CONFIDENTIAL

THE UNIVERSITY OF SYDNEY

TABLE NUMBER ……………………………………………………………

FAMILY NAME ……………………………………………………………

GIVEN NAMES ……………………………………………………………

STUDENT NUMBER ……………………………………………………………

FACULTY OF ENGINEERING

AMME3500/5501

Systems Dynamics and Control

Semester 1, 2010

Time allowed: 3 hours

There are six (6) questions in this exam. All questions count towards your final mark. These questions are to be completed in the **ANSWER BOOK PROVIDED**. Plotting pages for Questions 3, 4 & 6b are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period. Clearly label which question you are answering.

The total marks for this exam is **100**. This exam is designed to assess your understanding of the material covered by this Unit of Study. Be sure to read through the exam in its entirety before starting to work on the solutions.

This is a closed book exam.
This exam booklet is to be handed back with your answer book.
Non-Programmable calculators are allowed.

Section A – System Specifications [30 marks]

1. An LTI system has a closed loop transfer function given by:

$$T(s) = \frac{15(s+3)}{(s+3.1)(s^2+4s+13)(s^2+20s+109)}$$

[10 marks]

a. What is the steady state error for this system?

b. You are asked to make an estimate of the percentage overshoot for the step response of the system, and you do not have access to simulation software. What is your estimate? Provide a short sentence or two of explanation as part of your answer.

2. A biomedical engineer is looking to design a controller for a prosthetic knee. Ignoring gravity and assuming that the thigh is fixed, we wish to study the motion of the lower limb. The lower limb has a moment of inertia I=0.1kgm^2 and the knee features a rotational damper with coefficient b=0.25Nms/rad. The knee also has a motor embedded in the system that exerts a torque of T at the knee. Based on this system, answer the following questions

[20 marks]

a. Sketch a block diagram showing the relationship between the angular position of the lower limb and the applied torque at the knee. Find the transfer function between the torque T and the angular position θ of the knee.

b. An engineer decides to apply a simple proportional control law to regulate the angular position of the knee such that the applied torque is computed so that θ tracks a reference position θ_r according to the feedback law

$$T = K(\theta_r - \theta)$$

Sketch a block diagram of the resulting system. Find the transfer function between θ_r and θ .

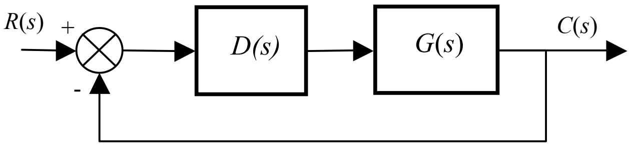
c. What is the value of K that can be used if we wish to have a settling time of 2s for a commanded change in angular position?

d. What is the maximum gain K that can be used such that we have an overshoot M_p<5% for a step change in desired position θ_r?

e. What is the peak torque required of the motor for a step change in desired angular position of 1.0 rad given the gain computed for part d.?

Section B – Root Locus and Bode plots [40 marks]

3. For a given system shown here



the transfer functions for the plant G(s) and controller D(s) is given by

$$G(s)=\frac{s^2+8s+80}{(s+1)(s+2)} \quad \text{and} \quad D(s)=\frac{K(s+4)}{s+10}$$

For this system, sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 7) and be sure to turn in your copy of the exam booklet with your answer book at the end of the exam. Based on your root locus sketch, estimate the gain K for which the system will have an overshoot of 5%. [15 marks]

4. Based on the system G(s) shown below, answer the following questions [25 marks]

$$G(s)=\frac{50(s+5)}{s(s+10)(s^2+16s+15)}$$

   a. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 8). Show any derivation you require in the answer book and include the asymptotes on the sketch itself.
   b. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin.
   c. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable.
   d. Estimate the change in Phase Margin that would result from the addition of a lead compensator of the form $D(s)=\dfrac{10(s+2)}{s+20}$.

