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Seat Number ________
Student Number |__|__|__|__|__|__|__|__|__|
ANONYMOUSLY MARKED
(Please do not write your name on this exam paper)

THE UNIVERSITY OF SYDNEY

CONFIDENTIAL EXAM PAPER

This paper is not to be removed from the exam venue

Aerospace, Mechanical and Mechatronic

EXAMINATION

Semester 1 - Main, 2019

AMME3500/AMME9501 System Dynamics and Control

EXAM WRITING TIME: 3 hours

READING TIME: 10 minutes

EXAM CONDITIONS:
This is a CLOSED book examination - no material permitted

During reading time - writing is not permitted at all

MATERIALS PERMITTED IN THE EXAM VENUE:
(No electronic aids are permitted e.g. laptops, phones)

Calculator - non-programmable

MATERIALS TO BE SUPPLIED TO STUDENTS:
1 x 16-page answer book

INSTRUCTIONS TO STUDENTS:

Please tick the box to confirm that your examination paper is complete. ☐

For Examiner Use Only

Q	Mark
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Total ________

First and Second Order Systems

1. [10 Marks] Consider the system

$$\dot{x} + ax = u.$$

   (a) [2 Marks] Let $u = 0$ for all time, and consider $x(t)$. If $x(0) = 2$ and $x(2) = e^{-8}$, what is the value of a?

   (b) [4 Marks] Now let a be the value found in Part (a). If $u(t) = \mathbf{1}(t)$ is the unit step function and $x(0) = 0$, derive the response of $x(t)$.

   (c) [4 Marks] For $a = 6$, what is the value of u, so that $x(t)$ approaches 2 as t tends to infinity for any initial value $x(0)$?

2. [10 Marks] Consider the following second-order system

$$\ddot{x} + 7\dot{x} + 12x = u.$$

   (a) [2 Marks] What are the poles of the system?

   (b) [2 Marks] What is the meaning that the system be stable in terms of system response $x(t)$? Is the system stable or not?

   (c) [6 Marks] Design a rate-feedback PD controller

$$u(t) = K_p(r - x) - K_d\dot{x}$$

   so that the system response to a step input has a settling time around 4 sec and an overshoot of about 5%. Show all working.

3. [10 Marks] The dynamics of a plane pendulum subject to some external force is described by

$$\ddot{x} + \frac{g}{\ell}\sin x = u(t),$$

where x is the angular displacement, ℓ is the length of its link, and g is the standard gravity.

   (a) [2 Marks] What is the equilibrium of the pendulum with $u(t) = F$ being a constant?

   (b) [2 Marks] Write down the linearized equation for small angular displacement.

   (c) [6 Marks] Derive the harmonic response $x(t) = X\sin(\omega t)$ subject to $u(t) = F\sin(\omega t)$ from the linearized equation.

State Space Control Design

4. [10 Marks] Let us reconsider the second-order system

$$\ddot{x} + 7\dot{x} + 12x = u.$$

   (a) [1 Mark] Rewrite the dynamics in state-space form.

   (b) [1 Mark] Is the system reachable?

   (c) [2 Marks] Compute the poles from the state-space form.

   (d) [2 Marks] Suppose the system output is given by

$$y(t) = \dot{x}.$$

   Is the system observable?

   (e) [4 Marks] Design a state feedback controller so that the closed-loop system has poles placed at $1 + j$ and $1 - j$.

5. [10 Marks] Consider the system given by

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, and $C \in \mathbb{R}^{1\times n}$.

   (a) [3 Marks] What is the meaning of the system being observable?

   (b) [3 Marks] What is the purpose of an observer?

   (c) [4 Marks] Consider an observer

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x})$$

   where $L \in \mathbb{R}^{n\times 1}$. What is the condition that L should satisfy for the observer to work? Explain your reasoning.

6. [10 Marks] Consider the following two single-input single-output subsystems:

$$S_1 : \begin{cases} \dot{x}_1 = A_1x_1 + B_1u_1 \\ y_1 = C_1x_1 \end{cases} \qquad S_2 : \begin{cases} \dot{x}_2 = A_2x_2 + B_2u_2 \\ y_2 = C_2x_2 \end{cases}$$

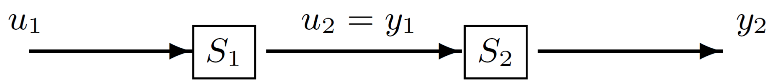

Figure 1: Block diagram of the system S.

