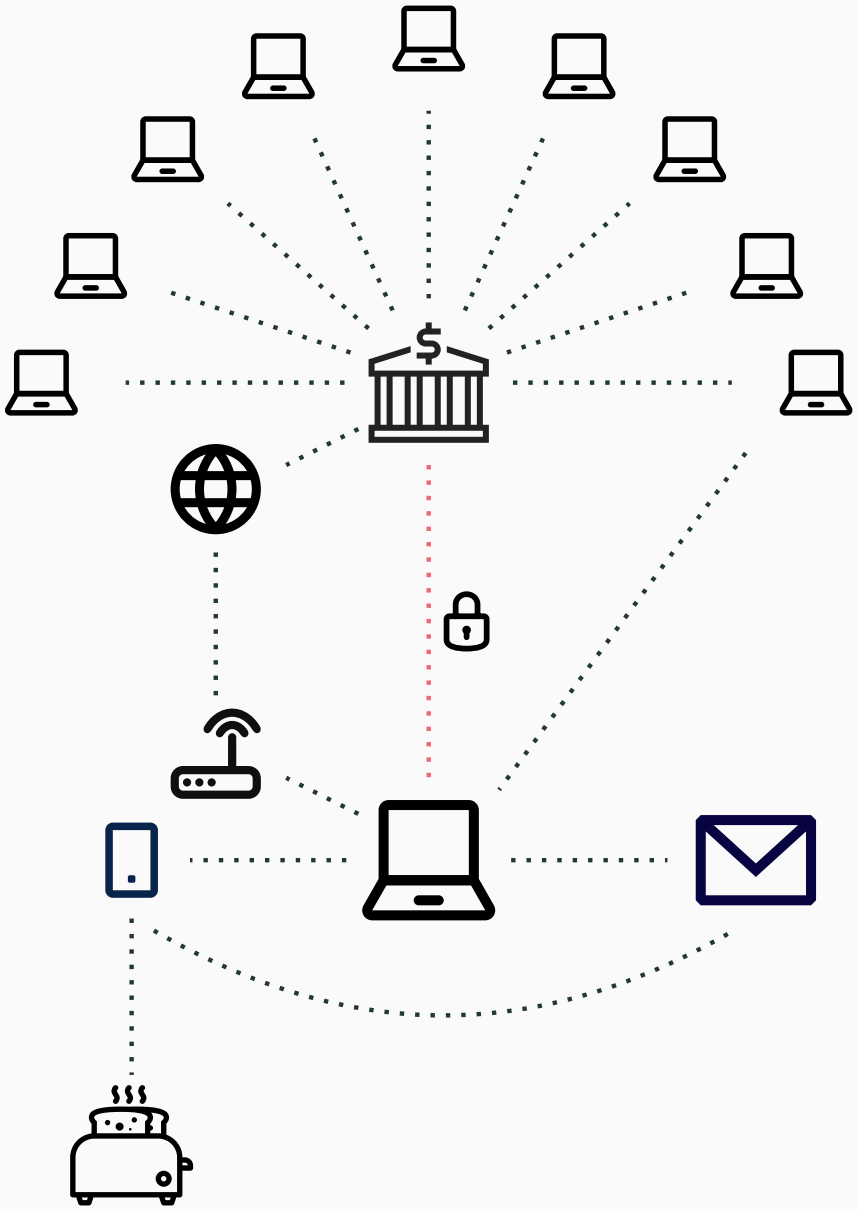

What do we need to prove about a computation in a vacuum so that it's still secure no matter what else is going on?

Universal Composability

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Due to Ran Canetti (2000).

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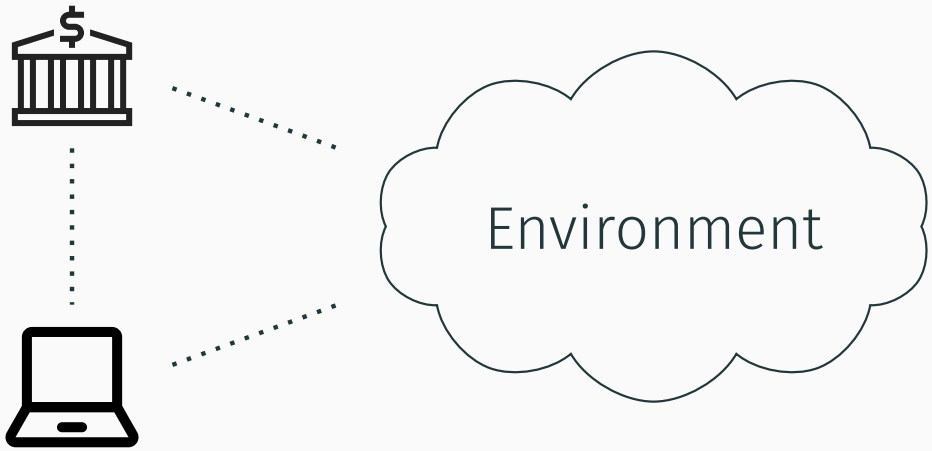

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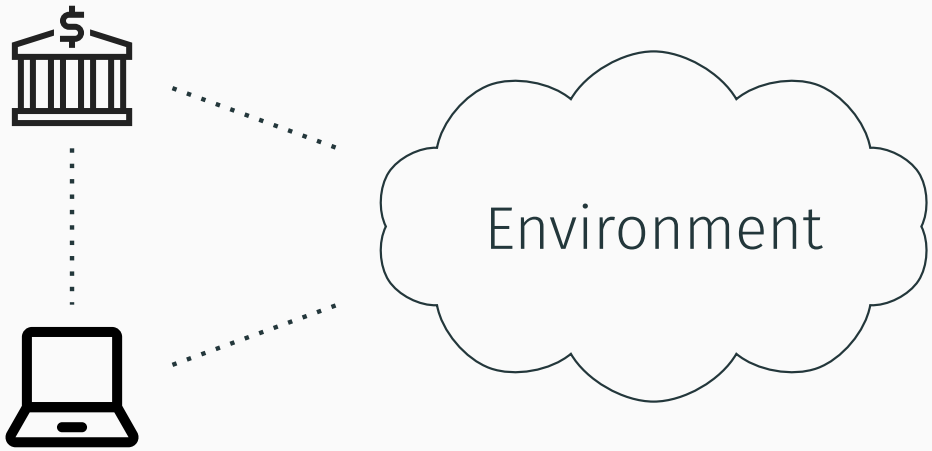

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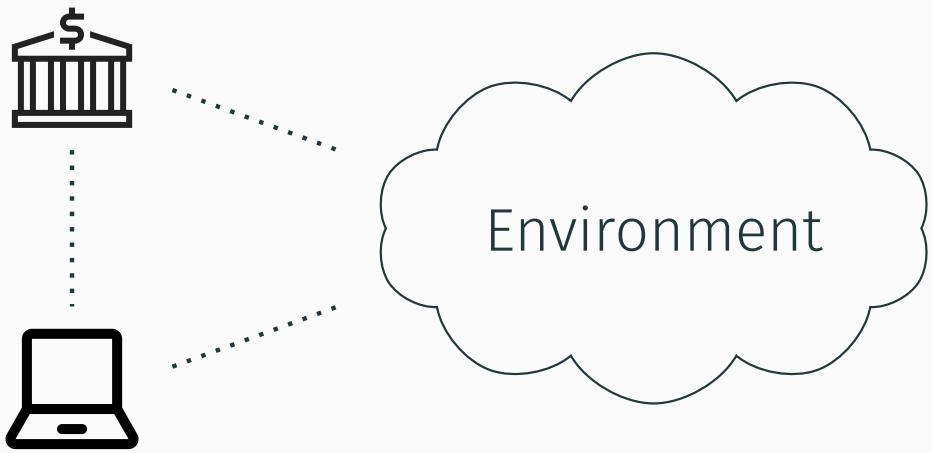

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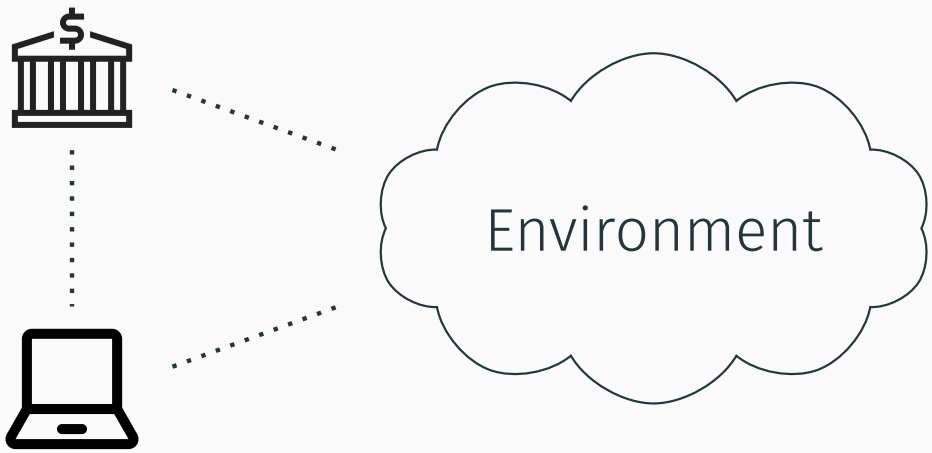

