## Final Exam AMME3500/8501/9501

(!) This is a preview of the published version of the quiz.

Started: May 31 at 16:47

## **Quiz Instructions**

This exam consists of three sections:

- Section 1 multiple choice questions,
- Section 2 written answer question, and
- · Section 3 analysis problems.

You will have 120 minutes to complete the exam.

- This is an open book exam: you may refer to any of your course notes or textbooks during the exam.
- You may use a programmable calculator or MATLAB/Octave (if desired) for calculations.
- You must complete this exam yourself without any assistance provided by any other person, including classmates.
- You will need to produce hand-written working for Section 3 analysis problems: Questions 16 to 19. You should properly write down your working with a pen on paper during the exam. You will need to scan or photograph this working and submit it via the Canvas assignment link provided with 30 minutes of completing the exam. Failure to provide handwritten working for these questions will result in an automatic zero mark for these questions, regardless of answers entered in Canvas.

### **Section 1: Mutiple Choice Questions**

Questions 1 to 10 are multiple choice questions. Click to select the most appropriate answer.

You do NOT need to provide any additional working for these questions at the end of the exam.

Question 1 1 pts

Consider the following first-order dynamical system

$$\dot{x} + ax = u$$

where x is the system state, and u is control input. Which of the following statements is true?

- $\bigcirc$  The system is stable if u = 0.
- $\bigcirc$  The system is stable regardless of the value of a.
- $\bigcirc$  The system is not stable if  $u = e^t$ .
- $\bigcirc$  The system is stable if a is positive.
- $\bigcirc$  The system is stable if a is negative.

Question 2 1 pts

Consider the first-order dynamical system

$$\dot{x} + ax = 0.$$

If x(0) = 1 and  $x(2) = e^{-4}$ , what is the value of a?

- $\bigcirc$  1
- 3
- $\bigcirc$  2
- **4**
- 0 8

**Question 3** 

1 pts

For the first-order dynamical system

$$\dot{x} + 3x = u$$

we know that x(t) approaches 2 when t tends to infinity. Then what is the value of u?

- $\bigcirc$  1.5
- $\bigcirc$  Not decidable since the value of u depends on x(0).
- $\bigcirc$  1
- O 6
- $\bigcirc$  4

Question 4 1 pts

Consider the following second-order system

$$\ddot{x} + 5\dot{x} + 6x = u.$$

The poles of the system are

- $\bigcirc$  -2 and 3
- $\bigcirc$  -2 and -3
- $\bigcirc$  2 and 3
- $\bigcirc$  2 and -3
- $\bigcirc$  1 and 5

Question 5 1 pts

For the following second-order system

$$\ddot{x} - 2\dot{x} - 3x = u$$

which statement of the following is true?

- $\bigcirc$  It is NOT possible to place the closed-loop poles at -1 and -10 by a PD controller  $u=-k_px-k_d\dot{x}$ .
- The system is open-loop stable.
- $\bigcirc$  It is possible to obtain a stable closed-loop system by a PD control  $u=-k_px-k_d\dot{x}$ .
- $\bigcirc$  It is possible to obtain a stable closed-loop system by a proportional feedback control  $u=-k_px$ .
- O It is NOT possible to for the closed-loop system to have an overshoot at 2% by a PD controller  $u = -k_p x k_d \dot{x}$ .

Question 6 2 pts

The dynamics of a plane pendulum with mass  $1kg\ \text{subject}$  to some external force is described by

$$\ddot{x} + \frac{g}{\ell} \sin x = u(t) \,,$$

where x is the angular displacement,  $\ell$  is the length of its link, and g is the standard gravity. Which statement of the following is true? Let  $\ell=1\mathrm{m}$ ,  $g=10\mathrm{m/s^2}$  and  $u=5\mathrm{N}$ . Then the corresponding equilibrium for x is

- $\bigcirc \frac{\pi}{2}$
- $\bigcirc \frac{\pi}{6}$
- $\bigcirc \frac{\pi}{3}$
- $\bigcirc 0$
- $\bigcirc \frac{\pi}{4}$

Question 7 2 pts

Let us reconsider the second-order system

$$\ddot{x} + 5\dot{x} + 6x = u.$$

Suppose the system output is  $y(t) = \dot{x}$ . For the state-space representation of the system, which statements of the following are true?

The resulting state-space model is observable.