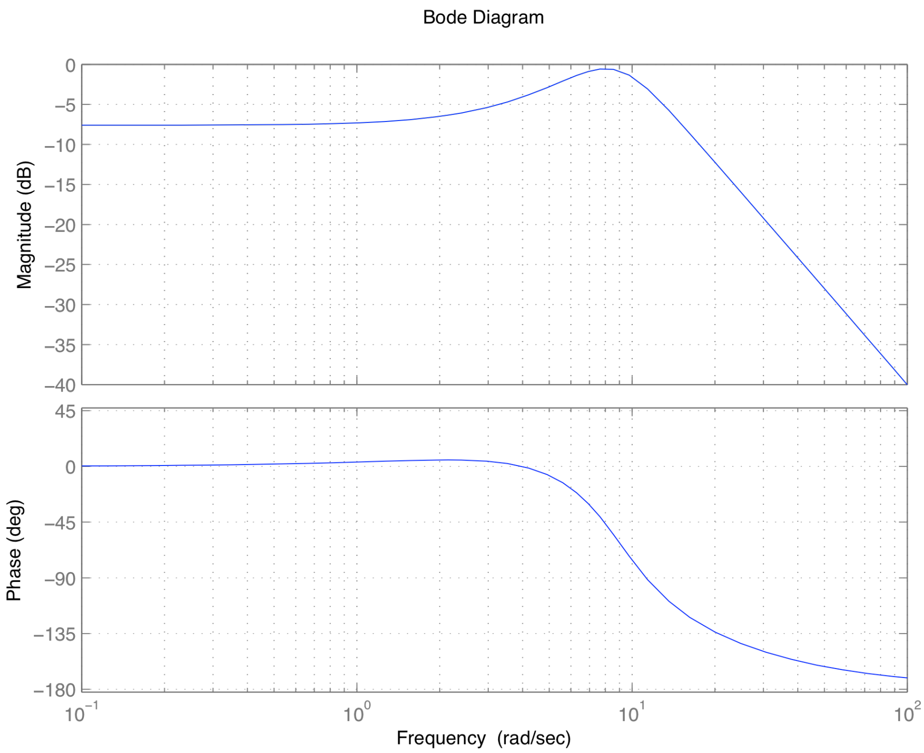
Section C – Controller Design [30 marks]

5. Given the second order system, answer the following [15 marks]

$$G(s) = \frac{1}{(s+1)(s+5)}$$

   a. Transform the tranfer function into the phase variable state space formulation.
   b. Design a full state feedback controller using pole placement (state space) to yield a 5% overshoot with a settling time of 1s.

6. In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions. [15 marks]
   a. For a PID controller, describe the effect that the proportional, integral and differential terms have on the transient response of a system. Use control terminology, sketches of step responses and any equations to help demonstrate your understanding of these controllers.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 9). Draw the response overlain on the input signals. Be sure to label your diagram showing important features such as the response magnitude and phase lag/lead.



   c. Name two advantages of modern control (state space) when compared with classical (frequency domain) methods.
   d. As we have seen, classical techniques are suitable for modelling and control design for Linear Time Invariant (LTI) systems. Briefly describe two system characteristics that are not easy to model using the LTI methods concentrated on in this course.

THERE ARE NO MORE QUESTIONS

Selected Equations

Time Response (First Order Systems)

$$G(s) = C_\infty \frac{1}{s+\sigma} \qquad t_r = \frac{2.2}{\sigma}$$

Time Response (Second Order Systems)

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} \qquad \sigma = \zeta\omega_n$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

$$t_r \cong \frac{1.8}{\omega_n} \qquad t_s \cong \frac{4}{\sigma}$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} \qquad M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$$

$$M_p \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \qquad e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

Root Locus

$$|KG(s)H(s)| = 1$$

$$\angle KG(s)H(s) = (2k+1)180^\circ$$

$$\sigma_a = \frac{\sum f\ inite\ poles - \sum\ f\ inite\ zeroe}{n-m}$$

$$\theta_a = \frac{(2k+1)\pi}{n-m}$$

Frequency Response

$$M = |G(j\omega)| = |G(s)|_{s=j\omega} = \sqrt{\left(\mathrm{Re}\left[G(j\omega)\right]\right)^2 + \left(\mathrm{Im}\left[G(j\omega)\right]\right)^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\mathrm{Im}\left[G(j\omega)\right]}{\mathrm{Re}\left[G(j\omega)\right]}\right]$$

$$GM = 20\log K \qquad PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

State Space

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Controllability

$$\mathbf{R} = \begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}$$

Observability

$$\mathbf{O} = \begin{bmatrix}\mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1}\end{bmatrix}$$

