COSC 221 - Introduction to Discrete Structures

Lecture - Logic-05

Readings

- Propositional Logic: Sections 3.1, 3.2, 3.3
- Predicate Logic: Sections 3.4
- Computer Science Connections
    1. Computational Complexity (Section 3.3)
    2. Modern Compilers (Section 3.3)
    3. Game Trees (Section 3.4)

Predicate Logic (Calculus)

- Predicate
- Universal Quantifier: $\forall$
- Existential Quantifier: $\exists$

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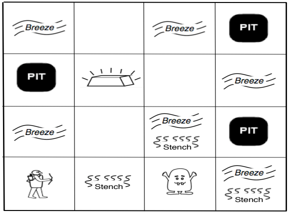
Wumpus World (Russell's AI textbook)



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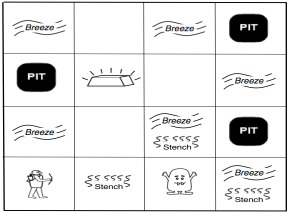

Modelling with Propositions

$$p_{i,j} = \text{“there is a pit in room (i, j)”}$$
$$w_{i,j} = \text{“wumpus is in room (i, j)”}$$
$$g_{i,j} = \text{“gold is in room (i, j)”}$$

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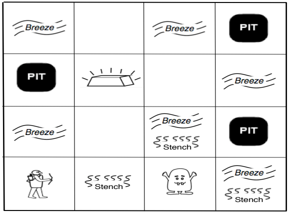

Gold in Row 3

- ▷ $g_{1,3} \lor g_{2,3} \lor g_{3,3} \lor g_{4,3}$

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Limitation of Propositional Logic

Not rich enough as a representation language

- ▷ "He is a student" is NOT a proposition
- ▷ Propositional symbols for everything.

What we need: HasGold(x)

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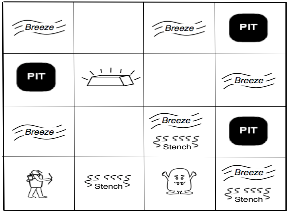

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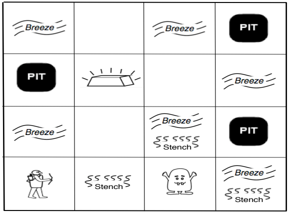

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Predicate and Proposition

- ▷ $P(x)$
- ▷ Domain (universe of discourse)
- ▷ Truth Set

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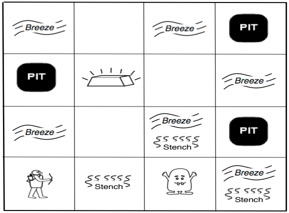

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Two parts of a statement

- ▷ P — Predicate (the verb)
- ▷ x — Argument (the Subject)

Predicate Logic (Calculus)

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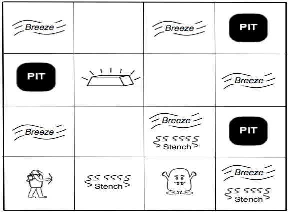

Proposition with "Blanks" — with subject removed

- ▷ __ is a student
- ▷ __ has gold
- ▷ __ implements __

Predicate and Proposition

- ▷ $P(x)$
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Predicate Logic (Calculus)

Predicates and Quantifiers

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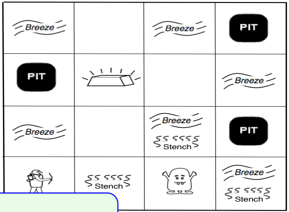

Property x (and y) may have

IsStudent(x)

HasGold(x)

Implements(x, y)

Predicate and Proposition

- ▷ $P(x)$
- ▷ Domain (universe of discourse)
- ▷ Truth Set

Two parts of a statement

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Predicate Logic (Calculus)

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```
% A Mini Knowledgebase about Java Classes

%Rules
is-a(X, Y):- extends(X, Y).
is-a(X, Y):- extends(X, Z), is-a(Z, Y).
is-a(X, Y):- interface(Y), implements(X, Y).