The resulting state-space model is reachable.

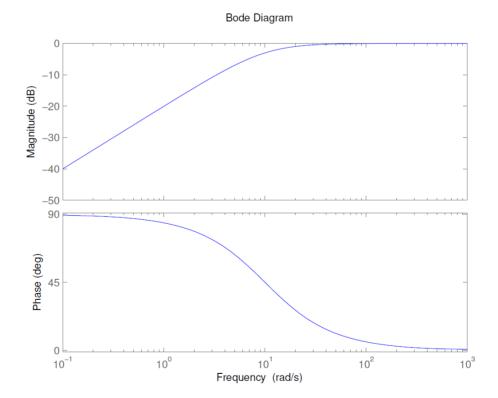
The resulting state-space model is not observable.

The resulting state-space model is not reachable.

The resulting state-space model is not reachable.

Question 8 2 pts

The Bode diagram of a transfer function H(s) is shown below.



The transfer function H(s) is

- $\bigcirc \frac{s}{s+10}$
- $\bigcirc \frac{-s}{s+10}$
- $\bigcirc \frac{1}{s-10}$

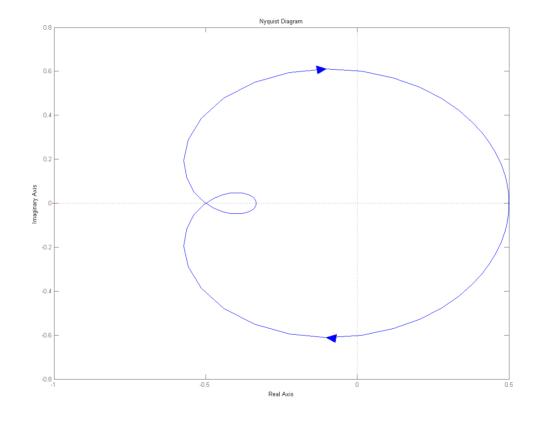
- $\bigcirc \frac{1}{s+10}$
- $\bigcirc \frac{s}{s-10}$

Question 9 2 pts

The Nyquist plot for the plant

$$G(s) = \frac{s^2 - 5s - 4}{2(s^2 + 5s + 6)}$$

(with gain K = 1) is given below.



The plant is placed in a unity feedback loop. Which statements of the following are true?

- ☐ The closed-loop system is unstable.
- ☐ The gain margin of the system is around 2.
- ☐ The closed-loop system is stable.
- ☐ The gain margin of the system is around 20.

Question 10 2 pts

Let a transfer function be  $G(s) = \frac{1}{s+1}$ . Let an input  $u(t) = \cos(t)$  enter the transfer function. The steady-state system output is close to

- $\bigcirc \cos(t-1)$
- $\bigcirc$  0.7 cos(t + 1)
- $\bigcirc 0.7\cos(t-\frac{\pi}{4})$
- $\bigcirc 0.7\cos(t-\frac{3\pi}{4})$
- $\bigcirc \cos(t+1)$

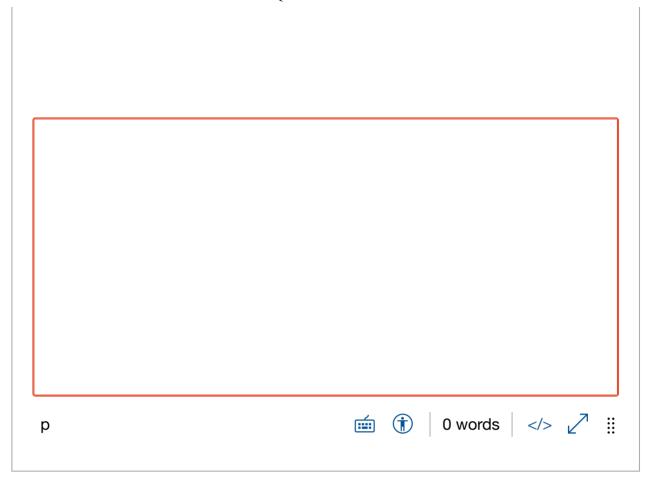
#### **Section 2: Written Answer Questions**

Questions 11 to 15 are written answer questions. Provide your answers using the textboxes provided. You may use text-based short hand notations to describe any equations or other symbols or formulas used as part of your explanations.

