CONFIDENTIAL

THE UNIVERSITY OF SYDNEY

TABLE NUMBER ……………………………………………………………………

FAMILY NAME ……………………………………………………………………

GIVEN NAMES ……………………………………………………………………

STUDENT NUMBER ……………………………………………………………………

FACULTY OF ENGINEERING

AMME3500/5501

Systems Dynamics and Control

Semester 1, 2014

Time allowed: 3 hours

There are six (6) questions in this exam. All questions count towards your final mark. These questions are to be completed in the **ANSWER BOOK PROVIDED**. Plotting pages for Questions 3, 4 & 6b are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period. Clearly label which question you are answering.

The total marks for this exam is **100**. This exam is designed to assess your understanding of the material covered by this Unit of Study. Be sure to read through the exam in its entirety before starting to work on the solutions.

This is a closed book exam.
This exam booklet is to be handed back with your answer book.
Non-Programmable calculators are allowed.

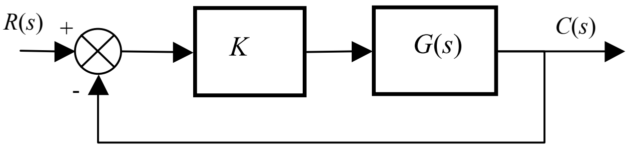
Section A – System Specifications [30 marks]

1. For the LTI system with an open-loop transfer function given by:

$$G(s) = \frac{s+7}{s(s+3)(s+5)}$$

[10 marks]

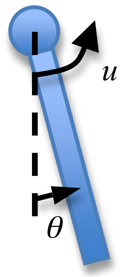
   a. Where are the poles and zeros of this system? Is the system stable in open-loop?
   b. What is the steady-state response from R to C of the closed-loop system shown below, with K=1, to a unit step input. You may assume the closed-loop system is stable. Explain this result in terms of the pole and zero locations



   c. Compute the steady-state error of the above system to a ramp input of unit-slope, with K = 1, 5, and 10. Again, you may assume the closed-loop system is stable.

2. You are part of a team designing a new walking humanoid robot, and your responsibility is to design a control system that will accurately place the robot's foot at a desired location. A greatly simplified model of a single leg control for a single angle is:

$$J\ddot{\theta} + D\dot{\theta} + mgl\sin\theta = u$$



where J=0.8 kgm^2 is the moment of inertia, D= 0.6 Nms/rad is a friction coefficient in the joints and actuators, m = 1.1kg is the mass, l = 0.65m is the distance from the joint to the centre of mass, and g=9.8m/s^2 is the gravitational constant, and u is the applied torque from a motor. [20 marks]

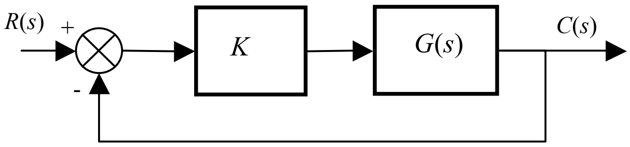
   a. Derive a transfer function for the effect of the control input u on the leg angle θ that is valid for small leg angles (near zero).
   b. Another subsystem of the robot computes the desired leg angle θ_R Design a PD controller:

      $u = K_p(\theta_R - \theta) + K_d(\dot{\theta}_R - \dot{\theta})$

      to achieve a settling time of 0.5 seconds and an overshoot of 5%, using second order assumptions.
   c. Derive the closed-loop transfer function. Do you expect the design requirements to be met perfectly? If not, why not?
   d. Using the transfer function model, what is steady state error for a reference angle of 0.2 rad given the gain computed in part d.? Would you expect the real steady-state error - for the true nonlinear model - to be larger or smaller than this?

Section B – Root Locus and Bode plots [40 marks]

3. A feedback system is shown below:



with the transfer functions for the plant G(s) given by:

$$G(s) = \frac{s-10}{(s^2+8s+20)(s+2)}$$

For this system, sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 7) and be sure to turn in your copy of the exam booklet with your answer book at the end of the exam. Could proportional control make this system unstable? If so, estimate the gain K at which the system is marginally stable. [15 marks]

4. Based on the system G(s) shown below, answer the following questions [25 marks]

$$G(s) = \frac{2000(s+0.05)}{(s^2+0.5s+0.1)(s^2+10s+1200)}$$

   a. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 8). Show any derivation you require in the answer book and include the asymptotes on the sketch itself.
   b. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin.
   c. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable.
   d. Suggest a control design that will increase the bandwidth of the closed-loop system (i.e. increase the maximum frequency of a reference command that can be accurately tracked) without increasing overshoot significantly. Include any formulas or sketches that are necessary to make your case. In your design, take into account the fact that sensor noise may be present at high frequencies, and discuss its influence.

