Confidential

SEAT NUMBER: ……………………………………………

STUDENT ID: ………………………………………………

SURNAME: …………………………………………………

GIVEN NAMES: ……………………………………..

AMME3500/9501
System Dynamics and Control

Final Examination
Semester 1, 2015

Time Allowed: Three Hours + 10 minutes reading time

This examination paper consists of 9 *pages*

INSTRUCTIONS TO CANDIDATES

1. This is a closed book exam.
2. A simple calculator (programmable versions and PDA's not allowed) may be taken into the exam room.

Calculator	Make	Model

3. The paper comprises 6 questions each with multiple parts. ANSWER ALL SIX QUESTIONS.
4. The mark to be awarded for each part is indicated. Marks total 100.
5. Answer all questions in the ANSWER BOOK PROVIDED. Plotting pages for Questions 3, 4 & 6b are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period.
6. The question paper must be returned with the answer script
7. Take care to write legibly. Write your final answers in ink, not pencil.

Section A – System Specifications [30 marks]

1. [10 marks] For the LTI system with an open-loop transfer function given by:

$$G(s) = \frac{10s^2 + 10s - 60}{(s^2 + 7s + 25)(s + 10)(s + 0.3)}$$

   a. Where are the poles and zeros of this system? Is the system stable in open-loop?
   b. Estimate the settling-time for this system, and comment on the expected accuracy of your estimate.
   c. What would be the steady-state response of this system to a unit step input?

2. [20 marks] You are designing a cruise-control system for a car. At high speeds on level ground, a dynamic model considering air resistance is:

$$m\dot{v} + \frac{1}{2}\rho C_D A v^2 = f$$

where v is the velocity in m/s, the mass m=1000 kg, the air mass density ρ = 1.2kg/m^3, the drag coefficient C_D = 0.4, the cross-sectional area A=2m^2 and f is the applied force from the engine. The drive-train from control input u to force f can be modelled as a first-order system with gain 100 and time-constant 2s.

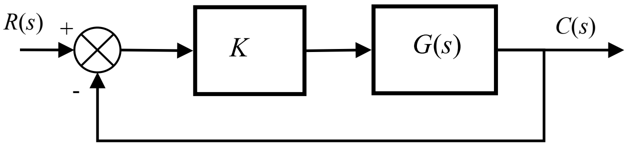
   a. Assuming the cruise speed is approximately 110km/h, derive the transfer function from u to v.
   b. Design a PD controller:

$$u = K_p(v_R - v) + K_d(\dot{v}_R - \dot{v})$$

   to achieve a settling time of 5 seconds and an overshoot of 5%, using second order assumptions.
   c. Derive the closed-loop transfer function. Do you expect the design requirements to be met perfectly? If not, why not?
   d. Would you expect this system to achieve zero steady-state error for this desired velocity? Discuss your reasons why/why not. If not, suggest an alternative controller structure that could be used.

Section B – Root Locus and Bode plots [40 marks]

1. [15 marks] A feedback system is shown below:



with K a constant gain and the transfer function for the plant G(s) the same as in Question 1:

$$G(s) = \frac{10s^2 + 10s - 60}{(s^2 + 7s + 25)(s + 10)(s + 0.3)}$$

For this system, sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 7) and be sure to turn in your copy of the exam booklet with your answer book at the end of the exam. Could proportional control make this system unstable? If so, estimate the gain K at which the system is marginally stable.

2. [25 marks] The transfer function below represents the elevator-to-pitch-angle response of an F-16 fighter at cruise:

$$G(s) = \frac{200(s + 0.1)(s + 0.02)}{(s^2 + 0.016s + 0.0064)(s^2 + 2.4s + 3.4)(s + 20)}$$

   a. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 8). Show any derivation you require in the answer book and include the asymptotes on the sketch itself. Be sure to mark the units on the graph.
   b. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin.
   c. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable (if any). Give an approximate value for the overshoot if this system were placed in a unity feedback loop.
   d. Give a detailed control design that will allow increased gain at low frequency and increased bandwidth, but a smaller overshoot, compared to unity feedback. Include any formulas or sketches that are necessary to make your case. In your design, take into account the fact that sensor noise may be present at high frequencies, and discuss its influence.

