Seat Number ________

Student Number |__|__|__|__|__|__|__|__|__|__|

Family Name ______________________

First Name ______________________

CONFIDENTIAL EXAM PAPER

This paper is not to be removed from the exam venue

Aerospace, Mechanical and Mechatronic

EXAMINATION

Semester 1 - Main, 2017

AMME3500/AMME9501 System Dynamics and Control

EXAM WRITING TIME: 3 hours

READING TIME: 10 minutes

EXAM CONDITIONS:
This is a closed book examination - no material permitted

During reading time - writing is not permitted at all

MATERIALS PERMITTED IN THE EXAM VENUE:
(No electronic aids are permitted e.g. laptops, phones)

Calculator - non-programmable

MATERIALS TO BE SUPPLIED TO STUDENTS:
1 x 16-page answer book

INSTRUCTIONS TO STUDENTS:

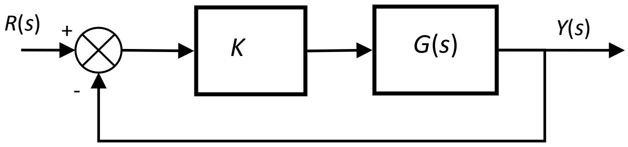
Please check your examination paper is complete (page numbers in footer of page) and indicate by signing below.

I have checked the examination paper and affirm it is complete.

Student Signature: ______________________________ *Date:* ______________

Section A – System Specifications [25 marks]

1. [10 marks] For the feedback system shown below:



with $G(s)=\dfrac{3(s+1)}{s(s^2+8s+20)}$

a. What are the open-loop poles and zeros?
b. If K=10, what is the closed-loop transfer function from R to Y? What are its poles and zeros?
c. What are the steady-state errors for a unit step input and ramp input with K=10? What about if K=100?

2. [15 marks] The equations of motion for the DC motor are given by the following equation:

$$J\ddot{\theta}_m+\left(B+\frac{K_bK_T}{R_a}\right)\dot{\theta}_m=\frac{K_T}{R_a}V_a$$

Assume the following motor parameters

$$J=0.01kg\cdot m^2,\quad B=0.001N\cdot m\cdot \sec$$
$$K_T=0.02N\cdot m/A,\quad K_b=0.02V\cdot\sec,\quad R_a=10\Omega$$

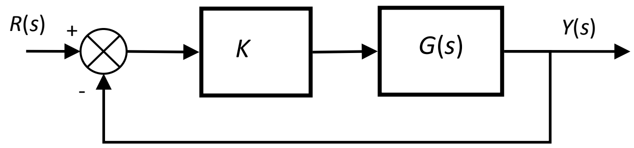
a. Find the transfer function between the applied voltage V_a and the motor shaft angle θ_m.
b. Suppose that feedback is added to the system to allow the position of the motor shaft to be controlled. Given a desired shaft position, θ_r, the applied voltage is given by:

$$V_a=K\left(\theta_r-\theta_m\right)$$

where K is the feedback gain. Find the transfer function from θ_r to θ_m.
c. What values of K will provide a rise time of less than 4 seconds?
d. What values of K can be used if an overshoot of less than 20% is desired? (Ignoring the rise-time constraint from c.)
e. Can both of these design specifications in c. and d. be achieved with the same value of K?

Section B – Root Locus and Bode plots [45 marks]

3. [20 marks] A feedback system for the force control in a robot manipulator has the following form:



with K a constant gain and the transfer function for the plant G(s) given by

$$G(s) = \frac{s+2.5}{(s^2+2s+2)(s^2+4s+5)}$$

a. Sketch the Root Locus. Show the asymptotes and the departure/arrival angles where appropriate. Produce your sketch on the s-plane sheet at the back of this exam booklet (pg. 7) and be sure to label units on the graph and submit your copy of the exam booklet with your answer book at the end of the exam.
b. Estimate the gain K that would result in an overshoot of 16%. Estimate the settling time with this gain.
c. Could the system become unstable if the gain is increased? If so, estimate the gain at which the system is marginally stable.

