CONFIDENTIAL

THE UNIVERSITY OF SYDNEY

TABLE NUMBER ……………………………………………………………………
FAMILY NAME ……………………………………………………………………
GIVEN NAMES ……………………………………………………………………
STUDENT NUMBER ……………………………………………………………………

FACULTY OF ENGINEERING

AMME3500

Systems Dynamics and Control

Semester 1, 2007

Time allowed: 3 hours

There are six (6) questions in this exam. All questions count towards your final mark. These questions are to be completed in the **ANSWER BOOK PROVIDED**. Plotting pages for Questions 1, 3, 4 & 6b are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period. Clearly label which question you are answering.

The total marks for this exam is **100**. This exam is worth **60%** of the final mark for this Unit of Study. Be sure to read through the exam in its entirety before starting to work on the solutions.

This is a closed book exam.
This exam booklet is to be handed back with your answer book.
Non-Programmable calculators are allowed.

Section A – System Specifications [30 marks]

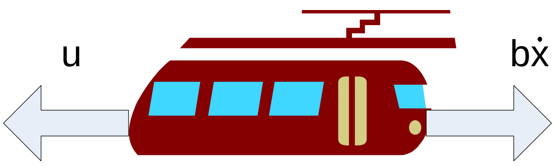
1. An LTI system has a transfer function given by:

$$G(s) = \frac{20(s+5.2)}{(s+5)(s^2+2s+5)(s^2+14s+50)}$$

[10 marks]

   a. What is the steady state value of the output for this system if the input is a unit step?
   b. You are asked to make an estimate of the percentage overshoot in the step response of the system, and you do not have access to simulation software. What is your estimate? Provide a short sentence or two of explanation as part of your answer.

2. A new light rail system has been proposed for inner city areas. The system dynamics can be derived by considering the following diagram where *u* is the force applied by the engines and *b* is the coefficient of friction. If the inertia of the wheels is neglected, and it is assumed that friction (which is proportional to the car's speed) opposes the motion of the car, then the problem is reduced to a relatively simple mass and damper system as shown. Assume that m=2500 kg, b=3.5Nsec/m and u_{max}=1500N. Based on this system, answer the following questions



[20 marks]

   a. Sketch a block diagram showing the relationship between the applied force and speed and position of the vehicle. Find the transfer function between the applied force *u* and the position of the car on the rail.
   b. A naïve control engineer decides to apply a simple proportional control law to move the train from one station to the next such that the applied force is computed so that x tracks a reference command x_r according to the feedback law

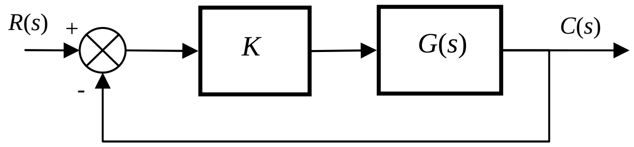
$$u = K_P(x_r - x)$$

Sketch a block diagram of the resulting system. Find the transfer function between x_r and x .

   c. What is the maximum value of K_P that can be used if we wish to have an overshoot M_p<5% for a step change in desired position x_r?
   d. For the controller gain found in c), what is the instantaneous demand force required of the engines if the next station is 500m away? Draw a block diagram showing the effect of the limited power available to the plant and briefly describe how this will affect the performance of the system.

Section B – Root Locus and Bode plots [40 marks]

3. For a given system shown here



the transfer function between is given by

$$G(s) = \frac{s^2 + 8s + 20}{(s-1)(s+2)(s+4)(s+6)}$$

For this system, sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 8) and be sure to turn in your copy of the exam booklet with your answer book at the end of the exam. Also answer the following questions (give a brief explanation) [15 marks]

   a. Is the system stable for small values of K?
   b. Is the system stable for very large values of K?

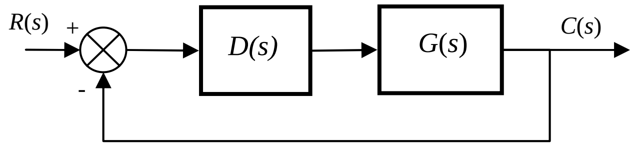
4. Based on the system G(s) shown below, answer the following questions [25 marks]

$$G(s) = \frac{100(s+20)}{s(s+50)(s^2 + 15s + 50)}$$

   a. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 9). Show any derivation you require in the answer book and include the asymptotes on the sketch itself.
   b. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin and -3dB bandwidth.
   c. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable.