%Facts
extends(mylist, arraylist).
extends(arraylist, abstractlist).
extends(error, throwable).
extends(exception, throwable).

implements(abstractlist, list).
implements(throwable, serializable).
interface(list).
```

Proposition with "Blanks" — with subject r

- __ is a student
- __ has gold
- __ implements __

Predicate and Proposition

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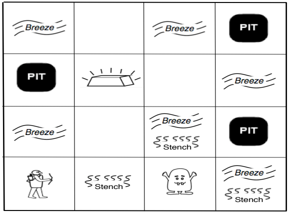

Predicate as a Boolean Function

HasGold(x): "Room x HasGold" where

$$x \in U = \{(1,1), \cdots, (4,4)\}$$

HasGold(x): $U \rightarrow \{T, F\}$

Predicate and Proposition

- ▷ $P(x)$
- ▷ Domain (universe of discourse)
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Two parts of a statement

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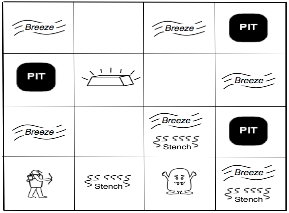

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Predicate and Proposition

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Truth Set: "Meaning" of a Predicate

$\{x \in U \mid P(x) \text{ is True}\}$

Predicate Logic (Calculus)

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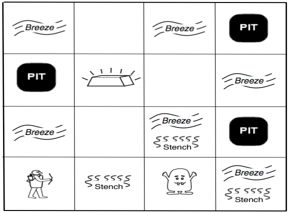

Q.1) Truth Set of IsSmelly(x) is

A) $\{(2,1)\}$ B) $\{(2,1),(3,2)\}$

C) $\{(3,2)\}$ D) $\{(2,1),(3,2),(4,1)\}$

Predicate and Proposition

- ▷ $P(x)$
- ▷ Domain (universe of discourse)
- ▷ Truth Set

Truth Set: "Meaning" of a Predicate

$$\{x \in U \mid P(x) \text{ is True}\}$$

Predicate Logic (Calculus)

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> ▷ for all (every, any, each)
>
> ▷ given any

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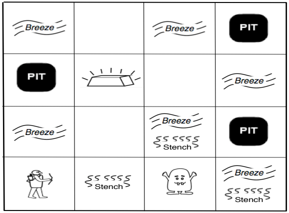

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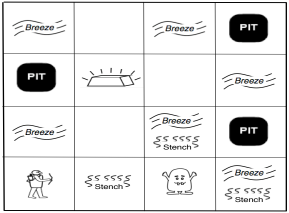

Game Rule: Smelly if wumpus nearby

$$\forall (x, y) \in U, \ \ Wumpus(x) \ \wedge \ Adjacent(x, y) \ \rightarrow \ IsSmelly(y)$$

$$U = \{(1,1), \cdots, (4,4)\}$$

Predicate Logic (Calculus)

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- Existential Quantifier: $\exists$

  - for all (every, any, each)
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  - there exists (is)
  - for some

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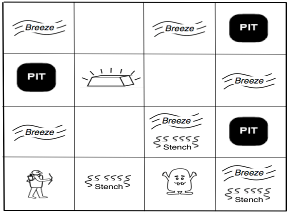

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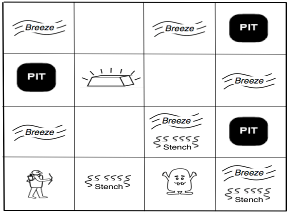

Game Rule: There is at least one Wumpus

$$\exists r \in U,\ Wumpus(r)$$

Predicate Logic (Calculus)

- Predicate
- Universal Quantifier: $\forall$
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Quantified Propositions

- $\forall x \in U, P(x)$
- $\forall x \in U, P(x) \;\Rightarrow\; Q(x)$
- $\exists x \in U, \text{ such that } Q(x)$

Universal (Conditional) Proposition

Predicate Logic (Calculus)

- Predicate
- Universal Quantifier: $\forall$