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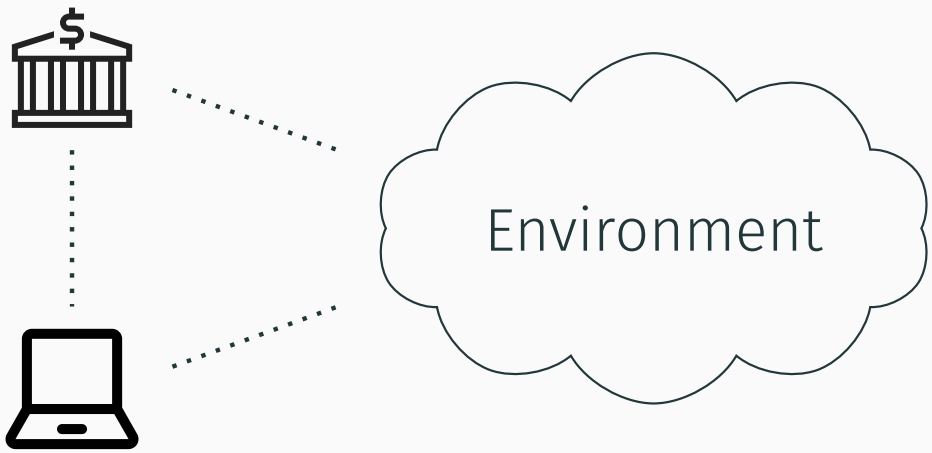

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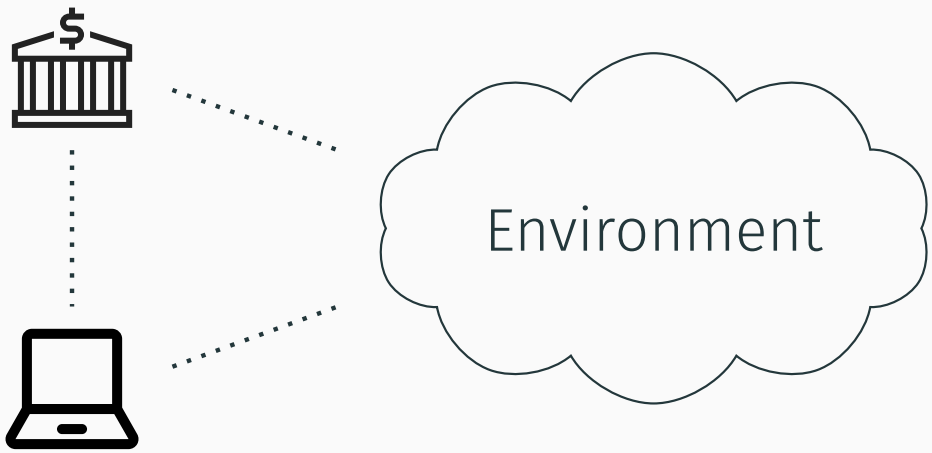
- 1 uses UC (Davies et al. 2023)



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Universal Composability

It is necessary that execution preserve security guarantees under concurrent composition. We refrain from proving UC security ... since such an analysis will be cumbersome. Instead, we prove the security of our protocols by constructing simulators and carefully arguing their security.

–David et al. 2023

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...**category theory** is an excellent candidate for such a theory.

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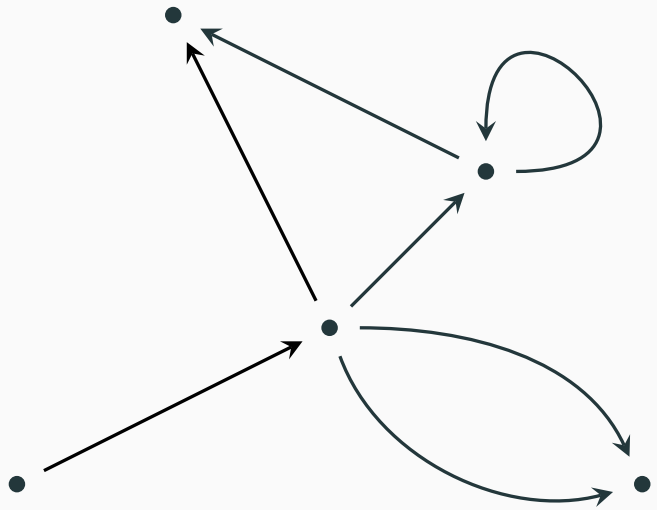

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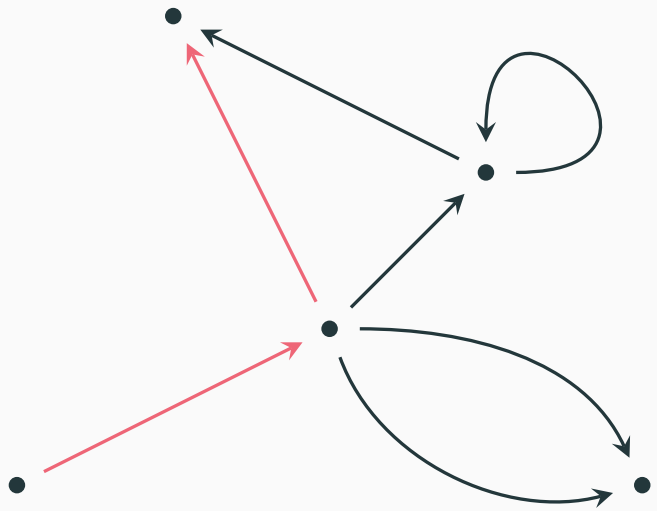

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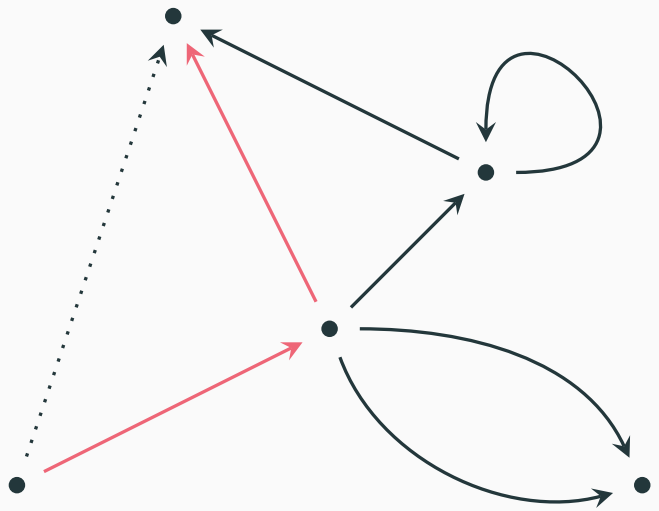

Categories and Cryptography

[Largely following Broadbent and Karvonen (2022)]

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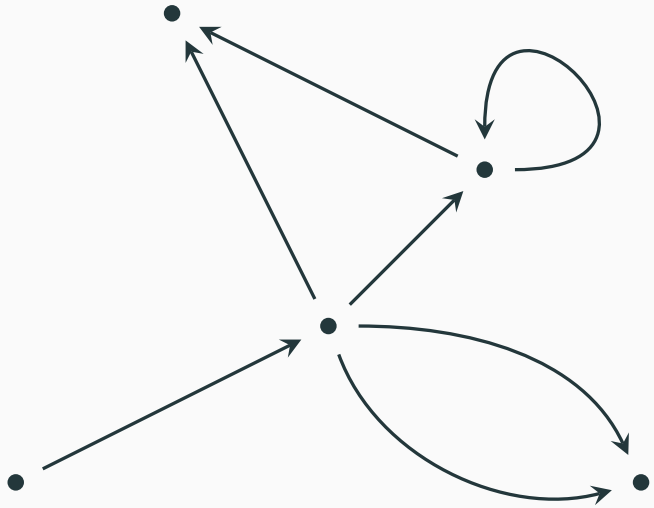

