CONFIDENTIAL

THE UNIVERSITY OF SYDNEY

TABLE NUMBER ………………………………………………………………

FAMILY NAME ………………………………………………………………

GIVEN NAMES ………………………………………………………………

STUDENT NUMBER ………………………………………………………………

FACULTY OF ENGINEERING

AMME3500/5501

Systems Dynamics and Control

Semester 1, 2012

Time allowed: 3 hours

There are six (6) questions in this exam. All questions count towards your final mark. These questions are to be completed in the **ANSWER BOOK PROVIDED**. Plotting pages for Questions 3, 4 & 6b are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period. Clearly label which question you are answering.

The total marks for this exam is **100**. This exam is designed to assess your understanding of the material covered by this Unit of Study. Be sure to read through the exam in its entirety before starting to work on the solutions.

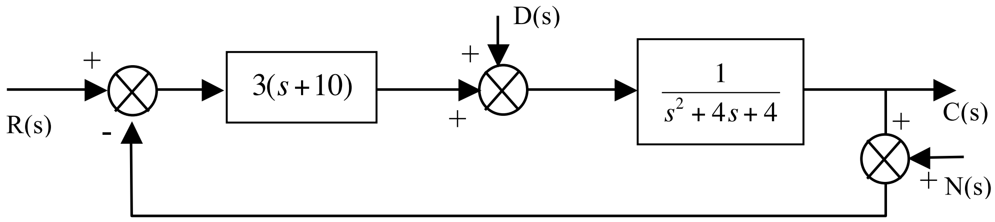
This is a closed book exam.

This exam booklet is to be handed back with your answer book.

Non-Programmable calculators are allowed.

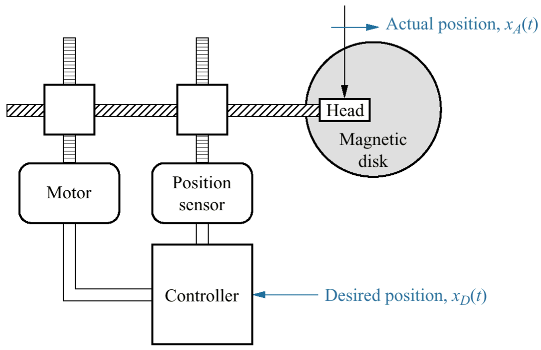
Section A – System Specifications [30 marks]

1. Consider the following block diagram [15 marks]:



   a. Derive the three closed-loop transfer functions from reference R(s), disturbance D(s), and measurement noise N(s) to the output C(s).
   b. Compute the steady-state output responses to a step input at the reference, and the disturbance, both of magnitude = 1.
   c. Your boss asks you to redesign the controller so that it has a steady-state response of zero for **both** a step disturbance, and a step change in measurement noise (e.g. due to a sensor offset). What is your reply? Give a brief mathematical justification of your answer.

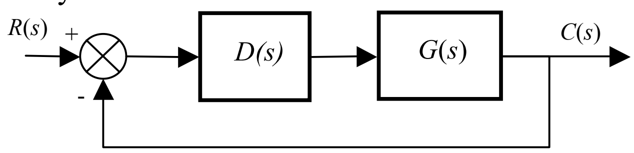
2. You are designing a control system for positioning a hard-disk drive read head (see figure right). Assume the head has a mass of 0.01Kg, and the gearing and bearings induce a friction force proportional to velocity with a coefficient of 0.5 Ns/m. Assume that the motor dynamics are sufficiently fast to ignore, and are effectively a static gain of 5. [15 marks]



   a. Write the equations of motion, sketch a block diagram for the system, indicating the controller (yet to be designed), the motor, the plant, the desired position, and the actual position.
   b. An engineer decides to apply a proportional+derivative (PD) controller:
$$u = K_P(x_D - x_A) + K_D(\dot{x}_D - \dot{x}_A)$$
   where u is the input to the motor, and x_D and x_A are the desired and actual position, respectively. Find gains K_P and K_D that would give exactly 5% overshoot and a settling time of 1 millisecond assuming second-order approximation. Would you expect these specifications to be achieved? Give a brief justification for your answer.
   c. What is the steady-state error to a reference command for this controller?
   d. Give the precise range of all possible gains K_P and K_D that will result in a stable closed-loop system.

