Confidential

THE UNIVERSITY OF SYDNEY

SEAT NUMBER: ……………………………………………

STUDENT ID: ………………………………………………

SURNAME: …………………………………………………

GIVEN NAMES: …………………………………..

AMME3500/9501
System Dynamics and Control

Final Examination
Semester 1, 2016

Time Allowed: Three Hours + 10 minutes reading time

This examination paper consists of 9 *pages*

INSTRUCTIONS TO CANDIDATES

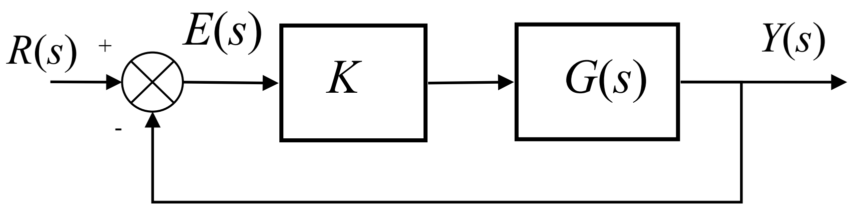
1. This is a closed book exam.
2. A simple calculator (programmable versions and PDA's not allowed) may be taken into the exam room.

Calculator	Make	Model

3. The paper comprises 6 questions each with multiple parts. ANSWER ALL SIX QUESTIONS.
4. The mark to be awarded for each part is indicated. Marks total 100.
5. Answer all questions in the ANSWER BOOK PROVIDED. Plotting pages for Questions 3, 4 & 6b are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period.
6. The question paper must be returned with the answer script
7. Take care to write legibly. Write your final answers in ink, not pencil.

Section A – System Specifications [25 marks]

1. [10 marks] For the feedback system shown below:



with $G(s) = \dfrac{4s}{(s^2 + 4s + 3)}$

   a. What are the open-loop poles and zeros?
   b. If K=5, what is the closed-loop transfer function from R to Y? What are its poles and zeros
   c. What is the closed-loop transfer function from R to E? What are its poles and zeros?

2. [15 marks] A model for the read head on a magnetic disk drive is given blow

$$G(s) = \frac{15}{s(\tau s + 1)}$$

where the time constant τ=0.001. Such systems need to be extremely fast and precise for high data throughput and reliable operation.

   a. Suppose a feedback controller was to be designed of the form

   $$u = K(r - y)$$

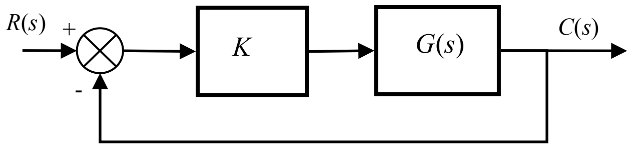
   to meet a specification of 0.005 second settling time and less than 5% overshoot. Either find a gain K which achieves this specification, or show why it is impossible.
   b. If you found that this specification cannot be met with proportional feedback, design a controller that meets them of Proportional + Derivative form:

   $$u = K_p(r - y) + K_d(\dot{r} - \dot{y})$$

   c. Derive the closed-loop transfer function. Do you expect the design requirements to be met perfectly? If not, why not?
   d. Calculate the steady-state error to a unit step and unit-slope ramp input. How can steady-state error be reduced or eliminated?

Section B – Root Locus and Bode plots [45 marks]

3. [20 marks] A feedback system for the pitch control system on a high-speed ferry is shown below:



with K a constant gain and the transfer function for the plant G(s) given by

$$G(s) = \frac{50}{s(s^2 + 80s + 2500)}$$

a. Sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 7) and be sure label units on the graph and submit your copy of the exam booklet with your answer book at the end of the exam.
b. Could proportional control make this system unstable? If so, estimate the gain K at which the system is marginally stable.
c. Estimate the gain K that will result in a settling time of 1 second.

4. [25 marks] The transfer function from motor current to joint position of a large flexible robot arm is given below:

$$G(s) = \frac{0.1(s + 500)}{s(s + 0.1)(s^2 + 2.4s + 6400)}$$

d. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 8). Show any derivation you require in the answer book and include the asymptotes on the sketch itself. and be sure label units on the graph and submit your copy of the exam booklet with your answer book at the end of the exam.
a. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin.
b. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable (if any). Give an approximate value for the overshoot if this system were placed in a unity feedback loop.
c. Give a detailed control design that will reduce settling-time to 10 seconds with less than 20% overshoot. What will the resulting steady-state error be for a unit step input? And for a ramp input of unit slope?

