Numerical Analysis

HW #2

소프트웨어학부
2018044720
석예림

1. Solve the following nonlinear system by the Newton method.

$$f(x,y,z) = x\,y\,z - 1 = 0$$
$$g(x,y,z) = x^2 + y^2 + z^2 - 4 = 0$$
$$h(x,y,z) = x^2 + 2\,y^2 - 3 = 0$$

Use as initial guess, x=1.0, y = 1.2, z = 1.4. At each iteration $X^{k+1} = X^k - J^{-1}F(X^k)$ you need to solve linear system

$$J\,\triangle X = F(X^k) \tag{1}$$

, rather than forming the inverse of the Jacobian.

To solve the linear system download the dgefa.f and dgesl.f from linpack in www.netlib.org. Also, they have dependencies, like daxpy, dscal and idamax, etc, from BLAS, also in linpack. But keep in mind that they are f77 routines, so you have to convert them into f90 routines.(Probably the only thing you need to do is to change the 'c' comment character into '!' in f90.)

Iterate until

$$\|X^{(k+1)} - X^k\|/\|X^{(k+1)}\| \leq 1.e^{-8} \tag{2}$$

Also, print out the values of f, g, and h at the solution.

Turn in your code listing and output.

```
program problem1
    implicit none
    call newton()
end program problem1

real * 8 function f(x,y,z)
    real * 8 :: x, y, z
    f = x * y * z - 1
end function

real * 8 function g(x,y,z)
    real * 8 :: x, y, z
    g = x**2 + y**2 + z**2 - 4
end function

real * 8 function h(x,y,z)
    real * 8 :: x, y, z
    h = x**2 + 2*y**2 - 3
end function

subroutine newton()
    implicit none
    real * 8 :: x, y, z, tol, eN, pxN

    real * 8, external :: f, g, h
    double precision, dimension(3, 1):: px,fx,before,error
    double precision, dimension(3, 3):: jacobi

    integer :: info, job
    integer, dimension(3) :: ipiv
```

```fortran
    double precision, dimension(3, 1) :: b
    x = 1
    y = 1.2
    z = 1.4
    b = reshape((/x,y,z/), (/3,1/))
    px = reshape((/x,y,z/), (/3,1/))

    tol = 1.0e-08
    10 fx = reshape((/f(x,y,z), g(x,y,z), h(x,y,z)/), (/3,1/))
    jacobi = reshape((/y*z, 2*x, 2*x, x*z, 2*y, 4*y, x*y, 2*z, real(0,8)/),(/3,3/))

    job = 0
    call dgefa(jacobi,3,3,ipiv,info)
    call dgesl(jacobi,3,3,ipiv,fx,job)

    before = px

    px = before - fx ! X(k+1)
    error = px-before
    eN = abs(error(1,1))+abs(error(2,1))+abs(error(3,1))
    pxN = abs(px(1,1))+abs(px(2,1))+abs(px(3,1))
    eN = eN/pxN

    x = px(1,1)
    y = px(2,1)
    z = px(3,1)
    if(eN > tol) then
        goto 10
    end if
    print *, "x =", x, "y =", y, "z =", z
    print *, "f =",f(x,y,z)
    print *, "g =",g(x,y,z)
    print *, "h =",h(x,y,z)
end subroutine
```

result)

```
yerim  ~/Downloads/2020-2 수치해석/HW #2
❯ gfortran -o 1.ex 1.f90 dgefa.f dgesl.f daxpy.f ddot.f dscal.f idamax.f
yerim  ~/Downloads/2020-2 수치해석/HW #2
❯ ./1.ex
 x =   0.56450806583699287       y =    1.1578710298658001       z =    1.5299233058563715

 f =    0.0000000000000000
 g =   -4.4408920985006262E-016
 h =    0.0000000000000000
```

2. For the following matrices compute by hand the LU factorizations.

a)

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 3 \end{pmatrix}$$

<LU 분해> 2. a)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ -1 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -\frac{1}{2} & 1 \end{bmatrix} A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}$$

b)

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ -1 & 3 & 3 & 0 \\ 2 & -2 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{pmatrix}$$

b)

