Macm 203 Assignment 7

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Part A

Here we consider shape optimization using MATLAB's Symbolic Math Toolbox. Specifically, we use MATLAB's Symbolic Math Toolbox to find the optimal dimensions of a cylindrical can (a cylinder used for storing goods, like a soda can) to minimize the surface area for a given volme. This problem involves seting up an equation for the surface area of a cylinder, differentiating it, and finding the minimum value under a volume constraint.

Background: The surface area S of a cylinder with radius r and height h is given by:

$$S = 2\pi r^2 + 2\pi \text{rh}$$

Where the first term represents the area of the top and bottom disks, and the second term represents the area of the side of the cylinder. The volume V of the cylinder is given by:

$$V = \pi r^2 h$$

Problem Statement: Given a fixed volume $V = 1000\,\text{cm}^3$, determine the dimensions (radius r and height h) of the cylinder that minimizes the surface area S.

Tasks: *Complete **all** of the following tasks using the Symbolic Math Toolbox (do not complete the tasks by hand). Use symbolic variables and expressions wherever possible.*

- Express the surface area S and volume V as symbolic expressions in terms of r and h.

```
syms r h
S = 2*pi*r^2 + 2*pi*r*h
```

S = $2\,\pi\,r^2 + 2\,h\,\pi\,r$

```
V = pi*r^2*h
```

V = $\pi\,h\,r^2$

- Given the volume $V = 1000$, solve the volume equations for h in terms of r, and substitue this back into the surface area formula to get S as a function of r only.

```
V = pi*r^2*h == 1000
```

V = $\pi\,h\,r^2 = 1000$

```
hV = solve(V,h)
```

hV =

$$\frac{1000}{r^2\,\pi}$$

```
S = subs(S,h,hV)
```

S =

$$2\,\pi\,r^2+\frac{2000}{r}$$

- Differentiate the surface area S with resepct to r and set the result equal to zero.

```
dS = diff(S,r) == 0
```

dS =

$$4\,\pi\,r-\frac{2000}{r^2}=0$$

- Determine the value of r that minimizes S by solving the equation obtained from differentiation.

```
rmin = solve(dS,'Real',true)
```

rmin =

$$\left(\frac{500}{\pi}\right)^{1/3}$$

- Ensure that you consider the second derivative test to confirm that it is indeed a minimum.

```
d2S = diff(S,2)
```

d2S =

$$4\,\pi+\frac{4000}{r^3}$$

```
subs(d2S,r,rmin) % d2S(0)>0, so r=(500/pi)^(1/3) is a minimum
```

ans = $12\,\pi$

- Output the dimensions (radius and height) of the optimal cylindrical can.

```
height = subs(hV,r,rmin)
```

height =

$$\frac{1000}{\pi\,\left(\frac{500}{\pi}\right)^{2/3}}$$

```
height = vpa(height)
```

height = 10.838521402785780174891227296593

```
radius = rmin
```

radius =

$$\left(\frac{500}{\pi}\right)^{1/3}$$

```
radius = vpa(rmin)
```

radius = 5.4192607013928900874456136482964

Part B

Here we implement the Bailey-Borwein-Plouffe (BBP) series to calculate π in Matlab using variable precision arithmetic (VPA). The BBP formula for π is given by:

$$\pi = \sum_{k=0}^{\infty} \left(\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \right)$$

This series allows for the calculation of π to high precision with fewer iterations to many other series.

Tasks:

- Write a Matlab function that calculates π using the BBP series. The function should take two arguments: the number of terms to sum, N, and the precision, P, for VPA.

- Calculate π using your function with $N = 50$ terms and $P = 100$ digits of precision.

- Output the approximation of π that you found. Find the error in your result by subtracting your computed value from the true value of π.

```
mypie = calculate(50,100)
```

mypie =
3.1415926535897932384357116792180407068930145885895449907274621769477808848862705310957919697

```
error = vpa(pi - mypie)
```

error =
0.00000000000000000002693170406146217730415481078556083024748241536003552139993846753224285550

```
function pie = calculate(N,P)
    % set the precision
    digits(P);
    % initialize the sum with precision
    pie = vpa(0);
    for k=0:N-1
        pie = pie + vpa(((1/(16^k)) * ...
            ((4/(8*k+1)) - (2/(8*k+4)) - (1/(8*k+5)) - (1/(8*k+6)))));
    end
end
```