Section C – Controller Design [30 marks]

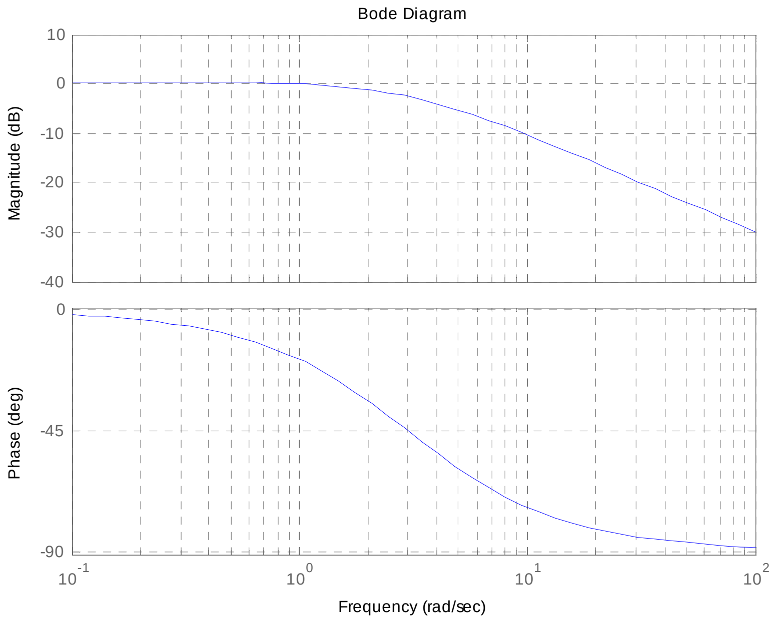
5. Given the following second order system, design a lead compensator D(s) to yield a 5% overshoot with a twofold reduction in settling time. Due to noise considerations, the lead pole should be place at s=-15 [15 marks]



where

$$G(s) = \frac{1}{(s+1)(s+5)}$$

6. In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions. Each response should take no more than half a page in your answer book. [15 marks]
   a. For a PID controller, describe the effect that the proportional, integral and differential terms have on the transient response of a system. Use control terminology, sketches of step responses and any equations to help demonstrate your understanding of these controllers.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 10). Draw the response overlain on the input signals. Be sure to label your diagram showing important features such as the response magnitude and phase lag/lead.



   c. Name two advantages of frequency response techniques over the root locus (for control system analysis and design).
   d. As we have seen, classical techniques are suitable for modelling and control design for Linear Time Invariant (LTI) systems. Briefly describe two system characteristics that are not easy to model using the LTI methods concentrated on in this course.

THERE ARE NO MORE QUESTIONS

Selected Equations

Time Response

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_r \cong \frac{1.8}{\omega_n}$$

$$t_s \cong \frac{4}{\sigma}$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}}$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$$

$$M_p \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \qquad e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

Root Locus

$$|KG(s)H(s)| = 1$$

$$\angle KG(s)H(s) = (2k+1)180^\circ$$

$$\sigma_a = \frac{\sum finite\ poles - \sum finite\ zeroes}{n-m}$$

$$\theta_a = \frac{(2k+1)\pi}{n-m}$$

Frequency Response

$$M = |G(j\omega)| = |G(s)|_{s=j\omega} = \sqrt{\left(\mathrm{Re}[G(j\omega)]\right)^2 + \left(\mathrm{Im}[G(j\omega)]\right)^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\mathrm{Im}[G(j\omega)]}{\mathrm{Re}[G(j\omega)]}\right]$$