4. [25 marks] To design the pitch control system for a commercial airliner, we make use of the transfer function from elevator deflection to pitch angle:

$$G(s) = \frac{12s+2}{s^3+0.75s^2+s}$$

a. Sketch the Bode plot for this system on the magnitude/phase diagram at the end of this exam booklet (pg. 8). Show any derivation you require in the answer book and include the asymptotes on the sketch itself. Be sure label units on the graph and submit your copy of the exam booklet with your answer book at the end of the exam.
b. From your Bode plot sketched in a), estimate the 0dB frequency, Gain Margin and Phase Margin.
c. Based on the Gain Margin found in b), determine the value of K for which the system is marginally stable (if any). Give an approximate value for the overshoot if this system were placed in a unity feedback loop.
d. Design a lead controller that will achieve less than 10% overshoot without reducing closed-loop bandwidth. What will the resulting steady-state error be for a unit step input and for a ramp input of unit slope?

Section C – Controller Design and Analysis [30 marks]

5. [15 marks] Given the system below, answer the following

$$G(s) = \frac{3s^2 + 2s + 1}{(s^2 + 1)(s + 3)}$$

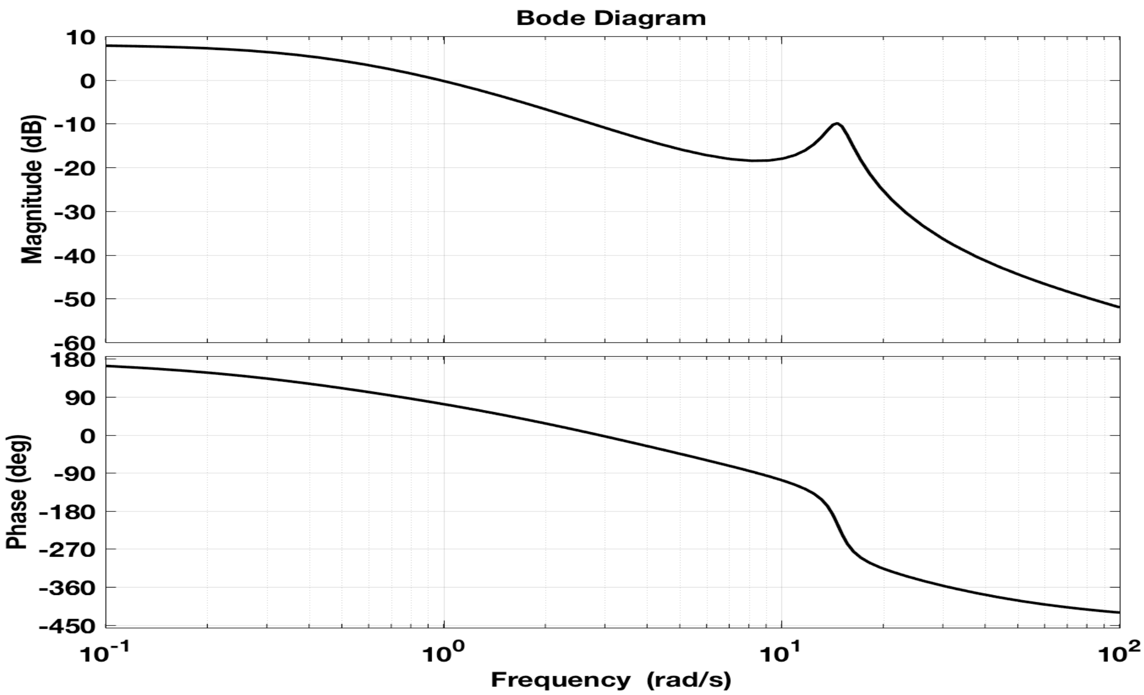
   a. Write the state-space equations for this system in phase-variable form.
   b. Design a state-feedback controller for overshoot of 20% and settling time of 1 second. Will it actually achieve these specifications accurately?
   c. Consider the following system with two state variables:

$$\dot{x} = \begin{bmatrix} -3 & 0 \\ 3 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, \quad y = [\ 3 \quad 1\ ] x + 5u$$

   Is it stable? controllable? observable? What is its transfer function?

   d. Describe in your own words the meaning of controllability and observability.