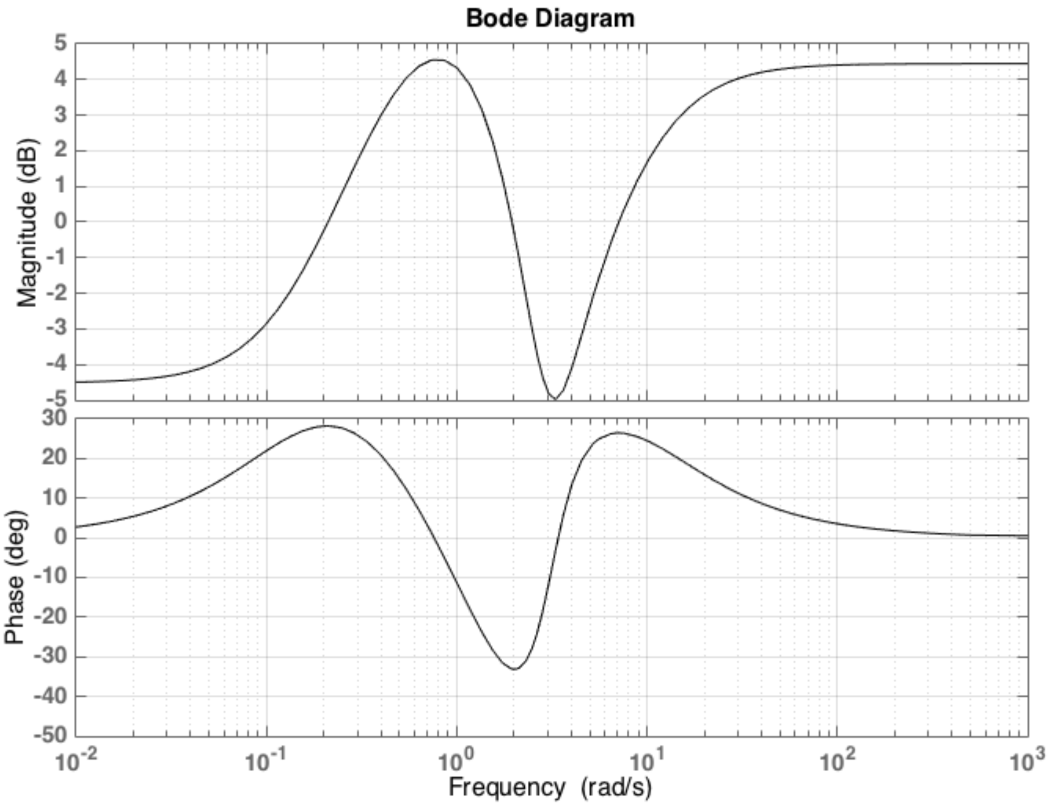
Section C – Controller Design and Analysis [30 marks]

5. [10 marks] Given the second order system, answer the following

$$G(s) = \frac{10s+5}{(s^2+4s+20)(s+3)}$$

   a. Transform the transfer function into phase-variable form
   b. Design a state-feedback controller that achieves an overshoot of 5% and settling time of 0.5 seconds.
   c. Design an observer and output-feedback controller achieving the same specifications.
   d. Describe in your own words one advantage and one disadvantage of state-space methods compared to classical (root locus, Bode) methods.

6. [20 marks] In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions.
   a. For a PID controller, describe role of each of the three terms, and describe a procedure for choosing the controller poles and zeros.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 9) – note the different time scales (units are seconds). Draw the response over the input signals. Label your diagram showing features such as the response magnitude and phase lag/lead.



   c. Prove that any state-space system written in phase-variable form is controllable.
   d. Derive the transfer function of a time-delay system: $y(t) = u(t-\tau)$, and sketch its Bode plot. What is the order of this system? Is it stable?

THERE ARE NO MORE QUESTIONS

Selected Equations

Time Response (First Order Systems)

$$G(s) = C_\infty \frac{1}{s+\sigma} \qquad t_s = \frac{4}{\sigma}$$

Time Response (Second Order Underdamped Systems)

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_r \cong \frac{1.8}{\omega_n} \qquad t_s \cong \frac{4}{\sigma}$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}}$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$$

$$M_p \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \qquad e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