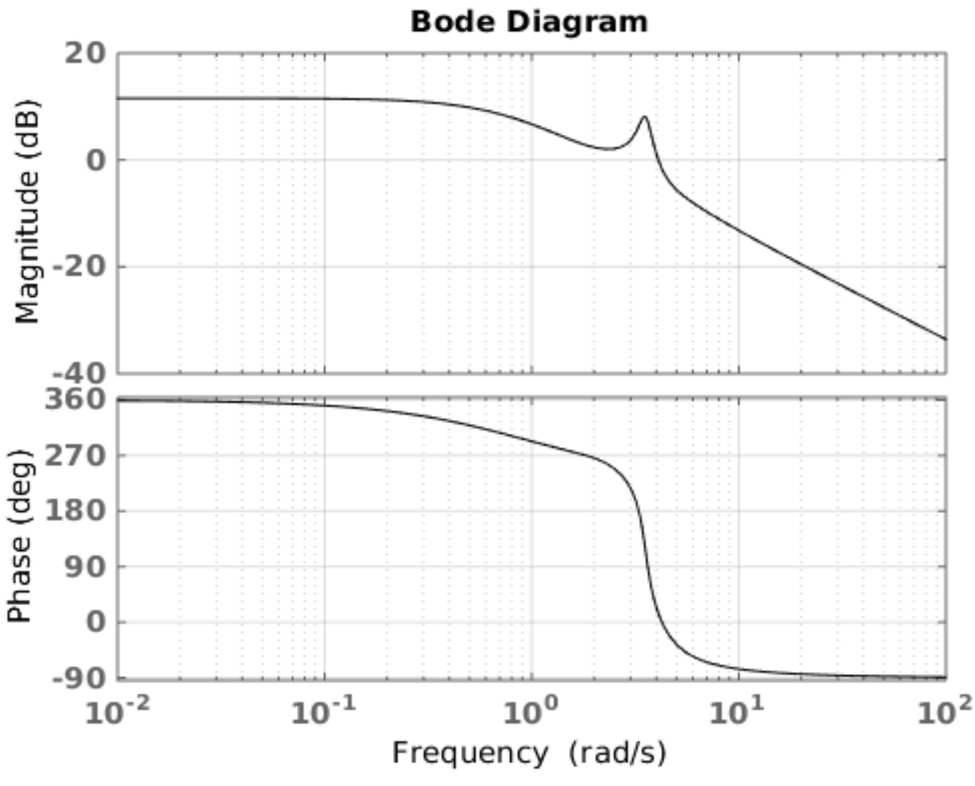
Section C – Controller Design and Analysis [30 marks]

3. [10 marks] Given the second order system, answer the following

$$G(s) = \frac{s+2}{(s^2+8s+15)(s+10)}$$

   a. Transform the transfer function into phase-variable form
   b. Design a state-feedback controller that achieves an overshoot of 10% and settling time of 0.5 seconds.

4. [20 marks] In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions.
   a. For a PID controller, describe the effect that the proportional, integral and differential terms have on the transient response of a system. Use any sketches or equations that you consider appropriate.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 9) – note the different time scales (units are seconds). Draw the response over the input signals. Label your diagram showing features such as the response magnitude and phase lag/lead.



   c. Given an open-loop stable system $G(s)$, comment on the fundamental limitations to control design, regardless of the controller structure, that are implied by Bode's Sensitivity Integral: $\int_0^\infty \log|S(j\omega)| d\omega = 0$.
   d. Prove that a necessary condition for stability of a linear system is that the denominator polynomial of the transfer function has all coefficients the same sign. Prove that for second-order systems this is also sufficient.

THERE ARE NO MORE QUESTIONS

Selected Equations

Time Response (First Order Systems)

$$G(s) = C_\infty \frac{1}{s+\sigma} \qquad t_r = \frac{2.2}{\sigma}$$

Time Response (Second Order Systems)

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} \qquad \sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_r \cong \frac{1.8}{\omega_n} \qquad t_s \cong \frac{4}{\sigma}$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} \qquad M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$$

$$M_p \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1+\lim\limits_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim\limits_{s\to 0} sG(s)} \qquad e_{parabola}(\infty) = \frac{1}{\lim\limits_{s\to 0} s^2 G(s)}$$

