CONFIDENTIAL

THE UNIVERSITY OF SYDNEY

TABLE NUMBER ………………………………………………………………………

FAMILY NAME ………………………………………………………………………

GIVEN NAMES ………………………………………………………………………

STUDENT NUMBER ………………………………………………………………………

FACULTY OF ENGINEERING

AMME3500/5501

Systems Dynamics and Control

Semester 1, 2013

Time allowed: 3 hours

There are six (6) questions in this exam. All questions count towards your final mark. These questions are to be completed in the **ANSWER BOOK PROVIDED**. Plotting pages for Questions 3, 4 & 6b are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period. Clearly label which question you are answering.

The total marks for this exam is **100**. This exam is designed to assess your understanding of the material covered by this Unit of Study. Be sure to read through the exam in its entirety before starting to work on the solutions.

This is a closed book exam.
This exam booklet is to be handed back with your answer book.
Non-Programmable calculators are allowed.

Section A – System Specifications [30 marks]

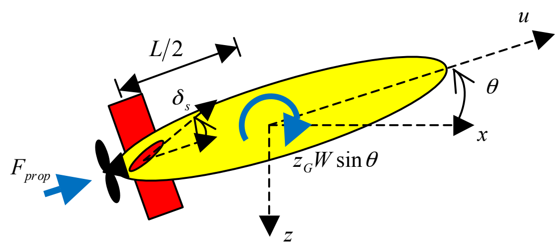
1. For the LTI system with (*closed loop*) transfer function given by:

$$T(s) = \frac{15(s+1.9)}{(s+2)(s^2+20s+125)(s^2+2s+50)}$$

[10 marks]

   a. Where are the poles and zeros of this system? Is the system stable?
   b. What is the steady-state response of the system to a step input of magnitude 1?
   c. Estimate the percentage overshoot for the step response of this system. Provide a short sentence or two justifying the assumptions you have made in your calculation.

2. There are many applications in which we wish to control the pitch of a vehicle. A common approach to depth controller design for Autonomous Underwater Vehicles (AUVs) is to assume the forward speed dynamics can be decoupled from the pitch/depth dynamics. Assume that a vehicle has the following simplified non-linear pitch dynamics:



$$I_y\ddot{\theta} + \frac{L}{4}\rho A_s C_L |u|\dot{\theta} = \frac{L}{4}\rho A_s C_L u|u|\delta_s - z_G W \sin\theta$$

with moment of inertia I_y=60 kg m^2, length L=1 m, water density ρ=1000 kg/m^3, frontal area A_s=0.06 m^2, drag coefficient C_L=0.8, hydrostatic moment arm z_G=0.006 m and weight W=400 N. If the vehicle is driven with speed u of 1.5m/s and is initially at zero pitch with zero pitch velocity and pitch is controlled using the tail fin deflection δ_s answer the following questions. [20 marks]

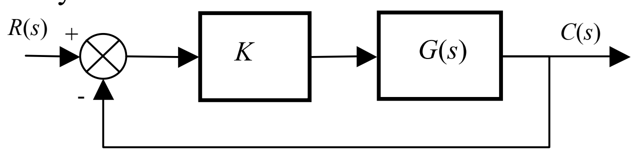
   a. Sketch a block diagram showing the relationship between the tail fin deflection and the pitch of the vehicle. Find a transfer function between the fin deflection δ_s and the pitch θ that is valid for small pitch angles.
   b. An engineer decides to apply a simple proportional control law to regulate the pitch of the vehicle. The fin deflection is computed so that θ tracks a reference pitch θ_r according to the feedback law

$$\delta_s = K(\theta_r - \theta)$$

      Sketch a block diagram of the resulting system. Find an approximate linear transfer function between θ_r and θ .
   c. What is the value of K that can be used if we wish to have a rise time of 5s for a commanded change in angular position?
   d. What is the maximum gain K that can be used such that we have an overshoot M_p<16% for a step change in desired pitch θ_r?
   e. What is the steady state error for a reference pitch of 0.3 rad given the gain computed in part d.?

Section B – Root Locus and Bode plots [40 marks]

3. For the system shown here



the transfer functions for the plant G(s) is given by

$$G(s) = \frac{2}{(s^2 + 6s + 10)(s + 5)}$$

For this system, sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 7) and be sure to turn in your copy of the exam booklet with your answer book at the end of the exam. Based on your root locus sketch, estimate the gain K for which the system will have an overshoot of 5%. [15 marks]

4. Based on the system G(s) shown below, answer the following questions [25 marks]

$$G(s) = \frac{500(s + 10)}{(s + 1)(s + 50)(s^2 + 26s + 25)}$$

a. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 8). Show any derivation you require in the answer book and include the asymptotes on the sketch itself.
b. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin.
c. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable.
d. Suggest a control design that will eliminate steady-state error but leave the phase margin, and hence transient behaviour, largely unchanged. Show a block diagram of the closed-loop system and give the location of any poles or zeros introduced in the controller, along with a few sentences justifying your choices.