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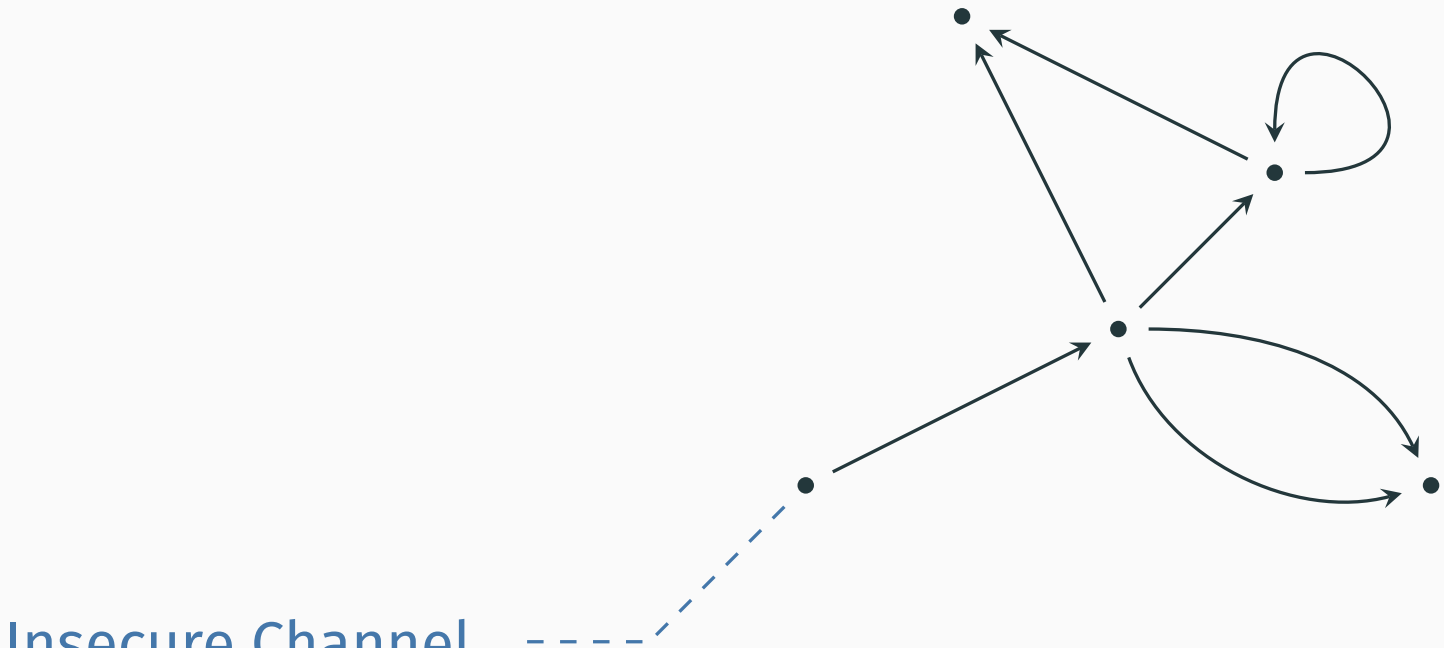

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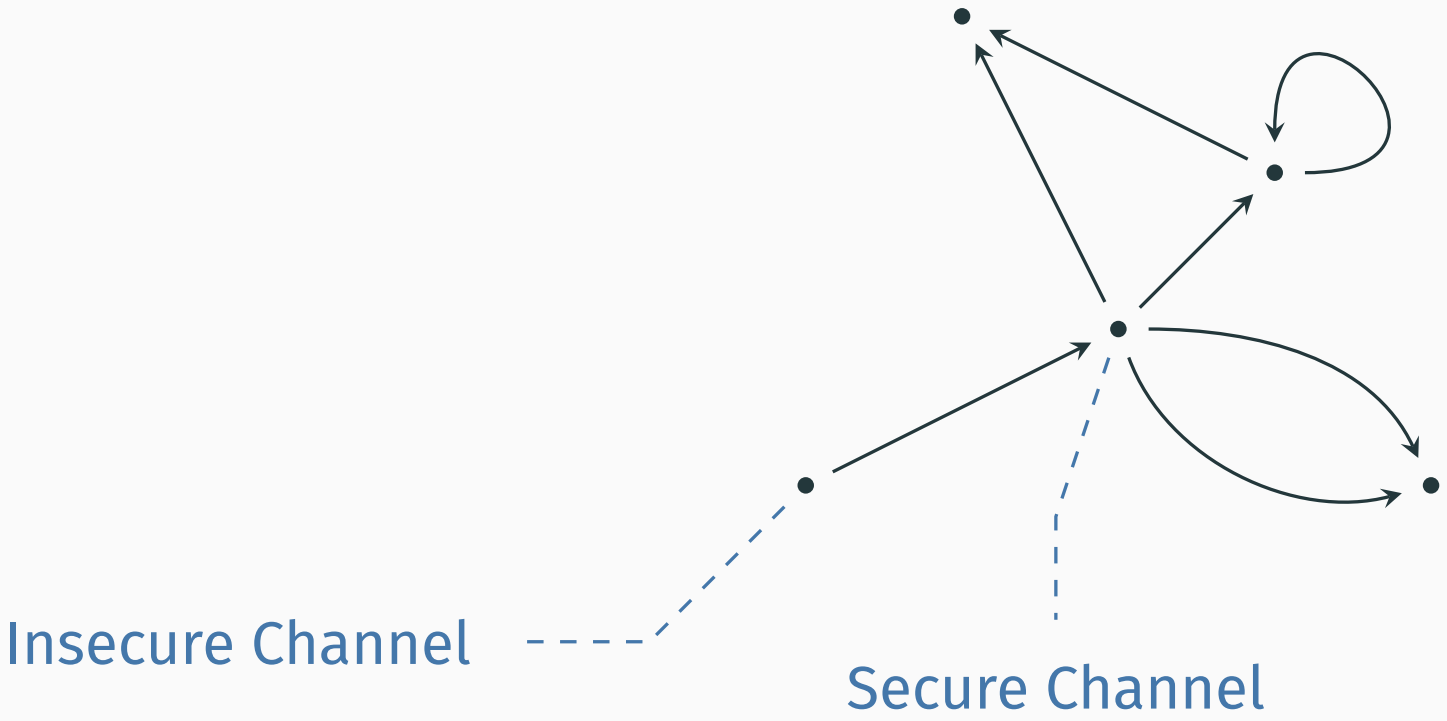

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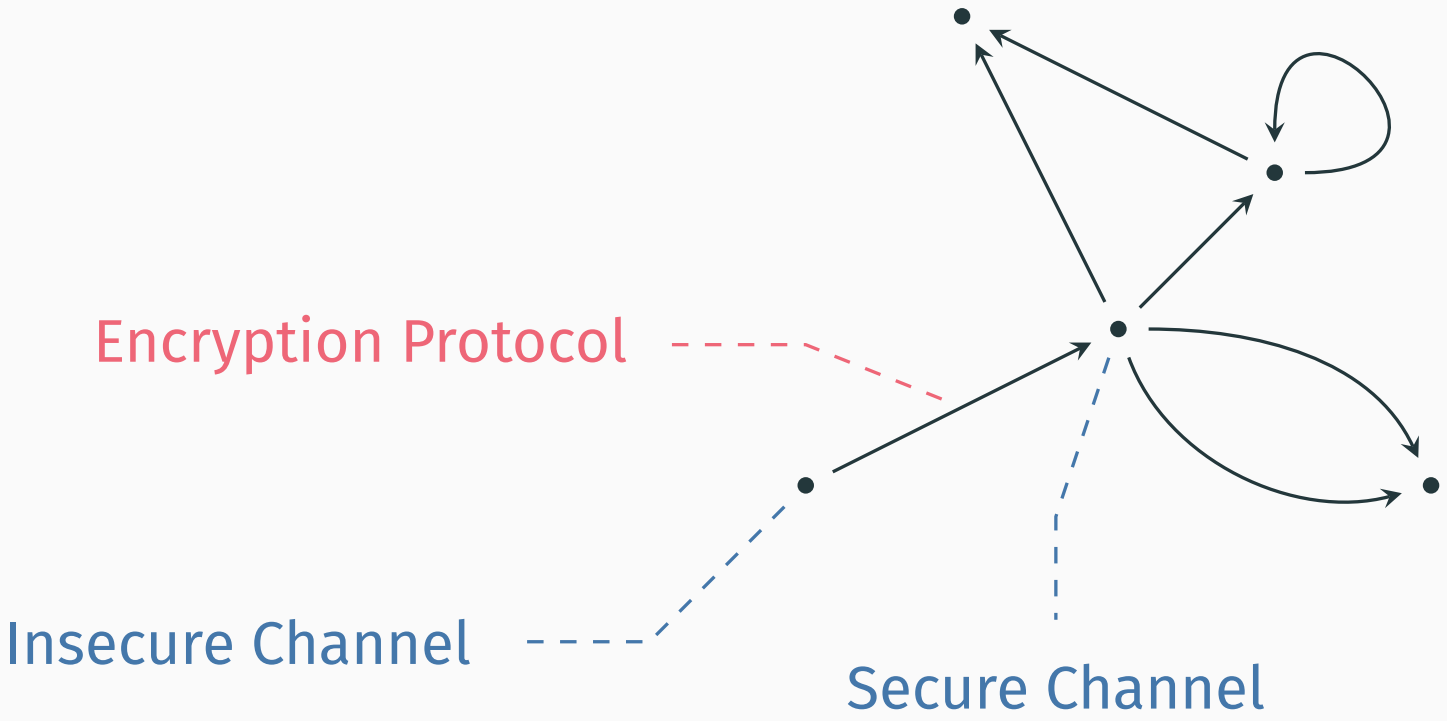