- Existential Quantifier: $\exists$

Example 1. $\forall x \in R, x^2 \geq 0$

Informally,

- Every real number has a nonnegative square
- $\forall$ real number x, its square is nonnegative

Quantified Propositions

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Example 2. isPrefixOf(x, y) (Example 3.35, Textbook)

(for two strings x and y with $|x| \leq |y|$)

$$\forall i \in Z^{>0},\ i \leq |x| \ \Rightarrow\ x_i = y_i$$

Quantified Propositions

- ▷ $\forall x \in U, P(x)$
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Example 3. isPrefixOf(x, y) (Example 3.35, Textbook)

(for two strings x and y with $|x| \leq |y|$)

$$\forall i \in Z^{>0},\ i \leq |x| \ \Rightarrow\ x_i = y_i$$

Or, we may write

$$\forall i \in \{i \in Z : 1 \leq i \leq |x|\},\ x_i = y_i$$

Quantified Propositions

- $\forall x \in U, P(x)$
- $\forall x \in U, P(x) \ \Rightarrow\ Q(x)$
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Universal (Conditional) Proposition

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- $\forall x \in U, P(x)$
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Existential Proposition

- Predicate
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- Existential Quantifier: $\exists$

Example 4.

$\exists\ n \in D \subseteq Z^{>0}$, such that $n^2 = n$

Quantified Propositions

- $\forall x \in U, P(x)$
- $\forall x \in U, P(x) \Rightarrow Q(x)$
- $\exists x \in U$, such that $Q(x)$

Existential Proposition

- Predicate
- Universal Quantifier: $\forall$
- Existential Quantifier: $\exists$

Example 5.

$\exists\ n \in D \subseteq Z^{>0}$, such that $n^2 = n$

Q.2) If $D = \{5, 6, 7, 8\}$, the truth value is

A) T

B) F

Quantified Propositions

- $\forall x \in U, P(x)$
- $\forall x \in U, P(x) \Rightarrow Q(x)$
- $\exists x \in U$, such that $Q(x)$

Existential Proposition

Predicate Logic (Calculus)

- Negation
- Contrapositive

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- Contrapositive

Universal Statement: $\forall x \in U, P(x)$

True iff Truth Set $= U$

Predicate Logic (Calculus)

- Negation
- Contrapositive

Universal Statement: $\forall x \in U, P(x)$

True iff Truth Set $= U$

$$\neg(\forall x \in D,\ P(x)) \equiv\ \exists\ x \in D \text{ such that } \neg P(x)$$

- ▷ Negation
- ▷ Contrapositive

$$
\begin{aligned}
LHS &\equiv \neg(P(d_1) \wedge P(d_2) \wedge \cdots \wedge P(d_n)) \\
&\equiv \neg P(d_1) \vee \neg P(d_2) \vee \cdots \vee \neg P(d_n)
\end{aligned}
$$

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Predicate Logic (Calculus)

- Negation
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Universal Conditional Statement: $\forall x \in U, P(x) \Rightarrow Q(x)$

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$$\neg(\forall x \in U, P(x) \Rightarrow Q(x)) \equiv \exists x \text{ such that } P(x) \wedge \neg Q(x)$$