Section B – Root Locus and Bode plots [40 marks]

3. For the system shown here



the transfer functions for the plant G(s) and compensator D(s) are given by

$$G(s) = \frac{2}{s^2 + 7s + 6} \quad \text{and} \quad D(s) = \frac{K(s+5)}{s}$$

For this system, sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 7) and be sure to turn in your copy of the exam booklet with your answer book at the end of the exam. Based on your root locus sketch, estimate the gain K for which the system will have an overshoot of 5%. [15 marks]

4. Based on the system G(s) shown below, answer the following questions [25 marks]

$$G(s) = \frac{50(s+5)}{(s+0.2)(s^2+10s+2500)}$$

   a. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 8). Show any derivation you require in the answer book and include the asymptotes on the sketch itself.
   b. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin.
   c. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable.
   d. We wish to add a lag compensator to this system to improve the steady state error. Propose the location of a lag compensator to improve the steady state error by a factor of 10. Your design should have minimal effect on phase margin. Remember that the ratio of the pole/zero position is related to the steady state error constant by the relationship $\frac{z_c}{p_c} = \frac{K_{pc}}{K_p}$ where z_c is the location of lag zero, p_c is the location of the lag pole, K_{pc} is the steady state error constant for the compensated system and K_p is the steady state error constant for the original system.

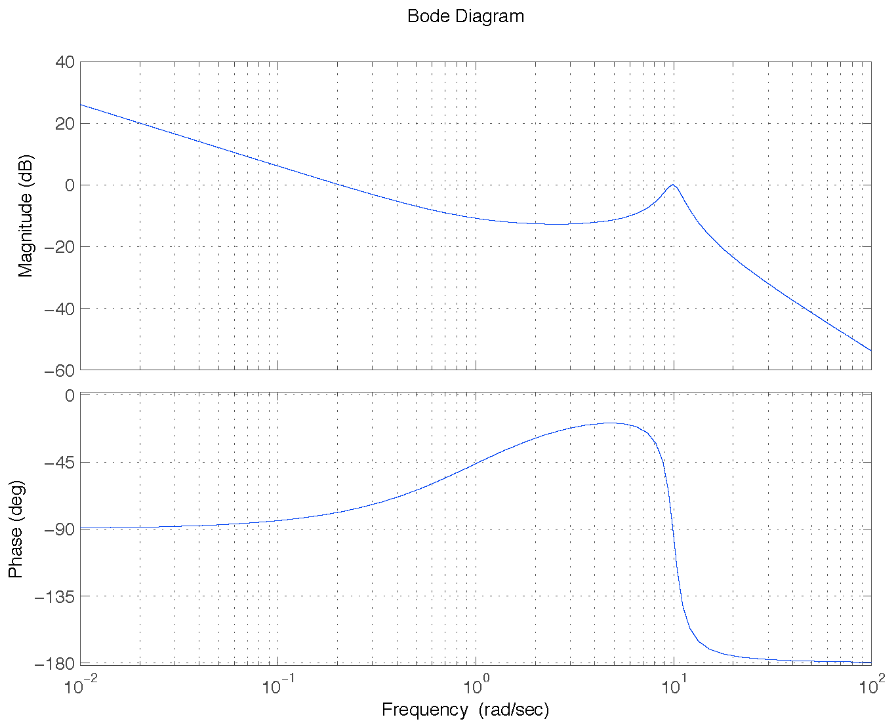
Section C – Controller Design [30 marks]

5. Given the second order system, answer the following [15 marks]

$$G(s) = \frac{1}{(s+2)(s+3)}$$

   a. Transform the transfer function into the phase variable state space formulation.
   b. Design a full state feedback controller using pole placement (state space) to yield a 5% overshoot with a settling time of 1s.