$$GM = 20\log K$$

$$PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

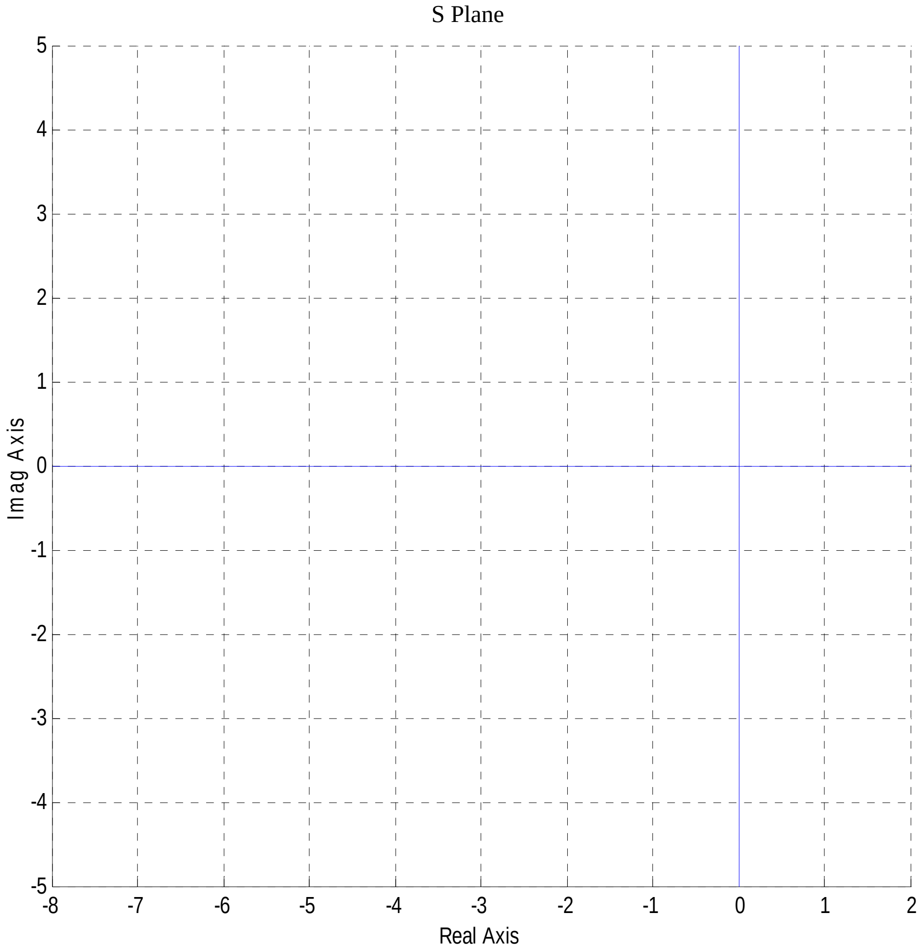
Laplace Transform Tables

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

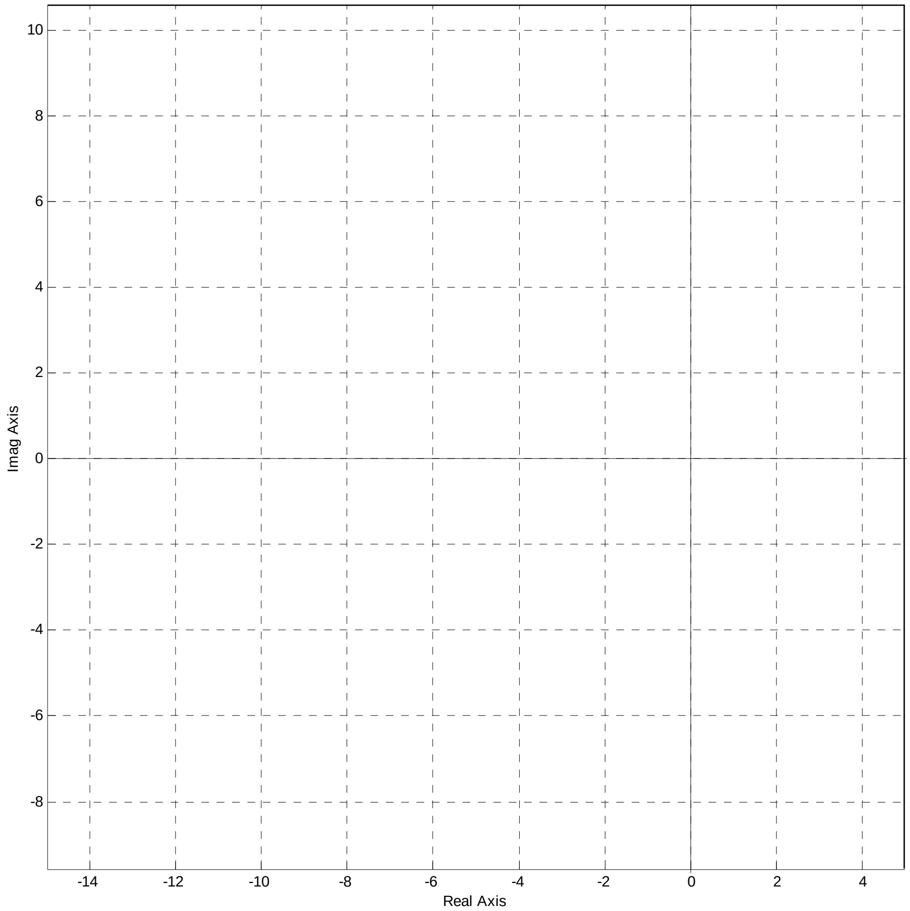
Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