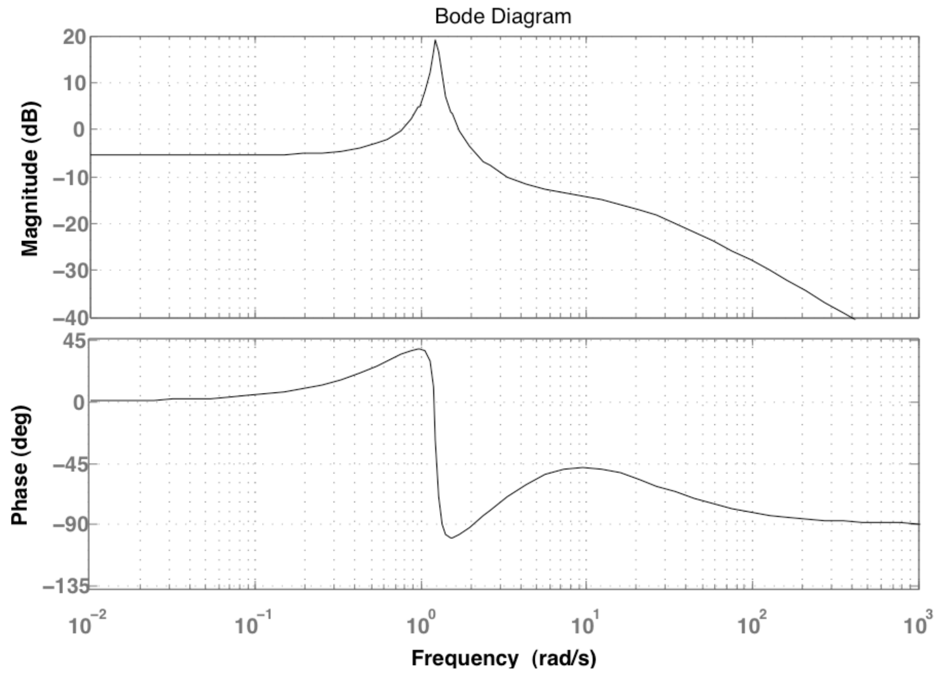
Section C – Controller Design and Analysis [30 marks]

5. Given the second order system, answer the following [15 marks]

$$G(s) = \frac{1}{(s+1)(s+5)}$$

   a. Transform the transfer function into the phase variable state space formulation.
   b. Design a full state feedback controller using pole placement to yield a 5% overshoot with a settling five times faster than $G(s)$.

6. In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions. [15 marks]
   a. For a PID controller, describe the effect that the proportional, integral and differential terms have on the transient response of a system. Use control terminology, sketches of step responses and any equations to help demonstrate your understanding of these controllers.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 9) – note the different time scales. Draw the response over the input signals. Be sure to label your diagram showing features such as the response magnitude and phase lag/lead.



   c. Describe in your own words the practical limitations imposed by the fact that the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$ satisfy the relation $S(s)+T(s)=1$.
   d. We have considered a number of non-linearities in systems that can affect system performance. Briefly describe the impact of (a) saturation (for example in an amplifier) and (b) sinusoidal terms in equations of motion. How are these systems approximated using LTI analysis techniques?

THERE ARE NO MORE QUESTIONS

Selected Equations

Time Response (First Order Systems)

$$G(s) = C_\infty \frac{1}{s+\sigma} \qquad t_r = \frac{2.2}{\sigma}$$

Time Response (Second Order Systems)

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_r \cong \frac{1.8}{\omega_n} \qquad t_s \cong \frac{4}{\sigma}$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} \qquad M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$$

$$M_p \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1+\lim_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \qquad e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