6. [15 marks] In order to demonstrate your understanding of the controllers we have studied in this course, provide short answers to the following questions.
   a. For a PID controller, describe in your own words the role of each of the three terms, and describe a procedure for choosing the controller gains.
   b. Given the Bode plot shown below, sketch the response of this system to the two inputs shown at the back of this booklet (pg. 9) – note the different time scales (units are seconds). Draw the response over the input signals. Label your diagram indicating magnitude and phase lag/lead.



   c. Describe in your own words the importance of Bode's sensitivity integral and Bode's gain-phase relation to practical engineering designs.
   d. Consider the time-delay system: $y(t) = u(t - \tau)$. Prove that this system is linear time-invariant and compute its transfer function. What are its poles and zeros?

Selected Equations

First order systems

$$G(s) = C_\infty \frac{a}{s+a} \qquad T_r \approx \frac{2.2}{a}, \quad T_s \approx \frac{4}{a}$$

Second order systems

$$G(s) = C_\infty \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \sigma = \zeta\omega_n, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$T_r \approx \frac{1.8}{\omega_n}, \quad T_s \approx \frac{4}{\sigma} \qquad \%OS = \begin{cases} 5\%, \zeta \approx 0.7, \theta \approx 45^\circ \\ 10\%, \zeta \approx 0.6, \theta \approx 53^\circ \\ 16\%, \zeta \approx 0.5, \theta \approx 60^\circ \\ 20\%, \zeta \approx 0.45, \theta \approx 63^\circ \end{cases}$$

Steady-State Error

$$e_{step}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} \qquad e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)}$$

Root Locus

$$|KG(s)H(s)| = 1, \quad \angle KG(s)H(s) = (2k+1)180^\circ \qquad \sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m}, \quad \theta_a = \frac{2(k+1)180^\circ}{n-m}$$

Frequency Response

$$PM \approx 100\zeta \quad (\zeta < 0.7), \quad T_s \approx \frac{10}{\omega_{BW}} \qquad \omega_{\max} = \frac{1}{T\sqrt{\beta}} \qquad \phi_{\max} = \sin^{-1}\frac{1-\alpha}{1+\alpha}, \qquad |U(j\omega_{\max})| = -10\log\alpha \text{ dB}$$

Fundamental Limitations

$$S = \frac{1}{1+GK}, \quad T = \frac{GK}{1+GK}, \quad \angle G(j\omega) \approx 90^\circ \frac{d\log|G(j\omega)|}{d\log\omega} \qquad \int_0^\infty \log|S(j\omega)|d\omega = \pi \sum_{Re(p_i)>0} Re(p_i)$$

State Space

$$\dot{x} = Ax + Bu, \quad y = Cx + Du, \quad u = -Kx + r \qquad R = [B \ AB \ A^2B \ \ldots \ A^{n-1}B], \quad O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

