Homework 2, Due Dec. 4

1. Solve the following nonlinear system by the Newton method.

$$f(x,y,z) = x\ y\ z - 1 = 0$$
$$g(x,y,z) = x^2 + y^2 + z^2 - 4 = 0$$
$$h(x,y,z) = x^2 + 2\ y^2 - 3 = 0$$

Use as initial guess, x=1.0, y = 1.2, z = 1.4. At each iteration $X^{k+1} = X^k - J^{-1}F(X^k)$ you need to solve linear system

$$J \triangle X = F(X^k) \qquad (1)$$

, rather than forming the inverse of the Jacobian.

To solve the linear system download the dgefa.f and dgesl.f from linpack in www.netlib.org. Also, they have dependencies, like daxpy, dscal and idamax, etc, from BLAS, also in linpack. But keep in mind that they are f77 routines, so you have to convert them into f90 routines.(Probably the only thing you need to do is to change the 'c' comment character into '!' in f90.)

Iterate until

$$\|X^{(k+1)} - X^k\|/\|X^{(k+1)}\| \leq 1.e^{-8} \qquad (2)$$

Also, print out the values of f, g, and h at the solution.

Turn in your code listing and output.

2. For the following matrices compute by hand the LU factorizations.

a)

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 3 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ -1 & 3 & 3 & 0 \\ 2 & -2 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{pmatrix}$$

3. For the following matrices compute by hand the eigenvalues and eigenvectors.

a)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 2 \\ 4 & -4 & 5 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} -4 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 3 \end{pmatrix}$$

4. For the following matrix compute by hand the QR factorization. a)

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}$$

b) For the above A matrix and $b = (3, 7, 8)^t$, compute by hand the least-squares solution to $A\ x = b$, using QR factorization.

5. The following is a fortran90 program segment that generates A matrix and b vector.

```
dimension a(1024,1024)
data A/1024*1024*0./
nx = 32
N = nx**2

do i = 1, n
          if(i .gt. 1) a(i,i-1) = -1.
   if(i .lt. n) a(i,i+1) = -1.
   a(i,i) = 4.5
   if(i+nx .le. n) a(i,i+nx) = -1.
   if(i-nx .ge. 1) a(i,i-nx) = -1.
   b(i) = cos(i+0.5)
        enddo
```

For the above program find the solution to $A\ x = b$ using Conjugate Gradient method. Use $X_0 = 0$ and terminate if $\|r_k\|_2 \ / \ \|b\|_2 < 10^{-8}$. Recall that $r_k = b - A\ x_k$. The Fortran90 intrinsic functions MATMUL and DOT_PRODUCT will be useful. Turn in the source code and final $x(1), x(1024)$, and the final number of iterations required.