Root Locus

$$|KG(s)H(s)| = 1$$

$$\angle KG(s)H(s) = (2k+1)180^\circ$$

$$\sigma_a = \frac{\sum finite\ poles - \sum finite\ zeroes}{n-m}$$

$$\theta_a = \frac{(2k+1)\pi}{n-m}$$

Frequency Response

$$M = |G(j\omega)| = |G(s)|_{s=j\omega} = \sqrt{\left(\mathrm{Re}[G(j\omega)]\right)^2 + \left(\mathrm{Im}[G(j\omega)]\right)^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\mathrm{Im}[G(j\omega)]}{\mathrm{Re}[G(j\omega)]}\right]$$

$$GM = 20\log K \qquad PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

State Space

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Controllability

$$\mathbf{R} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$$

Observability

$$\mathbf{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix}$$

Full State Feedback

$$\mathbf{u} = -\mathbf{Kx} + r$$

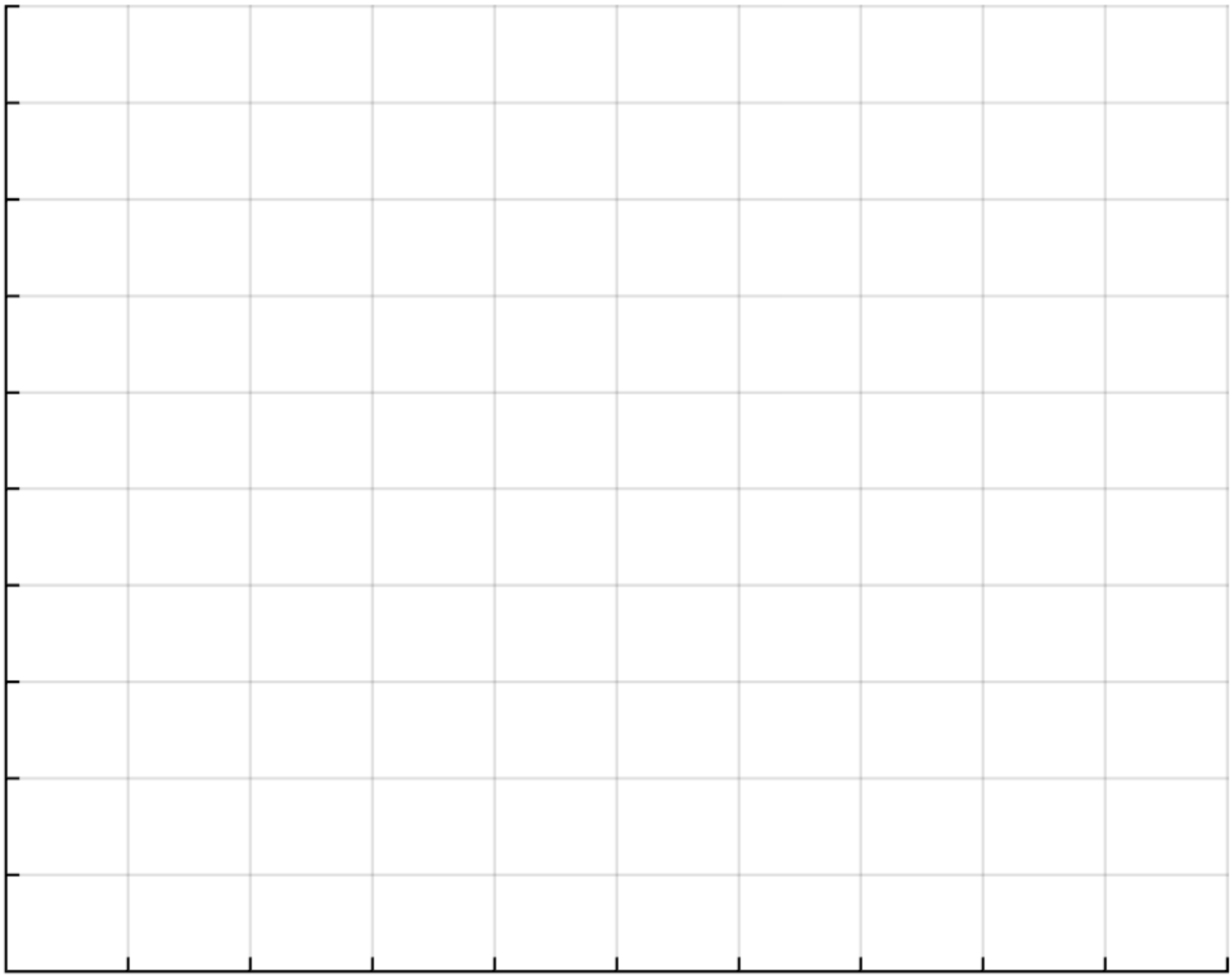
Laplace Transform Tables

Item no.	*f(t)*	*F(s)*
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

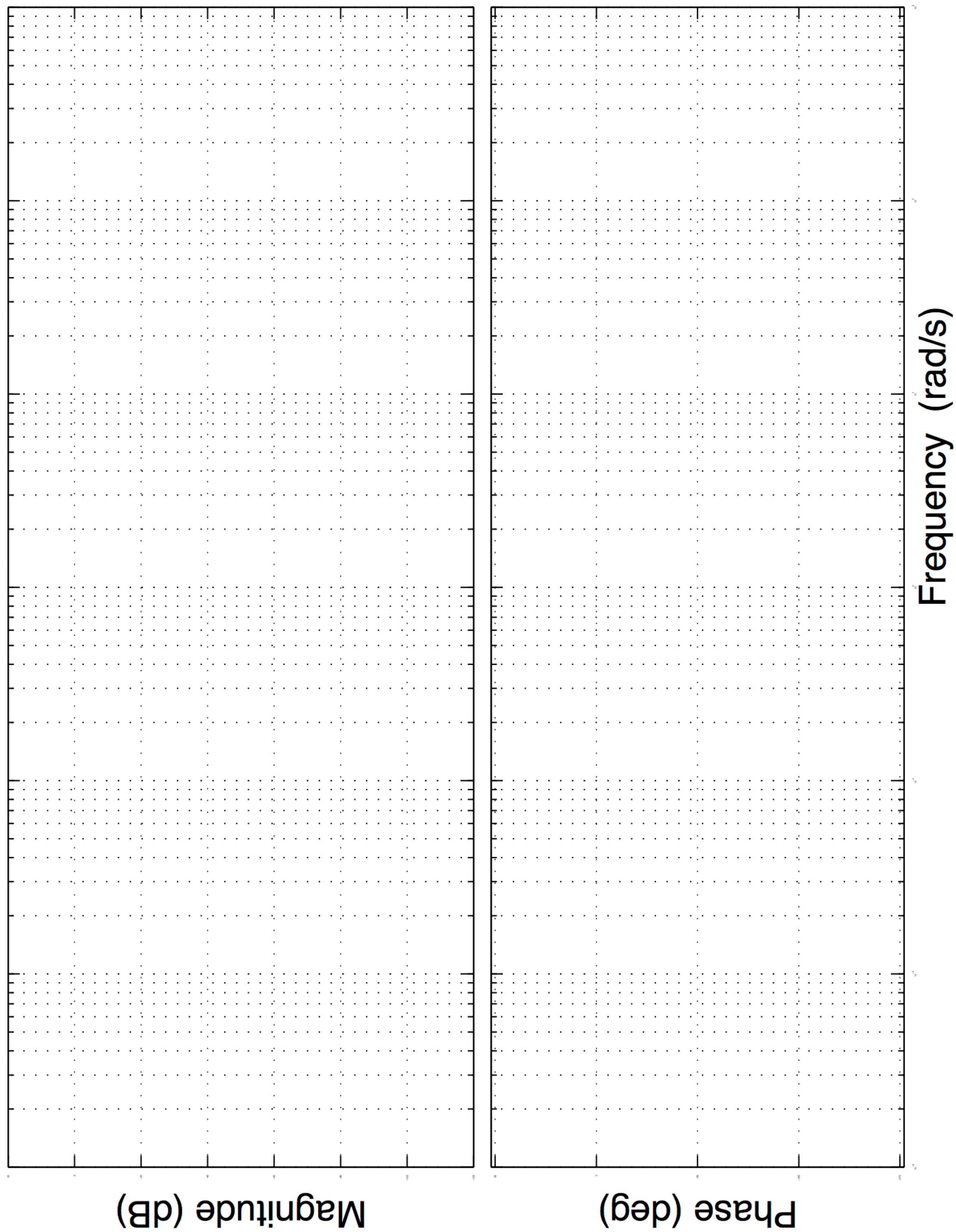
Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s)-sf(0-)-\dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s)-\sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to \infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