Predicate Logic (Calculus)

- Negation
- Contraposition

$$P(x) \Rightarrow Q(x) \equiv \neg P(x) \vee Q(x)$$

Universal Conditional Statement: $\forall x \in U, P(x) \Rightarrow Q(x)$

True iff Truth Set $= U$

$$\neg(\forall x \in U, P(x) \Rightarrow Q(x)) \equiv \exists x \text{ such that } P(x) \wedge \neg Q(x)$$

Predicate Logic (Calculus)

- Negation
- Contrapositive

Example

If the square of an integer n is even, then n is even.

$$\forall n \in Z, \text{EVEN}(n^2) \Rightarrow \text{EVEN}(n)$$

Negation:

$$\exists n \in Z \text{ such that } \text{EVEN}(n^2) \wedge \neg\text{EVEN}(n)$$

Universal Conditional Statement: $\forall x \in U, P(x) \Rightarrow Q(x)$

True iff Truth Set $= U$

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- Negation
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Universal Conditional Statement: $\forall x \in U, P(x) \ \Rightarrow \ Q(x)$

True iff Truth Set $= U$

$$\forall x \in D, P(x) \ \Rightarrow \ Q(x) \ \equiv \ \forall x \in D, \ \neg Q(x) \ \Rightarrow \ \neg P(x)$$

- ▷ Negation
- ▷ Contrapositive

Example

$\forall n \in Z, \text{EVEN}(n^2) \Rightarrow \text{EVEN}(n)$

$\forall n \in Z, \text{ODD}(n) \Rightarrow \text{ODD}(n^2)$

Universal Conditional Statement: $\forall x \in U, P(x) \Rightarrow Q(x)$

True iff Truth Set $= U$

$$\forall x \in D, P(x) \Rightarrow Q(x) \equiv \forall x \in D, \neg Q(x) \Rightarrow \neg P(x)$$

Predicate Logic (Calculus)

- Negation
- Contrapositive

Q.3) $\forall n \in Z^+, P(n) \;\rightarrow\; T(n) \;\wedge\; F(n)$

Its contrapositive is

A) $\forall n \in Z^+,\ \neg T(n) \;\vee\; \neg F(n) \rightarrow\; \neg P(n)$

B) $\forall n \in Z^+,\ \neg T(n) \;\wedge\; \neg F(n) \;\rightarrow\; \neg P(n)$

Universal Conditional Statement: $\forall x \in U, P(x) \;\Rightarrow\; Q(x)$

True iff Truth Set $= U$

$$\forall x \in D, P(x) \;\Rightarrow\; Q(x) \;\equiv\; \forall x \in D,\ \neg Q(x) \;\Rightarrow\; \neg P(x)$$

Predicate Logic (Calculus)

- Negation
- Contrapos

Q.4) If a graph is two-colorable, it has no odd cycle

Its negation is

A) There exists a graph that has odd cycles

B) If a graph has odd cycles, it is not two-colorable

C) There exists a graph that is not two-colorable

D) There exists a graph that is two-colorable, but has an odd cycle

Universal Conditional Statement: $\forall x \in U, P(x) \Rightarrow Q(x)$

True iff Truth Set $= U$

$$\forall x \in D, P(x) \Rightarrow Q(x) \equiv \forall x \in D, \neg Q(x) \Rightarrow \neg P(x)$$

Let $D = \{d_1, d_2, \cdots, d_n\}$. We have

$$\forall x \in D, Q(x) \equiv Q(d_1) \ \wedge \ Q(d_2) \ \wedge \ \cdots \ \wedge \ Q(d_n)$$

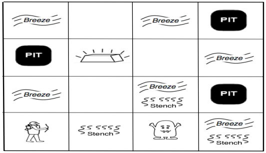
and

$$\exists x \in D, Q(x) \equiv Q(d_1) \ \vee \ Q(d_2) \ \vee \ \cdots \ \vee \ Q(d_n).$$

