FIITJEE

ALL INDIA TEST SERIES

PART TEST – II

JEE (Main)-2018-19

TEST DATE: 11-11-2018

Time Allotted: 3 Hours **Maximum Marks: 360**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each part has only one section: **Section-A**.

Section-A (01 – 30, 31 – 60, 61 – 90) *contains 90 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

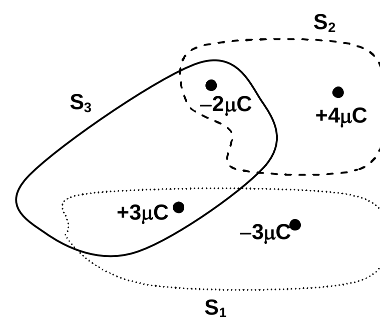
Physics

PART – I

SECTION – A
(One Options Correct Type)

This section contains **30 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. An array of point charges and a set of closed Gaussian surfaces S_1, S_2, and S_3 are illustrated in the figure. Choose the correct statement:



(A) Flux passing through S_1 is $\frac{1\mu C}{\varepsilon_0}$

(B) Flux passing through S_2 is $\frac{2\mu C}{\varepsilon_0}$

(C) Flux passing through S_3 is $\frac{4\mu C}{\varepsilon_0}$

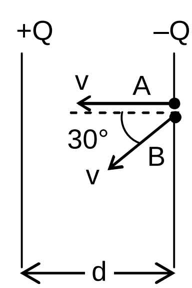
(D) Flux passing through S_1 is $\frac{2\mu C}{\varepsilon_0}$

Ans. B

Sol. Gauss' Law

flux = $\frac{q_{enclosed}}{\varepsilon_0}$

2. Two electrons enter a region at same point between charged capacitor plates with equal speed v. Electron A is moving horizontally to the left while electron B's velocity is directed at 30° below the horizontal. Each electron reaches left plate. Which one of the following choice best compares the speeds of electron upon arrival at the left plate? Ignore mutual interaction between electrons, gravity, any fringing effect of the fields and relativistic effects. Assume electric field between plates does not change due to electrons. (v_A : speed of electron A when it reaches left plate. v_B : speed of electron B when it reaches left plate.)



(A) $v_A > v_B$

(B) $v_A = v_B$

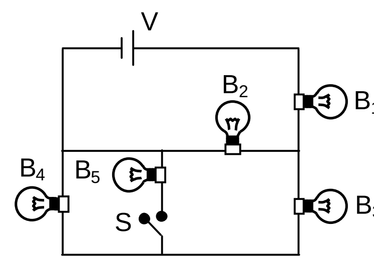
(C) $v_A < v_B$

(D) The answer depends on magnitude of the charge, Q on each plate.

Ans. **B**

Sol. Using conservation of mechanical energy for the particle field system as the particle cross the region between the plates, we write $\Delta KE + \Delta PE = 0$ where $\Delta PE = q\Delta V$. Since the potential difference between the plates will be the same for each charge, the change in PE of the charge field system is the same in each case, meaning the change in KE is the same for each charge. Hence, each charge reaches the other side with the same speed.

3. Five identical light bulbs are connected in a circuit as shown. All wires are ideal with no resistance, and the ideal battery has emf V. When the switch S in the circuit is closed, aside from bulb B_5, which of the other bulbs glow brighter than before.



(A) Only Bulb B_4

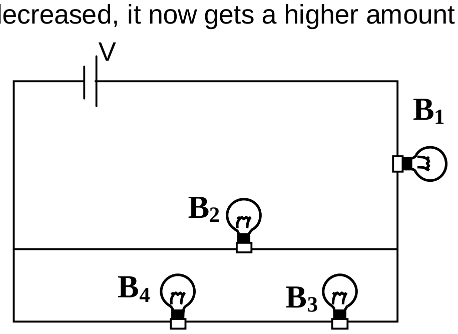
(B) Only Bulb B_1 and B_3

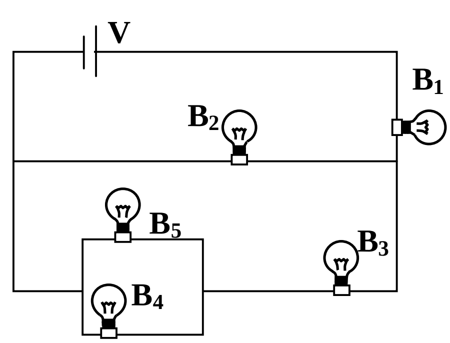
(C) Only Bulb B_3 and B_4

(D) Only Bulb B_2, B_3, and B_4

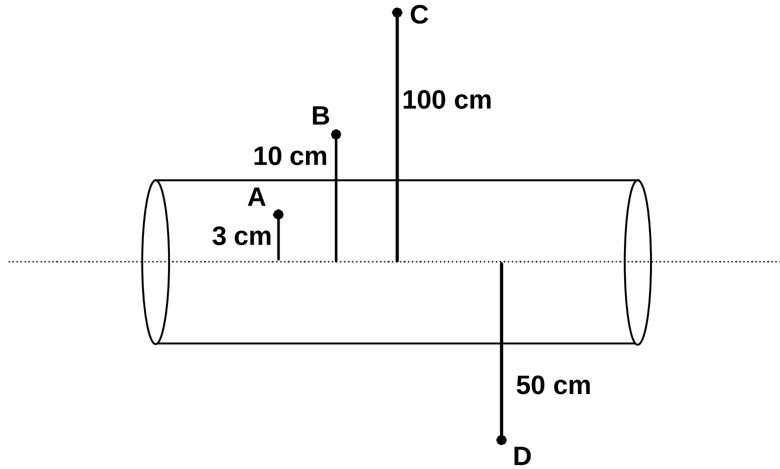
Ans. **B**

Sol. The equivalent circuit before and after the switch is closed for the resistors is shown in the figure. In words, by closing the switch the resistance of the entire circuit goes down since the resistance of the bottom branch drops from 2R to (3/2)R. Since there is less resistance in the circuit, there is more current, meaning that there is more current through bulb B_3 from Kirchhoff's loop rule with the battery and bulb B_1. Bulb B_2 is dimmer. Finally, since the resistance of the bottom branch decreased, it now gets a higher amount of current.





4. An infinite long cylindrical shell of radius 5 cm has a charge of 30 nC/m per unit axial length on its surface (uniformly distributed), as shown in the figure. Choose the **INCORRECT** statement.



(A) The electric field at point A at distance of 3 cm from axis of cylinder is 0 N/C

(B) The electric field at point B at distance of 10 cm from axis of cylinder is 5400 N/C

(C) The electric field at point C at distance of 100 cm from axis of cylinder is 540 N/C

(D) The electric field at point D at distance of 50 cm from axis of cylinder is 1800 N/C

Ans. **D**

Sol. $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{9\times10^9\times2\times30\times10^{-9}}{r} = \frac{540}{r} N/C$

5. Eight identical charged particles are located on a circle as shown in the figure-(1). Which vector shown in figure-(2), best represents the direction of net force acting on the charged particle-4 due to other charged particles.

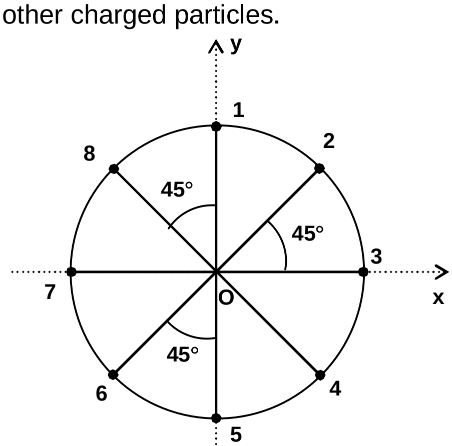


Figure 1

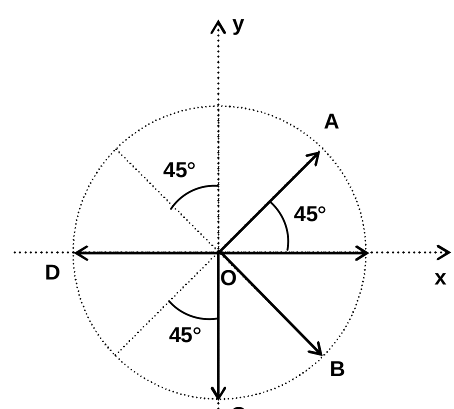


Figure 2

(A) A

(B) B

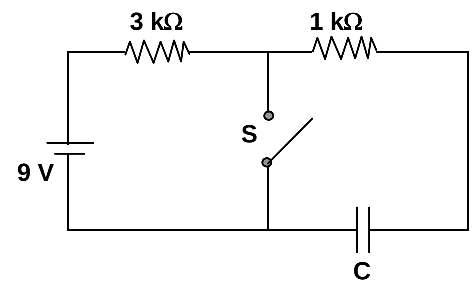
(C) C

(D) D

Ans. **B**

Sol. Since all of the charges around the circle are equal, the forces between them are repulsive. Particle 3 and 5 are at same distance away from particle-4 and repel the particle-4 with the same force in magnitude and resulting force will be along $\overrightarrow{OB}$. Likewise for particles 2 and 6 and particle 1 and 7. The force of particle-8 on particle-4 will be along $\overrightarrow{OB}$. Hence net force on particle-4 will be along $\overrightarrow{OB}$.

6. In the circuit shown, the switch S has been left open for a very long time. All circuit elements are considered to be ideal. Which one of the following statements best describes the behaviour of the current through the switch S once it is closed?



(A) The current initially is 12 mA and decreases to a steady 3 mA

(B) The current initially is 3 mA and increases to a steady 12 mA

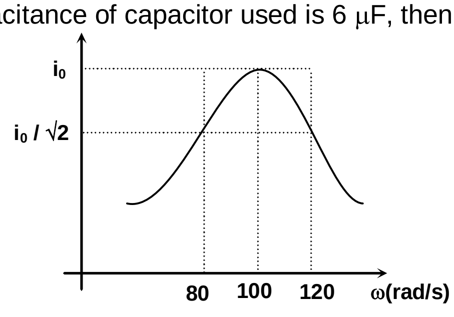
(C) The current initially is 9 mA and decreases to a steady 3 mA

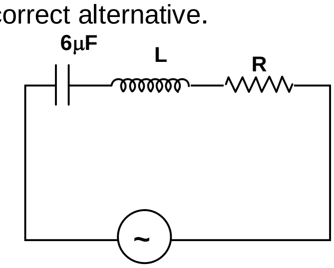
(D) The current initially is 6 mA and decreases to a steady 3 mA

Ans. **A**

Sol. Using Kirchoff loop Rule around the outside of the circuit reveals that the potential difference across the capacitor is 9V since there is no current after a long time. Once the switch is closed, there is a loop that encloses the battery and 3 kΩ resistor resulting in a total of 9 = 3000 I ⇒ I = 3 mA through the resistor. For the other branch on the right, there is a capacitor initially charged and a 1 kΩ resistor.