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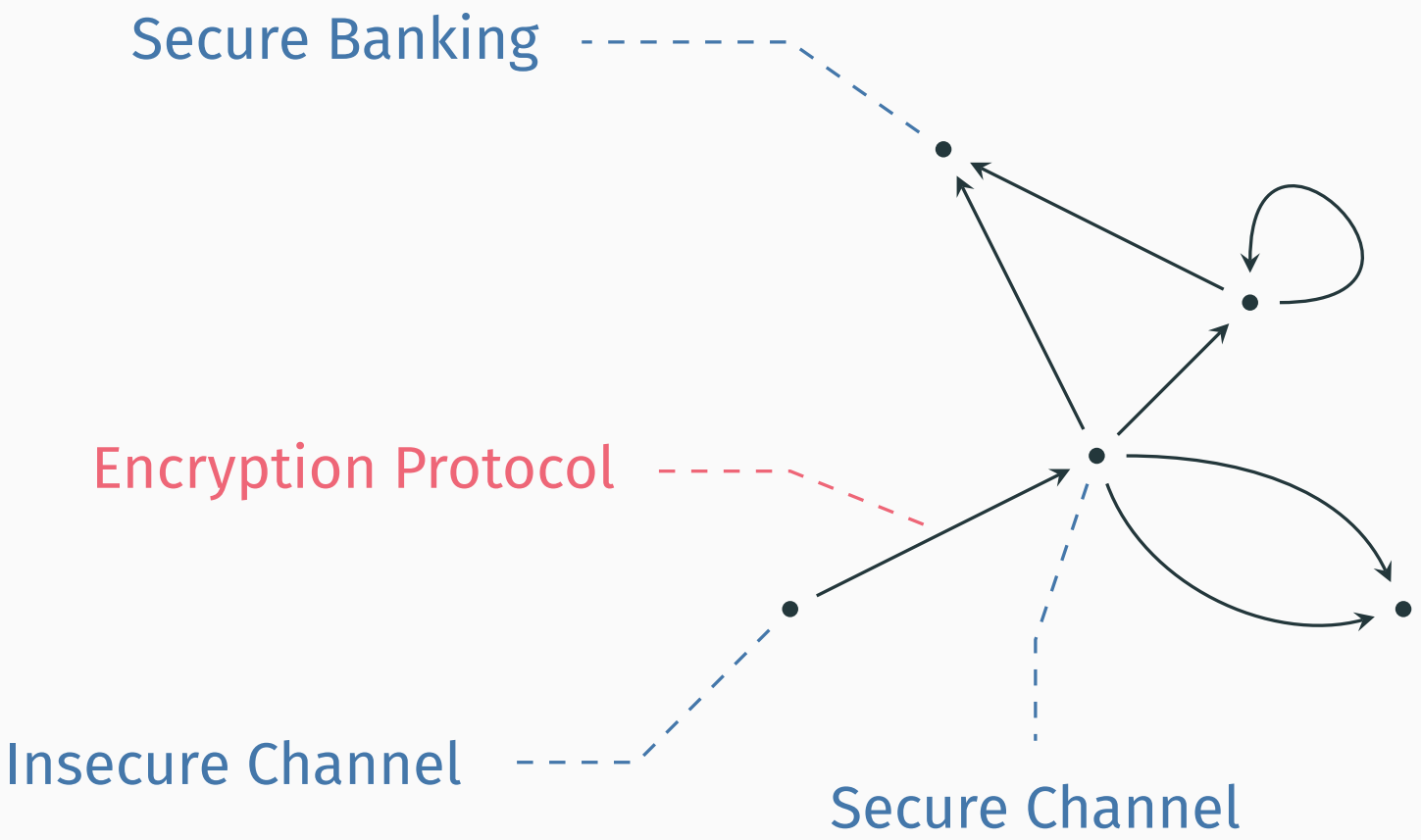

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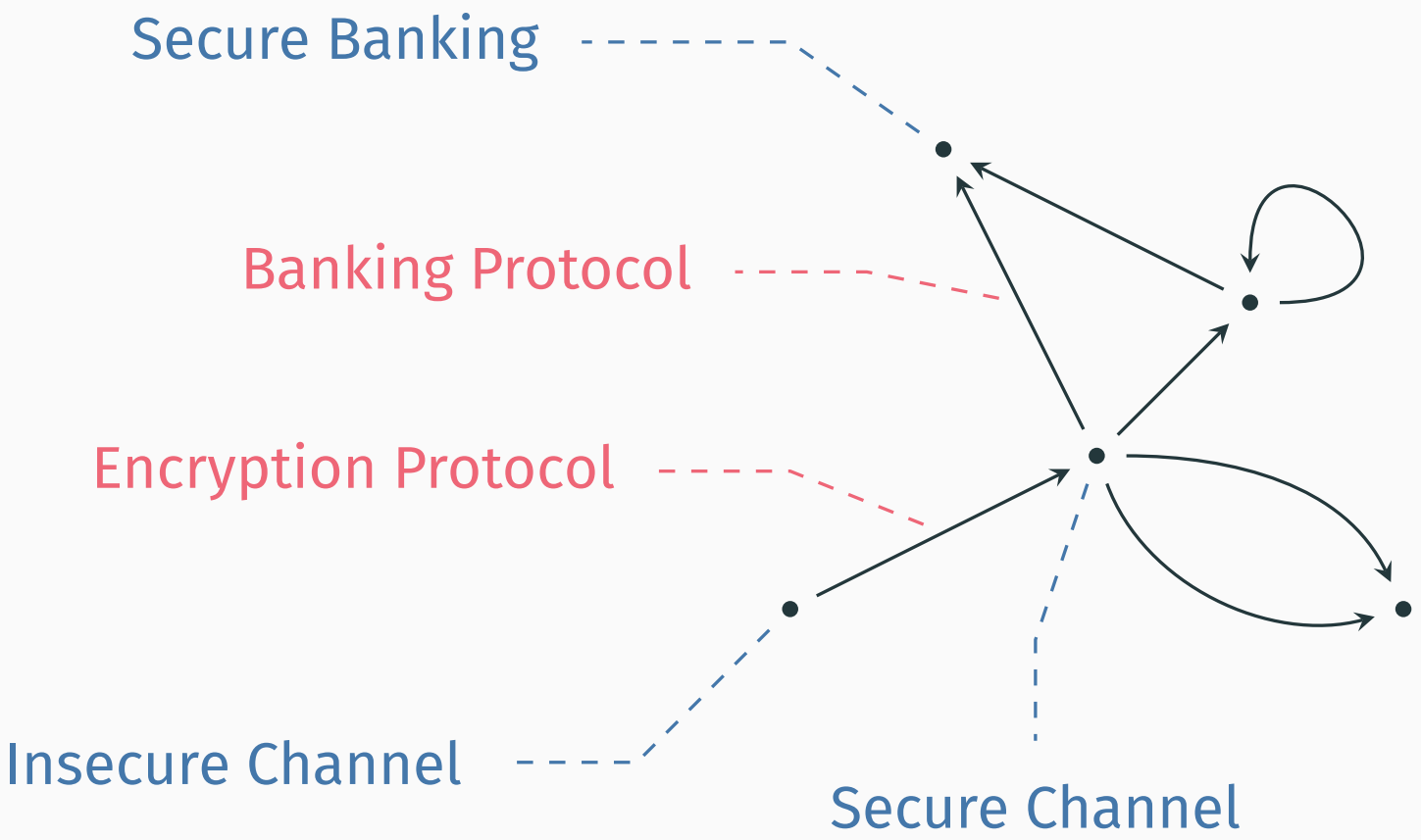