Therefore, by De Morgan's laws,

$$\neg(\forall x \in D, Q(x)) \equiv \exists x \in D, \ \neg Q(x).$$

$$\forall x, y \in D, Wumpus(x) \ \wedge \ Adjacent(x, y) \ \rightarrow \ IsSmelly(y).$$



Its contrapositive is

(A) $\forall\ x, y \ \in \ D, \ IsSmelly(y) \ \rightarrow \ Wumpus(x) \wedge Adjacent(x, y)$

(B) $\forall\ x, y \ \in \ D, \ \neg IsSmelly(y) \ \rightarrow \ \neg Wumpus(x) \ \vee \ \neg Adjacent(x, y)$

(C) $\exists\ x, y \ \in \ D, \ Wumpus(x) \ \wedge \ Adjacent(x, y) \ \rightarrow \ IsSmelly(y)$

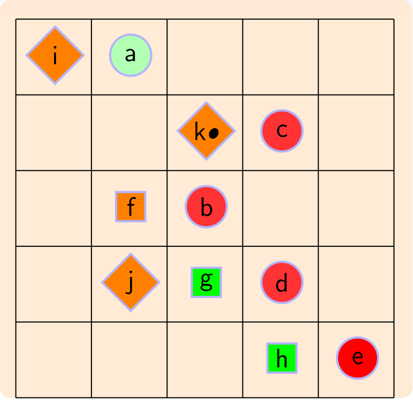
(D) $\exists\ x, y \ \in \ D, \ \neg IsSmelly(y) \ \rightarrow \ \neg Wumpus(x) \ \vee \ \neg Adjacent(x, y)$

Additional Notes

- $\triangleright$ Predicates, domain, truth set
- $\triangleright$ Quantified statements (Universal/Existential)
- $\triangleright$ Truth Value of a Universal/Existential Statement
- $\triangleright$ Negation and Contrapositive

Predicate Logic (Calculus)

iCliker Quiz: Tarski's World

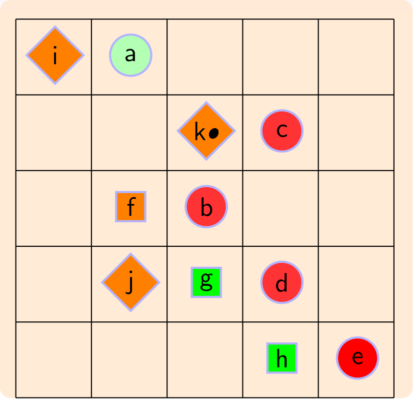


- D(x) (Diamond), C(x), S(x)
- O(x) (Orange), R(x), G(x)
- RightOf(x, y)
- 11 objects named a, ..., k

Tarski's World:

Computer program named after Alfred Tarski, a logician

iCliker Quiz: Tarski's World



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Q.1) "All diamonds are orange"

A) $\forall x, D(x) \Rightarrow O(x)$

B) $\forall x, O(x) \Rightarrow D(x)$

C) $\forall x, \text{ if } O(x) \text{ then } D(x)$

D) $\forall x, D(x) \ \wedge \ O(x)$

Predicate Logic (Calculus)

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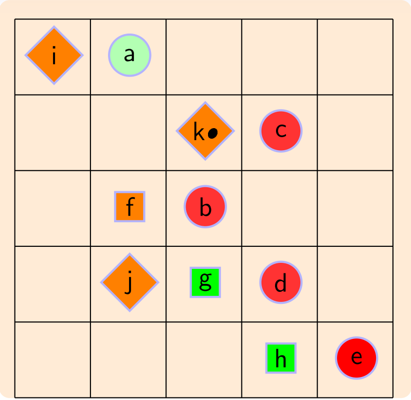


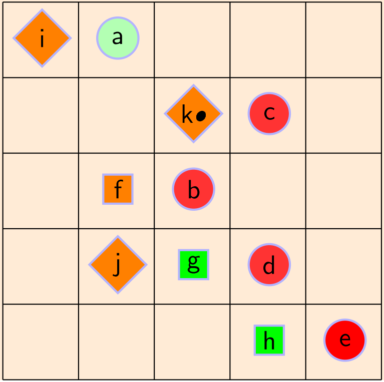
- D(x) (Diamond), C(x), S(x)
- O(x) (Orange), R(x), G(x)
- RightOf(x, y)
- 11 objects named a, ..., k

Q.2) The truth value of $\forall x, D(x) \Rightarrow O(x)$ is

A) T

B) F

Predicate Logic (Calculus)



- D(x) (Diamond), C(x), S(x)
- O(x) (Orange), R(x), G(x)
- RightOf(x, y)
- 11 objects named a, ..., k

Q.3) Every orange shape is a diamond

A. $\forall x, D(x) \Rightarrow O(x)$

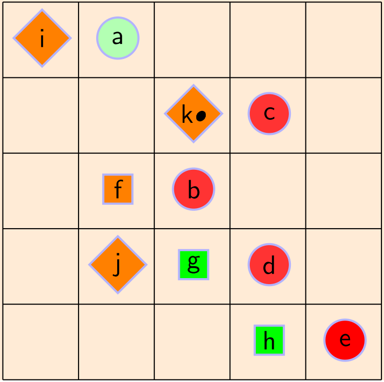
B. $\forall x, O(x) \Rightarrow D(x)$

C. $\forall x, \text{ if } \sim D(x) \text{ then } O(x)$

D. $\forall x, D(x) \wedge \neg O(x)$

Predicate Logic (Calculus)

iCliker Quiz: Tarski's World



- D(x) (Diamond), C(x), S(x)
- O(x) (Orange), R(x), G(x)
- RightOf(x, y)
- 11 objects named a, ..., k

Q.4) The truth set of $C(x) \Rightarrow R(x)$ is

A. $\{c, b, d, e\}$

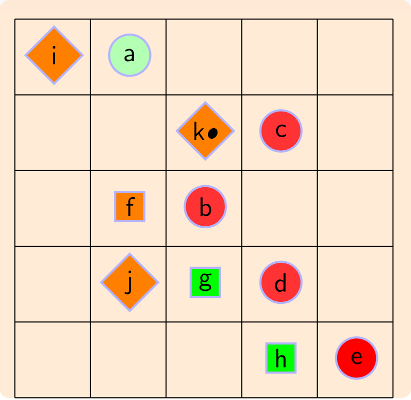
B. $\{f, g, h, i, j, k\}$

C. Everything but "a"

D. $\{a\}$

Predicate Logic (Calculus)

iCliker Quiz: Tarski's World



- D(x) (Diamond), C(x), S(x)
- O(x) (Orange), R(x), G(x)
- RightOf(x, y)
- 11 objects named a, ..., k

Q.5) A green shape is either a circle or a square

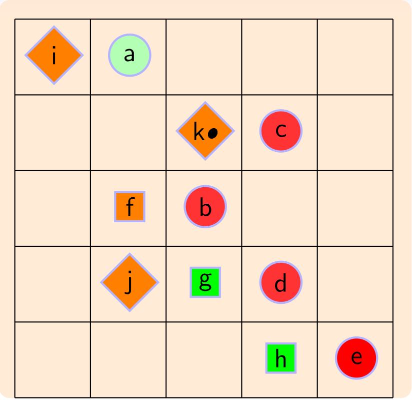