6. In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions. [15 marks]
   a. For a PID controller, describe the effect that the proportional, integral and differential terms have on the transient response of a system. Use control terminology, sketches of step responses and any equations to help demonstrate your understanding of these controllers.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 9). Draw the response overlain on the input signals. Be sure to label your diagram showing important features such as the response magnitude and phase lag/lead.



   c. Describe the "separation principle" in output-feedback state-space control. Use any equations and block diagrams that you need to explain your answer.
   d. Suppose an open-loop unstable system is to be stabilized with a feedback controller, but the actuator saturates at a certain level (e.g. due to a power amplifier clipping, or maximum angle of deflection of a tail fin). Describe how such a system could be designed using LTI methods, and the effect of the actuator saturation on stability.

THERE ARE NO MORE QUESTIONS

Selected Equations

Time Response (First Order Systems)

$$G(s) = C_\infty \frac{1}{s+\sigma} \qquad t_r = \frac{2.2}{\sigma}$$

Time Response (Second Order Systems)

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$t_r \cong \frac{1.8}{\omega_n} \qquad t_s \cong \frac{4}{\sigma}$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} \qquad M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$$

$$M_p \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \qquad e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

Root Locus

$$|KG(s)H(s)| = 1$$

$$\angle KG(s)H(s) = (2k+1)180^c$$

$$\sigma_a = \frac{\sum finite\ poles - \sum finite\ zeros}{n-m}$$

$$\theta_a = \frac{(2k+1)\pi}{n-m}$$

Frequency Response

$$M = |G(j\omega)| = |G(s)|_{s=j\omega} = \sqrt{\left(\mathrm{Re}\left[G(j\omega)\right]\right)^2 + \left(\mathrm{Im}\left[G(j\omega)\right]\right)^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\mathrm{Im}\left[G(j\omega)\right]}{\mathrm{Re}\left[G(j\omega)\right]}\right]$$

$$GM = 20\log K \qquad PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

State Space

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Controllability

$$\mathbf{R} = [\mathbf{B}\ \ \mathbf{AB}\ \ \mathbf{A^2B} \cdots \mathbf{A^{n-1}B}]$$

Observability

$$\begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA^2} \\ \vdots \\ \mathbf{CA^{n-1}} \end{bmatrix}$$