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ -1 & 3 & 3 & 0 \\ 2 & -2 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 3 & 0 \\ 2 & -2 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 3 & 0 \\ 0 & -3 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ -1 & \frac{6}{7} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 3 & 0 \\ 0 & 0 & \frac{25}{7} & 4 \\ -2 & 2 & 2 & 5 \end{bmatrix}$$

$-3 + \frac{7}{2} \times x = 0$

$\frac{7}{2}x = 3$

$x = \frac{6}{7}$

$\frac{18}{7} + \frac{7}{7} = \frac{25}{7}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ -1 & \frac{6}{7} & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 3 & 0 \\ 0 & 0 & \frac{25}{7} & 4 \\ 0 & 3 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ -1 & \frac{6}{7} & 1 & 0 \\ 1 & -\frac{6}{7} & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 3 & 0 \\ 0 & 0 & \frac{25}{7} & 4 \\ 0 & 0 & -\frac{4}{7} & 5 \end{bmatrix}$$

$-\frac{18}{7} + \frac{14}{7} = -\frac{4}{7}$

$\frac{25}{7} \times \frac{4}{25} - \frac{4}{7} = 0$

$-\frac{24}{7} + \frac{30}{7} = \frac{6}{7}$

$\frac{16}{25} + \frac{125}{25} = \frac{141}{25}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ -1 & \frac{6}{7} & 1 & 0 \\ 1 & -\frac{6}{7} & \frac{4}{25} & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 3 & 0 \\ 0 & 0 & \frac{25}{7} & 4 \\ 0 & 0 & 0 & \frac{141}{25} \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 1 & -\frac{6}{7} & 1 & 0 \\ -1 & \frac{6}{7} & -\frac{4}{25} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 0 & 0 \\ 0 & 0 & \frac{25}{7} & 4 \\ 0 & 0 & 0 & \frac{141}{25} \end{bmatrix}$$

3. For the following matrices compute by hand the eigenvalues and eigenvectors.

a)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 2 \\ 4 & -4 & 5 \end{pmatrix}$$

<고유값과 고유벡터> 3. a)

I = Identity matrix : 단위행렬

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 2 \\ 4 & -4 & 5 \end{bmatrix}$$

$\lambda^2 - 4\lambda + 3$

$-5 + \lambda - 5\lambda + \lambda^2 + 8.$

$$\det(A - \lambda I) = 0 = \det \begin{bmatrix} 1-\lambda & 0 & 0 \\ 2 & -1-\lambda & 2 \\ 4 & -4 & 5-\lambda \end{bmatrix} = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & -1-\lambda & 2 \\ 4 & -4 & 5-\lambda \end{vmatrix} = (1-\lambda)\big((-1-\lambda)(5-\lambda) + 8\big)$$

$$= (1-\lambda)(\lambda-1)(\lambda-3) = -(\lambda-1)^2(\lambda-3)$$

$\therefore \lambda_1 = 1 \quad \lambda_2 = 3$

1) $\lambda_1 = 1$

$$A - \lambda_1 I = \begin{bmatrix} 0 & 0 & 0 \\ 2 & -2 & 2 \\ 4 & -4 & 4 \end{bmatrix}$$

$$(A - \lambda_1 I) \cdot v = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 2 & -2 & 2 & 0 \\ 4 & -4 & 4 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow v_1 - v_2 + v_3 = 0$$

$$\Rightarrow \begin{pmatrix} v_2 - v_3 \\ v_2 \\ v_3 \end{pmatrix} = v_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + v_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

if) $v_2 = 1,\ v_3 = 0 \quad v = [1, 1, 0]^T.$

$v_2 = 0,\ v_3 = 1 \quad v = [-1, 0, 1]^T$

2) $\lambda_2 = 3$

$$(A - \lambda_2 I) \cdot v = \left[\begin{array}{ccc|c} -2 & 0 & 0 & 0 \\ 2 & -4 & 2 & 0 \\ 4 & -4 & 2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -4 & 2 & 0 \\ 0 & -4 & 2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} v_1 = 0 \\ v_2 - \frac{1}{2}v_3 = 0 \\ 2v_2 = v_3 \end{array}$$

$$\Rightarrow \begin{pmatrix} 0 \\ \frac{1}{2}v_3 \\ v_3 \end{pmatrix} = v_3 \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

