Saturated Transfer Systems: Homework I

Mathcamp 2025

Problem 1 (recommended). Enumerate all the saturated transfer systems on $\underline{2} \times \underline{2}$, and describe the corresponding interior operators and submonoids.

Problem 2 (recommended). Give a closed form for $s(m, 2)$. (Hint: think about saturated transfer systems.)

Problem 3 (optional). Construct a map from the set of join-closed subsets of $\underline{m} \times \underline{n}$ to the set of submonoids of $\underline{m} \times \underline{n}$. Show that this map sends exactly two join-closed subsets to each submonoid.

Problem 4 (optional). Verify that if $\circ$ is an interior operator, then $\text{Fix}(\circ)$ is a submonoid.

Problem 5 (optional). Verify that if R is a saturated transfer system, then $\circ_R$ is an interior operator.

Problem 6 (optional). Write in detail the proof that the constructions sending a saturated transfer system to its characteristic interor operator and sending an interior operator to its associated saturated transfer system are inverses of each other.

Problem 7 (optional, if you know what a topological space is). Verify that the ordinary interior forms an interior operator on the power set of a topological space.