You do NOT need to provide any additional working for these questions at the end of the exam.

Question 11 10 pts

Describe the benefits and limitations of PID control as opposed to state-feedback control for feedback systems. Use your experiences in designing controllers for the cruise control in Design Project 1 and for the crane control in Simulink Lab 4, to help explain the respective advantages of PID and state-feedback control, and their suitable systems.

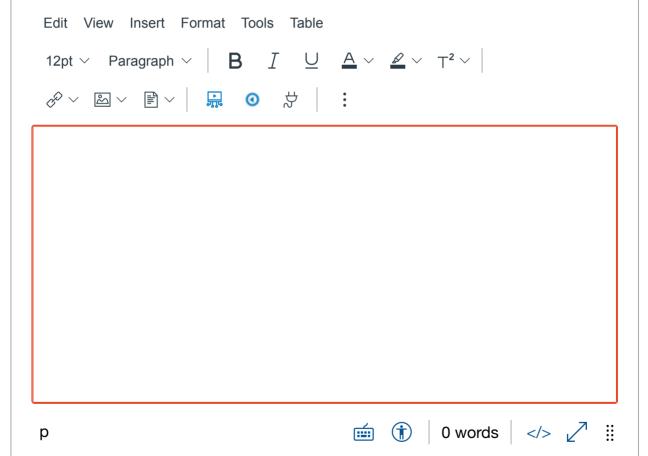


#### Question 12 10 pts

Describe the purpose and implication of observability and observer for a dynamical system, respectively, and in particular, what is their relationship? You may use examples to help explain your points.

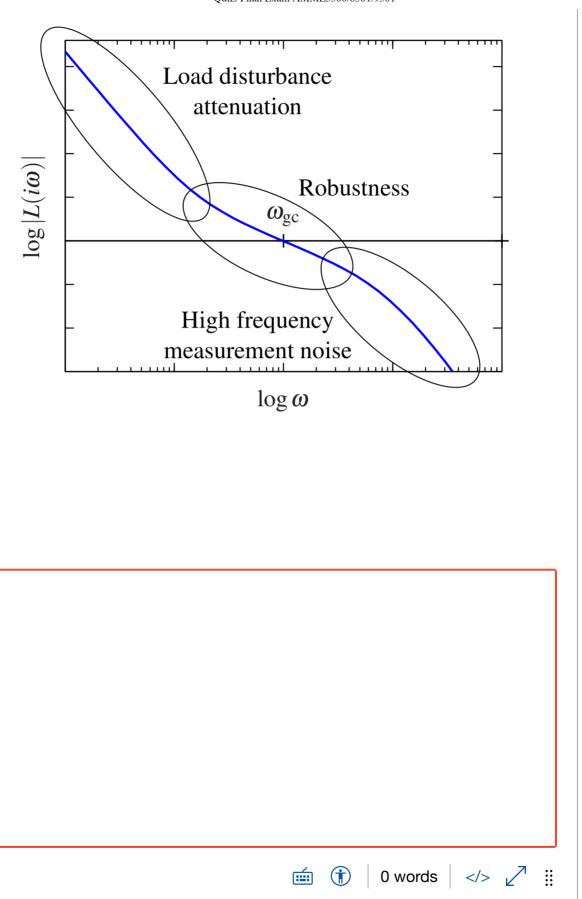
# Question 13 5 pts

Explain the significance of sensitivity functions in terms of plant uncertainty, and how the sensitivity function and complementary sensitivity function impose limitations to the performance of feedback controllers in the presence of disturbances and noises.



### Question 14 10 pts

Describe the motivation and the procedure of loop shaping for frequency-domain design of feedback controllers: WHY and HOW do we carry out loop shaping?



