Q1) there are 312 uniques strings. This is because simply, for the first letter there are only 3 ways to place it and there are 2 options. For the second letter there are only 2 ways to place it. And for the last space there are 26 available options. Which gives, 26*2*(2*3) = 312

Q2) there are 125 (5^3) ways of distributing the passengers. When Alice gets off on the 3rd floor by herself, with Carlos, bob or both. Which means removing 4 combinations from 125 giving us 121 different ways.

Q3)
Part 1:
There are 3 cards with 1 or 0 as fire or 0 and 1 as water, the rest are 9C3
Two are any other and 1 is fire. Gives 9c2*5c1; two are any other and 1 is water is 9c2*7c1.
Each type has 7c1*5c1*9c1
Which gives total of 381

Part 2:
10 cards of 3 type will give us 3^10

Q4) when we have 6 stars/4 bars we have 9c4 giving us 126. When all stars are in one box, there are 4 ways to place the copies in copier. This gives us 126-4 = 122 ways.

Q5)
The 2 node trees has 2 ways, 3 node trees have 5 ways, 4 node trees has 14 ways and 5 node trees has 42 ways. BST trees root is 8, left child is 5 in value and right side could be 9,10,11, or 12. Left side left child is size 5 and right child of size 2. Multiplying this we get 14*42*2 giving us 1176 ways.