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 1 \end{bmatrix},$$
$$A_2 = -1, \qquad B_2 = 1, \qquad C_2 = 1.$$

The two subsystems are cascaded forming an overall system S, as shown in Figure 1.

(a) [3 Marks] Denote

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Write down the state-space equation of the overall system S with input u_1, output y_2, and state x.

(b) [3 Marks] Verify reachability (controllability) and observability of the system S.

(c) [4 Marks] Calculate the transfer function from u_1 to y_2 of the system S.

Frequency-Domain Control Design

7. [10 Marks] The Bode diagram of a transfer function $H(s)$ is shown in Figure 2.

(a) [2 Marks] Estimate the number of zeros and poles for $H(s)$ from the Bode gain plot. Explain your reasoning.

(b) [4 Marks] Estimate the transfer function $H(s)$ from both the gain and phase plots. Explain your reasoning.

(c) [4 Marks] Plot the steady-state response of the system with an input signal $u(t) = \sin(2t)$. Clearly show key features.

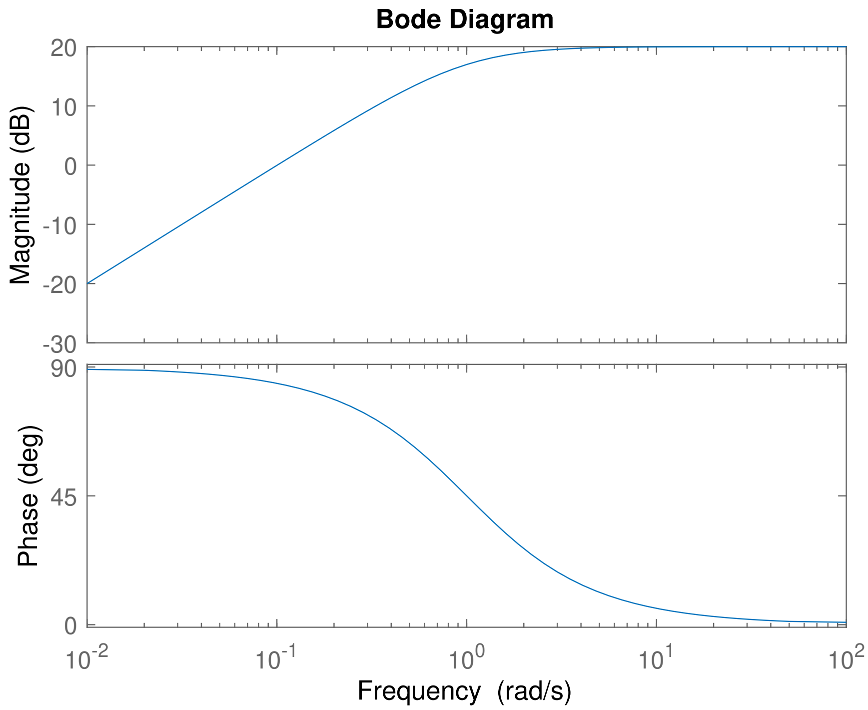

Figure 2: The Bode diagram for Problem 7.

8. [10 Marks] Consider the unity feedback system shown in Figure 3. The Nyquist locus for the plant

$$G(s) = \frac{s^2 - 5s - 4}{2(s^2 + 5s + 6)}$$

(with gain $K = 1$) is given in Figure 4.

(a) [3 marks] Is the closed loop system stable for $K = 1$? *Explain your answer.*

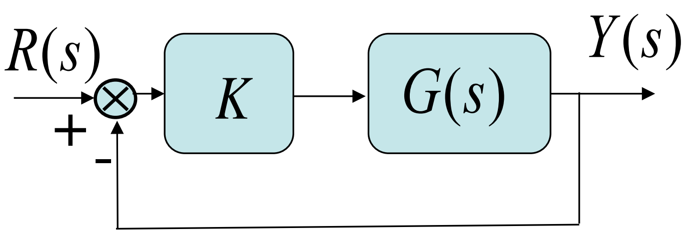

Figure 3: Unity feedback system for Question 8.