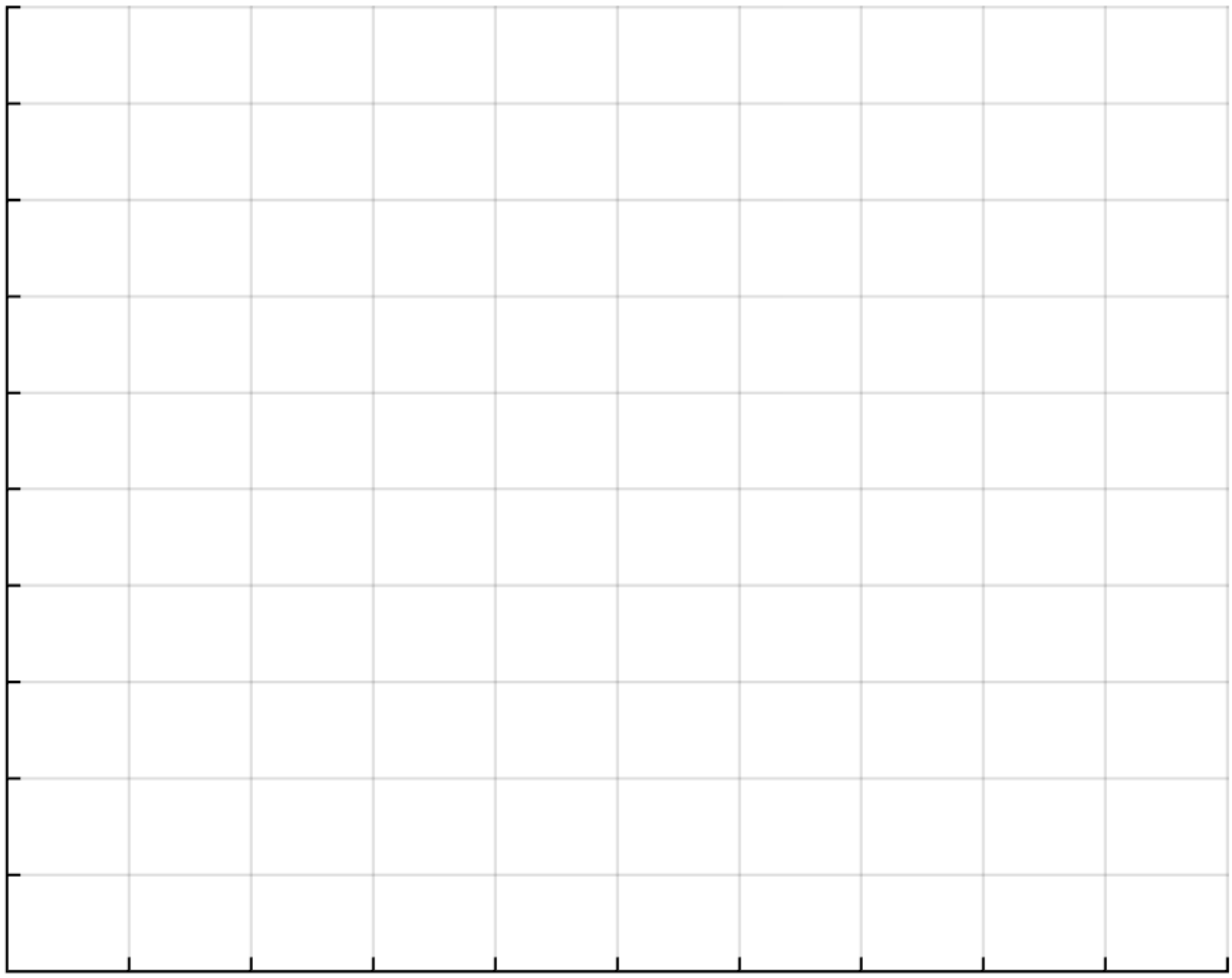
Laplace Transform Tables

Item no.	*f(t)*	*F(s)*
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

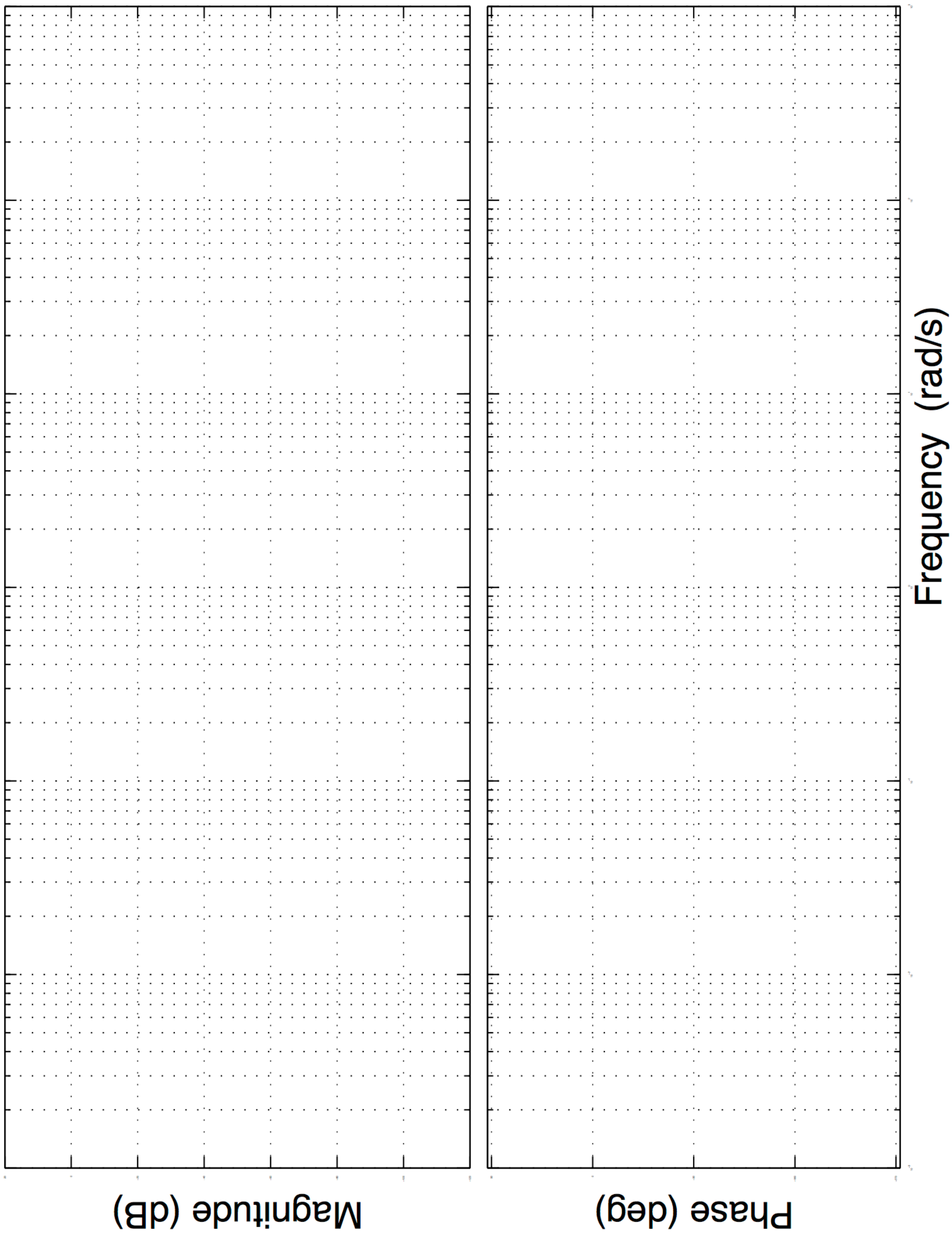
Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s)-sf(0-)-\dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s)-\sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to \infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