Full State Feedback

$$\mathbf{u} = -\mathbf{Kx} + r$$

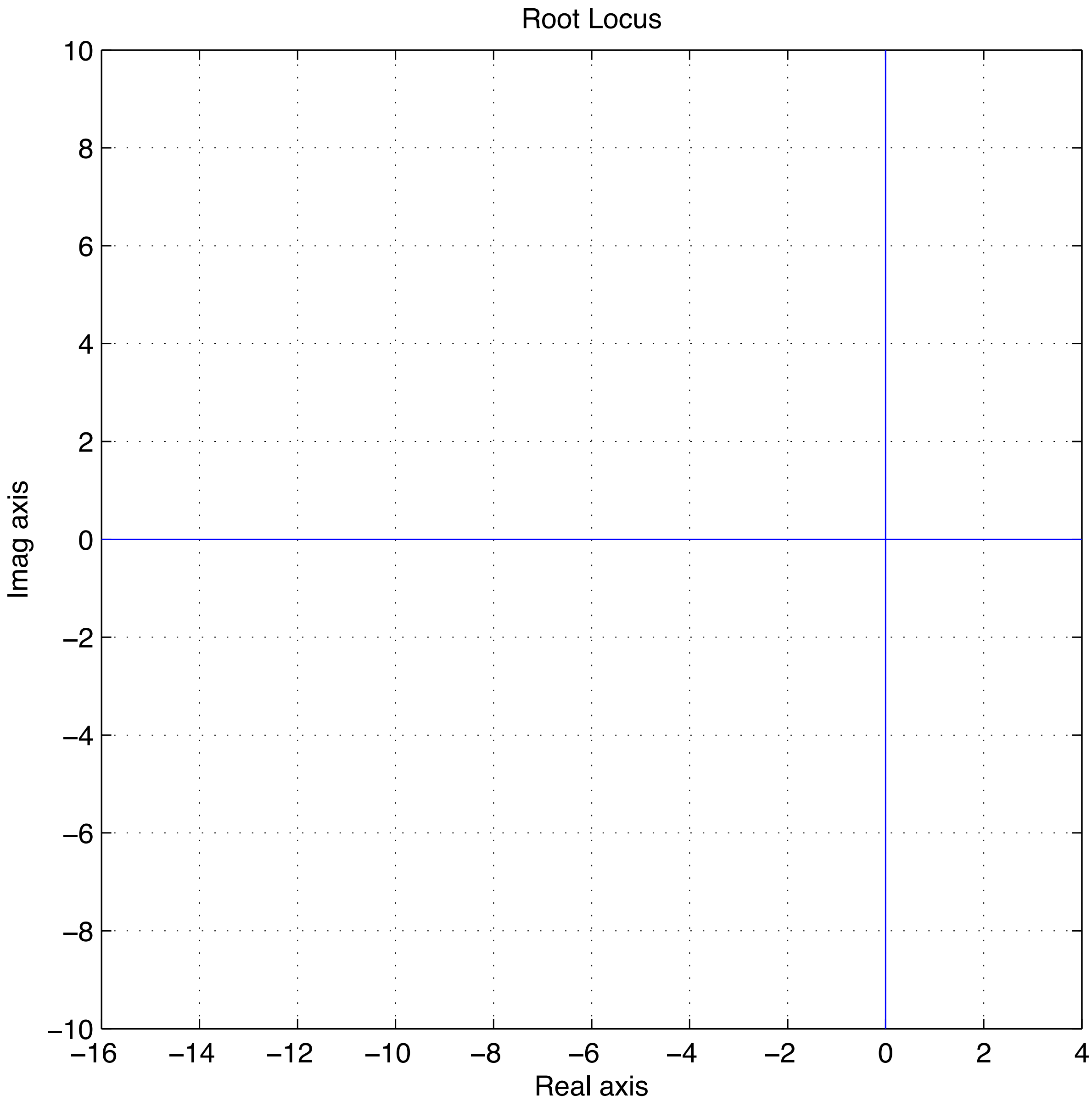
Laplace Transform Tables

Item no.	*f(t)*	*F(s)*
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