7. An LCR series circuit is connected to a sinusoidal AC voltage of variable angular frequency and the graph between peak current and angular frequency is plotted as shown in the figure. If capacitance of capacitor used is 6 μF, then choose the correct alternative.





(A) L = 16.67 H, R = 1.3 kΩ

(B) L = 3.34 H, R = 12 kΩ

(C) L = 16.67 H, R = 0.67 kΩ

(D) can't be calculated

Ans. C

Sol. $Q = \frac{\omega}{2\Delta\omega} = \frac{100}{40} = \frac{5}{2}$

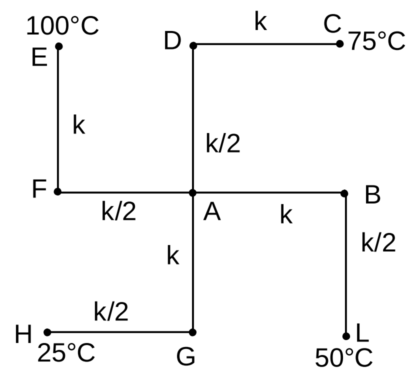
$Q = \frac{\omega_0 L}{R} \Rightarrow \frac{5}{2}\omega_0 C = \frac{1}{R}$

$R = \frac{2}{5} \times \frac{1}{100 \times 6 \times 10^{-6}} \Rightarrow R = \frac{2}{30} \times 10^4$

$= \frac{2}{3} \times 10^3 = 0.67\ k\Omega$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C} = \frac{10^6}{10^4 \times 6} = 16.67H$

8. All rods have identical length and cross-sectional area. Find the temperature at junction A. The thermal conductivity of rod-AF, rod-GH rod-BL & rod-AD are $\frac{k}{2}$ and the thermal conductivity of rod-EF, rod-CD, rod-AB & rod-AG are k.



(A) $\frac{125°C}{4}$

(B) $\frac{125°C}{2}$

(C) Zero

(D) 125°C

Ans. B

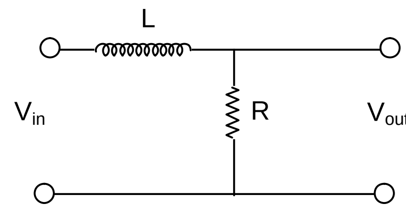
Sol. $\frac{x-100}{3R} + \frac{x-75}{3R} + \frac{x-25}{3R} + \frac{x-50}{3R} = 0$

$4x - 250 = 0$

$x = \frac{250°C}{4}$

9. An L–R circuit is made so that the AC - input (angular frequency : ω) is applied across the combination of L and R. The output is taken across R. The circuit works as a filter circuit. If the time constant of the DC (L – R) circuit is τ, then the ratio: $\left|\frac{V_{out}}{V_{in}}\right|$ equals

(where V_{out} and V_{in} are peak values of output and input voltage respectively.)



(A) $\frac{1}{\sqrt{1+(\omega\tau)^2}}$

(B) $\omega\tau$

(C) $\sqrt{1+(\omega\tau)^2}$

(D) $\frac{1}{\omega\tau}$

Ans. **A**

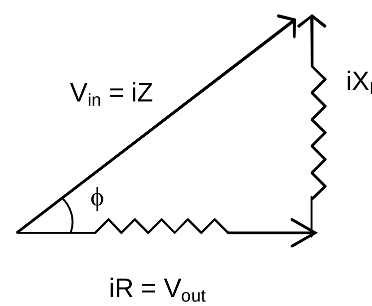
Sol. The phasor diagram of the circuit, in operation, is shown in the adjacent figure

$Z = \sqrt{R^2 + x_1^2}$,

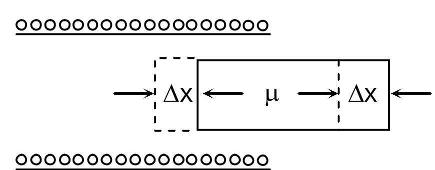
where $X_L = \omega L$

The ratio $\left|\frac{V_{out}}{V_{in}}\right|$

$= \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}}$



10. A solenoid of cross sectional area α, N turns and length ℓ carries a constant current I. An iron rod of constant permeability μ and cross sectional area α is partially inserted in the solenoid along its axis. Calculate the force acting on the iron rod. (Neglect the end effects and assume that when the iron rod is moved slightly through a distance Δx from its position, the field structure remains the same.)



(A) $\frac{(\mu - \mu_0)N^2I^2\alpha}{\ell^2}$

(B) $\frac{(\mu - \mu_0)N^2I^2\alpha}{2\ell^2}$

(C) $\frac{(\mu-\mu_0)N^2I^2\alpha}{4\ell^2}$

(D) $\frac{2(\mu-\mu_0)N^2I^2\alpha}{\ell^2}$

Ans. **B**

Sol. The only difference is that a length Δx of the rod is effectively transferred from the extreme right hand end of the rod (outside the field region) to the uniform field region within the solenoid. The difference in the magnetic energy of the two configurations is

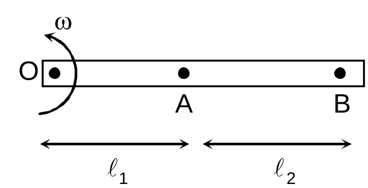
$$\Delta U = U(x+\Delta x) - U(x) = \frac{1}{2}(\mu-\mu_0)\int_v H^2 dV, \qquad \text{(i)}$$

Where $V = \alpha\Delta x$ and H = NI/ℓ = the magnetic field intensity inside the solenoid. Since H is constant, we obtain from (i)

$$\Delta U = \frac{1}{2}(\mu-\mu_0)\frac{N^2I^2}{\ell^2}\alpha\Delta x.$$

The force on the rod is $F_x = \left.\frac{\Delta U}{\Delta x}\right|_I = \frac{1}{2}(\mu-\mu_0)\frac{N^2I^2\alpha}{\ell^2}$

11. A conducting rod of length ℓ rotates about its one end with angular velocity ω. Find the potential difference between A and B (e : charge on electron, m : mass of electron).



(A) $\frac{m\omega^2(\ell^2-\ell_1^2)}{e}$

(B) $\frac{m\omega^2(\ell^2-\ell_2^2)}{e}$

(C) $\frac{m\omega^2(\ell^2-\ell_1^2)}{2e}$

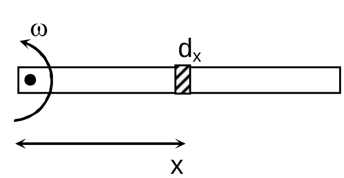
(D) $\frac{m\omega^2(\ell^2-\ell_2^2)}{2e}$

Ans. **C**

Sol. $V_{AB} = \frac{W_{AB}}{q}$

dV = $m\omega^2$xdx/e

$dV = \frac{mw^2xdx}{e}$

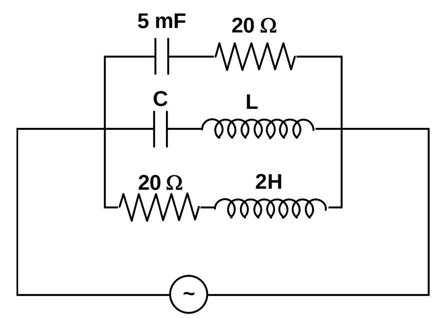


$$\int_0^{\omega} dV = \frac{m\omega^2}{e}\int_{\ell_1}^{\ell} x\,dx$$

$$V = \frac{m\omega^2}{2e}\left(\ell^2 - \ell_1^2\right)$$

$$\therefore \quad V_{AB} = \frac{m\omega^2\left(\ell^2 - \ell_1^2\right)}{2e}$$

12. In the circuit shown, find the value of C and L if net current from the source is to lead net voltage by a time $\frac{\pi}{40}$ seconds.



(A) 2.5 mF, 2H

(B) 5 mF, 2H

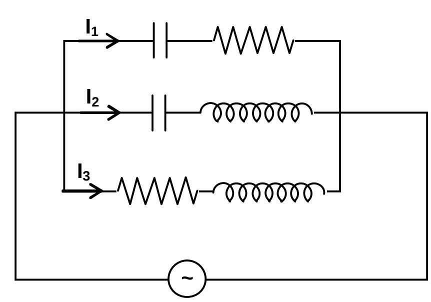
(C) 5 mF, 1 H

(D) 2.5 mF, 1 H

Ans. **A**

Sol. Phasor diagram :

To lead by $\frac{\pi}{40}\,s\left(\equiv \frac{\pi}{4}\right)$,

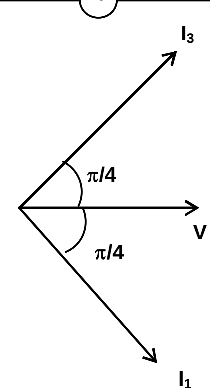


$$I_3 = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$I_2 = \frac{V}{\sqrt{(X_C - X_L)^2}}$$

$$I_1 = \frac{V}{\sqrt{R^2 + X_C^2}}$$

$$I_2 = \sqrt{I_1^2 + I_3^2}$$



13. A metallic charged ball, having charge q_0 is immersed into the liquid of dielectric constant K and low resistivity ρ. Find the time constant of disappearance of charge on the ball

(A) $\frac{1}{2}\varepsilon_0\rho K$

(B) $\varepsilon_0\rho K$

(C) $2\varepsilon_0\rho K$

(D) $\frac{3}{2}\varepsilon_0\rho K$

Ans. B

Sol. As we know that

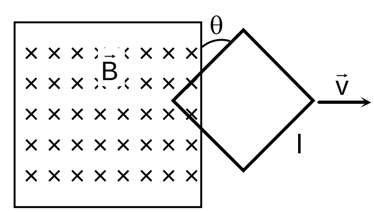
$$J = \frac{E}{\rho} = \frac{q}{4\pi\varepsilon_0 Kr^2\rho}$$

$$I = s \times J = 4\pi r^2 \times \frac{q}{4\pi\varepsilon_0 Kr^2\rho} = \frac{q}{\varepsilon_0 K\rho}$$

$$\Rightarrow \quad I_0 = \frac{q_0}{\varepsilon_0 K\rho}$$

$$\tau = \frac{q_0}{I_0} = \varepsilon_0 K\rho$$

14. An external force is applied to move a square loop of dimension 5 cm × 5 cm and resistance 10Ω at a constant speed across a region of uniform magnetic field B = 0.20 Tesla. The side of the square loop makes an angle θ = 45° with the boundary of the field region, as shown in figure. At t = 0, the loop is completely inside the field region, with its right vertex at the boundary. Total charge flown through loop when loop is completely outside the magnetic region.