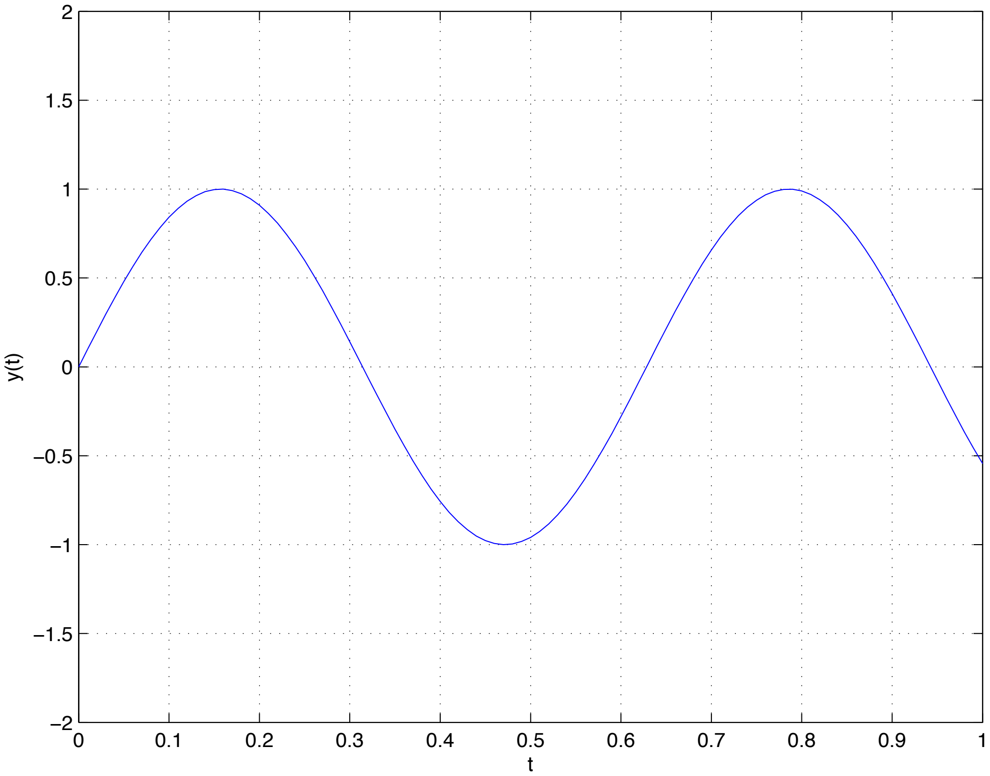
Root Locus for Question 3

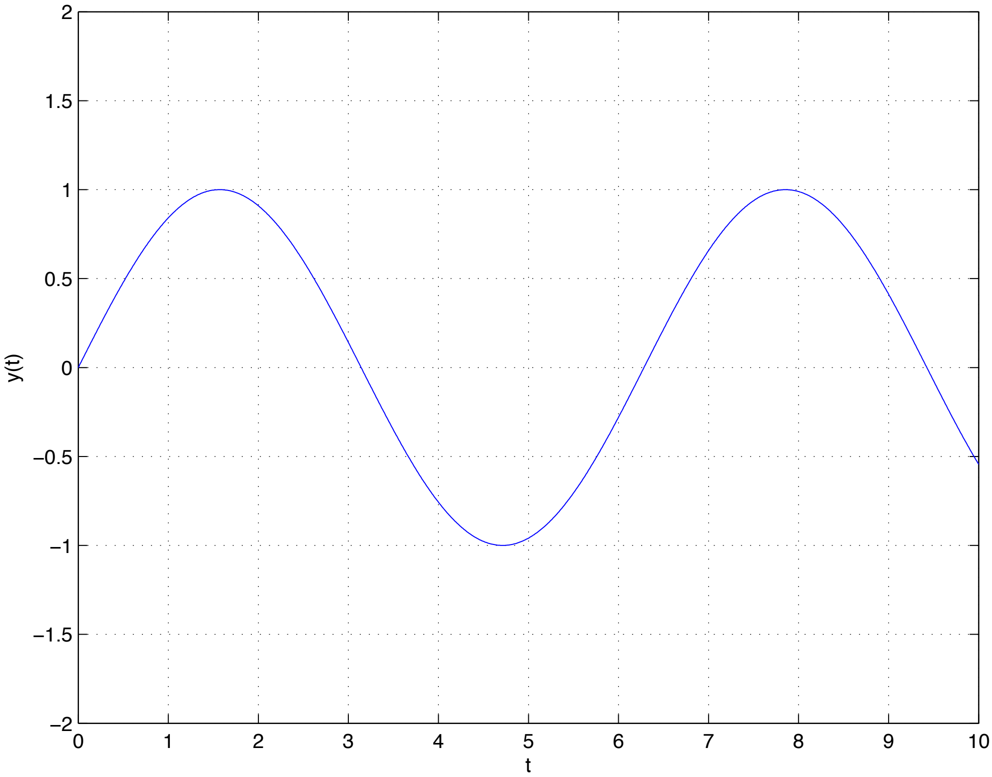


Empty Bode Plot for Question 4



Input plots for Question 6b)





END OF EXAMINATION