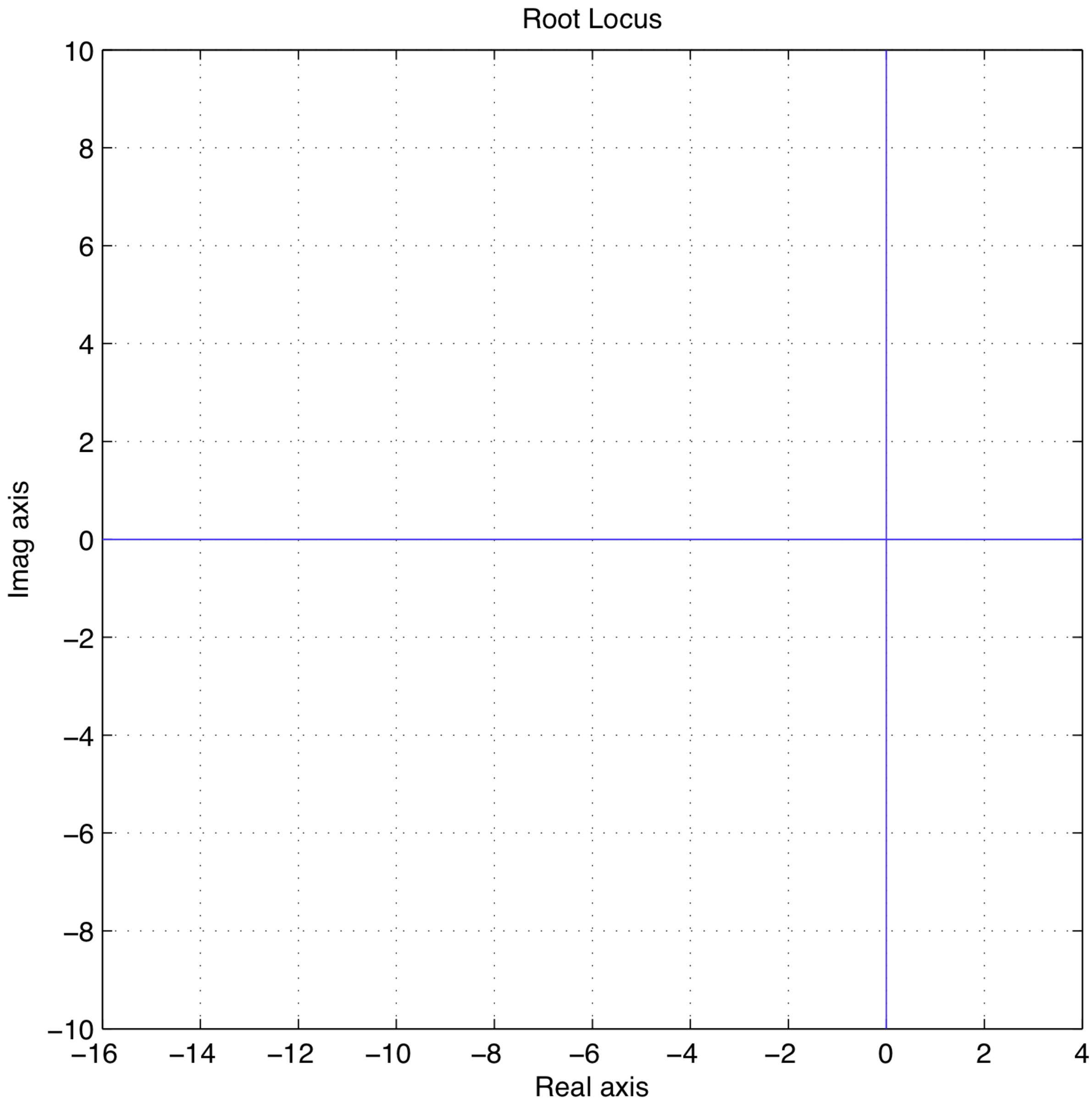
Laplace Transform Tables

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