(A) 0.05 milli coulomb

(B) 0.10 milli coulomb

(C) 0.4 milli coulomb

(D) 1 milli coulomb

Ans. A

Sol. $\Delta Q = \frac{\Delta\phi}{R} = \frac{Ba^2}{R}$

15. Two bar magnets having same geometry with magnetic moments 1.5 M and 2.5 M are placed in such a way that their similar poles are on the same side, and its time period of oscillations is T_1. Now if the polarity of one of the magnets is reversed, keeping other quantities same, then the time period of oscillation is T_2. Choose the correct option.

(A) $\frac{T_1}{T_2} = \frac{1}{2}$

(B) $\frac{T_1}{T_2} = \frac{2}{1}$

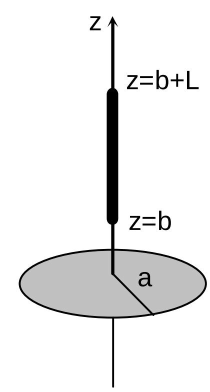
(C) $\frac{T_1}{T_2} = \frac{1}{4}$

(D) $\frac{T_1}{T_2} = \frac{4}{1}$

Ans. **A**

Sol. $T \propto \frac{1}{\sqrt{M}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{2.5M - 1.5M}{2.5M + 1.5M}} = \frac{1}{2}$

16. A uniform charged (thin) non-conducting rod is located on the central axis a distance b from the center of an uniformly charged non-conducting disk. The length of the rod is L and has a linear charge density λ. The disk has radius a and a surface charge density σ. The electrostatic force on the rod due to disc is



(A) $\vec{F} = \frac{\lambda\sigma}{2\varepsilon_0}\left(L + \sqrt{a^2 + b^2} - \sqrt{(b+L)^2 + a^2}\right)\hat{k}$

(B) $\vec{F} = \frac{\lambda\sigma a^2 L}{4\varepsilon_0 b^2}\hat{k}$

(C) $\vec{F} = \frac{\lambda\sigma a^2 L}{8\varepsilon_0 b^2}\hat{k}$

(D) $\vec{F} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{L}{\sqrt{L^2 + a^2}}\right)\hat{k}$

Ans. **A**

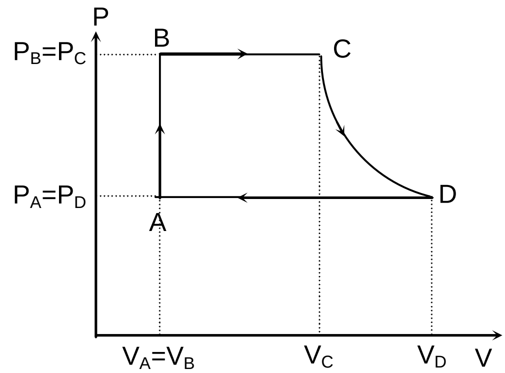
Sol. The electric field created at any point along the central axis is given by

$$\vec{E}(z) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right)\hat{k}$$

Breaking up the rod into an infinite number of infinitesimally small point charges dq, we have that the net force on each tiny charge is $d\vec{F} = dq\vec{E}(z)$. Summing up all these contributions, and using the fact that dq = λ dz gives

$$\vec{F} = \int dq \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{a^2+z^2}}\right)\hat{k} = \frac{\lambda\sigma}{2\varepsilon_0}\hat{k}\int_b^{b+L} dz\left(1 - \frac{z}{\sqrt{a^2+z^2}}\right)$$
$$= \frac{\lambda\sigma}{2\varepsilon_0}\left(L - \sqrt{a^2+(b+L)^2} + \sqrt{a^2+b^2}\right)\hat{k}$$

17. A cycle followed by an engine (using one mole of an ideal gas in a cylinder with a piston) is shown in figure. Heat exchanged by the engine, with the surroundings for each section of the cycle is (C_v = (3/2) R).
AB : constant volume
BC : constant pressure
CD : adiabatic
DA : constant pressure



(A) $Q_{AB} = \frac{5}{2}V_A(P_B - P_A)$, $Q_{BC} = (3/2)P_B(V_C - V_A)$, $Q_{CD} = 0$, $Q_{DA} = (5/2)P_A(V_A - V_D)$

(B) $Q_{AB} = \frac{3}{2}V_A(P_B - P_A)$, $Q_{BC} = (5/2)P_B(V_C - V_A)$, $Q_{CD} = 0$, $Q_{DA} = (5/2)P_A(V_A - V_D)$

(C) $Q_{AB} = \frac{3}{2}V_A(P_B - P_A)$, $Q_{BC} = (5/2)P_B(V_C - V_A)$, $Q_{CD} = 0$, $Q_{DA} = (3/2)P_A(V_A - V_D)$

(D) $Q_{AB} = \frac{5}{2}V_A(P_B - P_A)$, $Q_{BC} = (5/2)P_B(V_C - V_A)$, $Q_{CD} = 0$, $Q_{DA} = (5/2)P_A(V_A - V_D)$

Ans. **B**

Sol. $Q_{AB} = U_{AB} = \frac{3}{2}R(T_B - T_A) = \frac{3}{2}V_A(P_B - P_A)$

$Q_{BC} = U_{BC} + W_{BC}$

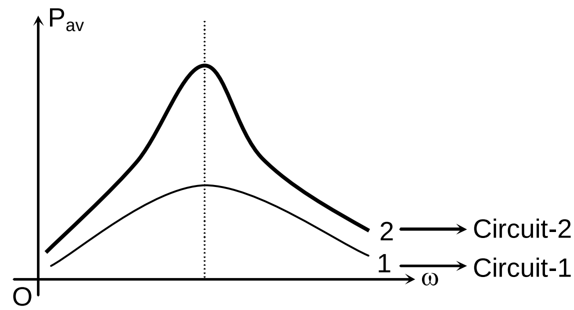
$= (3/2)P_B(V_C - V_B) + P_B(V_C - V_B)$

$= (5/2)P_B(V_C - V_A)$

$Q_{CA} = 0$

$Q_{DA} = (5/2)P_A(V_A - V_D)$

18. The figure shows the variation of average power versus angular frequency of source voltage of two series. LCR-Circuits connected to same voltage source (the circuit-1 has resistance R_1, capacitance C_1, inductance L_1 and quality factor Q_1 while circuit-2 has resistance R_2, capacitance C_2, inductance L_2 and quality factor Q_2 respectively.
Choose the correct option:



(A) $Q_1 < Q_2$ and $R_1 < R_2$, $L_1L_2 = C_1C_2$

(B) $Q_1 > Q_2$ and $R_1 < R_2$, $L_1C_2 = L_2C_1$

(C) $Q_1 < Q_2$ and $R_1 > R_2$, $L_1C_1 = L_2C_2$

(D) $Q_1 > Q_2$ and $R_1 > R_2$, $L_1 / L_2 = C_1 / C_2$

Ans. C

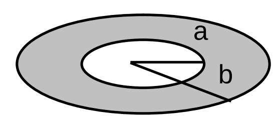
Sol. Quality Factor = $Q = \frac{\omega}{2\Delta\omega}$

$Q = \frac{\omega_0 L}{R}$ $Q_1 < Q_2$

$(P_{av})_{max} = \frac{V_0}{R} \Rightarrow R_1 > R_2$

$\omega_{10} = \omega_{20} \Rightarrow L_1C_1 = L_2C_2$

19. The figure shows a non-conducting (thin) disk with a hole. The radius of the disk is b and the radius of the hole is a. A total charge Q is uniformly distributed on its surface. Assuming that the electric potential at infinity is zero, what is the electric potential at the center of the disk?



(A) $\frac{Q}{2\pi\varepsilon_0(b+a)}$

(B) $\frac{Q}{2\pi\varepsilon_0(b-a)}$

(C) $\frac{Q}{2\pi\varepsilon_0(b^2-a^2)}$

(D) 0

Ans. A

Sol. The potential produced by a charged disk of radius R, at a distance z from it, along its central axis was

$$\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-z\right) \quad \ldots(1)$$

By superposition, we can think of the potential created by disk with the hole as the sum of two disks, with the same but opposite surface densities:

$$V(z)=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+b^2}-z\right)-\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+a^2}-z\right)=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+b^2}-\sqrt{z^2+a^2}\right)$$

Since we are only interested at the center, we have

$$V(0)=\frac{\sigma}{2\varepsilon_0}(b-a)$$

The total area of the disk with the hole is $A=\pi\left(b^2-a^2\right)$, thus

$$\sigma=\frac{Q}{\pi\left(b^2-a^2\right)}$$

Finally

$$V(0)=\frac{1}{2\varepsilon_0}(b-a)\frac{Q}{\pi\left(b^2-a^2\right)}=\frac{2kQ(b-a)}{(b-a)(b+a)}=\frac{2kQ}{b+a}$$

20. Choose the **INCORRECT** Statement

(A) The paramagnetic material displays greater magnetization when cooled, for the same magnetizing field.

(B) The diamagnetic property of material is almost independent temperature.

(C) If a toroid uses bismuth for its core, the magnetic field in the core will be slightly greater than when core is empty.

(D) The permeability of a ferromagnetic material depends on the magnetic field.

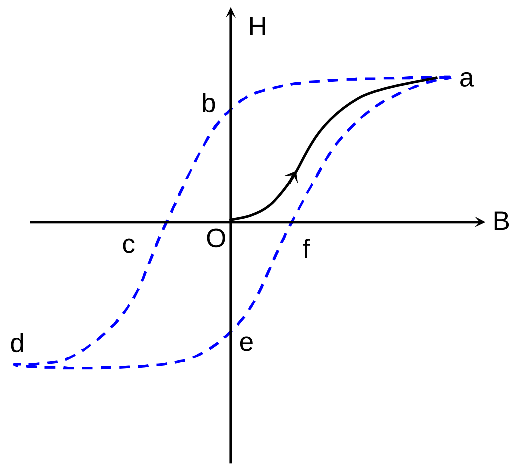
Ans. **C**

Sol. (A) On cooling, the tendency of thermal agitations to disrupt the alignment of magnetic dipoles decreases in case of paramagnetic materials. These they display greater amount of magnetism.

(B) on placing a sample of diamagnetic material in magnetic field, the magnetization (Induced dipole moment) is opposite direction of magnetizing field. Thus it does not affected by temperature.

(C) The field in core will be slightly less than when core is empty as Bismuth is a diamagnetic material ($B = \mu_0\mu_r\, nI$).

(D) $\mu = \frac{B}{H}$, so permeability of ferromagnetic material is dependent on the applied magnetic field. As shown in graph for B & H, B is larger for smaller value of H, thus permeability is greater for lower fields.



21. If 100 gm water at 20°C is mixed with 100 gm ice at –20°C. Let in final state system consists of m_1 gm ice and m_2 gm water. Choose the correct option.
Specific heat capacity of ice is 0.5 cal/g/°C.
Specific heat capacity of water is 1 cal/g/°C.
Latent heat of fusion of water is a 80 cal/g.
Latent heat of vaporisation of water is a 540 cal/g.