if) $v_3 = 1 \quad v = [0, \frac{1}{2}, 1]^T$

b)

$$A = \begin{pmatrix} -4 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 3 \end{pmatrix}$$

b)

$$A = \begin{bmatrix} -4 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= -(\lambda-1)\left(\lambda - \frac{1 \pm \sqrt{89}}{2}\right) = -(\lambda-1)\left(\lambda - \frac{\sqrt{89}+1}{2}\right)\left(\lambda + \frac{\sqrt{89}-1}{2}\right)$$

$$= (\lambda-1)(-\lambda^2 + \lambda + 22)$$

$$= (-4-\lambda)(\lambda-1)(\lambda-5) + 2(\lambda-1)$$

$$= (-4-\lambda)(\lambda^2-6\lambda+5) + (\lambda-1) + (+\lambda-1)$$

$$(-4-\lambda)(\lambda^2-6\lambda+5) + (\lambda-3)+2 + (2+\lambda-3)$$

$$\lambda^2-6\lambda+9-4$$

$$\det(A-\lambda I) = \det\begin{bmatrix} -4-\lambda & -1 & 1 \\ -1 & 3-\lambda & -2 \\ 1 & -2 & 3-\lambda \end{bmatrix} = \begin{vmatrix} -4-\lambda & -1 & 1 \\ -1 & 3-\lambda & -2 \\ 1 & -2 & 3-\lambda \end{vmatrix} = (-4-\lambda)((3-\lambda)^2-4) - ((\lambda-3)+2) + (2+\lambda-3)$$

$$\Rightarrow \lambda_1 = 1,\ \lambda_2 = \frac{\sqrt{89}+1}{2},\ \lambda_3 = \frac{\sqrt{89}-1}{2}$$

i) $\lambda_1 = 1$

$$(A-\lambda_1 I)\cdot V = \left[\begin{array}{ccc|c} -5 & -1 & 1 & 0 \\ -1 & 2 & -2 & 0 \\ 1 & -2 & 2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} -5 & -1 & 1 & 0 \\ -1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ -1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & \frac{11}{5} & -\frac{11}{5} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} V_1 = 0 \\ V_2 - V_3 = 0 \\ V_3 = V_3 \end{array}$$

$$V = \begin{bmatrix} 0 \\ V_3 \\ V_3 \end{bmatrix} = V_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

if) $V_3 = 1$ $V = [0, 1, 1]^T$

ii) $\lambda_2 = \frac{\sqrt{89}+1}{2}$

$$(A-\lambda_2 I)\cdot V = \left[\begin{array}{ccc|c} \frac{-\sqrt{89}-9}{2} & -1 & 1 & 0 \\ -1 & \frac{-\sqrt{89}+5}{2} & -2 & 0 \\ 1 & -2 & \frac{-\sqrt{89}+5}{2} & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{\sqrt{89}-9}{4} & \frac{-\sqrt{89}+9}{4} & 0 \\ -1 & \frac{-\sqrt{89}+5}{2} & -2 & 0 \\ 1 & -2 & \frac{-\sqrt{89}+5}{2} & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{\sqrt{89}-9}{4} & \frac{-\sqrt{89}+9}{4} & 0 \\ 0 & \frac{-\sqrt{89}+1}{4} & \frac{-\sqrt{89}+1}{4} & 0 \\ 1 & -2 & \frac{-\sqrt{89}+5}{2} & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{\sqrt{89}-9}{4} & \frac{-\sqrt{89}+9}{4} & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -2 & \frac{-\sqrt{89}+5}{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{\sqrt{89}-9}{4} & \frac{-\sqrt{89}+9}{4} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & \frac{-\sqrt{89}+1}{4} & \frac{-\sqrt{89}+1}{4} & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{\sqrt{89}-9}{4} & \frac{-\sqrt{89}+9}{4} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & \frac{-\sqrt{89}+9}{2} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} V_1 + \frac{-\sqrt{89}+9}{2} V_3 = 0 \\ V_2 + V_3 = 0 \end{array} \Rightarrow V = \begin{bmatrix} +\frac{\sqrt{89}-9}{2} V_3 \\ -V_3 \\ V_3 \end{bmatrix} = V_3 \begin{bmatrix} \frac{\sqrt{89}-9}{2} \\ -1 \\ 1 \end{bmatrix}$$