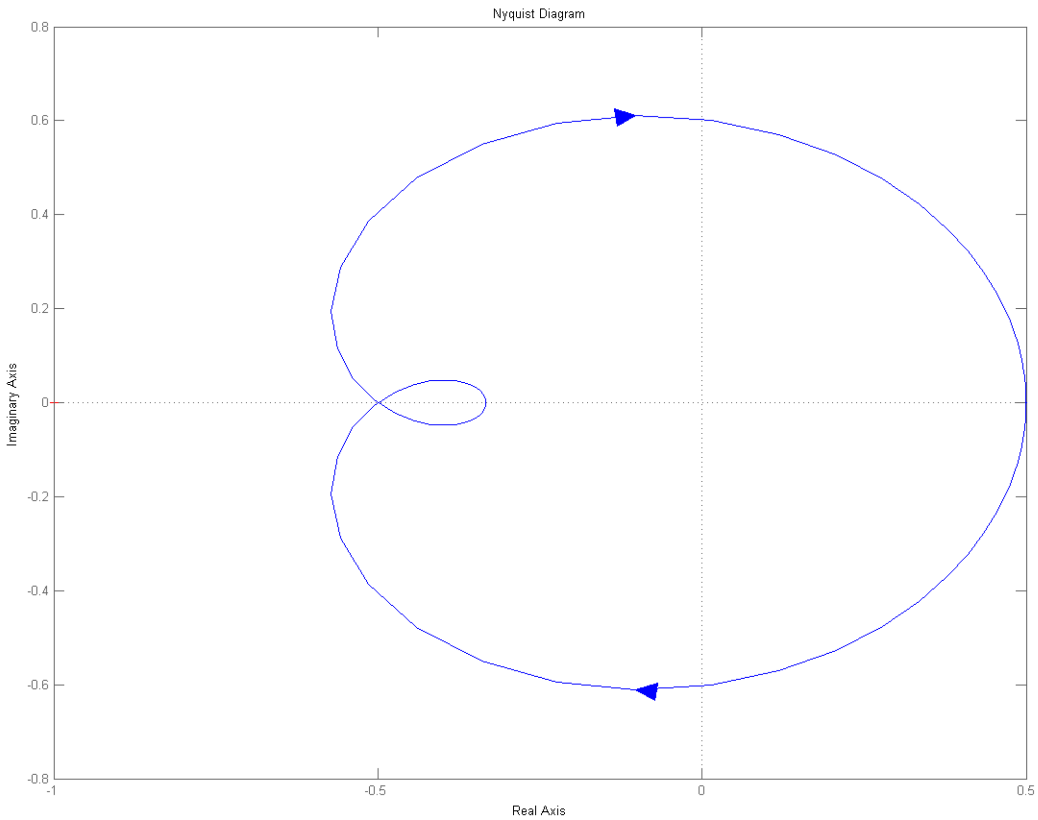

Figure 4: Nyquist plot for Question 8.

(b) [3 marks] What is the definition of gain margin? Particularly, what is the gain margin of the system with $K = 1$? *Explain your answer.*

(c) [4 marks] The Nyquist plot goes through the point $(-0.5, 0)$. Find the value of s in $G(s)$ that corresponds to that point.

9. [10 Marks] Consider a plant with transfer function

$$P(s) = \frac{1}{s(s+1)}.$$

(a) [4 Marks] For a PD control $C(s) = 2s + 1$, what are the sensitivity and complementary sensitivity functions?

(b) [2 Marks] Explain the significance of sensitivity functions in terms of plant uncertainty.

(c) [4 Marks] Explain the idea of loop shaping for frequency-domain design of feedback controllers.

Real-world Dynamics and Control

10. [10 Marks] In *vivo*, e.g., within a single living cell, various types of proteins are interacting with each other, which are in the meantime continuously synthesized by genes through mRNA. A full description of such a system would involve thousands of signals, which is difficult to achieve. However, we can simplify our world by focusing on the relationship between one particular type of protein and one particular type of mRNA.

Let $x_1(t)$ be the concentration of the mRNA, and $x_2(t)$ be the concentration of the protein, respectively. The interaction between the two **states** is described by

$$\dot{x}_1 = p_1 u - k_1 x_1 \tag{1}$$

$$\dot{x}_2 = p_2 x_1 - k_2 x_2 \tag{2}$$

where p_1, p_2 and k_1, k_2 are parameters for the production rates and degradation rates, and u is our **input**.

(a) [3 Marks] Let us for the moment assume x_1 is a constant, i.e., $x_1(t) = a$ for all t. Thus, we can just look at the Eq. (2). Determine the *sign* of k_2, that is, it should be positive or negative, from the physical interpretation of the system. Show your reasoning.

(b) [4 Marks] Suppose we have sensors that can measure the real-time concentration of the protein $x_2(t)$. What would be the approach you would take for the design of a feedback controller, under which $x_2(t)$ can hopefully track a reference signal r? Show the main steps of your design.

(c) [3 Marks] Explain possible *uncertainties* for the equation (1)-(2) considering the real world scenario as described from the beginning.

END OF EXAM.