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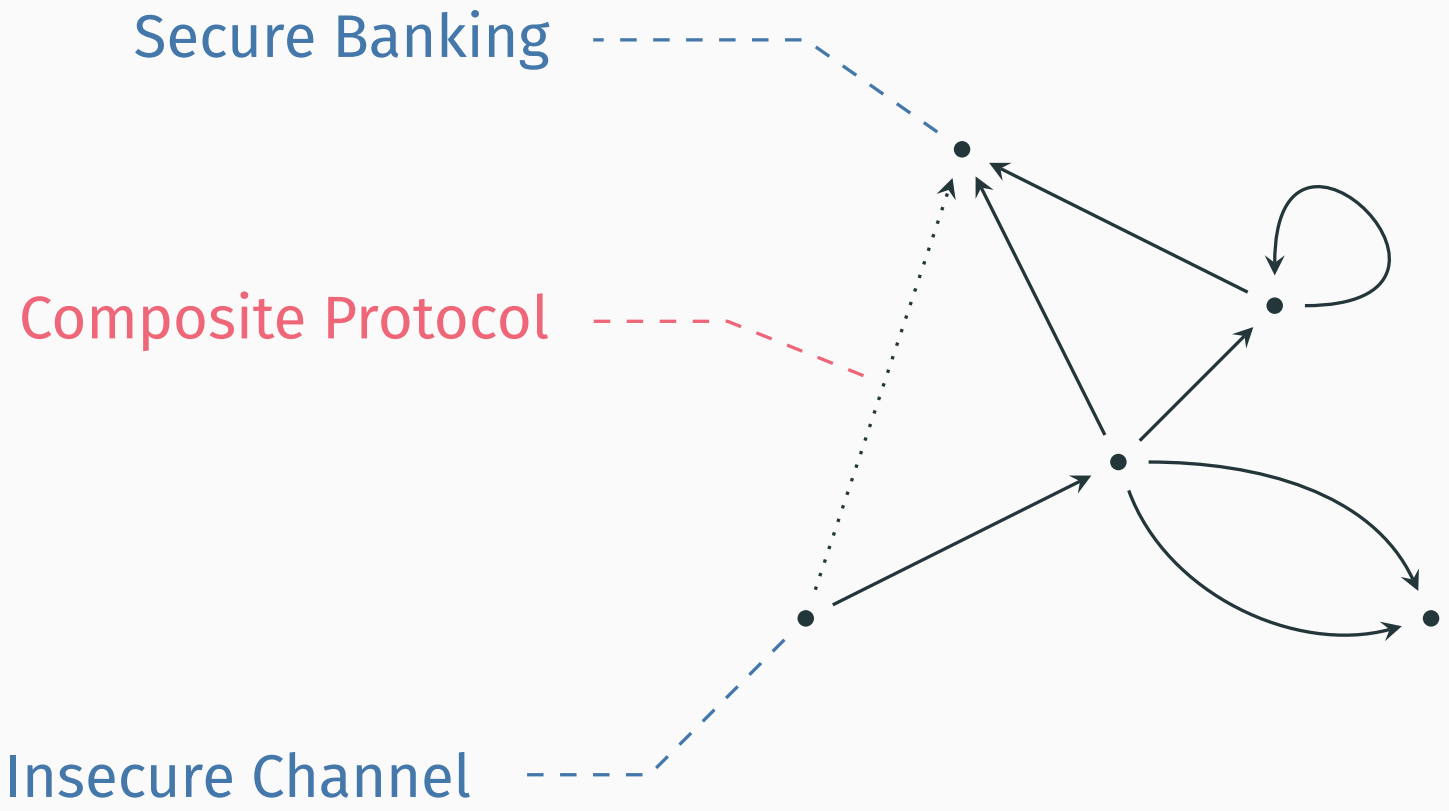

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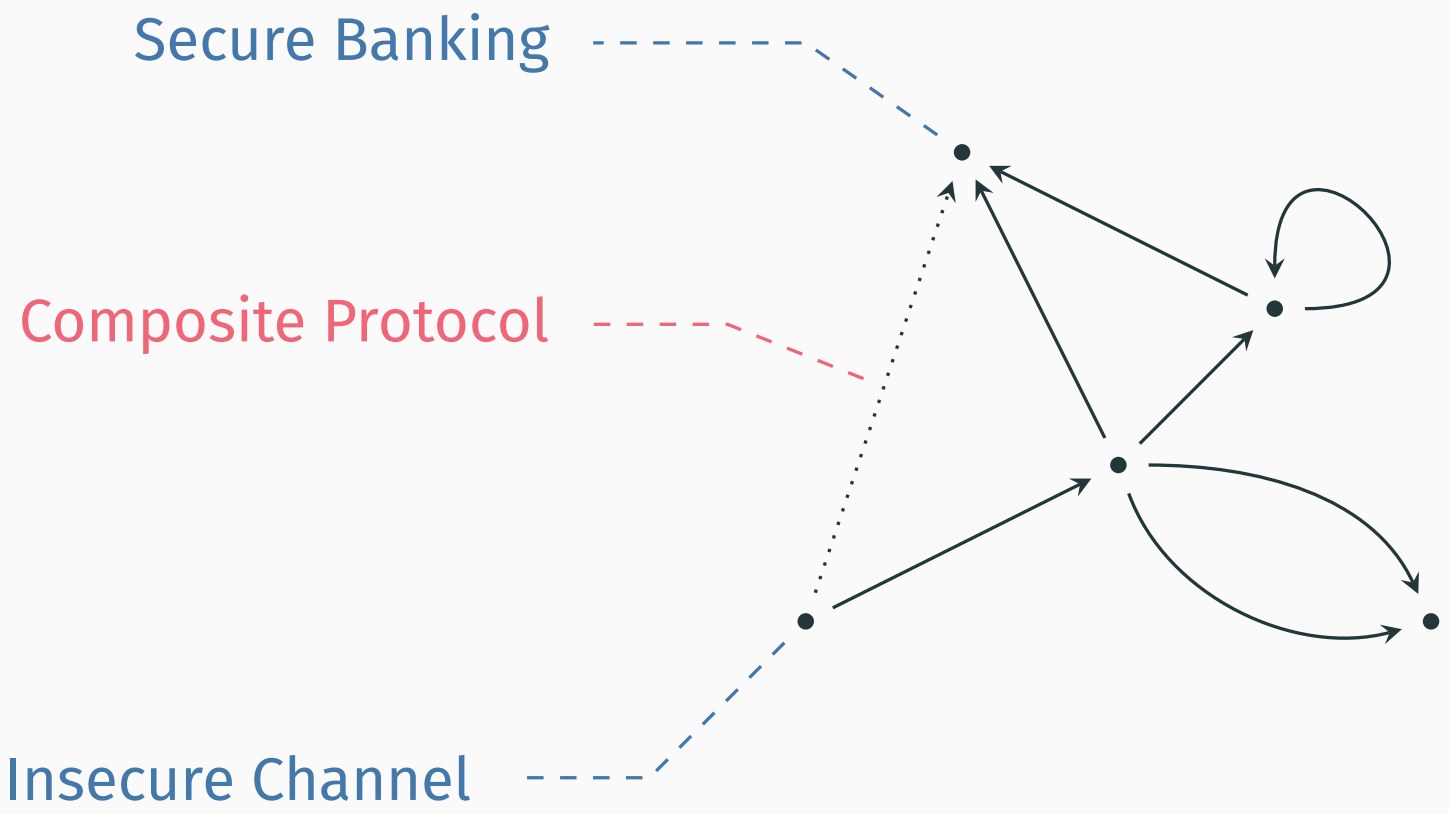

The challenge is to encode cryptographic objects as a category.

Security is Approximate

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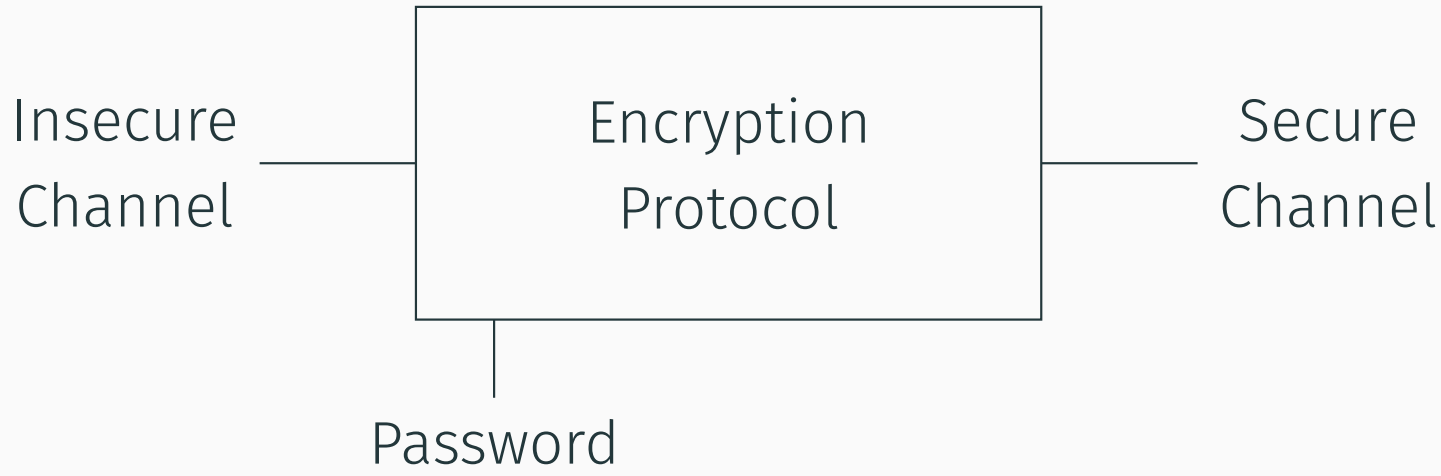