(A) m_1 = 200 gm, m_2 = 0 gm, θ = –5°C

(B) m_1 = 87.5 gm, m_2 = 112.5 gm, θ = 0°C

(C) m_1 = 112.5 gm, m_2 = 87.5 gm, θ = 0°C

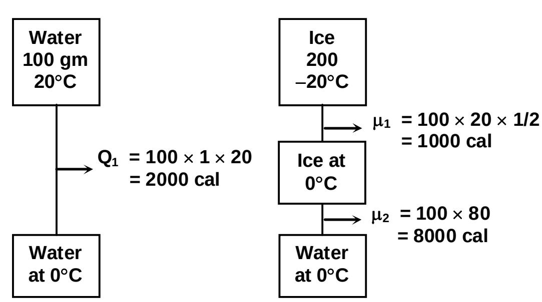
(D) m_1 = 200 gm, m_2 = 0 gm, θ = –3°C

Ans. **B**

Sol. All ice will not melt. It means in final state, the system will contain both ice and water at 0°C. Let x gm of ice melts due excess heat.

$$x = \frac{1000}{80} = 12.5\text{gm}$$

Water in final state = 112.5 gm
ice in final state = 87.5 gm

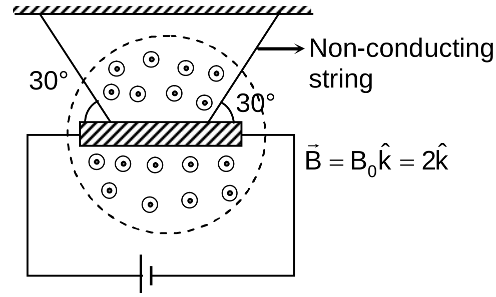


Second method : Let whole system is at 0°C in the state of water so.
Heat constant = $Q_1 - H_1 - H_2$ = –7000 cal
It means the heat available will force the water to freeze to compensate the –7000 cal (negative heat). Let x gm of water to fulfil the condition so

$$x = \frac{7000}{80} = 87.5 \text{ gm}$$

Water = 112.5 gm

22. A conducting rod of mass = 1 kg, length = 1m suspended in uniform magnetic field $\vec{B} = (2T)\hat{k}$. The rod is attached with two different string as shown in the figure. Tension in each string is 20 N. Find the value of current in the wire. (g = 10 m/s^2).



(A) 10 Amp

(B) 5 Amp

(C) 2.5 Amp

(D) 0.5 Amp

Ans. **B**

Sol. $2T\sin 30° = B_0 i\ell + mg$

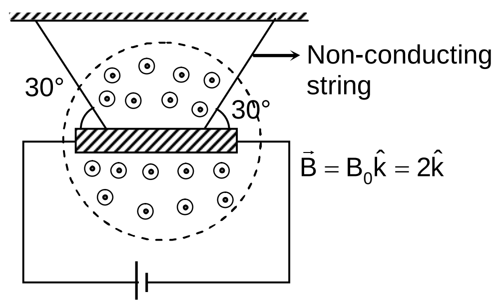
$\Rightarrow \quad T = B_0 i\ell + mg$

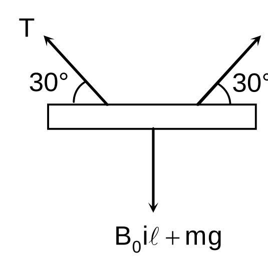
$\Rightarrow \quad 20 = B_0 i\ell + 10$

$\Rightarrow \quad B_0 i\ell = 10$

$\Rightarrow \quad i = \dfrac{10}{B_0 \times \ell} = \dfrac{10}{B_0}$

= 5A.





23. An ideal mutual inductor is made from a primary coil of inductance 5mH and a secondary coil of inductance 10 mH. Find the value of the Mutual inductance (Approximately).
Assume no flux leakage.

(A) 28 mH

(B) 21 mH

(C) 14 mH

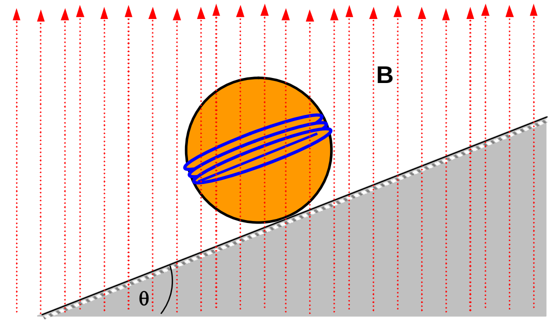
(D) 7 mH

Ans. **D**

Sol. $M = \sqrt{L_p L_s} = \sqrt{5 \times 10^{-3} \times 10 \times 10^{-3}} = \sqrt{50} \times 10^{-3}H$ = 7 mH

24. A non-conducting sphere has mass m = 88 g and radius r = 20.0 cm. A flat compact coil of wire with 5 turns is wrapped tightly around it, with each turn concentric with the sphere, as shown. The sphere is placed on a rough inclined plane that slopes downward to the left, making an angle θ with the horizontal, so that the coil is parallel to the inclined plane. A uniform magnetic field of 0.350 T vertically upward exists in the region of the sphere.

Take $\pi = \frac{22}{7}; g = 10\,m/s^2$.



(A) The current in the coil, that will enable the sphere to rest in equilibrium on the inclined plane is 2.5 A.

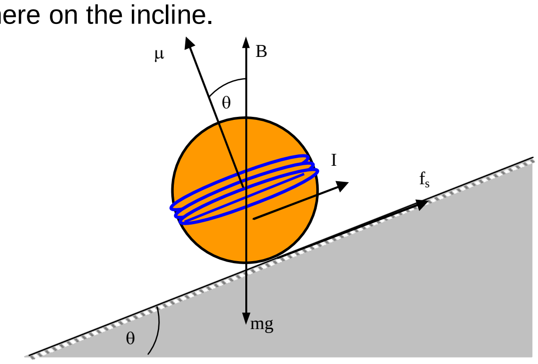
(B) The current in the coil, that will enable the sphere to rest in equilibrium on the inclined plane is 1.25 A.

(C) The current in the coil, that will enable the sphere to rest in equilibrium on the inclined plane is 0.8 A.

(D) The current in the coil, that will enable the sphere to rest in equilibrium on the inclined plane is 0.6 A.

Ans. C

Sol. The diagram shows a free-body diagram for the sphere on the incline.
We will use the following notation: f is the force of static friction; B is the external magnetic filed given as B = 0.350 T; I is the unknown current in the coil; μ is the magnetic moment of the current in the coil; m= 0.088 kg is the mass of the sphere; g = 10 m/s^2 is the free fall acceleration; r = 0.200 m is the radius of the sphere; and N = 5 is the number of turns in the coil. We will use everywhere three significant digits as given in the problem statement.



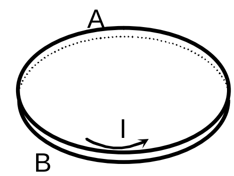
The sphere is in translational equilibrium, thus
$f_s - mg\sin\theta = 0$ ……………….. (1)
The sphere is in rotational equilibrium. If torques are taken about the center of the sphere, the magnetic field produces a clockwise torque of magnitude
μB sinθ, and the frictional force a counter-clockwise torque of magnitude f_s R. Thus:
f_s R – μBsinθ = 0 …………………. (2)

From (1): fs= Mg sinθ. Substituting this in (2) and cancelling out sinθ , one obtains

μB = MgR $\Rightarrow I = \frac{mgr}{N\pi r^2 B} = \frac{mg}{N\pi rB} = \frac{\left(88\times10^{-3}\right)\times10}{5\times\left(22/7\right)\times0.2\times.35} = 0.8\,\text{A}$

The current must be counter-clockwise as seen from above.
The result does not depend upon the angle of inclination θ.

25. Two circular loops A and B have their planes parallel to each other, as shown in figure. Loop A has a current flowing in the counterclockwise direction, viewed from above.



(A) If the current in loop A decreases with time, the two loops attract each other.

(B) If the current in loop A increases with time, the two loops attract each other

(C) If current in loop A decreases current in B is clockwise.

(D) If current in loop A increases, current in B will be anticlockwise.

Ans. **A**

Sol. Apply Lenz's law

26. If magnetic dip in India is found to be 18°. One of the option, that is closer to the magnetic dip in Britain, is

(A) 18°

(B) 70°

(C) –18°

(D) 16°

Ans. **B**

Sol. Britain is closer to magnetic north.

27. In the system of three charged particles, each charge particle is in equilibrium under their electrostatic forces only. Choose the **INCORRECT** option

(A) The particle must be collinear.

(B) All the charges cannot have the same magnitude.

(C) All the charges cannot have the same sign.

(D) The equilibrium is stable.

Ans. **D**

Sol. Basic concept

28. In a moving coil galvanometer the number of turns N = 30, area of the coil $A = 4 \times 10^{-3}\ m^2$ and the magnetic field strength B = 0.3T. (The resistance per unit length of the wire of coil is λ). Choose the **INCORRECT** option

(A) To increase its voltage sensitivity by 50% we increase number of turns to 60

(B) To increase its voltage sensitivity by 50% we increase area to $9 \times 10^{-3}\ m^2$

(C) To increase its voltage sensitivity by 50% we increase magnetic field to 0.45 T

(D) To increase its voltage sensitivity by 50% we change the material of wire such that its specific resistance would be $2/3^{rd}$ of the specific resistance of the present wire.

Ans. A

Sol. BINA = Cθ

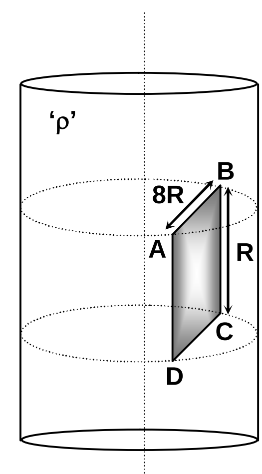
$$I = \frac{C\theta}{BNA}$$

$$V = IR = \frac{C\theta R}{BNA} \quad \ldots(1)$$

$$R = \frac{\rho\ell}{a} = \lambda\ell = C_1\lambda\sqrt{A}\,N$$ where C_1 is a constant.

$$V = \frac{\lambda\theta C_1 C}{B\sqrt{A}} \Rightarrow S_V = \frac{d\theta}{dV} = \frac{B\sqrt{A}}{\lambda C C_1}.$$

29. A non conducting solid cylinder of infinite length having uniform charge density ρ and radius of cylinder is 5R. Find the flux passing through the surface ABCD as shown in figure.