Root Locus

$$|KG(s)H(s)| = 1$$

$$\angle KG(s)H(s) = (2k+1)180^\circ$$

$$\sigma_a = \frac{\sum \textit{finite poles} - \sum \textit{finite zeroes}}{n-m}$$

$$\theta_a = \frac{(2k+1)\pi}{n-m}$$

Frequency Response

$$M = |G(j\omega)| = |G(s)|_{s=j\omega} = \sqrt{\left(\mathrm{Re}\left[G(j\omega)\right]\right)^2 + \left(\mathrm{Im}\left[G(j\omega)\right]\right)^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\mathrm{Im}\left[G(j\omega)\right]}{\mathrm{Re}\left[G(j\omega)\right]}\right]$$

$$GM = 20\log K \qquad PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

State Space

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Controllability

$$\mathbf{R} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$$

Observability

$$\mathbf{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix}$$

Full State Feedback

$$\mathbf{u} = -\mathbf{Kx} + r$$

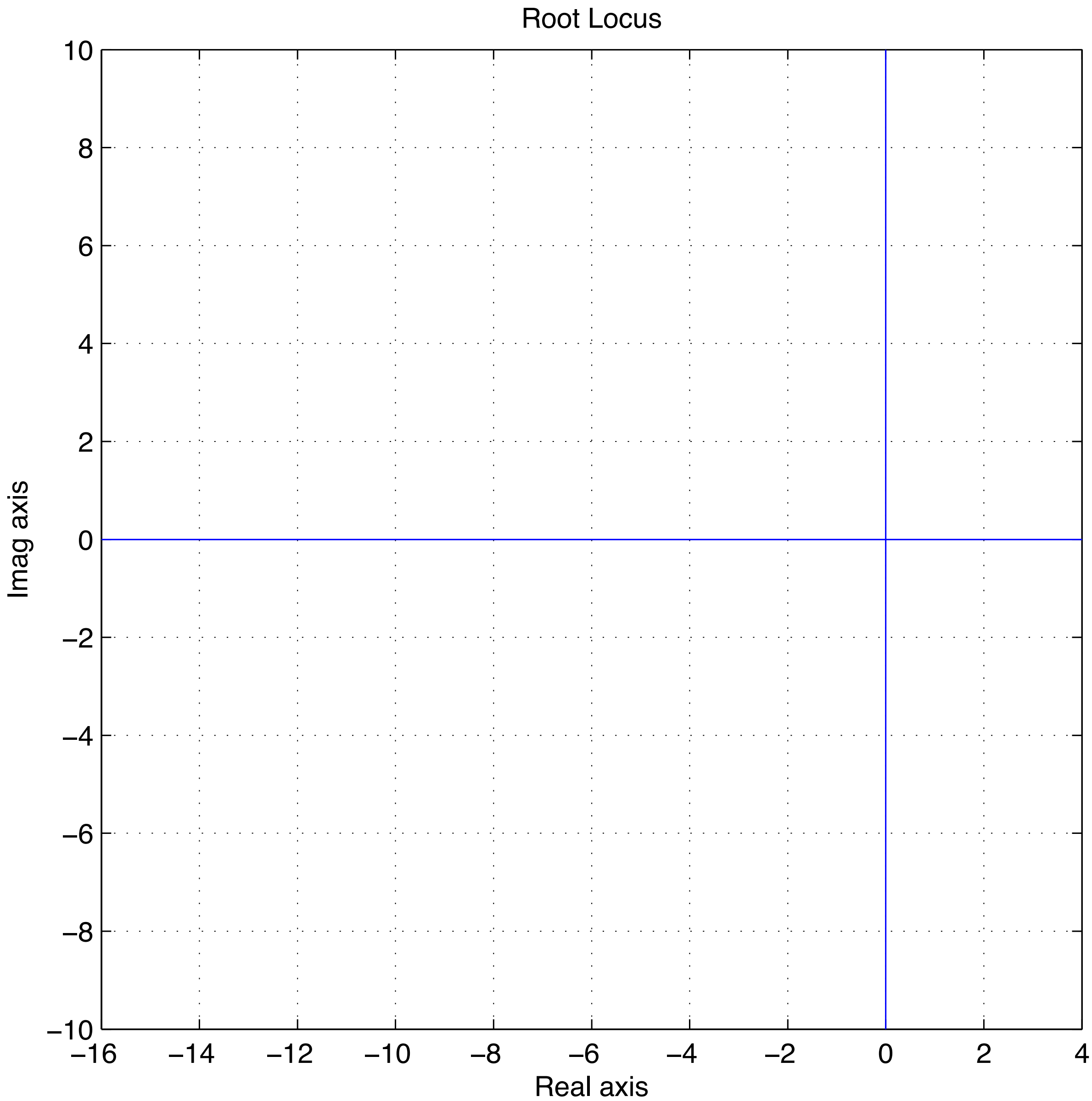
Laplace Transform Tables

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

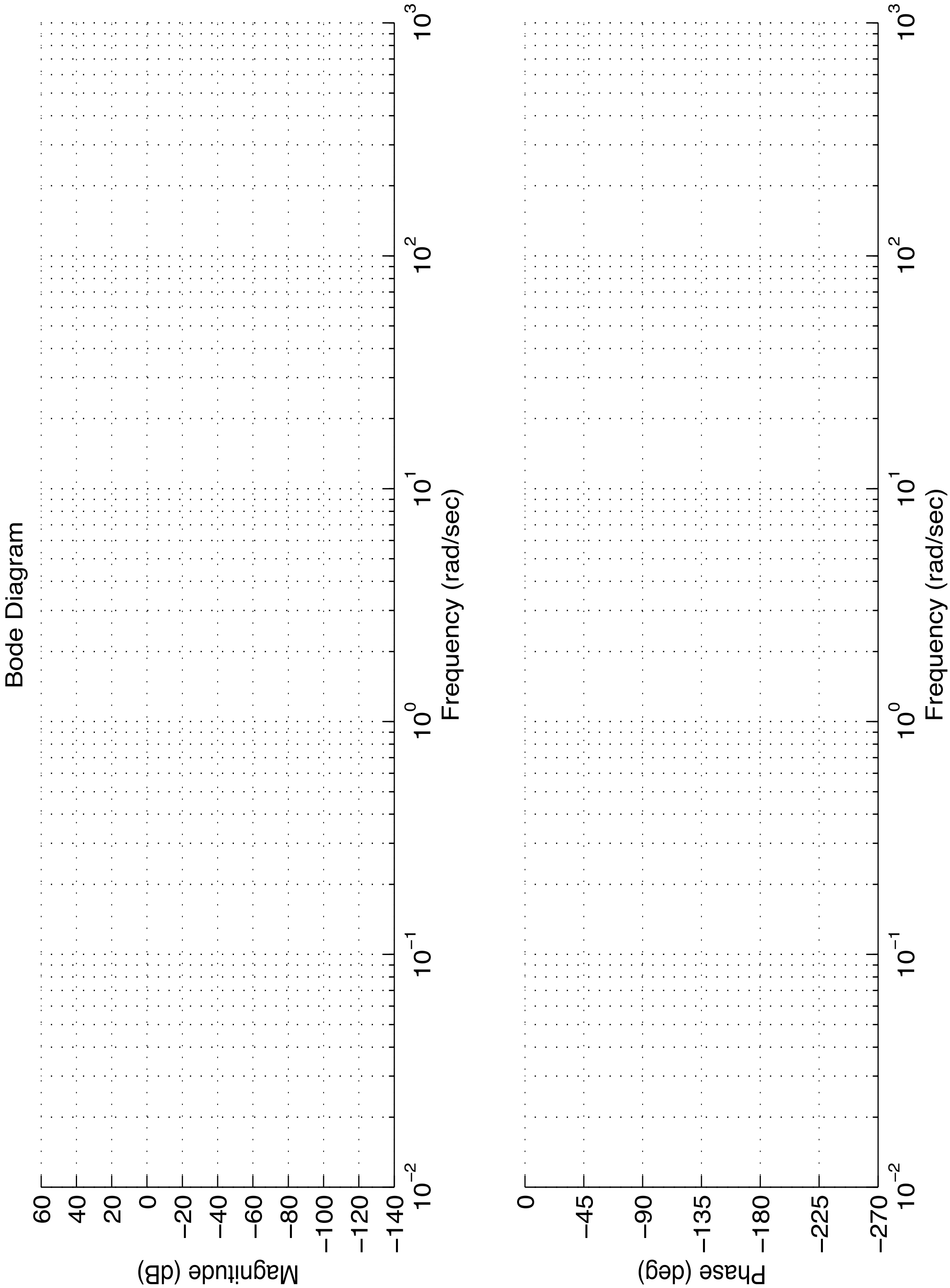
Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)]$	$= F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