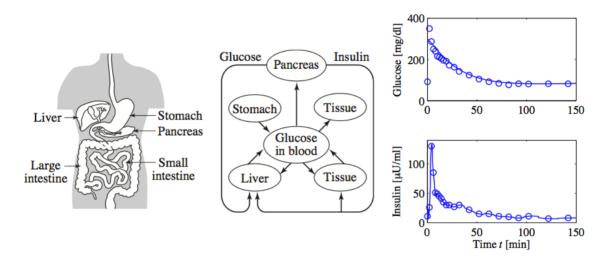


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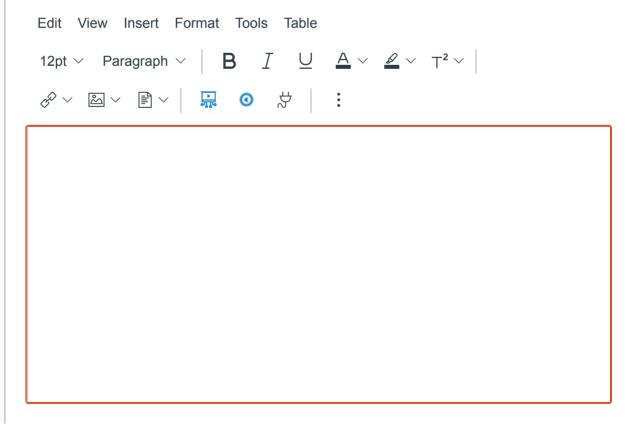
Insulin/Glucose dynamics is of fundamental importance for us to understand and develop treatment for diabetes. Let y(t) be the blood glucose concentration ( mg/dL), and u(t) be the intraperitoneal insulin (U/h). Then from clinical data, the continuous transfer function from u to y is

$$G(s) = \frac{-12000(TDI)^{-1}}{(247s+1)(17s+1)^2}$$

where TDI is a positive constant representing the total daily insulin dose of the patient.



Assume y(t) starts from y(0) = 0. Describe the main features that we can expect from the response of y(t) when u(t) is a unit step input, and explain how these features are determined from the transfer function.



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#### **Section 3: Analysis Problems**

Questions 16 to 19 are analysis problems involving numerical calculation. Enter your final answer in the input boxes provided and make sure to follow instructions on units and decimal places required.

You will need to produce hand written working, including all relevant stepby-step calculations, while completing these questions.

At the completion of the quiz, you will have 30 minutes to scan/photograph and submit your working for these questions. Failure to provide handwritten working for these questions will result in an automatic zero mark for these questions, regardless of answers entered in Canvas.

**Question 16** 10 pts

Consider the second-order system  $\dot{x} = Ax + Bu$  with

$$A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Design a state feedback controller  $u = -Kx = -[k_1 \ k_2]x$  so that the closedloop system has poles placed at -1+j and -1-j. Provide your answer for  $k_1$ and  $k_2$  as integers.

$$k_1 =$$
  $k_2 =$ 

Question 17 10 pts

Consider again the second-order system  $\dot{x} = Ax + Bu$  with

$$A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Now suppose the system has an output  $y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$ . Design an observer

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x})$$

with

$$L = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix},$$

so that the error dynamics  $\dot{\tilde{x}}=(A-LC)\tilde{x}$  have poles placed at -1+j and -1-j. Provide your answer for  $l_1$  and  $l_2$  as integers.

$$l_1 =$$
  $l_2 =$ 

Question 18 10 pts

The Nyquist diagram for the transfer function

$$G(s) = \frac{s^2 - 5s - 4}{2(s^2 + 5s + 6)}$$

goes through the point (-0.5, 0). Find a value of s in G(s) that corresponds to this point (-0.5, 0). Provide your answer as a complex number in the form of real and imaginary parts.

$$s =$$
  $+$   $j$ 

Question 19 10 pts

Consider the following two single-input single-output subsystems:

$$S_1: \left\{ \begin{array}{l} \dot{x}_1 = A_1 x_1 + B_1 u_1 \\ y_1 = C_1 x_1 \end{array} \right. \qquad S_2: \left\{ \begin{array}{l} \dot{x}_2 = A_2 x_2 + B_2 u_2 \\ y_2 = C_2 x_2 \end{array} \right.$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 1 \end{bmatrix},$$
  

$$A_2 = -3, \quad B_2 = 1, \quad C_2 = 1.$$

The two systems are cascaded forming an overall system S, as shown in the following block diagram.

$$u_1 \longrightarrow S_1 \longrightarrow S_2 \longrightarrow y_2$$

Calculate the transfer function G(s) from  $u_1$  to  $y_2$  of the system S. Provide the components of your answer as integers for the numerator and denominator of G.

Numerator of 
$$G(s) = s^3 + s^2 + s^3 + s^4 + s^$$

Denominator of 
$$G(s) = s^3 + s^2 + s^3 + s^2 + s^3 + s^3 + s^4 +$$

**END of EXAM.** 

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