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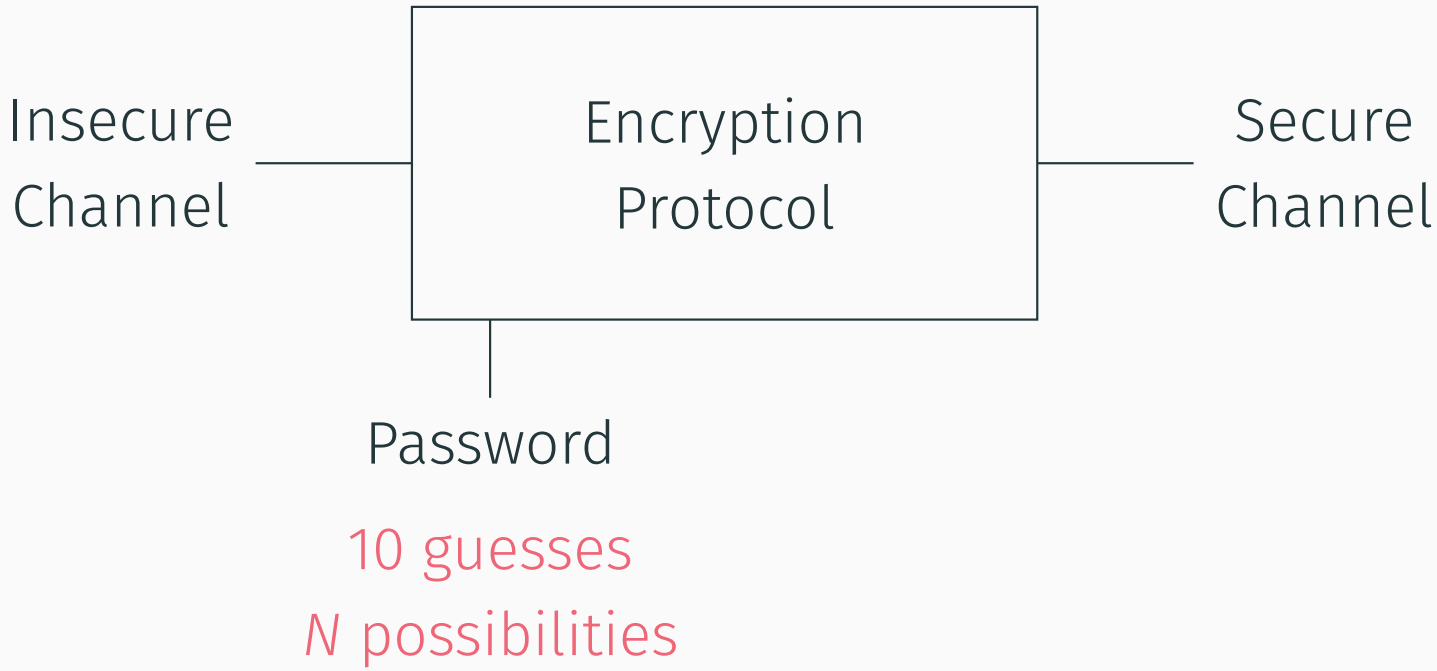

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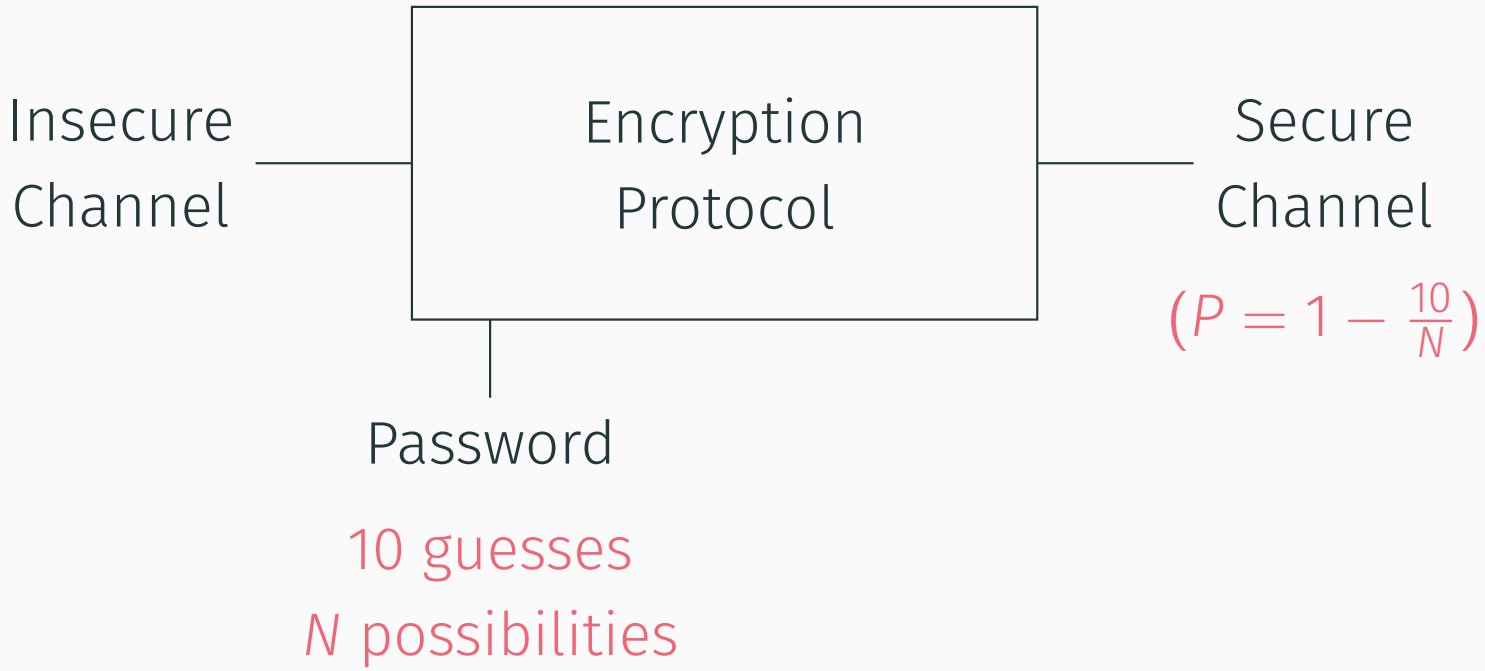

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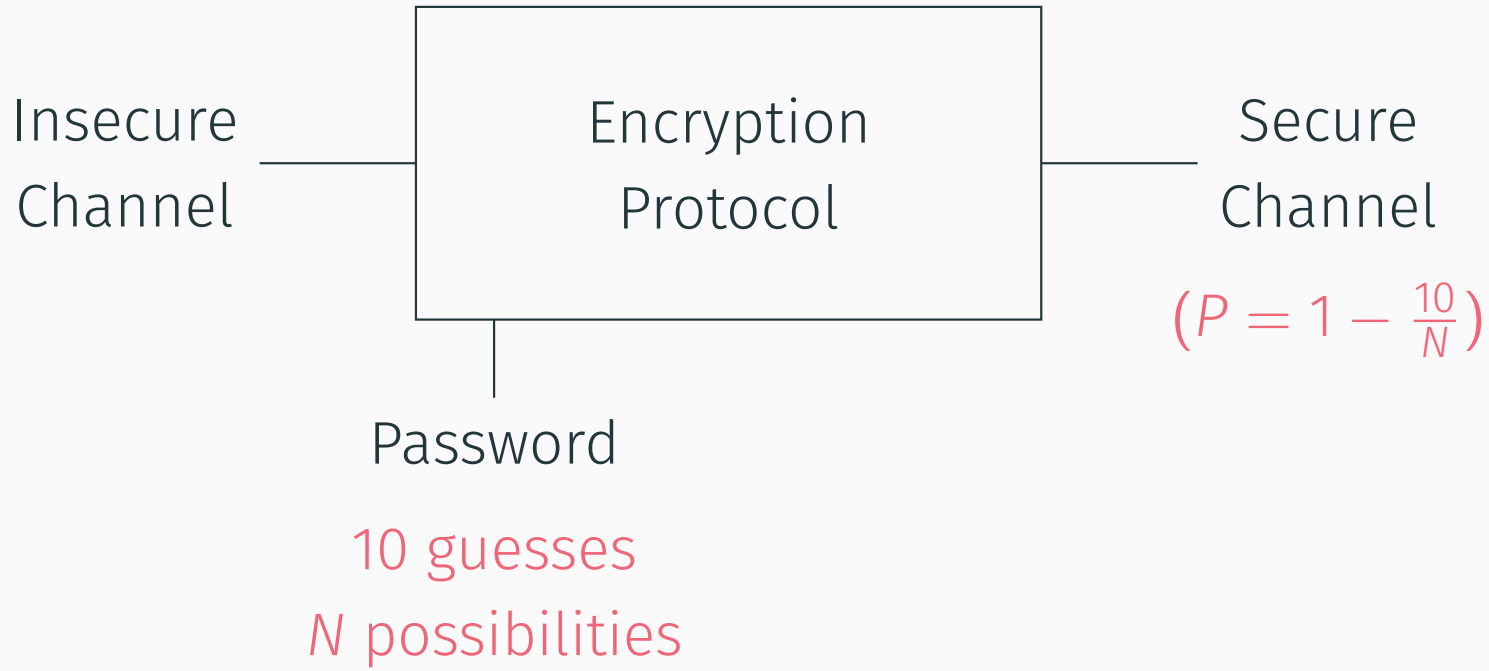
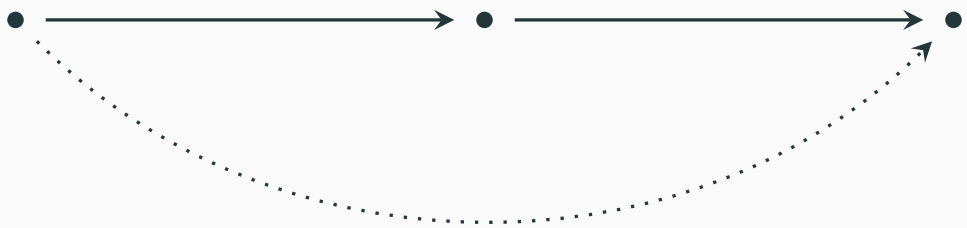