A) $\forall x, C(x) \wedge S(x)$

B) $\forall x, G(x) \Rightarrow C(x) \vee S(x)$

C) $\forall x, C(x) \vee S(x) \Rightarrow G(x)$

D) $\forall x, \neg S(x) \Rightarrow C(x)$

Predicate Logic (Calculus)



- D(x) (Diamond), C(x), S(x)
- O(x) (Orange), R(x), G(x)
- RightOf(x, y)
- 11 objects named a, ..., k

Q.6) The truth value of

$\forall x, D(x) \rightarrow (\exists y \text{ such that } C(y) \wedge \mathit{RightOf}(y, x))$

A) T   B) F

Predicate Logic (Calculus)

- Universal Instantiation
- Universal Modus Ponens
- Universal Modus Tollens
- Transitivity

Valid Argument Form

The conclusion is true whenever all premises are true.

- ▷ Universal Instantiation
- ▷ Universal Modus Ponens
- ▷ Universal Modus Tollens
- ▷ Transitivity

Universal Instantiation

$$\begin{array}{rl} & \forall x \in D, P(x) \\ \hline \therefore & P(o) \end{array}$$

Valid Argument Form

The conclusion is true whenever all premises are true.

Predicate Logic (Calculus)

- Universal Instantiation
- Universal Modus Ponens
- Universal Modus Tollens
- Transitivity

Universal Instantiation

$$\begin{array}{rl} & \forall x \in D, P(x) \\ \hline \therefore & P(o) \end{array}$$

Universal Modus Ponens

$$\begin{array}{rl} & \forall x \in D, P(x) \rightarrow Q(x) \\ & P(o) \text{ for a particular } o \\ \hline \therefore & Q(o) \end{array}$$

Valid Argument Form

The conclusion is true whenever all premises are true.

- Universal Instantiation
- Universal Modus Ponens
- Universal Modus Tollens
- Transitivity

Universal Instantiation

$$\begin{array}{ll} & \forall x \in D, P(x) \\ \hline \therefore & P(o) \end{array}$$

Universal Modus Tollens

$$\begin{array}{ll} & \forall x \in D, P(x) \rightarrow Q(x) \\ & \neg Q(o) \text{ for a particular } o \\ \hline \therefore & \neg P(o) \end{array}$$

Universal Modus Ponens

$$\begin{array}{ll} & \forall x \in D, P(x) \rightarrow Q(x) \\ & P(o) \text{ for a particular } o \\ \hline \therefore & Q(o) \end{array}$$

Valid Argument Form

The conclusion is true whenever all premises are true.

Predicate Logic (Calculus)

Arguments Using Quantified Statements

Prove EVEN(k^2) — **Given: EVEN(k)**

(for a particular even number $k > 0$.)

Prove EVEN(k^2) — **Given: EVEN(k)**

(for a particular even number $k > 0$.)

$$
\begin{array}{ll}
 & \forall n \in Z^+, \text{EVEN}(n) \Rightarrow (\exists\, m \in Z^+ \text{ such that } n = 2m). \\
\therefore & \text{EVEN}(k) \Rightarrow (\exists\, m \in Z^+ \text{ such that } k = 2m).
\end{array}
$$

Prove EVEN(k^2) **Given: EVEN(k)**

(for a particular even number $k > 0$.)

$$\forall n \in Z^+, \text{EVEN}(n) \Rightarrow (\exists\, m \in Z^+ \text{ such that } n = 2m).$$
$$\therefore \quad \text{EVEN}(k) \Rightarrow (\exists\, m \in Z^+ \text{ such that } k = 2m).$$

Q.7) The inference rule used is