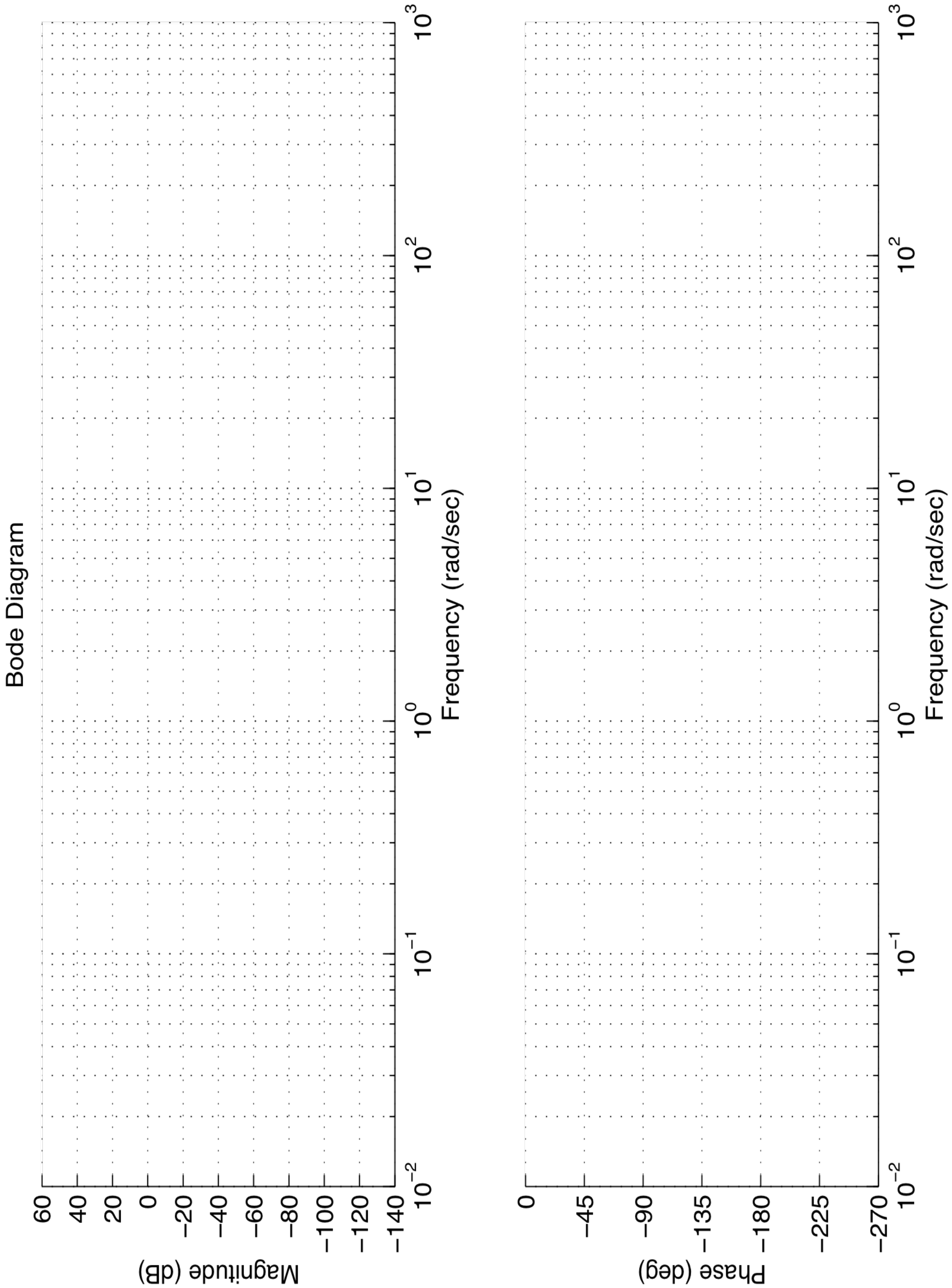
Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