Cryptographers are very good at dealing with this.

Approximate Composition



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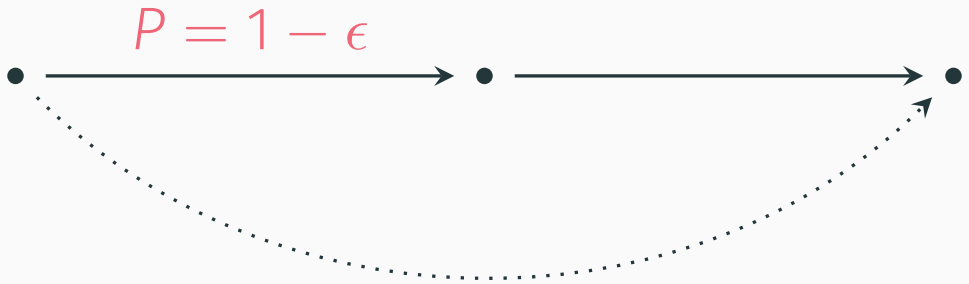

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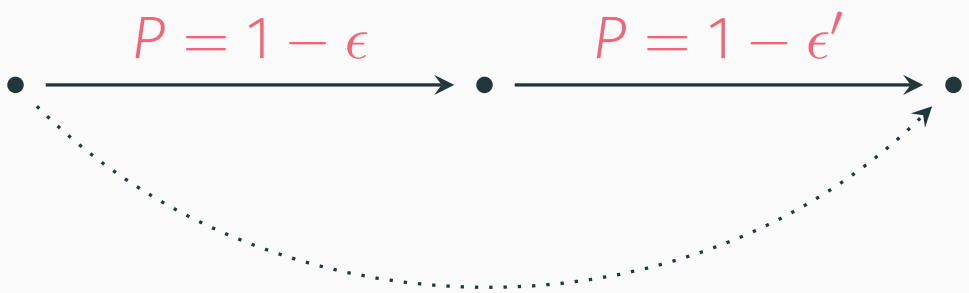

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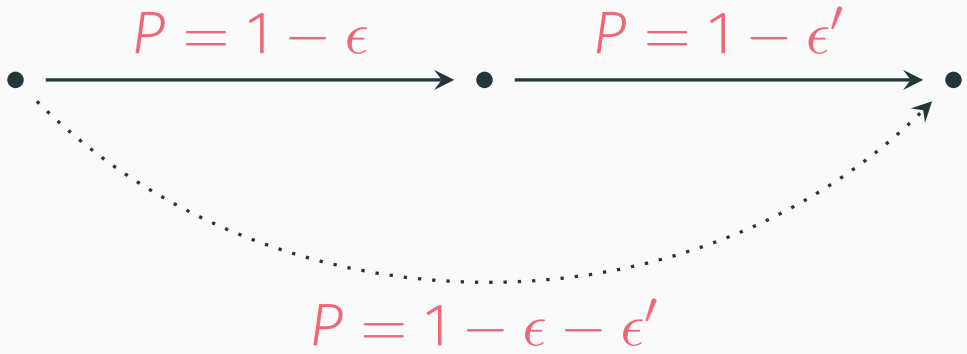

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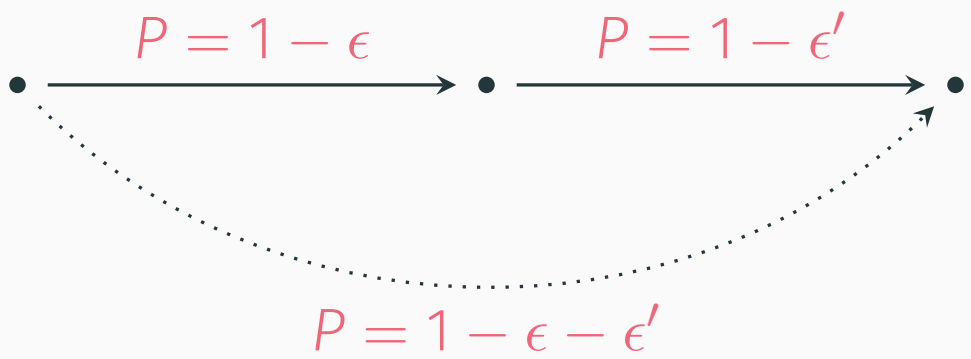

Small probabilities compound under composition.

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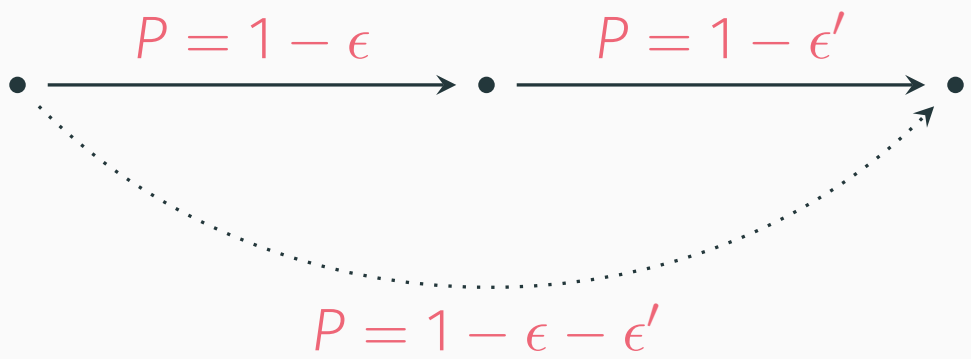

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Our idea is to work in *monoidal categories enriched over symmetric monoidal bicategories*.

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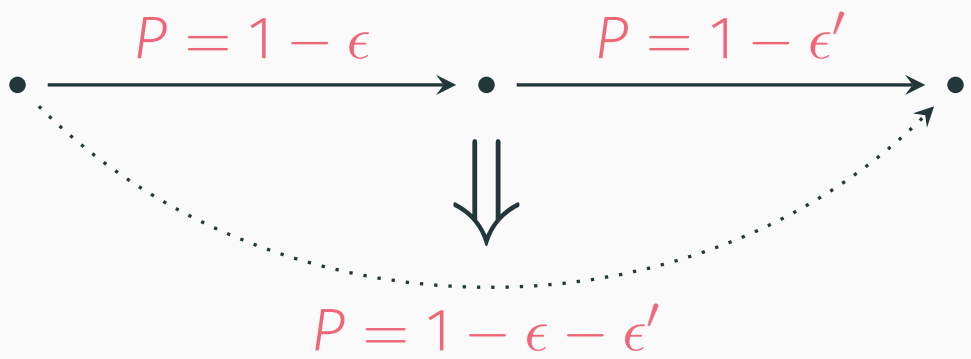

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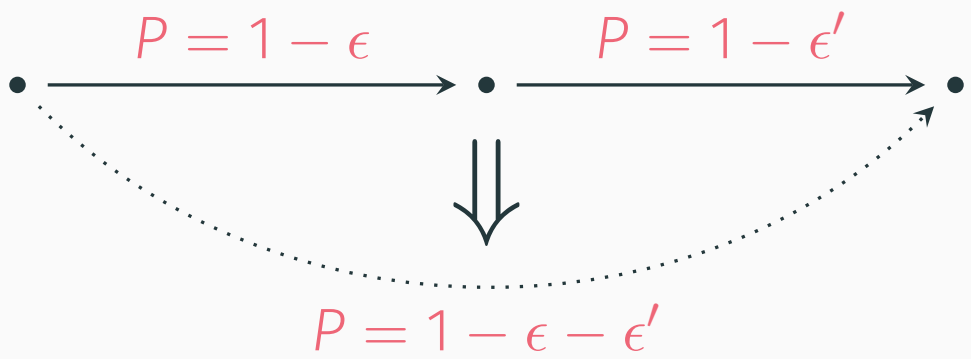

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There are complexity costs to this.