(A) $\frac{12\rho R^3}{\varepsilon_0}$

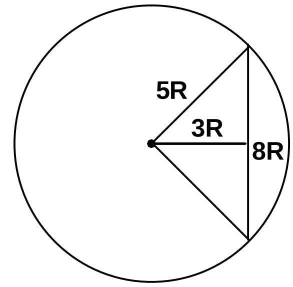
(B) $\frac{6\rho R^3}{\varepsilon_0}$

(C) $\frac{\rho R^3}{\varepsilon_0}$

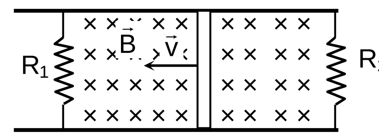
(D) $\frac{3\rho R^3}{\varepsilon_0}$

Ans. A

Sol. $\phi = \dfrac{q_{in}}{\varepsilon_0} = \dfrac{\rho\left(\dfrac{1}{2}8R\cdot 3R\right)R}{\varepsilon_0} = \dfrac{12\rho R^3}{\varepsilon_0}$



30. A conducting rod of length 25 cm is free to slide on two parallel conducting bars as shown in the figure. In addition, two resistors 5Ω and 10Ω are connected across the ends of the bars. There is a uniform magnetic field 0.02 tesla pointing inside the plane of paper. Suppose an external agent pulls the bar towards the left at a constant speed 5 ms^{-1}.



Sliding rod

(A) Current through 5Ω resistor is 5 mA

(B) Current through 10 Ω resistor is 10 mA

(C) Force applied by agent to maintain constant 5 ms^{-1} velocity is 20 N

(D) Power delivered by external agent is 50 watt.

Ans. **A**

Sol. $B\ell v = iR$

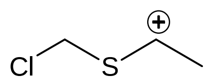
$0.02 \times \dfrac{1}{4} \times 5 = i \times 5$

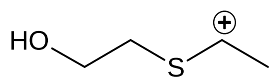
Chemistry PART – II

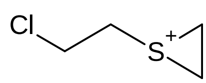
SECTION – A
Straight Objective Type

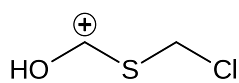
This section contains **30 multiple choice** questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

31. Hydrolysis of mustard gas give HCl and a diol. Which of the following intermediate will form during the hydrolysis

(A) 

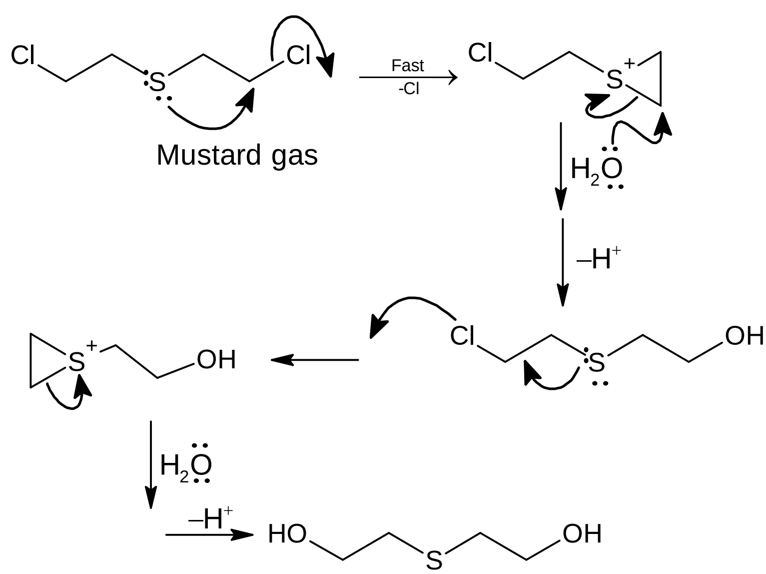
(B) 

(C) 

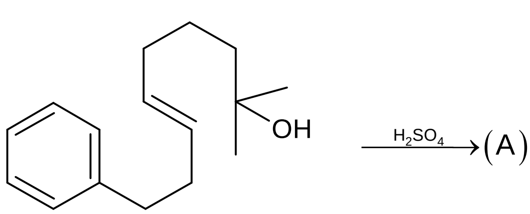
(D) 

Ans. **C**

Sol.

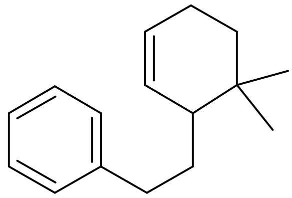


32.

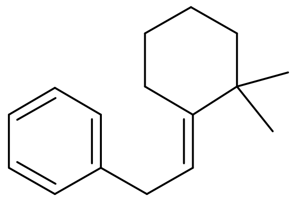


Major product formed in the above reaction (A) is:

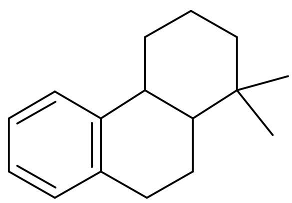
(A)



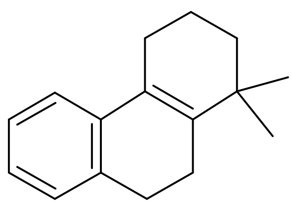
(B)



(C)

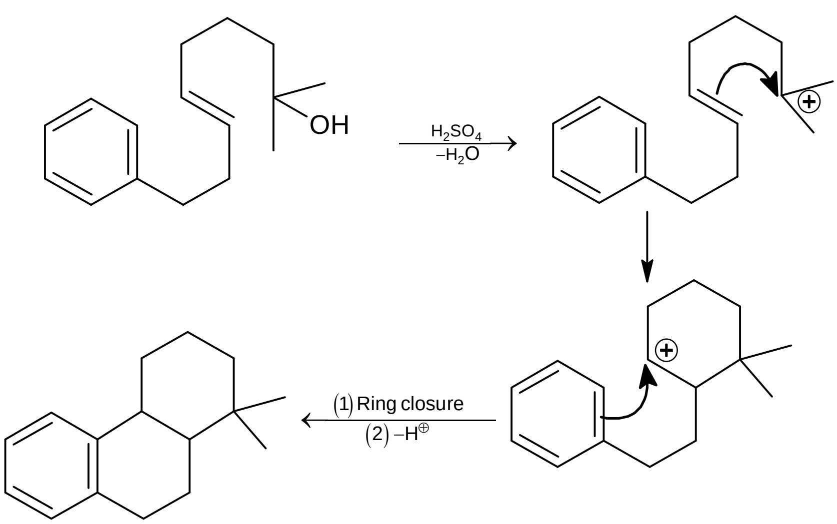


(D)

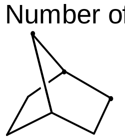


Ans. **C**

Sol.



33. Number of monochloro structural isomers form when A undergoes monchlorination

 $\xrightarrow[\text{(Monochlorination)}]{Cl_2/h\nu}$ Products

(A) 1

(B) 2

(C) 3

(D) 7

Ans. **C**

34. Which of the following is correct?

(A) An achiral molecule which is superimposable on its mirror image cannot exist as two enantiomers.

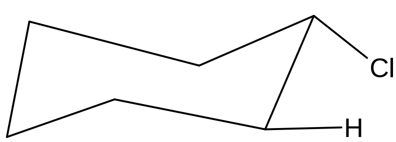
(B) Changing the configuration of a molecule always means that bonds are broken.

(C) Both (A) and (B)

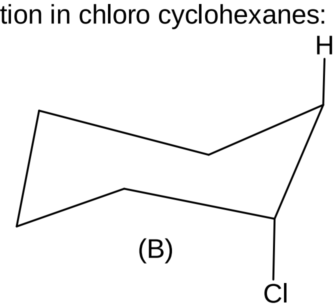
(D) None of these

Ans. **C**

35. Which of the following correct for E2 elimination in chloro cyclohexanes:



(A)



(B)

(A) Rate of E_2 elimination in (A) is faster than (B).

(B) Rate of E_2 elimination in (B) is faster than (A).

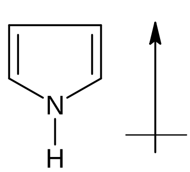
(C) (B) can undergo E_2 elimination only if (B) undergoes ring inversion.

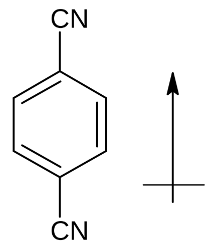
(D) Both (A) and (B) are correct.

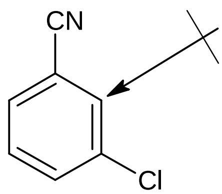
Ans. **B**

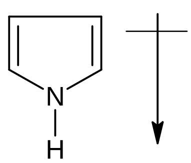
36. Which of the following molecule showing the correct direction of dipole moment:

(A)



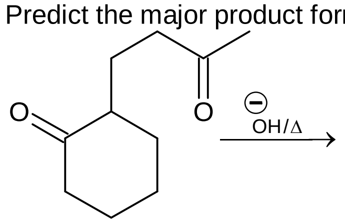
(B) 

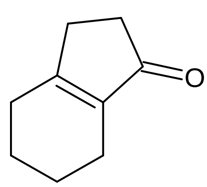
(C) 

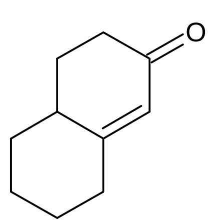
(D) 

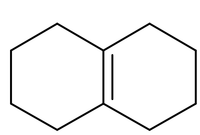
Ans. A

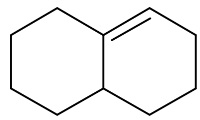
37. Predict the major product formed when the following reaction occurs in dilute base:



(A) 

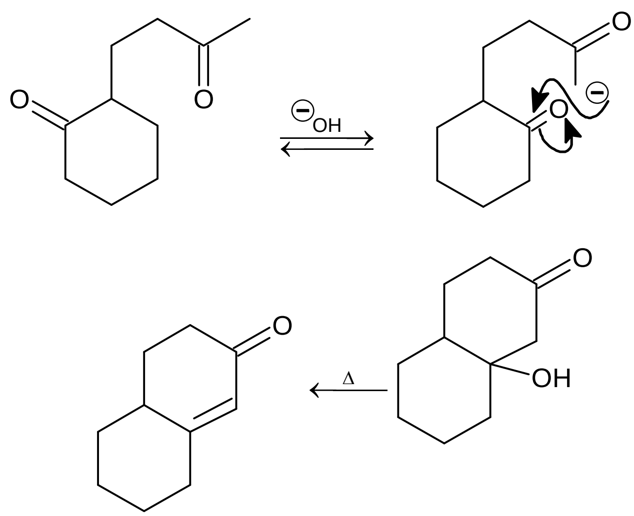
(B) 

(C) 

(D) 

Ans. **B**

Sol.