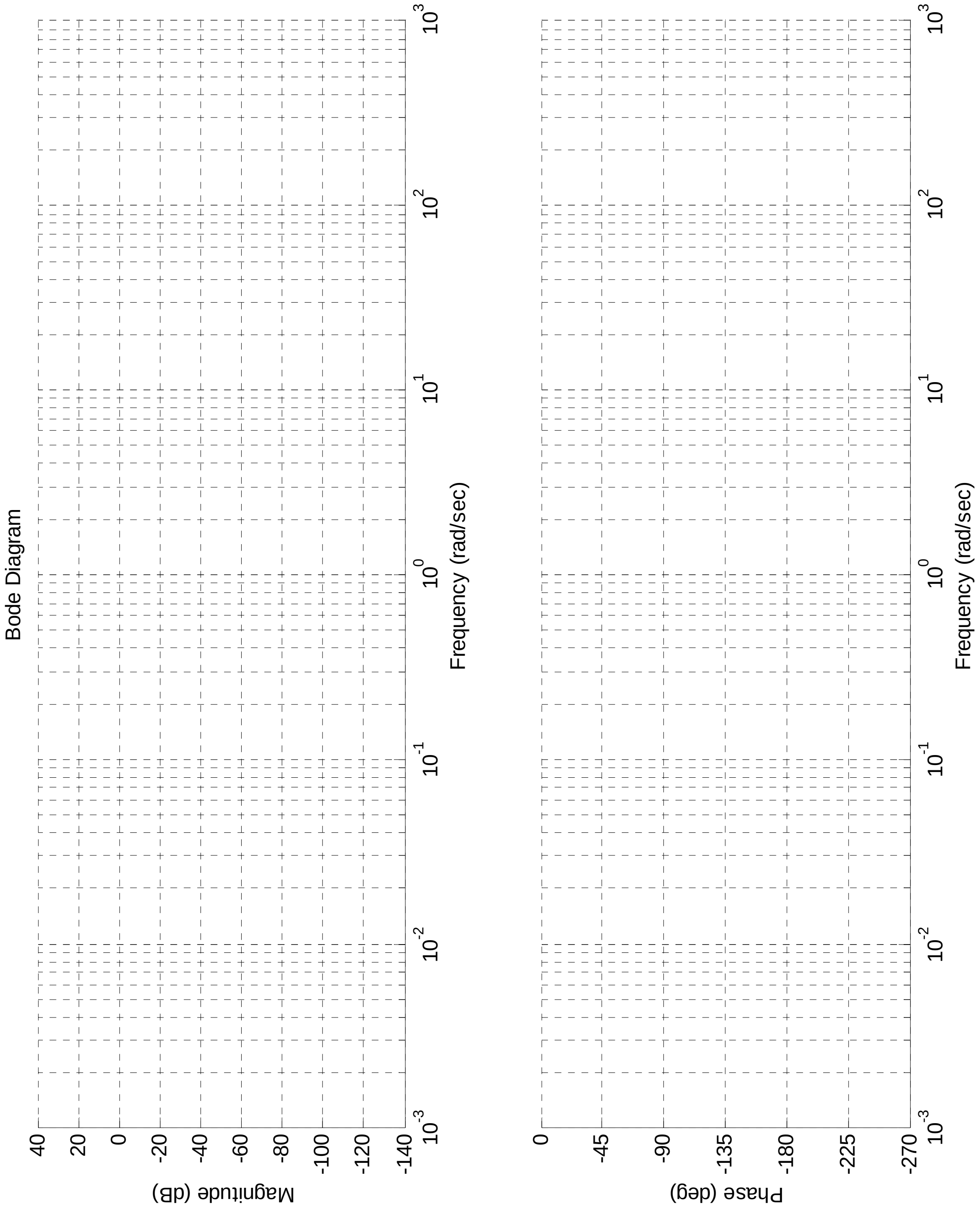
S-plane for Question 1



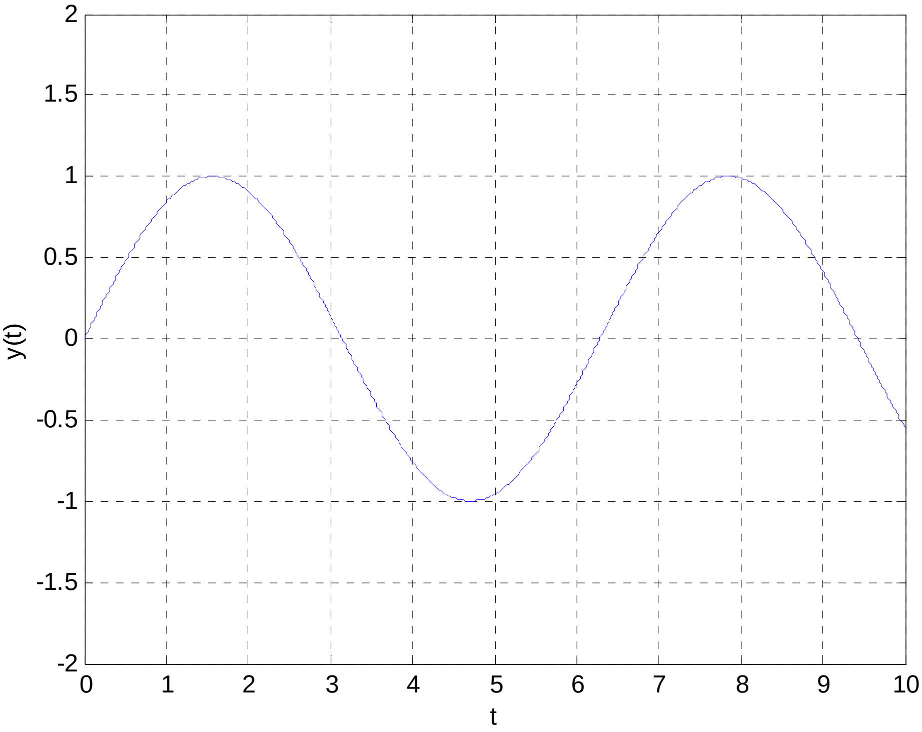
Root Locus for Question3