Breaking the Type System

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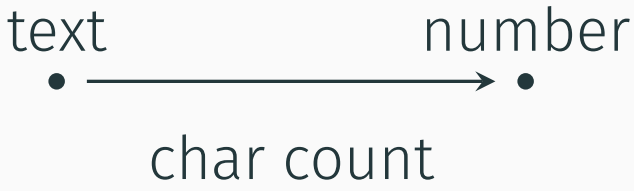

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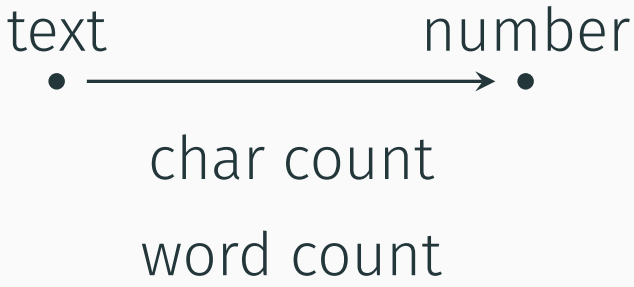


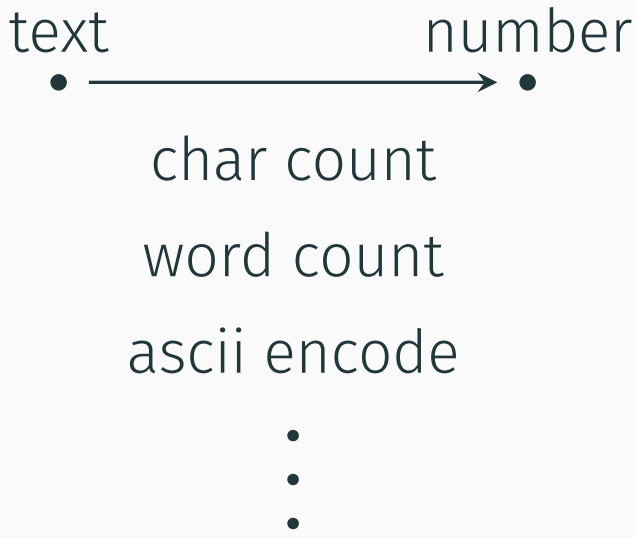
Breaking the Type System

char count

word count

ascii encode

Breaking the Type System



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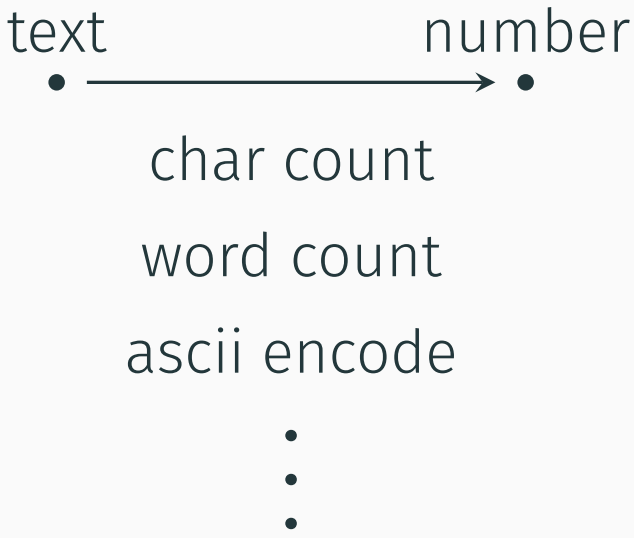

In a category, we *have* to get a number at the end.

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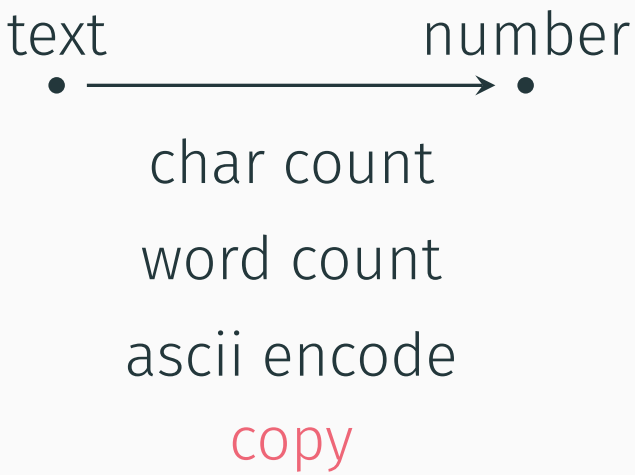

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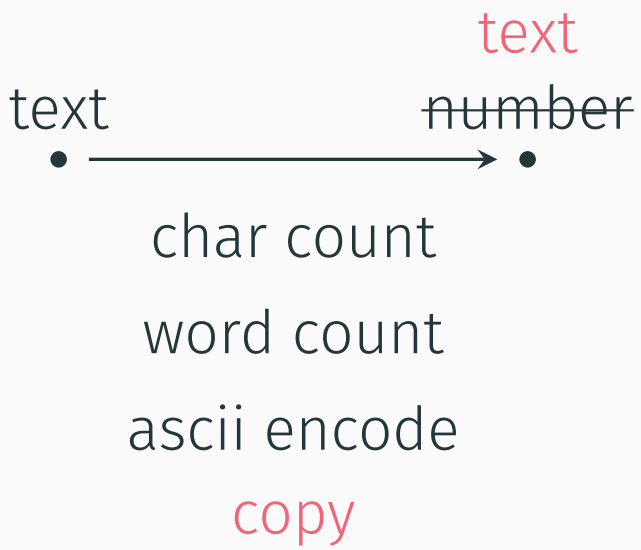

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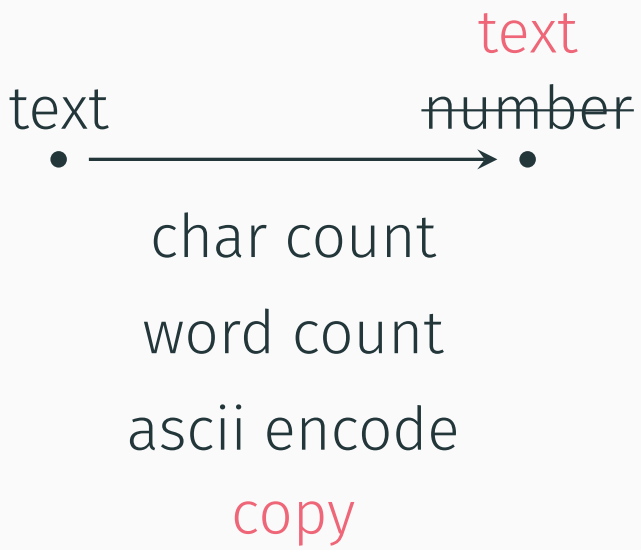

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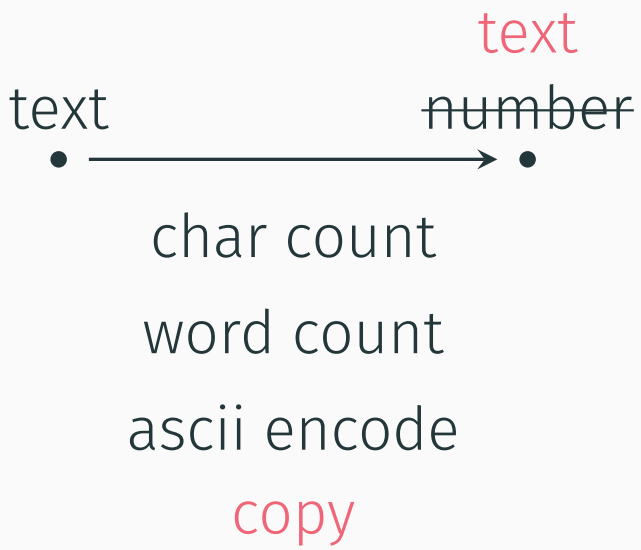
In cryptography, this isn't always true.

Breaking the Type System



In a category, we *have* to get a number at the end.

In cryptography, this isn't always true.

B&K propose a workaround, but it's somewhat artificial.

Outlook

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Thanks for your time!

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