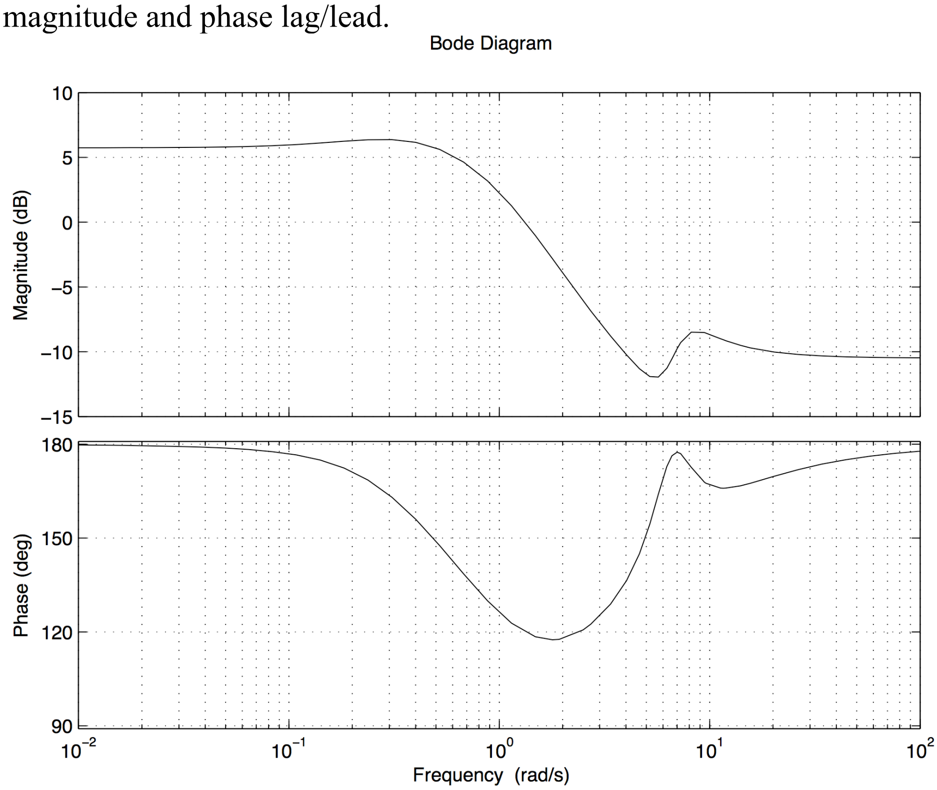
Section C – Controller Design and Analysis [30 marks]

5. Given the second order system, answer the following [15 marks]

$$G(s) = \frac{s+2}{s^2 + 3s + 15}$$

   a. Transform the transfer function into the phase variable state space formulation.
   b. Design a full state feedback controller using pole placement to yield closed-loop poles at -5 and -10.

6. In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions. [15 marks]
   a. For a PID controller, describe the effect that the proportional, integral and differential terms have on the transient response of a system. Use control terminology, sketches of step responses and any equations to help demonstrate your understanding of these controllers.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 9) – note the different time scales. Draw the response over the input signals. Be sure to label your diagram showing features such as the response magnitude and phase lag/lead.



   c. Describe the "separation principle" in output-feedback state-space control. Use any equations and block diagrams that you need to explain your answer.
   d. Describe in your own words the practical difficulties in control design that are a result of Bode's sensitivity integral: $\int_0^\infty \log|S(j\omega)|d\omega = 0$, for stable systems with at least two more poles than zeros.

THERE ARE NO MORE QUESTIONS

Selected Equations

Time Response (First Order Systems)

$$G(s) = C_\infty \frac{1}{s+\sigma} \qquad t_r = \frac{2.2}{\sigma}$$

Time Response (Second Order Systems)

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_r \cong \frac{1.8}{\omega_n} \qquad t_s \cong \frac{4}{\sigma}$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} \qquad M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$$

$$M_p \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \qquad e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

