Totient Function

What is Euler's Totient Function?

Number theory is one of the most important topics in the field of Math and can be used to solve a variety of problems. Many times one might have come across problems that relate to the prime factorization of a number, to the divisors of a number, to the multiples of a number and so on.

Euler's Totient function is a function that is related to getting the number of numbers that are coprime to a certain number X that are less than or equal to it. In short , for a certain number X we need to find the count of all numbers Y where $gcd(X, Y) = 1$ and $1 \le Y \le X$.

A naive method to do so would be to **Brute-Force** the answer by checking the gcd of X and every number less than or equal to X and then incrementing the count whenever a GCD of 1 is obtained. However, this can be done in a much faster way using Euler's Totient Function.

According to Euler's product formula, the value of the Totient function is below the product over all prime factors of a number. This formula simply states that the value of the Totient function is the product after multiplying the number N by the product of $(1 - (1/p))$ for each prime factor of N.

So,
$\phi(n) = n \prod_{p \text{ prime } p|n} \left(1 - \frac{1}{p}\right)$

Algorithm steps:

- Generate a list of primes.
- While dealing with a certain N, check and store all the primes that perfectly divide N.
- Now, it is just needed to use these primes and the above formula to get the result.

Implementation:

```
set<> primes;
static void mark(int num,int max,int[] arr)
{
    int i=2,elem;
    while((elem=(num*i))<=max)
    {
        arr[elem-1]=1;
        i++;
    }
}
GeneratePrimes()
{
    int arr[max_prime];
    for(int i=1;i<arr.length;i++)
    {
        if(arr[i]==0)
        {
            list.add(i+1);
            mark(i+1,arr.length-1,arr);
        }
    }
}
main()
{
    GeneratePrimes();
    int N=nextInt();
    int ans=N;
    for(int k:set)
    {
```

```
            if(N%k==0)
            {
                ans*=(1-1/k);
            }
        }
        print(ans);
}
```

There are a few subtle observations that one can make about Euler's Totient Function.

- The sum of all values of Totient Function of all divisors of N is equal to N.
- The value of Totient function for a certain prime P will always be $P-1$ as the number P will always have a GCD of 1 with all numbers less than or equal to it except itself.
- For 2 number A and B, if $GCD(A, B) == 1$ then $Totient(A) \times Totient(B)$ = $Totient(A \cdot B)$.

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