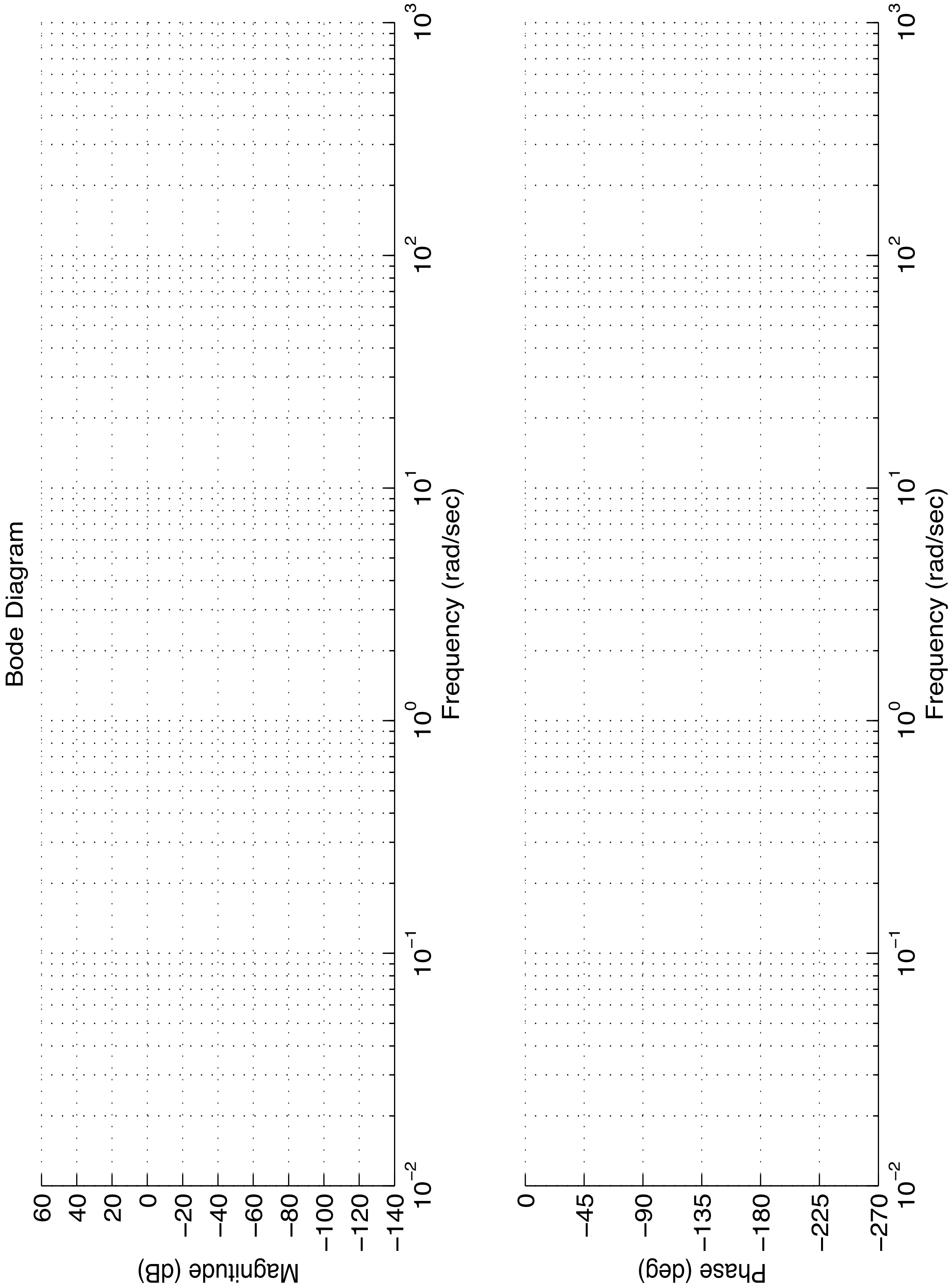
Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)]$	$= F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