Root Locus

$$|KG(s)H(s)| = 1$$

$$\angle KG(s)H(s) = (2k+1)180^\circ$$

$$\sigma_a = \frac{\sum f\ inite\ poles - \sum f\ inite\ zeroe}{n-m}$$

$$\theta_a = \frac{(2k+1)\pi}{n-m}$$

Frequency Response

$$M = |G(j\omega)| = |G(s)|_{s=j\omega} = \sqrt{\left(\mathrm{Re}[G(j\omega)]\right)^2 + \left(\mathrm{Im}[G(j\omega)]\right)^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\mathrm{Im}[G(j\omega)]}{\mathrm{Re}[G(j\omega)]}\right]$$

$$GM = 20\log K \qquad PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

State Space

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Controllability

$$\mathbf{R} = \begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}$$

Observability

$$\mathbf{O} = \begin{bmatrix}\mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1}\end{bmatrix}$$

Full State Feedback

$$\mathbf{u} = -\mathbf{Kx} + r$$

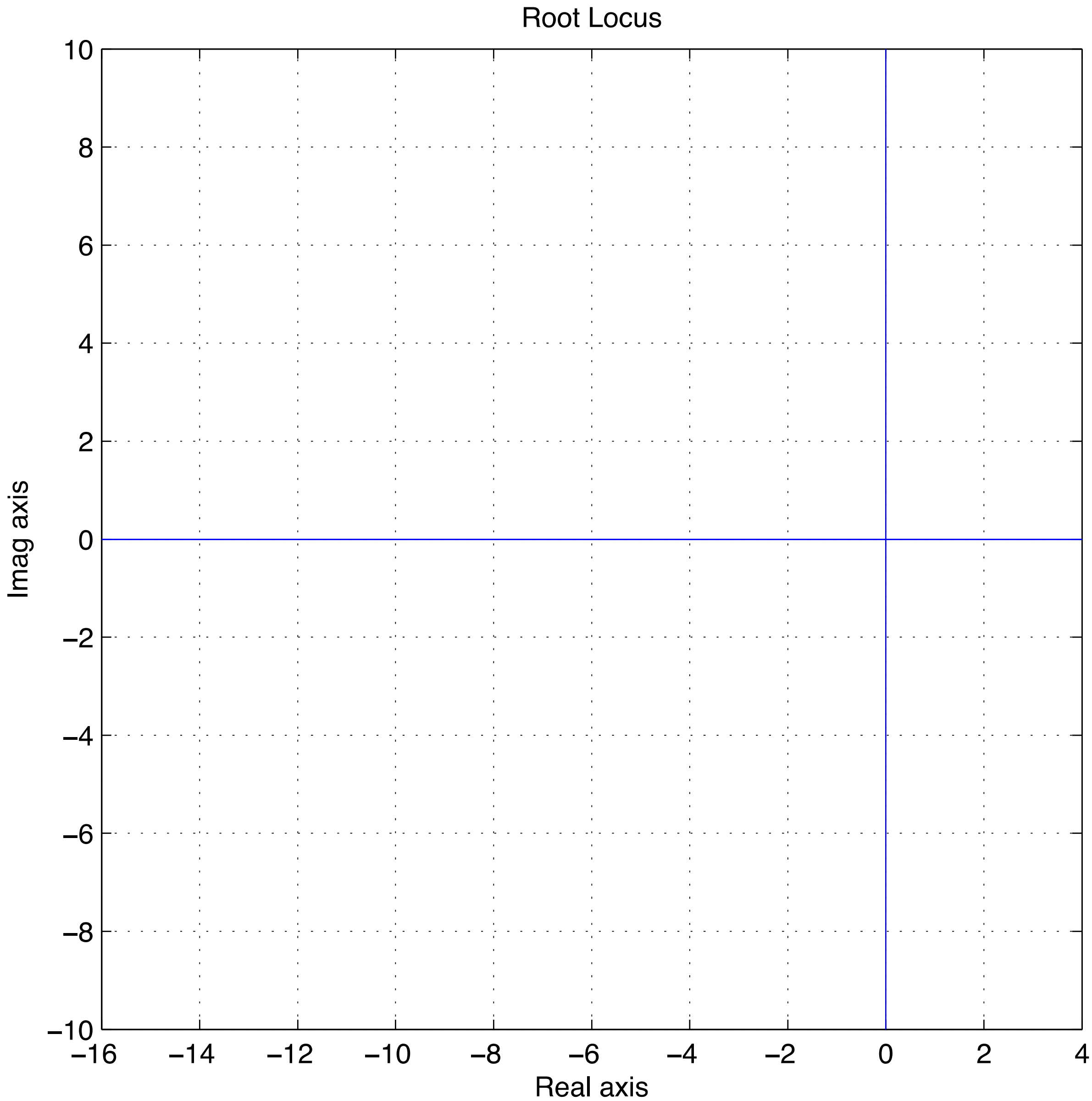
Laplace Transform Tables

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

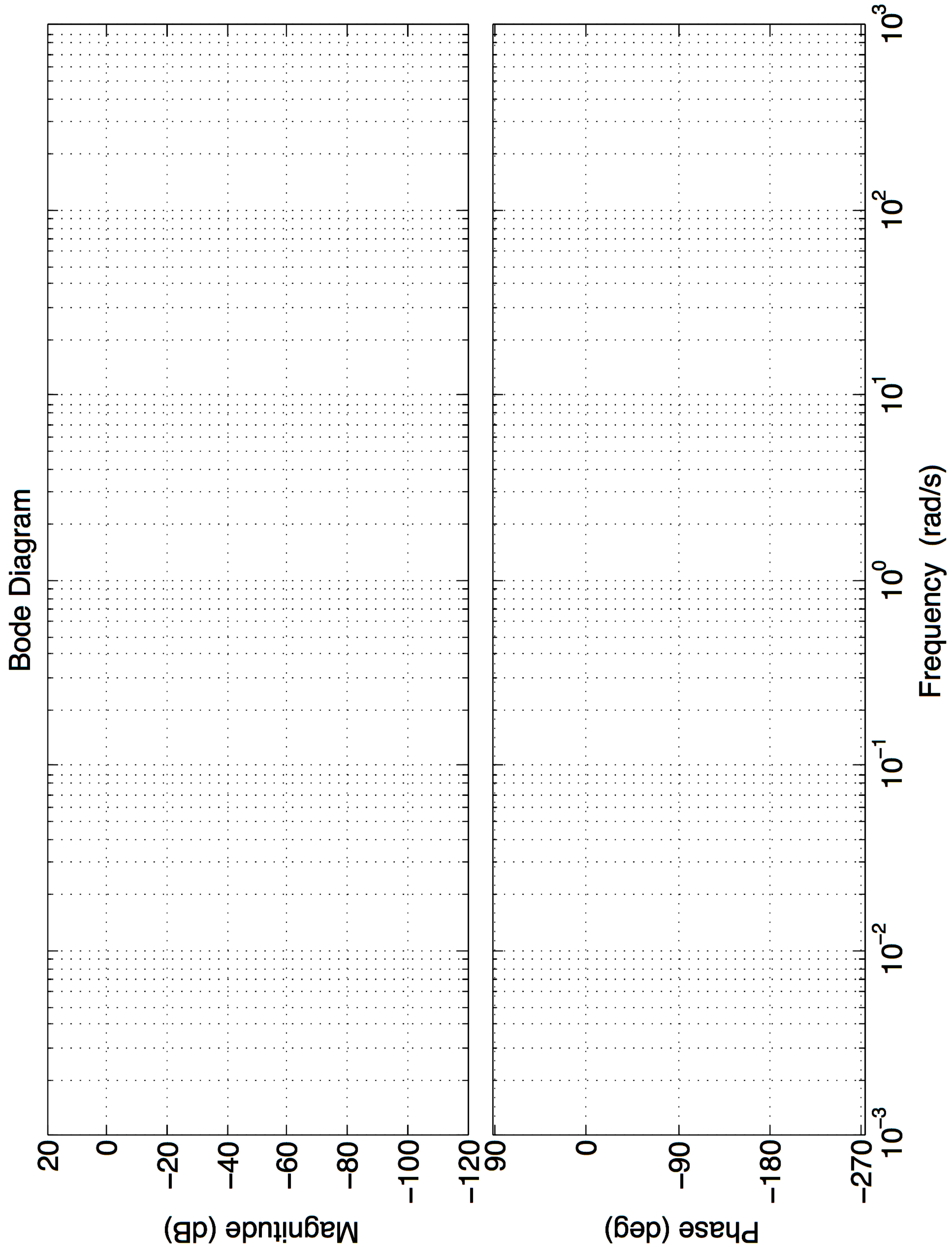
Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)]$	$= F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