38. Propanol is more volatile as compared to glycerol because of

(A) Less extent of hydrogen bonding

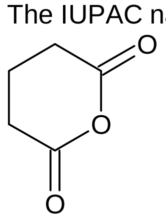
(B) High molar mass of propanol

(C) Hybridization

(D) All of the above

Ans. **A**

39. The IUPAC name of the following compound is:



(A) Cyclopentanedioic anhydride

(B) Pentanedicarboxylic anhydride

(C) Pentanedioic anhydride

(D) Dicyclopentane anhydride

Ans. **C**

40. Predict the product of the following reaction:

$$\underset{1:1}{RCOOAg + I_2} \longrightarrow (A) + CO_2 + AgI$$

(A) Alkyl iodide

(B) Carboxylic acid

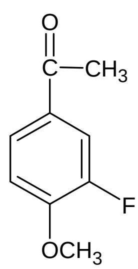
(C) Ester

(D) Alcohol

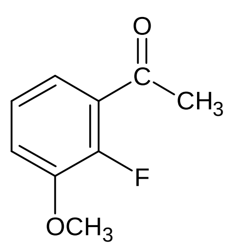
Ans. A

41. Which of the following compound will form when acetic anhydride reacts with o-Fluoroanisole in presence of dry $AlCl_3$.

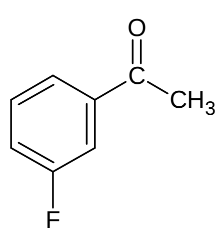
(A)



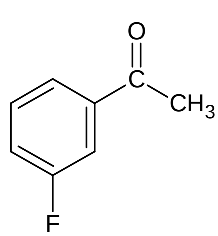
(B)



(C)

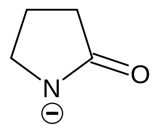


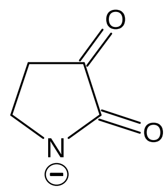
(D)

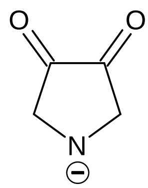


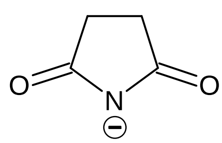
Ans. A

42. Which of the following species is most stable?

(A) 

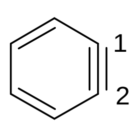
(B) 

(C) 

(D) 

Ans. D

43. The hybridization of C-1 and C-2 carbon atoms marked in the following structure of benzyne intermediate?



(A) sp^2 and sp^2

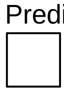
(B) sp and sp

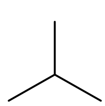
(C) sp and sp^2

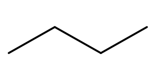
(D) None of these

Ans. A

44. Predict the major product formed in the following reaction:

 $+ H_2 \xrightarrow[200^0 C]{Ni}$

(A) 

(B) 

(C) C_2H_6

(D) No reaction

Ans. **B**

45. $\text{Alcohol} \xrightarrow[\text{(ii) AgNO}_2]{\text{(i) I}_2\text{/Red P}} (\text{A})$

Compound A does not react with HNO_2 and gives a colourless solution with NaOH.
Degree of the alcohol taken initially

(A) 1

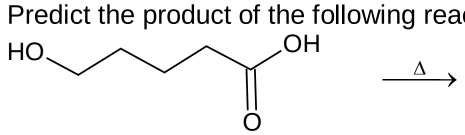
(B) 2

(C) 3

(D) None of these

Ans. **C**

46. Predict the product of the following reaction:



(A) Lactone

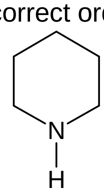
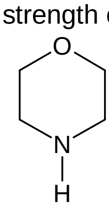
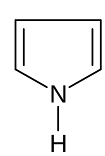
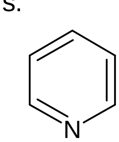
(B) Anhydride

(C) Alcohol

(D) Alkene

Ans. **A**

47. The correct order of basic strength of the following compound is:

(1)  (2)  (3)  (4) 

(A) 1 > 2 > 3 > 4

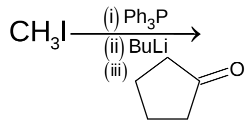
(B) 1 > 3 > 2 > 4

(C) 4 > 3 > 2 > 1

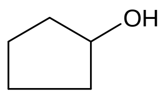
(D) 1 > 2 > 4 > 3

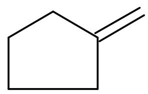
Ans. **D**

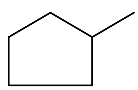
48. Predict the product of the following reaction?



(A) CH_3CHO

(B) 

(C) 

(D) 

Ans. C

49. Which mechanism the following reaction will follows:

$$C_2H_5OH \xrightarrow[\text{Pyridine}]{SOCl_2} C_2H_5Cl + SO_2$$

(A) S_N1

(B) S_N2

(C) S_Ni

(D) None of the above

Ans. B

50. In the following reaction

$$CH_2 = CH_2 \xrightarrow{Ag_2O} (A) \xrightarrow{H^+/H_2O} (B) \xrightarrow{PCl_3} (C)$$

The compound (C) is:

(A) $CH_3 - CH_3$

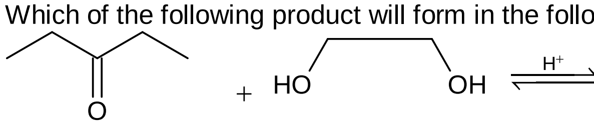
(B) $CH_3 - CH_2 - Cl$

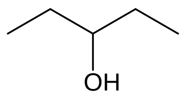
(C) $\underset{\displaystyle Cl}{\underset{|}{CH_2}} - \underset{\displaystyle Cl}{\underset{|}{CH_2}}$

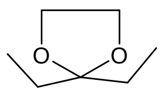
(D) $CH_3 - Cl$

Ans. C

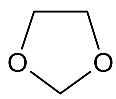
51. Which of the following product will form in the following reaction:



(A) 

(B) 

(C) $\begin{array}{c} CHO \\ | \\ CHO \end{array}$

(D) 

Ans. B

52. The possible product of the following reaction:

$$H_3C-CH_2-COOH \xrightarrow[\text{(ii)}\, H_3O^+]{\text{(i)}\, CH_3MgBr}$$

(A) CH_3-CH_3

(B) CH_3CH_2OH

(C) $CH_3-CH_2-\overset{\overset{\large O}{||}}{C}-CH_3$

(D) CH_4

Ans. D

53. $CH_3CHO \xrightarrow[KCN]{NH_4Cl} (A) \xrightarrow[\text{(ii)}\ \overset{-}{O}H]{\text{(i)}\ H_2O,\, HCl,\, \Delta} (B)$

Compound B formed in the above reaction is:

(A) Glycine

(B) Alanine

(C) Threonine

(D) Serine

Ans. B

54. The only two pK_a values of an amino acid are 2.32 and 9.62. The isoelectric point of the amino acid is:

(A) 7.3

(B) 11.94

(C) 5.97

(D) 9.62

Ans. **C**

55. Which of the following products will form when cellulose undergoes complete hydrolysis:

(A) D-fructose

(B) D-ribose

(C) L-glucose

(D) D-glucose

Ans. **D**

56. Neoprene is prepared by polymerization of:

(A) 2-Chloro-1,3-butadiene

(B) Butadiene

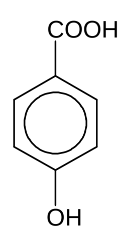
(C) Acrylonitrile

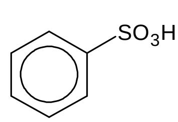
(D) Styrene

Ans. **A**

57. An organic compound 'X' gives CO_2 gas on reaction with $NaHCO_3$. The compound 'X' is:

(A) Picric acid

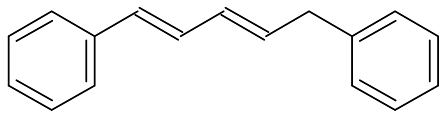
(B) 

(C) 

(D) All of these

Ans. **D**

58. The degree of unsaturation in the following compound is:



(A) 8

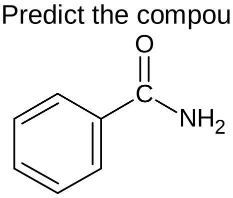
(B) 10

(C) 2

(D) 6

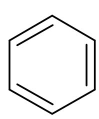
Ans. B

59. Predict the compound 'X' in the following reaction:

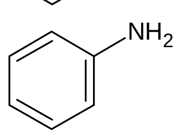


$$+Br_2 + NaOH \xrightarrow{\Delta} X + Na_2CO_3 + 2NaBr + 2H_2O$$

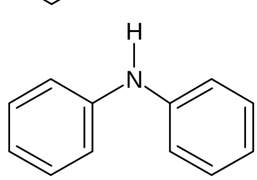
(A)



(B)



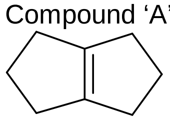
(C)



(D) CH_3NH_2

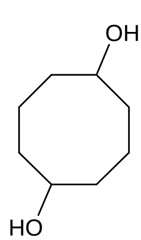
Ans. B

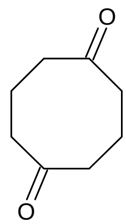
60. Compound 'A' on reductive ozonolysis will give:

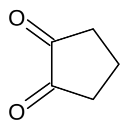


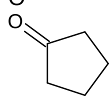
$$\xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i) } O_3}$$

(A)



(B) 

(C) 

(D) 

Ans. **B**

Mathematics

PART – III

SECTION – A

Straight Objective Type

This section contains **30 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which only **ONE** is correct

61. Normals having slope m_1, m_2, m_3; ($m_1 < 0$) are drawn at points P(a, b), Q, R respectively on the curve $y^2 - 6y - 16x + 73 = 0$ so as to intersect at point S(19, 6), then a + b is equal to

(A) 24

(B) 28

(C) 32

(D) 36

Ans. **B**

Sol. Roots of the equation $4m^3 - 7m + 3 = 0$ are m_1, m_2, m_3 solving $m_1 = -\frac{3}{2}$

$\therefore\ a = 4\left(1 + \frac{9}{4}\right),\ b = 3 - 8\left(-\frac{3}{2}\right)$

a = 13, b = 15

62. The locus of the centroid of triangle PSQ, where PQ is any chord of the parabola $y^2 = 8(x + 2)$ subtending right angle at the vertex and S be its focus is also a parabola whose latus rectum is equal to

(A) $\frac{1}{3}$

(B) $\frac{4}{3}$

(C) $\frac{8}{3}$

(D) 2

Ans. **C**

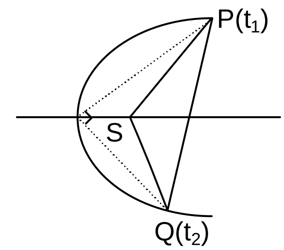
Sol. $t_1t_2 = -4$

If (h, k) is the centroid of ΔPSQ,

Then $\frac{3h+4}{2} = t_1^2 + t_2^2$, $\frac{3k}{4} = t_1 + t_2$

$\Rightarrow k^2 = \frac{8}{3}(h-4)$

$\therefore$ Latus rectum = $\frac{8}{3}$



63. A hyperbola having foci A(4, –1) and B(4, 5) has x + y – 7 = 0 as one of its tangent, then the point of contact of this tangent is