Root Locus

$$|KG(s)H(s)| = 1 \qquad \sigma_a = \frac{\sum finite\ poles - \sum finite\ zeroes}{n-m}$$

$$\angle KG(s)H(s) = (2k+1)180^\circ \qquad \theta_a = \frac{(2k+1)\pi}{n-m}$$

Frequency Response

$$M = |G(j\omega)| = |G(s)|_{s=j\omega} = \sqrt{\left(\mathrm{Re}\left[G(j\omega)\right]\right)^2 + \left(\mathrm{Im}\left[G(j\omega)\right]\right)^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\mathrm{Im}\left[G(j\omega)\right]}{\mathrm{Re}\left[G(j\omega)\right]}\right]$$

$$GM = 20\log K \qquad PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

State Space

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Controllability

$$\mathbf{R} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$$

Observability

$$\mathbf{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix}$$

Full State Feedback

$$\mathbf{u} = -\mathbf{Kx} + r$$

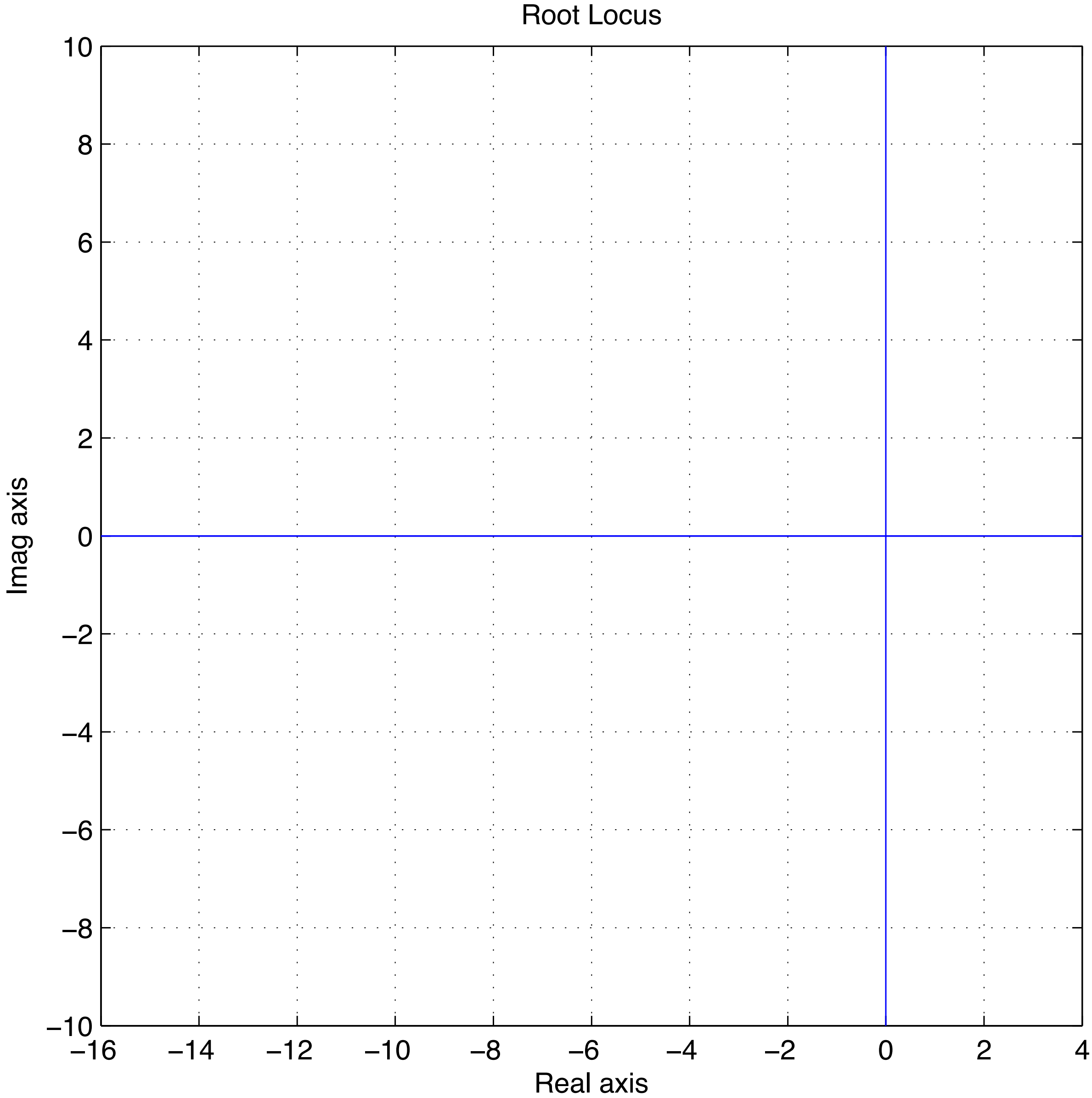
Laplace Transform Tables

Item no.	*f(t)*	*F(s)*
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

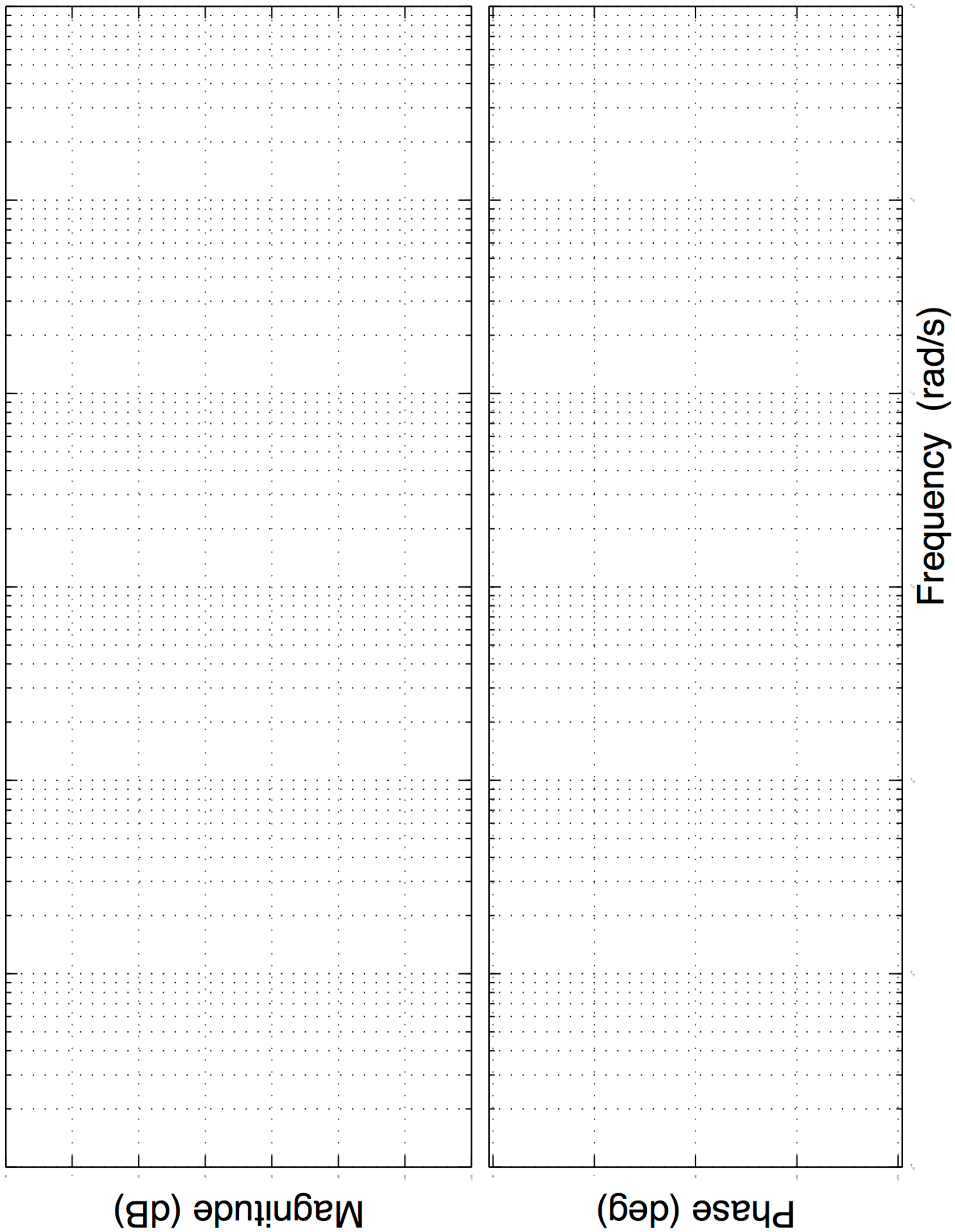
Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s)-sf(0-)-\dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s)-\sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

