Assignment 1

1. (a) *It will either be sunny and warm, or it will be sunny. If it is not sunny then it will not be cloudy. Therefore it will be cloudy.*

   i. *Translate the argument into the formal language for propositional logic.*

$$\begin{aligned} \text{Let } S &= \text{It will be sunny} \\ \text{Let } W &= \text{It will be warm} \\ \text{Let } C &= \text{It will be cloudy} \end{aligned}$$

$$(S \wedge W) \vee C, \neg C \to \neg S \vdash C \tag{1}$$

   ii. *The argument is propositionally valid; provide a natural deduction proof.*

$$\dfrac{\dfrac{\dfrac{(S \wedge W) \vee C \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{S \wedge W}\,1}{S}\,\wedge E}{S \vee C}\,\vee I}{(S \wedge W) \to (S \vee C)}\,\to I,1 \qquad \dfrac{\dfrac{\dfrac{}{C}\,2}{S \vee C}\,\vee I}{C \to (S \vee C)}\,\to I,2}{S \vee C}\,\vee E \qquad \dfrac{(C \to \bot) \to (S \to \bot) \qquad \dfrac{}{C \to \bot}\,3}{S \to \bot}\,\to E \qquad \dfrac{}{C \to \bot}\,3}{\bot}\,\vee E}{\mathrm{C}}\,\bot, 3$$

   iii. *Alter, add, or remove premises (or change the conclusion) in the argument to make it formally invalid, for the same narrative.*
   Change the conclusion to be $\neg C$.

   iv. *Give an explicit counterexample to your argument in (iii), and explain why it is a counterexample.*

   If we alter the conclusion to be $\neg C$, we have a counterexample as marked on the truth table below. This case is a counterexample because all of the premises are true, but the conclusion ($\neg C$) is not.

	S	W	C	$\neg C$	$\neg S$	$(S \wedge W) \vee C$	$\neg C \to \neg S$
	0	0	0	1	1	0	1
$\to$	0	0	1	0	1	1	1
	0	1	0	1	1	0	1
	0	1	1	0	1	1	1
	1	0	0	1	0	0	0
	1	0	1	0	0	1	1
	1	1	0	1	0	1	0
	1	1	1	0	0	1	1

(b) *Consider the sequent $\neg\alpha \vee \beta \vdash \alpha \to \beta$.*

   i. *Provide a natural deduction proof.*

$$\dfrac{\dfrac{(\alpha \to \bot) \vee \beta \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\alpha}\,1 \qquad \dfrac{}{\alpha \to \bot}\,2}{\bot}\,\to E}{\beta}\,\bot}{(\alpha \to \bot) \to \beta}\,\to I, 2 \qquad \dfrac{\dfrac{}{\beta}\,3}{\beta \to \beta}\,\to I, 3}{\beta}\,\vee E}{\alpha \to \beta}\,\to I, 1$$