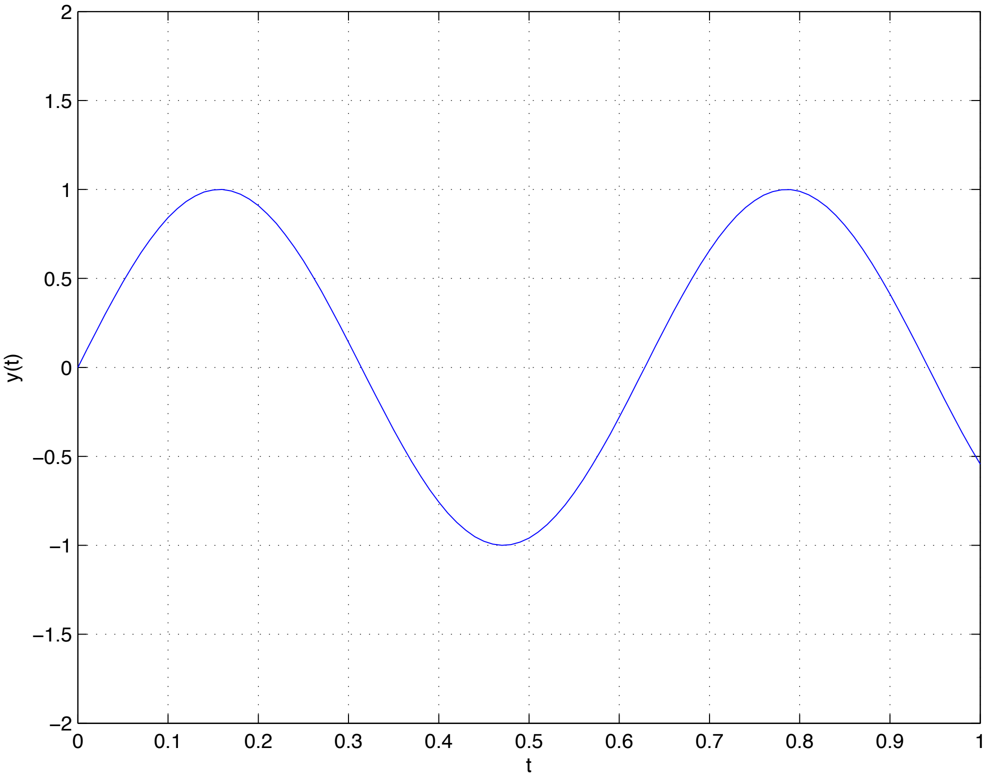
Root Locus for Question 3

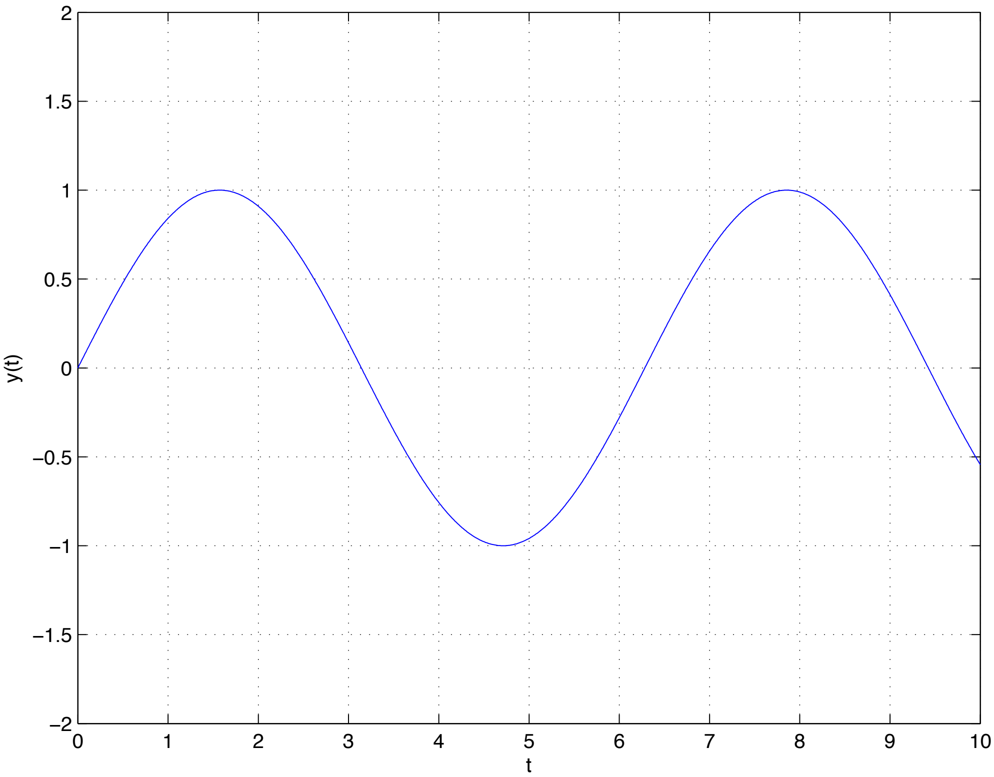


Bode Plot for Question 4



Input plots for Question 6b)





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