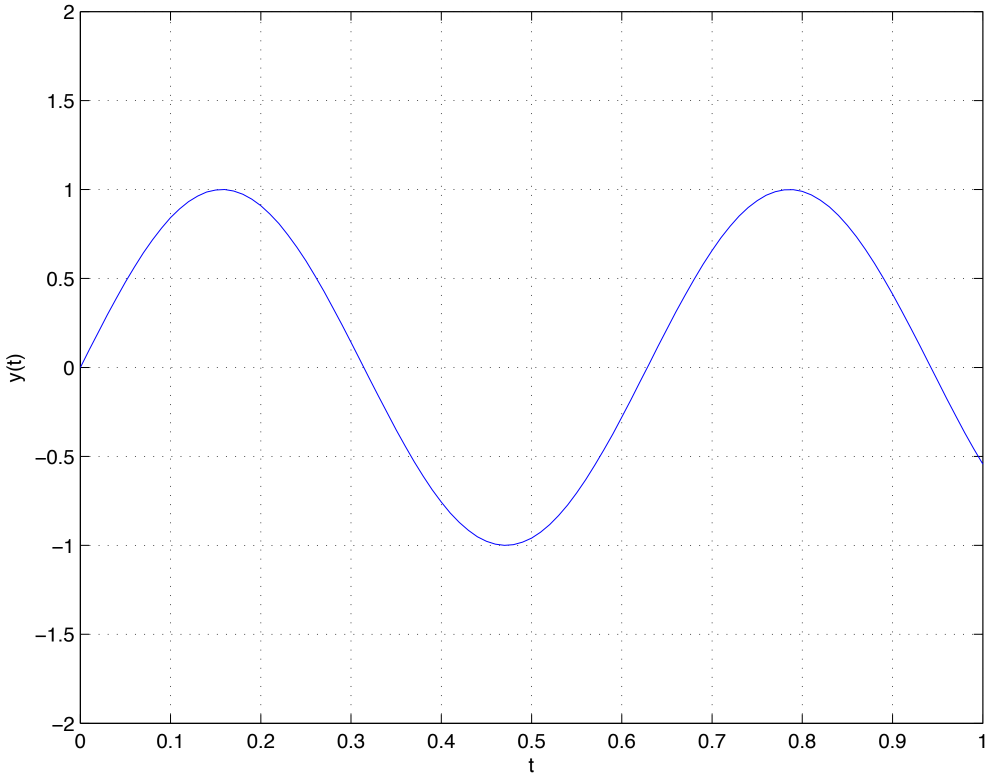
Root Locus for Question 3

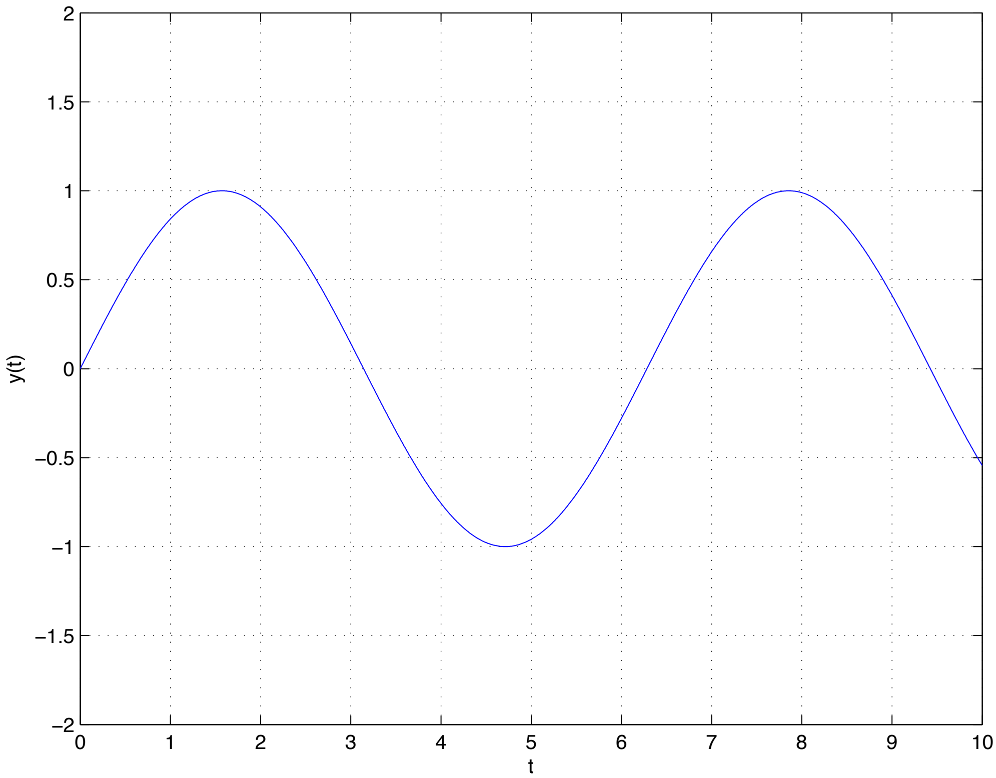


Empty Bode Plot for Question 4



Input plots for Question 6b)





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