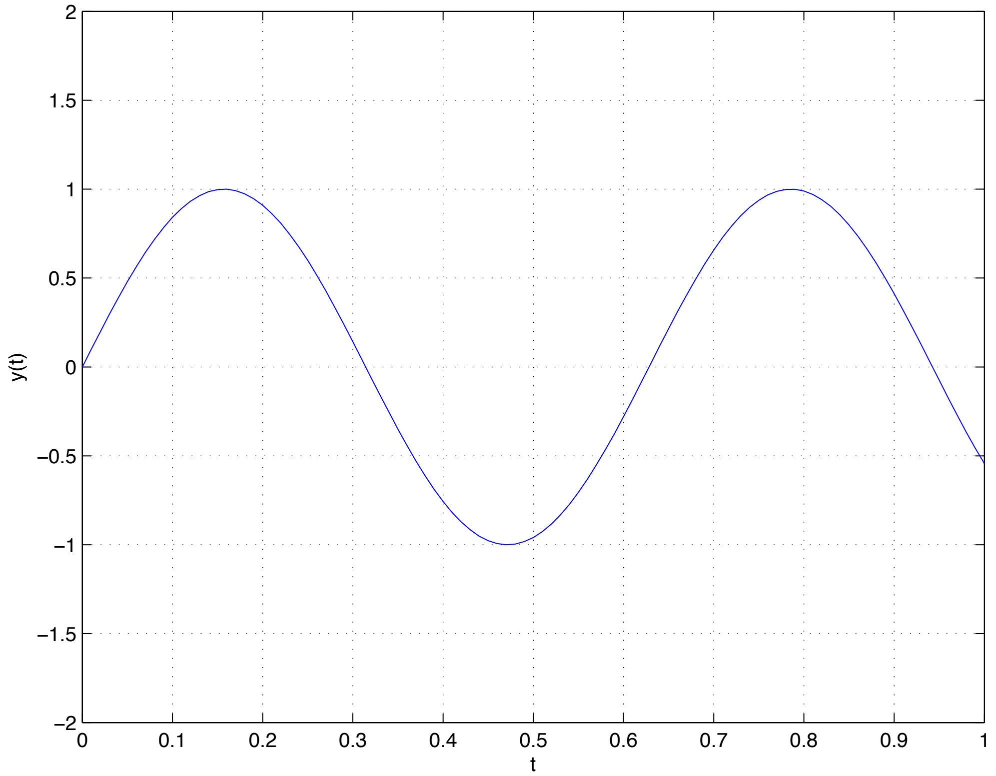
Root Locus for Question 3

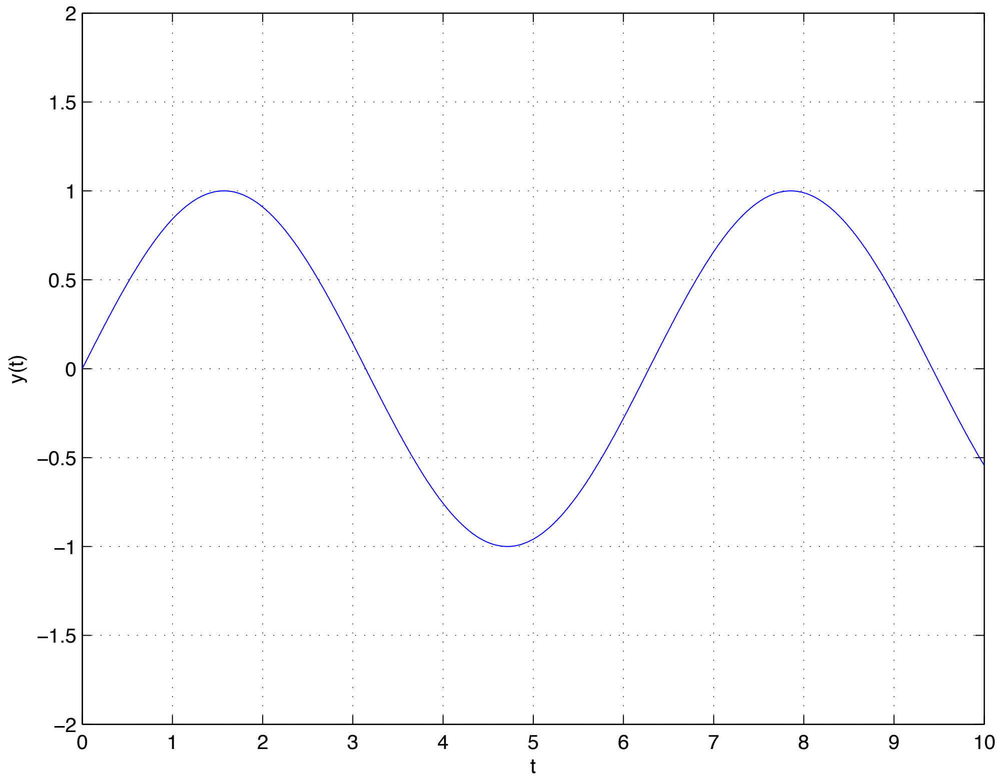


Empty Bode Plot for Question 4



Input plots for Question 6b)





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