if) $V_3 = 1$, $V = \left[\frac{\sqrt{89}-9}{2}, -1, 1\right]^T$

$\frac{2 \quad 10}{4} \quad -\frac{4}{4}$

iii) $\lambda_3 = \frac{\sqrt{89}+1}{2}$

$$(A-\lambda_3 I)\cdot v = \left[\begin{array}{ccc|c} \frac{\sqrt{89}-9}{2} & -1 & 1 & 0 \\ -1 & \frac{\sqrt{89}+5}{2} & -2 & 0 \\ 1 & -2 & \frac{\sqrt{89}+5}{2} & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{-\sqrt{89}-9}{4} & \frac{\sqrt{89}+9}{4} & 0 \\ -1 & \frac{\sqrt{89}+5}{2} & -2 & 0 \\ 1 & -2 & \frac{\sqrt{89}+5}{2} & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{-\sqrt{89}-9}{4} & \frac{\sqrt{89}+9}{4} & 0 \\ 0 & \frac{\sqrt{89}+1}{4} & \frac{\sqrt{89}+1}{4} & 0 \\ 1 & -2 & \frac{\sqrt{89}+5}{2} & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{-\sqrt{89}-9}{4} & \frac{\sqrt{89}+9}{4} & 0 \\ 0 & \frac{\sqrt{89}+1}{4} & \frac{\sqrt{89}+1}{4} & 0 \\ 0 & \frac{\sqrt{89}+1}{4} & \frac{\sqrt{89}+1}{4} & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{-\sqrt{89}-9}{4} & \frac{\sqrt{89}+9}{4} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & \frac{\sqrt{89}+1}{4} & \frac{\sqrt{89}+1}{4} & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{-\sqrt{89}-9}{4} & \frac{\sqrt{89}+9}{4} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & \frac{\sqrt{89}+9}{2} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_1 + \frac{\sqrt{89}+9}{2} v_3 = 0$, $v_2 = -v_3$, $v_3 = v_3$

$$\Rightarrow v = \begin{bmatrix} \frac{-\sqrt{89}-9}{2} v_3 \\ -v_3 \\ v_3 \end{bmatrix} = v_3 \begin{bmatrix} \frac{-\sqrt{89}-9}{2} \\ -1 \\ 1 \end{bmatrix}$$

if) $v_3 = 1$, $v = \left[\frac{-\sqrt{89}-9}{2}, -1, 1\right]^T$

4. For the following matrix compute by hand the QR factorization. a)

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}$$

<QR분해> 4. a)

$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$

1) $Q = (u_1, u_2)$

$u_1 = \frac{1}{\sqrt{14}}(1,2,3)$

$w_2 = (1,1,1) - \langle u_1 \cdot v_2 \rangle u_1 = (1,1,1) - \frac{6}{\sqrt{14}}(1,2,3) \times \frac{1}{\sqrt{14}}$

$= (1,1,1) - \frac{3}{7}(1,2,3)$

$= \frac{1}{7}(4,1,-2)$

$\frac{21}{\frac{2}{42}}$

$u_2 = \frac{1}{\sqrt{16+1+4}}(4,1,-2) = \frac{1}{\sqrt{21}}(4,1,-2)$

$\therefore Q = \frac{1}{\sqrt{42}}\begin{pmatrix} \sqrt{3} & 4\sqrt{2} \\ 2\sqrt{3} & \sqrt{2} \\ 3\sqrt{3} & -2\sqrt{2} \end{pmatrix}$

2) R

$R = \begin{pmatrix} \sqrt{14} & \frac{3\sqrt{14}}{7} \\ 0 & \frac{\sqrt{21}}{7} \end{pmatrix}$

$u_1 \cdot a_1 = (1+4+9)\frac{1}{\sqrt{14}} = \frac{14}{\sqrt{14}} = \sqrt{14}$

$u_1 \cdot a_2 = (1+2+3) \cdot \frac{1}{\sqrt{14}} = \frac{6}{\sqrt{14}} = \frac{3}{7}\sqrt{14}$