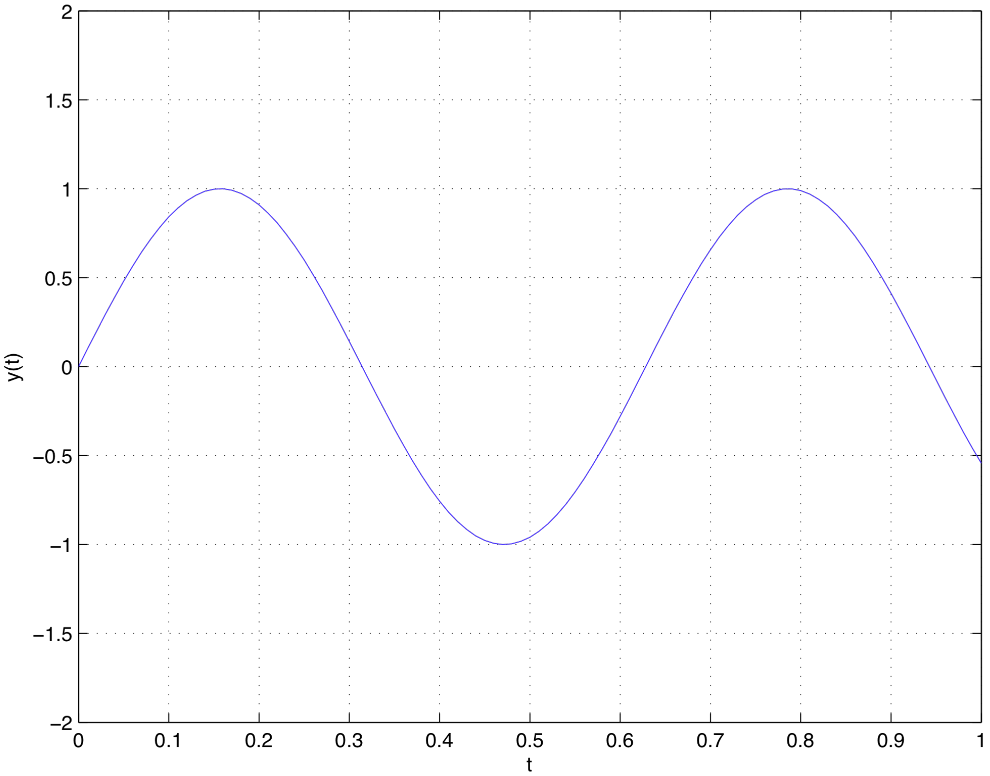
Root Locus for Question 3

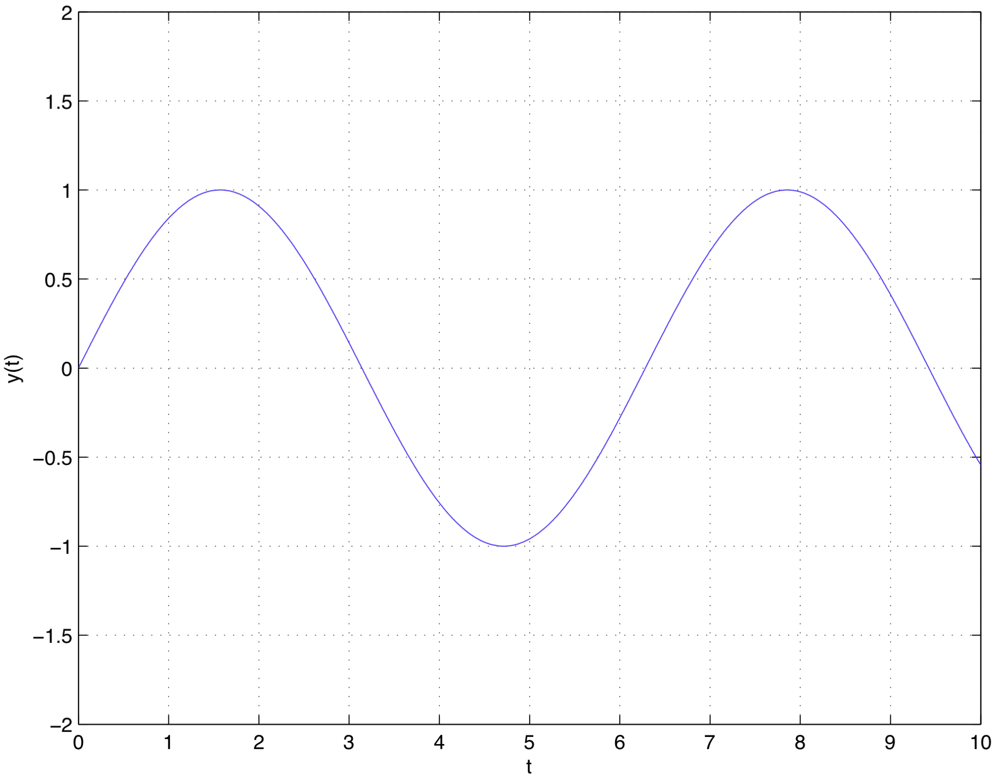


Bode Plot for Question 4



Input plots for Question 6b)





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