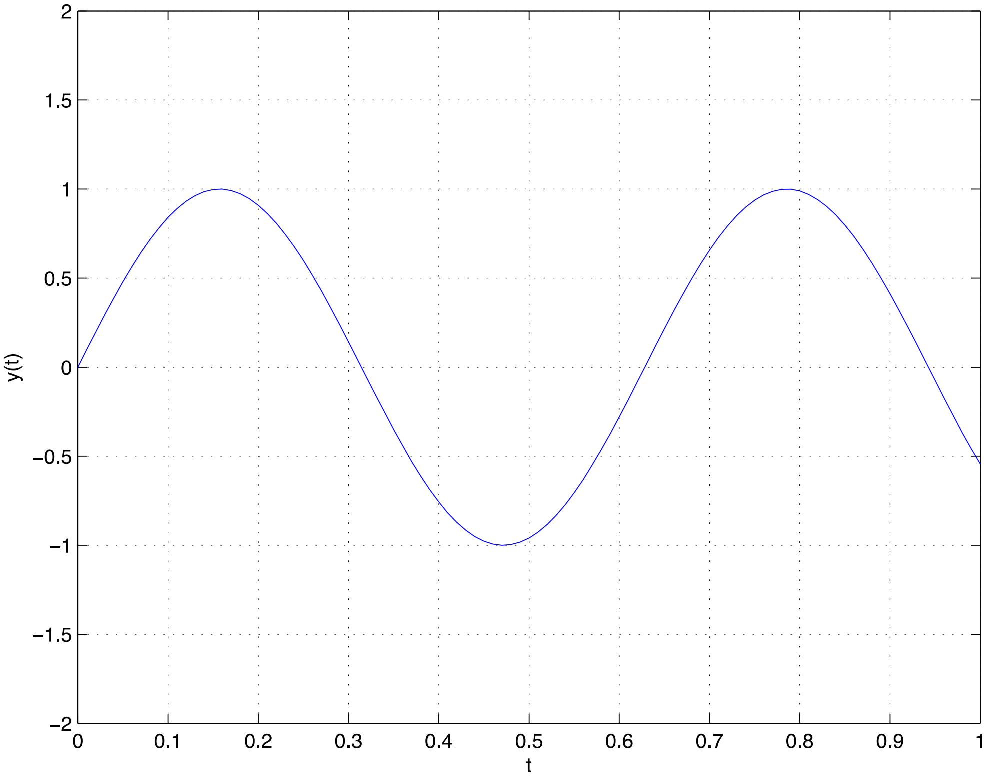
Root Locus for Question 3

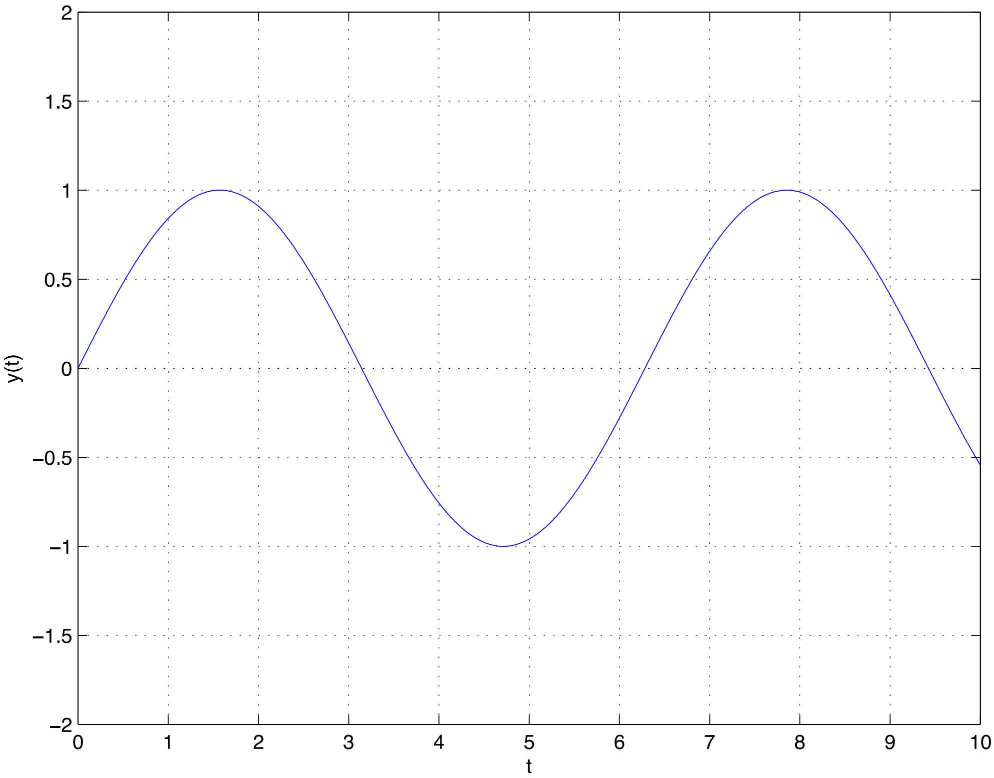


Bode Plot for Question 4



Input plots for Question 6b)





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