

1) $P(\text{no student answering more twice})$

$$\frac{\frac{15!}{7!}}{158} \Rightarrow \frac{259459200}{2562890625} = 0.1012 = 10.12\%$$

2) Digits (unique)

$$\frac{\text{Unique combos}}{\text{Total combos}} = \frac{5 \cdot 4 \cdot 7 \cdot 6 \cdot 5}{10^5} = \frac{4200}{100000} = 0.042$$

3) $A = \text{at least 2 dice} \geq 4$

$B = \text{all 3 dice show}$

$A_1 = \text{only 2 dice showing} \geq 4$

$A_2 = \text{all 3 dice showing} \geq 4$

$$\Rightarrow P(A) = P(A_1) + P(A_2) = \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 + \binom{3}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0$$
$$= \frac{1}{2}$$

$$P(B) = \frac{6}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$P(A \cap B) = \frac{3}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{72}$$

$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72}$$

$$P(A) \cdot P(B) = P(A \cap B) \text{ independent}$$

1) geometric distribution

$$P(f) = \binom{4}{1} \cdot \binom{13}{5} = \frac{4!}{3!} \cdot \frac{13!}{5!8!}$$

$$= 4 \cdot 1287 = 5148 \text{ hands}$$

$$P(h) = \binom{52}{5} = \frac{52!}{5!47!} = 2598960 \text{ hands}$$

$$P = \frac{5148}{2598960} = 0.00198$$

$$E[X] = \frac{1}{P} = 505 \text{ hands}$$

2) ~~if (= team winning)~~

$$P(s) = 0.75$$

$$P(s') = 0.25$$

$P(w)$ = winning team
[when $P(s)$ is stays played
 s' = didn't play]

$$P(s|w) = \frac{P(w|s) \cdot P(s)}{P(w|s) \cdot P(s) + P(w|s') \cdot P(s')}$$

Binomial distribution \Rightarrow

$$P(w|s) = \binom{5}{4} (0.70)^4 (0.30)^1 = 0.36015$$

$$P(w|s') = 0.15625$$

$$\Rightarrow \frac{0.36015 \times 0.75}{0.36015 \times 0.75 + 0.25 \times 0.15625}$$

$$\Rightarrow 0.87$$

\therefore there is a 87% chance