A) Universal Instantiation

B) Universal Modus Ponens

Prove EVEN(k^2) **Given: EVEN(k)**

(for a particular even number $k > 0$.)

$$\begin{array}{ll} & \forall n \in Z^+, \text{EVEN}(n) \Rightarrow (\exists\ m \in Z^+ \text{ such that } n = 2m). \\ \therefore & \text{EVEN}(k) \Rightarrow (\exists\ m \in Z^+ \text{ such that } k = 2m). \end{array}$$

$$\begin{array}{ll} & \text{EVEN}(k) \rightarrow (\exists\ m \in Z^+ \text{ such that } k = 2m). \\ & \text{EVEN(k)} \\ \therefore & k = 2m \text{ for some } m \in Z^+. \end{array}$$

Prove EVEN(k^2) **Given: EVEN(k)**

(for a particular even number $k > 0$.)

$$\forall n \in Z^+, \text{EVEN}(n) \Rightarrow (\exists\ m \in Z^+ \text{ such that } n = 2m).$$
$$\therefore \quad \text{EVEN}(k) \Rightarrow (\exists\ m \in Z^+ \text{ such that } k = 2m).$$

$$\text{EVEN}(k) \rightarrow (\exists\ m \in Z^+ \text{ such that } k = 2m).$$
$$\text{EVEN(k)}$$
$$\therefore \quad k = 2m \text{ for some } m \in Z^+.$$

Q.8) The inference rule used is

A. Universal Modus Ponens

B. Modus Ponens

Prove EVEN(k^2) Given: EVEN(k)

(for a particular even number $k > 0$.)

$$\begin{array}{ll} & \forall n \in Z^+, \text{EVEN}(n) \ \Rightarrow \ (\exists\ m \in Z^+ \text{ such that } n = 2m). \\ \therefore & \text{EVEN}(k) \ \Rightarrow \ (\exists\ m \in Z^+ \text{ such that } k = 2m). \end{array}$$

$$\begin{array}{ll} & \text{EVEN}(k) \ \rightarrow \ (\exists\ m \in Z^+ \text{ such that } k = 2m). \\ & \text{EVEN(k)} \\ \therefore & k = 2m \text{ for some } m \in Z^+. \end{array}$$

$$\begin{array}{ll} & \forall n \in N, (n = 2m \text{ for some } m \in N) \ \rightarrow \ \text{EVEN}(n). \\ & k^2 = 4m^2 = 2 * (2m^2) \\ \therefore & \text{EVEN}(k^2). \end{array}$$

Prove EVEN(k^2) Given: EVEN(k)

(for a particular even number $k > 0$.)

$$\begin{array}{ll} & \forall n \in Z^+, \text{EVEN}(n) \Rightarrow (\exists \\ \therefore & \text{EVEN}(k) \Rightarrow (\exists\ m \in Z^+ \\ & \\ & \text{EVEN}(k) \rightarrow (\exists\ m \in Z^+ \\ & \text{EVEN(k)} \\ \therefore & k = 2m \text{ for some } m \in Z^+. \end{array}$$

$$\begin{array}{ll} & \forall n \in N, (n = 2m \text{ for some } m \in N) \rightarrow \text{EVEN}(n). \\ & k^2 = 4m^2 = 2 * (2m^2) \\ \therefore & \text{EVEN}(k^2). \end{array}$$

Q.9) The inference rule used is

A. Universal Modus Ponens
B. Modus Ponens

- Mathematical Definitions
- Programming: Nested Loops
- Game Representation

Predicate Logic (Calculus)

- ▷ Mathematical Definitions
- ▷ Programming: Nested Loops
- ▷ Game Representation

Example: Definition of Limit $\lim_{n\to\infty} a_n = +\infty$

$\forall M \in Z^+, \exists N \in Z^+$ such that $\forall n \geq N, a_n > M$

Predicate Logic (Calculus)

- Mathematical Definitions
- Programming: Nested Loops
- Game Representation

Behavior of Nested Loops