(A) $\left(\frac{9}{2}, \frac{5}{2}\right)$

(B) (1, 6)

(C) (0, 7)

(D) (2, 5)

Ans. **C**

Sol. Image of A(4, –1) about the tangent x + y – 7 = 0 is P(8, 3)

$\because$ Equation of line passing through P(8, 3) and B(4, 5) is 2y + x – 14 = 0

Solving this with tangent, we get point of contact

64. The equation of circle touching the parabola $y = 1 - x^2$ at the point (2, –3) and having its centre on the line y + x = 0 is

(A) $(x-3)^2 + (y+3)^2 = 1$

(B) $\left(x - \frac{14}{5}\right)^2 + \left(y + \frac{14}{5}\right)^2 = \frac{17}{25}$

(C) $\left(x + \frac{14}{25}\right)^2 + \left(y - \frac{14}{25}\right)^2 = \frac{17}{25}$

(D) $(x-4)^2 + (y+4)^2 = 5$

Ans. **B**

Sol. Normal to the parabola $y = 1 - x^2$ at (2, –3) is 4y – x + 14 = 0

Let centre of circle is $(\alpha, -\alpha)$ which lies on the normal

$\therefore \alpha = \frac{14}{5}$

$\therefore$ Radius is $\frac{\sqrt{17}}{5}$

65. If the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$, b > 0 and the hyperbola $\frac{x^2}{81}-\frac{y^2}{63}=\frac{1}{16}$ intersect orthogonally, then the value of b^2 is

(A) 5

(B) 7

(C) 9

(D) $\frac{81}{7}$

Ans. B

Sol. Foci of the two curves are coincident

66. If the line joining the foci of the hyperbola $S_1 \equiv \frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$ does not subtend a right angle at any point on the hyperbola $S_2 \equiv \frac{x^2}{4a^2}-\frac{y^2}{b^2}=1$, then number of integral values of $4e^2$ is/are (e is eccentricity of $S_2 = 0$)

(A) 1

(B) 2

(C) 3

(D) 4

Ans. B

Sol. Circle having foci of the hyperbola S_1 as the extremities of diameter should not intersect the hyperbola S_2 at real points
$\Rightarrow a^2 + b^2 < 4a^2$
$\Rightarrow b^2 < 3a^2$
$\Rightarrow e^2 < 7/4$
$\Rightarrow 4 < 4e^2 < 7$

67. One of the sides of a triangle is divided into segments of 4 and 6 units by the point of tangency of the inscribed circle which has radius $2\sqrt{2}$ units, then the largest side of triangle is

(A) 10

(B) $\frac{21}{2}$

(C) $\frac{43}{4}$

(D) 11

Ans. **D**

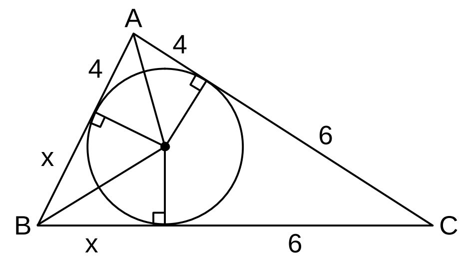
Sol. $\tan\frac{A}{2} = \frac{1}{\sqrt{2}}$, $\tan\frac{C}{2} = \frac{\sqrt{2}}{3}$

$\therefore\ \tan\frac{(A+C)}{2} = \frac{5}{2\sqrt{2}}$

$\Rightarrow\ \cot\frac{B}{2} = \frac{x}{2\sqrt{2}} = \frac{5}{2\sqrt{2}}$

$\Rightarrow x = 5$

Alternate: Use $\Delta = rs$



68. Let $S_n = 1^3 + 2^3 + 3^3 + ... + n^3$; $n \in N$, then $64S_n$ is always less than

(A) $(2n - 1)^4$

(B) $(n + 2)^4$

(C) $(2n + 1)^4$

(D) $2(n + 1)^4$

Ans. **C**

Sol. $64\sum n^3 = 16\left(n(n+1)\right)^2 < 16\left(n^2 + n + \frac{1}{4}\right)^2$

69. A line is drawn from A(–4, 0) to intersect the curve $\frac{x^2}{8} + \frac{y^2}{4} = 1$ at P and Q above x-axis. If $\frac{1}{AP} + \frac{1}{AQ} \ge \frac{\sqrt{3}}{2}$, then the maximum value of the slope of line is

(A) $2\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{4}{5}$

(D) $\sqrt{3}$

Ans. **B**

Sol. AP and AQ are the roots of the equation $(r\cos\theta - 4)^2 + 2(r\sin\theta)^2 = 8$

$\Rightarrow r^2(\cos^2\theta + 2\sin^2\theta) - 8\cos\theta\, r + 8 = 0$

$\therefore\ \frac{1}{AP} + \frac{1}{AQ} = \left|\frac{8\cos\theta}{8}\right| \ge \frac{\sqrt{3}}{2}$

$\Rightarrow\ |\cos\theta| \ge \frac{\sqrt{3}}{2} \Rightarrow\ \tan\theta \le \frac{1}{\sqrt{3}}$

70. If two distinct chords drawn from the point $\left(2\sqrt{2}\sin\theta, \frac{1}{\sqrt{2}}\right)$ to the circle $x^2 + y^2 = 2\sqrt{2}\sin\theta\, x + \frac{1}{\sqrt{2}}y$, ($\theta$ is a parameter) are bisected by x-axis, then the exhaustive set in which θ lies is

(A) $\left(n\pi + \frac{\pi}{4}, n\pi + \frac{3\pi}{4}\right); n \in I$

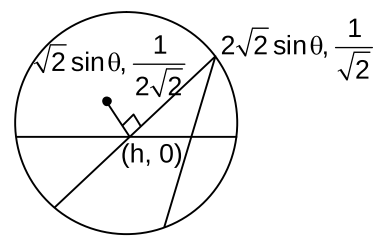
(B) $\left(n\pi + \frac{\pi}{6}, n\pi + \frac{5\pi}{6}\right); n \in I$

(C) $\left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right); n \in I$

(D) $(2n\pi, (2n + 1)\pi)$; $n \in I$

Ans. A

Sol. $\dfrac{\frac{1}{2\sqrt{2}}}{\sqrt{2}\sin\theta - h} \times \dfrac{1}{\sqrt{2}\left(2\sqrt{2}\sin\theta - h\right)} = -1$



$\Rightarrow h^2 - 3\sqrt{2}\sin\theta \cdot h + 4\sin^2\theta + \frac{1}{4} = 0$

For distinct real roots

$D > 0 \Rightarrow \sin^2\theta > \frac{1}{2} \Rightarrow \theta \in \left(n\pi + \frac{\pi}{4}, n\pi + \frac{3\pi}{4}\right); n \in I$

71. Let A be (100, 50), a point B on the line y = x and point C on x-axis such that AB + BC + CA is minimum, then the coordinates of C is

(A) (50, 0)

(B) $\left(\frac{200}{3}, 0\right)$

(C) $\left(\frac{250}{3}, 0\right)$

(D) $\left(\frac{400}{3}, 0\right)$

Ans. C

Sol. Image of A(100, 50) about x-axis is P(100, –50) and about y = x is Q(50, 100)
Equation of line PQ is y + 3x – 250 = 0
Putting y = 0, $x = \frac{250}{3}$

72. A line $L_1 \equiv \frac{x}{10} + \frac{y}{8} = 1$ intersects the coordinate axes at points A and B. Another line L_2 perpendicular to L_1 intersects the coordinate axes at C and D. The locus of circumcentre of ΔABD is

(A) $5x - 4y = 9$

(B) $5x - 4y = 18$

(C) $4x - 5y = 9$

(D) $4x - 5y = 18$

Ans. **A**

Sol. Circumcentre will lie on the perpendicular bisector of AB

73. The value of $\cot^4\frac{\pi}{16} - 4\cot^3\frac{\pi}{16} - 6\cot^2\frac{\pi}{16} + 4\cot\frac{\pi}{16} + 2$ is

(A) 0

(B) –1

(C) 2

(D) 1

Ans. **D**

Sol. $\cot 4A = \frac{1-\tan^2 2A}{2\tan 2A} \Rightarrow 1 = \frac{\left(1-\tan^2 A\right)^2 - 4\tan^2 A}{4\tan A\left(1-\tan^2 A\right)}$; $A = \frac{\pi}{16}$

$\Rightarrow \tan^4\frac{\pi}{16} + 4\tan^3\frac{\pi}{16} - 6\tan^2\frac{\pi}{16} - 4\tan\frac{\pi}{16} + 1 = 0$

74. The equation of two sides of a triangle are $3x + 4y - 12 = 0$, $2x + y - 8 = 0$. If the circumcentre is (0, 3), then the centroid of the triangle is

(A) $\left(0, \frac{7}{3}\right)$

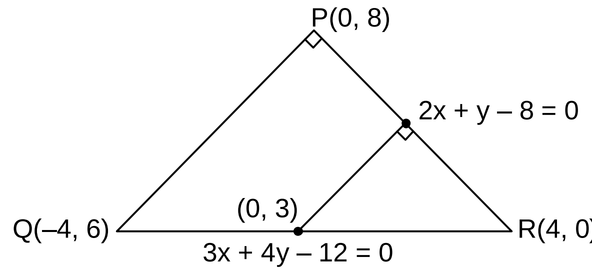
(B) $\left(0, \frac{14}{3}\right)$

(C) $\left(0, \frac{16}{3}\right)$

(D) (0, 6)

Ans. **B**

Sol. Foot of perpendicular from (0, 3) on the line
2x + y – 8 = 0 is (2, 4)
Hence, P is (0, 8) which is orthocentre
$\therefore$ Centroid is $\left(0, \frac{14}{3}\right)$



75. The value of $\sec^{-1}(\text{cosec } A) + \tan^{-1}(\cot^3 A) - \cot^{-1}\left(\frac{\cot A}{\cot^2 A - 1}\right)$, $0 < A < \frac{\pi}{6}$ is

(A) 0

(B) $3 \tan^{-1} 1$

(C) $3\tan^{-1}\frac{1}{\sqrt{3}}$

(D) $\tan^{-1} 1$

Ans. C

Sol. $-\cot^{-1}\left(\frac{\cot A}{\cot^2 A - 1}\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$

$= -\left(\pi - \cot^{-1}\left(\frac{\cot A}{1 - \cot^2 A}\right)\right) + \pi + \tan^{-1}\frac{\cot A}{1 - \cot^2 A} = \frac{\pi}{2}$

