Macm 203 Assigment 8

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Optimal Distribution Planning for Minimized Transportation Costs

Part A (5 marks)

Background

A logistics company is responsible for transporting goods from a set of warehouses to a set of retail stores. Each warehouse has a limited supply of goods, and each store has a certain demand that must be met. The transportation cost per unit of goods varies for each warehouse-store pair due to factors like distance, transportation mode, and contractual agreements. The objective is to minimize the total transportation cost whil meeting the demands of all stores without exceeding the supply capacity of any warehouse.

Problem Statement

The company operates 3 warehouses (W1, W2, W3) and needs to distribute goods to 4 retail stores (S1, S2, S3, S4). The supply capacity of the warehouses and the demand of the stores are as follows:

Warehouse Supply: W1: 500 units, W2: 400 units, W3: 300 units

Store Demand: S1: 200 units, S2: 300 units, S3: 250 units, S4: 150 units

The cost of transporting a single unit of goods from each warehouse to each store is given in the following table (in dollars):

To/From	W1	W2	W3
S1	2	3	1
S2	4	1	2
S3	5	3	4
S4	2	3	2

Formulate the Linear Programming Problem: Clearly define the decision variables, the objective function to minimize the total transportation cost, and the constraints based on warehouse supplies and store demands.

The linear programming problem is:

Minimize

$$2 * x_{11} + 4 * x_{12} + 5 * x_{13} + 2 * x_{14} + 3 * x_{21} + x_{22} + 3 * x_{23} + 3 * x_{24} + x_{31} + 2 * x_{32} + 4 * x_{33} + 2 * x_{34}$$

subject to

$x_{11} + x_{12} + x_{13} + x_{14} \leq 500$ # w1 supply

$x_{21} + x_{22} + x_{23} + x_{24} \leq 400$ # w2 supply

$x_{31} + x_{32} + x_{33} + x_{34} \leq 300$ # w3 supply

$x_{11} + x_{21} + x_{31} \geq 200$ # s1 demand

$x_{12} + x_{22} + x_{32} \geq 300$ # s2 demand

$x_{13} + x_{23} + x_{33} \geq 250$ # s3 demand

$x_{14} + x_{24} + x_{34} \geq 150$ # s4 demand

$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34} \geq 0$ # non-negative constraints

where decision variable $x_{i,j}$ is the number of single unit goods transported from warehouse W_i to store S_j.

Part B (5 marks)