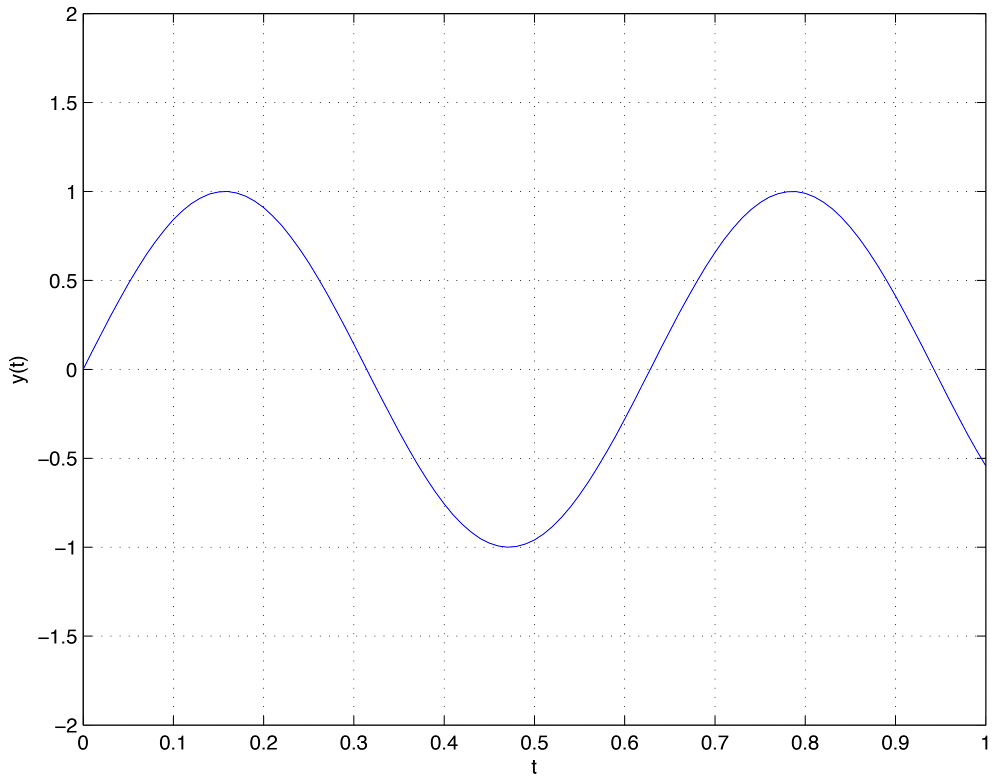
Root Locus for Question 3

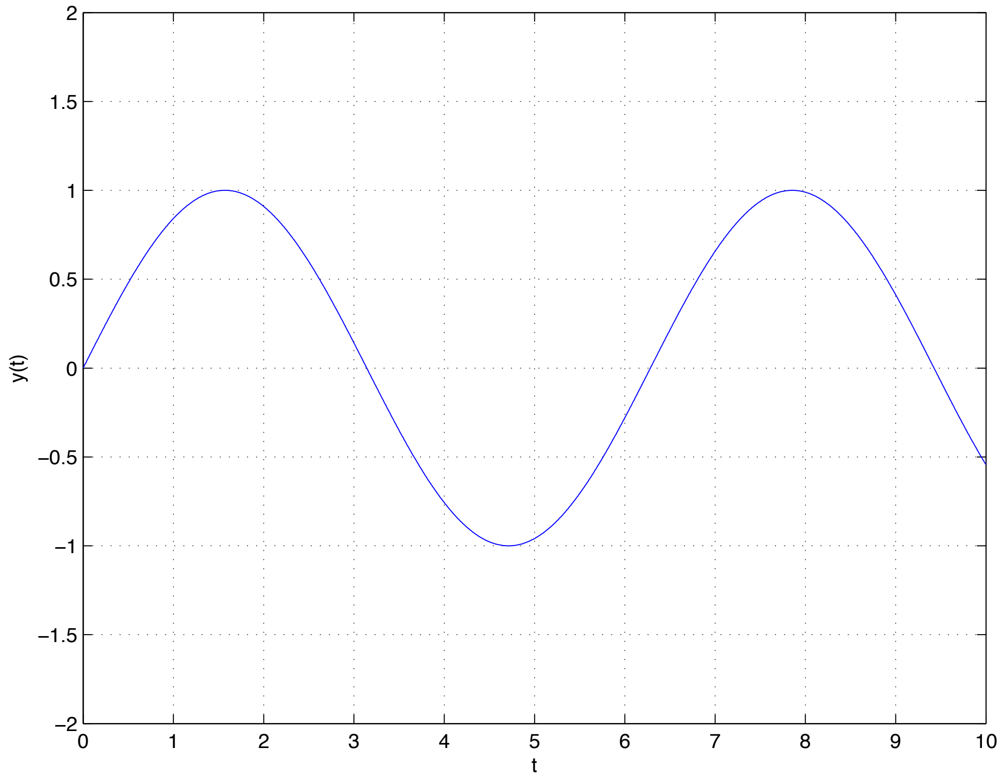


Bode Plot for Question 4



Input plots for Question 6b)





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