$u_2 \cdot a_2 = (4+1-2) \cdot \frac{1}{\sqrt{21}} = \frac{3}{\sqrt{21}} = \frac{\sqrt{21}}{7}$

$\therefore A = \frac{1}{\sqrt{42}}\begin{bmatrix} \sqrt{3} & 4\sqrt{2} \\ 2\sqrt{3} & \sqrt{2} \\ 3\sqrt{3} & -2\sqrt{2} \end{bmatrix}\begin{bmatrix} \sqrt{14} & \frac{3\sqrt{14}}{7} \\ 0 & \frac{\sqrt{21}}{7} \end{bmatrix}$

b) For the above A matrix and $b = (3,7,8)^t$, compute by hand the least-squares solution to $A\,x = b$, using QR factorization.

〈최소제곱문제〉

b)

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \underbrace{\frac{1}{\sqrt{42}} \begin{bmatrix} \sqrt{3} & 4\sqrt{2} \\ 2\sqrt{3} & \sqrt{2} \\ 3\sqrt{3} & -2\sqrt{2} \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} \sqrt{14} & \frac{3\sqrt{14}}{7} \\ 0 & \frac{\sqrt{21}}{7} \end{bmatrix}}_{R} \qquad b = \begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix}$$

$Ax = b$

$Rx = Q^T b$

$x = R^{-1} Q^T b$

$R \cdot R^{-1} = I$

$$\left[\begin{array}{cc|cc} \sqrt{14} & \frac{3\sqrt{14}}{7} & 1 & 0 \\ 0 & \frac{\sqrt{21}}{7} & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & \frac{3}{7} & \frac{1}{\sqrt{14}} & 0 \\ 0 & \frac{\sqrt{21}}{7} & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & \frac{3}{7} & \frac{1}{\sqrt{14}} & 0 \\ 0 & 1 & 0 & \frac{7}{\sqrt{21}} \end{array}\right]$$

$$\frac{\cancel{49}^{7}\sqrt{21}}{3\cancel{\sqrt{14}}\,\cancel{21}_{3}}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{21}} \\ 0 & 1 & 0 & \frac{7}{\sqrt{21}} \end{array}\right]$$

$$-\frac{49}{3\sqrt{21}}$$

$$\frac{3\cancel{\sqrt{14}}}{7} \times \frac{1}{\cancel{\sqrt{14}}} = \frac{3}{7}$$

$$R^{-1} = \begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{21}} \\ 0 & \frac{7}{\sqrt{21}} \end{bmatrix}$$

$$-\frac{3}{\cancel{7}} \times \frac{\cancel{7}}{\sqrt{21}} = -\frac{3}{\sqrt{21}}$$

$$Q^T = \frac{1}{\sqrt{42}} \begin{bmatrix} \sqrt{3} & 2\sqrt{3} & 3\sqrt{3} \\ 4\sqrt{2} & \sqrt{2} & -2\sqrt{2} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{21}} \\ 0 & \frac{7}{\sqrt{21}} \end{bmatrix} \cdot \frac{1}{\sqrt{42}} \begin{bmatrix} \sqrt{3} & 2\sqrt{3} & 3\sqrt{3} \\ 4\sqrt{2} & \sqrt{2} & -2\sqrt{2} \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix}$$

$3\sqrt{3} + 14\sqrt{3} + 24\sqrt{3}$

$= 41\sqrt{3}$

$12\sqrt{2} + 7\sqrt{2} - 16\sqrt{2}$

$$= \begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{21}} \\ 0 & \frac{7}{\sqrt{21}} \end{bmatrix} \cdot \frac{1}{\sqrt{42}} \begin{bmatrix} 41\sqrt{3} \\ 3\sqrt{2} \end{bmatrix}$$

$$\frac{41\sqrt{3}}{\sqrt{14}} - \frac{9\sqrt{2}}{\sqrt{21}} = \frac{123-18}{\sqrt{42}} = \frac{105}{\sqrt{42}}$$

$$\frac{21\sqrt{2}}{\sqrt{21}} = \frac{42}{\sqrt{42}}$$

$$= \frac{1}{\sqrt{42}} \begin{bmatrix} \frac{105}{\sqrt{42}} \\ \frac{42}{\sqrt{42}} \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$$