76. If the incircle of a triangle ABC passes through the circumcentre, then value of $(\cos A + \cos B + \cos C)^2$ is

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) $\frac{9}{4}$

Ans. C

Sol. Distance between circumcentre and incentre = $\sqrt{R^2 - 2Rr}$

$\therefore \sqrt{R^2 - 2Rr} = r$

$\Rightarrow \frac{r}{R} = \sqrt{2} - 1$

77. The minimum value of the expression $(t^2 + 1 - \alpha)^2 + (2t - \alpha - 4)^2$, $(t, \alpha \in R)$ is

(A) 2

(B) 3

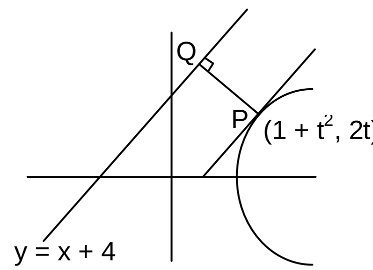
(C) 8

(D) 10

Ans. C

Sol. Slope of tangent $\frac{1}{t} = 1 \Rightarrow t = 1$

$\therefore$ P(2, 2) $\Rightarrow$ PQ $= \left|\frac{2-2+4}{\sqrt{2}}\right| = 2\sqrt{2}$



78. In ΔABC, $\angle A = \frac{\pi}{6}$, then the maximum value of $\sin^2 B + \sin^2 C$ is

(A) $\sqrt{3} - 1$

(B) $1 + \frac{\sqrt{3}}{2}$

(C) $\frac{3}{2}$

(D) $2 - \frac{\sqrt{3}}{2}$

Ans. B

Sol. $\sin^2 B + \sin^2\left(\frac{5\pi}{6} - B\right) = 1 + \frac{\sqrt{3}\sin 2B - 3\cos 2B}{4}$

79. If $8\cos y = \frac{4x^2 + 4x + 7}{x^2 + x + 1}$; $x \in R$, then the range of $\sin^2 y + \cos y + 1$ is

(A) $\left[1, \frac{9}{4}\right]$

(B) $\left[1, \frac{13}{4}\right)$

(C) $\left[2, \frac{9}{4}\right)$

(D) $\left[3, \frac{13}{4}\right)$

Ans. C

Sol. $8\cos y = 4 + \frac{3}{x^2+x+1} \Rightarrow \frac{1}{2} < \cos y \le 1$

80. Two tangents to the hyperbola $\frac{x^2}{100} - \frac{y^2}{81} = 1$ having slopes m_1 and m_2 cuts the coordinate axes at four concyclic points. If m_1 and m_2 satisfy the equation $2\alpha^2 - 5\alpha + k = 0$, then the value of k is

(A) 1

(B) 2

(C) $\frac{3}{2}$

(D) $\frac{5}{4}$

Ans. B

Sol. $m_1m_2 = 1$

81. A parabola is drawn through two given points A(2, 0) and B(–2, 0) such that its directrix always touch the circle $x^2 + y^2 = 16$, then locus of focus of the parabola is

(A) $3x^2 + 4y^2 = 48$

(B) $4x^2 + 3y^2 = 48$

(C) $3x^2 + 4y^2 = 60$

(D) $4x^2 + 3y^2 = 60$

Ans. A

Sol. Let focus be S(h, k), then
$(h - 2)^2 + k^2 = 4(\cos\theta - 2)^2$ ..... (1)
$(h + 2)^2 + k^2 = 4(\cos\theta + 2)^2$ ..... (2)
$\Rightarrow \cos\theta = \frac{h}{4}$
$\Rightarrow (h-2)^2 + k^2 = 4\left(\frac{h}{4} - 2\right)^2 \Rightarrow \frac{h^2}{16} + \frac{k^2}{12} = 1$

82. The locus of foot of perpendicular from (1, 2) on each member of the family of lines
$(1 + 2t)x + (1 - t)y + t - 1 = 0$; $t \in R$ is

(A) $x^2 + y^2 + x - 3y + 2 = 0$

(B) $x^2 + y^2 - x - 3y + 2 = 0$

(C) $x^2 + y^2 - x - 3y - 4 = 0$

(D) none of these

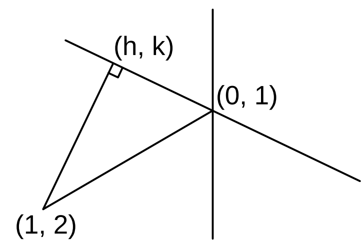
Ans. **B**

Sol. Given family passes through the point (0, 1)

$\Rightarrow \frac{k-2}{h-1} \times \frac{k-1}{h} = -1$

$x(x - 1) + (y - 1)(y - 2) = 0$

$x^2 + y^2 - x - 3y + 2 = 0$



83. Consider point P with ordinate t lying on the curve $\frac{x^2}{4} - y^2 = 1$; $t \in N$. If S_n represents the minimum distance from point P to the line 2y – x = 0, then $\lim_{t\to\infty}(tS_n)$ is

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{2}{\sqrt{5}}$

Ans. **C**

Sol. $P\left(2\sqrt{1+t^2}, t\right)$

$\therefore\ S_n = \frac{2\left(\sqrt{1+t^2} - t\right)}{\sqrt{5}} = \lim_{t\to\infty} t \cdot S_n = \lim_{t\to\infty} \frac{2t\left(\sqrt{1+t^2} - t\right)}{\sqrt{5}} = \frac{1}{\sqrt{5}}$ (By rationalizing)

84. In ΔABC, if medians from B and C are mutually perpendicular, then the possible value of cot B + cot C is

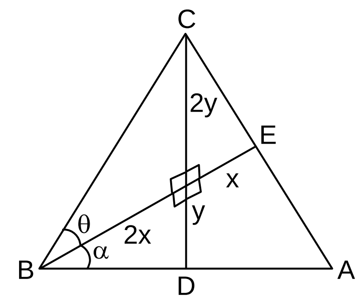
(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

Ans. **D**



Sol. $\tan\theta = \frac{y}{x} \Rightarrow \tan\alpha = \frac{y}{2x}$

$\cot B = \frac{1-\frac{y}{x}\cdot\frac{y}{2x}}{\frac{y}{x}+\frac{y}{2x}} = \frac{2x^2-y^2}{3xy}$

Similarly $\cot C = \frac{2y^2-x^2}{3xy}$

$\therefore\ \cot B + \cot C = \frac{x^2+y^2}{3xy} \geq \frac{2}{3}$

85. If $\sum_{n=1}^{89}\frac{1}{\sin((n+1)k)\sin nk} + \frac{\cot 90k}{\sin k} = \frac{2}{3}$, then k is given by

(A) $n\pi \pm \frac{\pi}{3}; n \in I$

(B) $2n\pi \pm \frac{\pi}{3}; n \in I$

(C) $n\pi \pm \frac{\pi}{6}; n \in I$

(D) $2n\pi \pm \frac{\pi}{6}; n \in I$

Ans. **B**

Sol. $\sum_{n=1}^{89}\frac{1}{\sin(n+1)k\sin nk} = \frac{1}{\sin k}\sum_{k=1}^{89}\frac{\sin((n+1)k-nk)}{\sin(n+1)k\sin nk}$

$= \frac{1}{\sin k}\sum_{n=1}^{89}\cot(nk)-\cot(n+1)k = \frac{\cot k}{\sin k} - \frac{\cot 90k}{\sin k}$

$\Rightarrow \frac{\cos k}{\sin^2 k} = \frac{2}{3}\ \therefore\ k = 2n\pi \pm \frac{\pi}{3};\ n \in I$

86. Let two tangents 3x – 4y + 20 = 0 and x – y – 3 = 0 of a parabola intersect the tangent at vertex at points P(0, 5) and Q(3, 0) respectively, then the length of latus rectum is

(A) $\frac{12}{\sqrt{34}}$

(B) $\frac{18}{\sqrt{34}}$

(C) $\frac{24}{\sqrt{34}}$

(D) $\sqrt{34}$

Ans. **C**

Sol. Focus is (6, –3) and equation of tangent at vertex is 5x + 3y – 15 = 0

$\therefore$ LR = $4\left|\frac{30-9-15}{\sqrt{34}}\right| = \frac{24}{\sqrt{34}}$

87. From point P(8, t); t being the parameter, tangents are drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{12}=1$ so as to touch the curve at A and B. The locus of the image of P about AB is

(A) x = –4

(B) x + y = 2

(C) x = –8

(D) x + y = 4

Ans. **A**

Sol. $\frac{h+8}{2}=2$, $\frac{k+t}{2}=0 \Rightarrow h = -4$

88. Let the equation of incircle of ΔABC be $4(x^2 + y^2) = 25$ which touches the sides BC, CA, and AB at D, E and F respectively. If BD, CE and AF are in AP with common difference 2.5, then the circumradius of ΔABC is

(A) 5

(B) 6.25

(C) 7.5

(D) 10

Ans. **B**

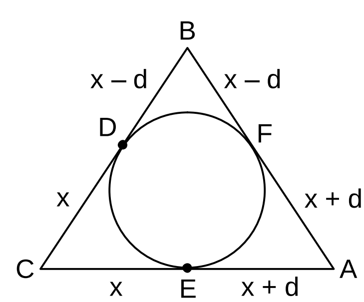
Sol. s = 3x

$r=\frac{\Delta}{s}=\frac{\sqrt{3x^2\left(x^2-d^2\right)}}{3x}=2.5$

$\Rightarrow$ x = 5

$\therefore$ Sides are 7.5, 10, 12.5

$\therefore$ $R=\frac{7.5\times10\times12.5}{4\times37.5}=6.25$



89. The number of solutions of the equation $\sin^{-1}(\cos 3x) + \cos^{-1}(\sin 3x) = \frac{\pi}{2}$; $x \in [-\pi, \pi]$ is/are

(A) 2

(B) 3

(C) 5

(D) 6

Ans. **D**

Sol. $\sin^{-1}(\cos 3x) = \sin^{-1}(\sin 3x) \Rightarrow \cos 3x - \sin 3x = 0 \Rightarrow \cos\left(3x + \frac{\pi}{4}\right) = 0$

$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{12}$; $n \in I$

90. The lower one-fifth portion of a vertical tower subtends an angle $\alpha = \tan^{-1}\frac{1}{5}$ at a point A in the horizontal plane through its foot and at a distance 100 m from the foot. If the angle subtended by the upper four fifth portion of tower at point A is β, then $\beta - \alpha$ is

(A) $\tan^{-1}\frac{5}{17}$

(B) $\tan^{-1}\frac{17}{7}$

(C) $\tan^{-1}\frac{7}{17}$

(D) $\tan^{-1}\frac{11}{17}$

Ans. **C**

Sol. $\because \tan\alpha = \frac{h}{500} = \frac{1}{5}$

$\Rightarrow h = 100$ m

Hence $\tan(\beta + \alpha) = 1$

$\Rightarrow \frac{\tan\beta + \frac{1}{5}}{1 - \frac{\tan\beta}{5}} = 1 \Rightarrow \tan\beta = \frac{2}{3}$

Also, $\tan(\beta - \alpha) = \frac{\frac{2}{3} - \frac{1}{5}}{1 + \frac{2}{15}} = \frac{7}{17}$