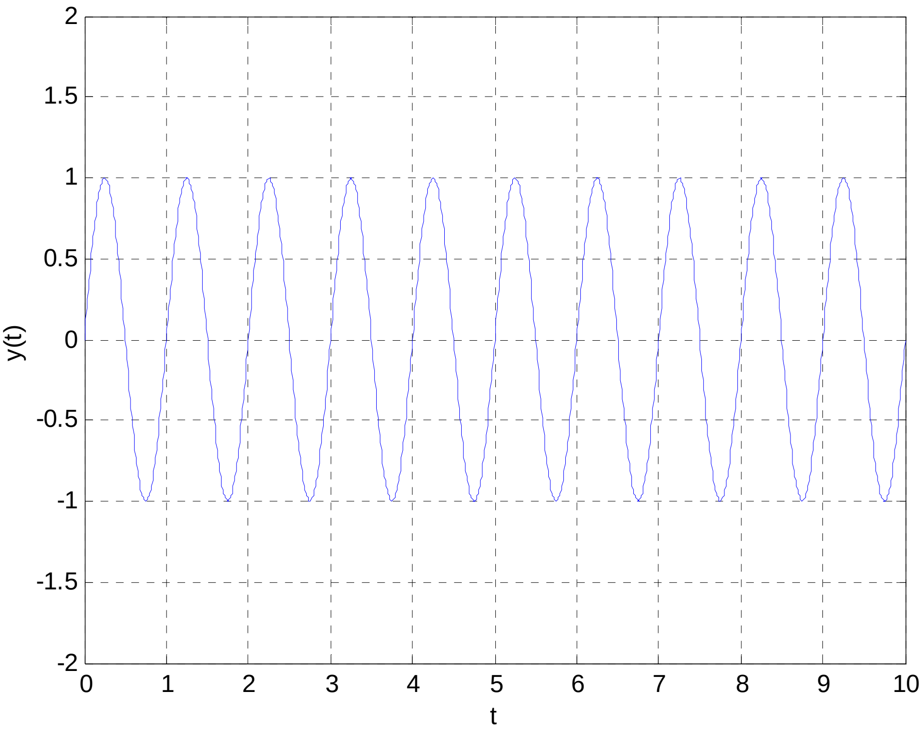


Bode Plot for Question 4



Input plots for Question 6b)





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