```
for(int i = 0; i < n; i++) {
    for(int j = 0; j < n; j++){
        if(A[i, j] > 0)

 return True;
    }
}
return False;
```

$\exists i, \exists j,$ such that $A[i,j] > 0$

- ▷ Mathematical Definitions
- ▷ Programming: Nested Loops
- ▷ Game Representation

Pic from internet

Game: Tic-Tac-Toe

- X and O: two players. O goes first.
- Predicate Variables for X: $x_1, \cdots, x_5$
- Predicate Variables for O: $o_1, \cdots, o_4$

Predicate Logic (Calculus)

Statements with Multiple Quantifiers

- ▷ Mathematical Definitions
- ▷ Programming: Nested Loops
- ▷ Game Representation

Pic from internet

Game: Tic-Tac-Toe

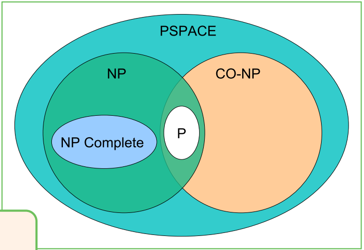
- X and O: two players. O goes first.
- Predicate Variables for X: $x_1, \cdots, x_5$
- Predicate Variables for O: $o_1, \cdots, o_4$

Quantified Statement of the Game: X's Winning Strategy

$$\forall o_1 \, \exists x_1 \, \forall o_2 \, \exists \, x_2 \, \forall o_3 \, \exists \, x_3 \, \forall o_4 \, \exists \, x_4$$

$$\text{NOT-ALL-EQUAL}(x_i, o_i) \, \wedge \text{NO-THREE-IN-A-LINE}(o_i)$$

- Mathematical Definitions
- Programming: Nested Loops
- Game Representation



Generic PSPACE-Complete Problems

Hardest problems solvable using a polynomial amount of space (i.e., main memory).

Quantified Statement of the Game: X's Winning Strategy

$$\forall o_1\ \exists x_1\ \forall o_2\ \exists\ x_2\ \forall o_3\ \exists\ x_3\ \forall o_4\ \exists\ x_4$$
$$\text{NOT-ALL-EQUAL}(x_i, o_i)\ \wedge \text{NO-THREE-IN-A-LINE}(o_i)$$

References I