Solve the problem in Matlab. Print an optimal distribution plan and the total transportation cost from this optimal solution. Vectorize your code wherever possible.

```
% Create optimization variables
x11 = optimvar("x11","Type","integer","LowerBound",0);
x12 = optimvar("x12","Type","integer","LowerBound",0);
x13 = optimvar("x13","Type","integer","LowerBound",0);
x14 = optimvar("x14","Type","integer","LowerBound",0);
x21 = optimvar("x21","Type","integer","LowerBound",0);
x22 = optimvar("x22","Type","integer","LowerBound",0);
x23 = optimvar("x23","Type","integer","LowerBound",0);
x24 = optimvar("x24","Type","integer","LowerBound",0);
x31 = optimvar("x31","Type","integer","LowerBound",0);
x32 = optimvar("x32","Type","integer","LowerBound",0);
x33 = optimvar("x33","Type","integer","LowerBound",0);
x34 = optimvar("x34","Type","integer","LowerBound",0);

% Set initial starting point for the solver
initialPoint.x11 = zeros(size(x11));
initialPoint.x12 = zeros(size(x12));
initialPoint.x13 = zeros(size(x13));
initialPoint.x14 = zeros(size(x14));
initialPoint.x21 = zeros(size(x21));
initialPoint.x22 = zeros(size(x22));
initialPoint.x23 = zeros(size(x23));
initialPoint.x24 = zeros(size(x24));
initialPoint.x31 = zeros(size(x31));
initialPoint.x32 = zeros(size(x32));
initialPoint.x33 = zeros(size(x33));
initialPoint.x34 = zeros(size(x34));

% Create problem
cost = optimproblem;

% Define problem objective
cost.Objective = 2*x11 + 4*x12 + 5*x13 + 2*x14 + 3*x21 + x22 + 3*x23 + 3*x24 + x31 + 2*x32 + 4*x33 + 2*x34;

% Define problem constraints
cost.Constraints.constraint1 = x11 + x12 + x13 + x14 <= 500;
cost.Constraints.constraint2 = x21 + x22 + x23 + x24 <= 400;
cost.Constraints.constraint3 = x31 + x32 + x33 + x34 <= 300;
cost.Constraints.constraint4 = x11 + x21 + x31 >= 200;
cost.Constraints.constraint5 = x12 + x22 + x32 >= 300;
cost.Constraints.constraint6 = x13 + x23 + x33 >= 250;
cost.Constraints.constraint7 = x14 + x24 + x34 >= 150;

% Set nondefault solver options
options = optimoptions("intlinprog","Display","final");

% Display problem information
show(cost);
```

```
OptimizationProblem :

       Solve for:
      x11, x12, x13, x14, x21, x22, x23, x24, x31, x32, x33, x34
       where:
      x11, x12, x13, x14, x21, x22, x23, x24, x31, x32, x33, x34 integer

       minimize :
      2*x11 + 4*x12 + 5*x13 + 2*x14 + 3*x21 + x22 + 3*x23 + 3*x24 + x31 + 2*x32 + 4*x33 + 2*x34


       subject to constraint1:
      x11 + x12 + x13 + x14 <= 500

       subject to constraint2:
```

```
% Solve problem
```

```
[solution,objectiveValue,reasonSolverStopped] = solve(cost,initialPoint,...
    "Solver","intlinprog","Options",options);
```

```
Solving problem using intlinprog.
Warning: x0 is infeasible. Ignoring x0.
Optimal solution found.

Intlinprog stopped at the root node because the objective value is within a gap tolerance of the optimal value, options
```

```
% Display results
solution;
reasonSolverStopped;
objectiveValue;

% Print an optimal distribution plan and the total transportation cost from
% this optimal solution.
if reasonSolverStopped == 1
    fprintf('The optimal distribution plan is:\n');
    fprintf('Distribute from W1 to S1: %.0f\n', solution.x11);
    fprintf('Distribute from W1 to S2: %.0f\n', solution.x12);
    fprintf('Distribute from W1 to S3: %.0f\n', solution.x13);
    fprintf('Distribute from W1 to S4: %.0f\n', solution.x14);
    fprintf('Distribute from W2 to S1: %.0f\n', solution.x21);
    fprintf('Distribute from W2 to S2: %.0f\n', solution.x22);
    fprintf('Distribute from W2 to S3: %.0f\n', solution.x23);
    fprintf('Distribute from W2 to S4: %.0f\n', solution.x24);
    fprintf('Distribute from W3 to S1: %.0f\n', solution.x31);
    fprintf('Distribute from W3 to S2: %.0f\n', solution.x32);
    fprintf('Distribute from W3 to S3: %.0f\n', solution.x33);
    fprintf('Distribute from W3 to S4: %.0f\n', solution.x34);
    fprintf('Total cost: %.2f\n', objectiveValue);
else
    fprintf('No optimal distribution plan found.\n');
end
```

```
The optimal distribution plan is:
Distribute from W1 to S1: 50
Distribute from W1 to S2: -0
Distribute from W1 to S3: -0
Distribute from W1 to S4: 150
Distribute from W2 to S1: -0
Distribute from W2 to S2: 150
Distribute from W2 to S3: 250
Distribute from W2 to S4: -0
Distribute from W3 to S1: 150
Distribute from W3 to S2: 150
Distribute from W3 to S3: -0
Distribute from W3 to S4: -0
Total cost: 1750.00
```