COSC 221 - Introduction to Discrete Structures

Lecture - Logic-01

Readings

- Propositional Logic: Sections 3.1, 3.2, 3.3
- Predicate Logic: Sections 3.4
- Computer Science Connections
  1. Computational Complexity (Section 3.3)
  2. Modern Compilers (Section 3.3)
  3. Game Trees (Section 3.4)

Propositional Logic

- Propositions
- Connectives

A statement that is either true or false

Propositional Logic

- Propositions
- Connectives

A statement that is either true or false

Propositions and Their Truth Value

1. It is snowing
2. It is July 1
3. He is a student
4. $2 + 2 = 5$
5. $2 + 2 = 4$

Propositional Logic

A statement that is either true or false

Propositions and Their Truth Value

1. It is snowing
2. It is July 1
3. He is a student
4. $2 + 2 = 5$
5. $2 + 2 = 4$

Q.1) Which one is NOT a proposition?

A) 1 B) 2

C) 3 D) 4

Propositional Logic

- Propositions
- Connectives

A statement that is either true or false

Propositions and Their Truth Value

1. It is snowing
2. It is July 1
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- Conceptually indivisible
- Truth Value: Truth (T, or 1)
  Falsity (F, 0)
- Boolean Variable

Propositional Logic

- Propositions
- Connectives

A statement that is either true or false

p = "It is snowing"

- p = F today, but
- p = T on a snowing day

Propositions and Their Truth Value

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Propositions and Their Truth Value

- Atomic propositions
- Compound propositions

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  Falsity (F, 0)
- Boolean Variable

Propositional Logic

- Propositions
- Connectives

Operators for building Compound Propositions

Propositions and Their Truth Value

- Atomic propositions
- Compound propositions

Logic Connectives

$\neg p$	(negation)
$p \wedge q$	(and, conjunction)
$p \vee q$	(or, disjunction)
$p \Rightarrow q$	(implications)

Propositional Logic

- Propositions
- Connectives

Example 1. Compound Proposition

p : "John is a CS major", q : "John is a Math major"

- $p \wedge q$, $p \vee q$
- $p \wedge \neg q$ (p, but not q)
- $\neg p \wedge \neg q$ (neither p, nor q)

Propositions and Their Truth Value

- Atomic propositions
- Compound propositions

Logic Connectives

$\neg p$	(negation)
$p \wedge q$	(and, conjunction)
$p \vee q$	(or, disjunction)
$p \Rightarrow q$	(implications)

Propositional Logic

- Propositions
- Connectives

Q.2) Is the assertion correct?

"A compound proposition is either T or F, but not both."

A) No B) Yes

C) Depends D) Hard to Tell

Propositions and Their Truth Value

- Atomic propositions
- Compound propositions

Logic Connectives

$\neg p$	(negation)
$p \wedge q$	(and, conjunction)
$p \vee q$	(or, disjunction)
$p \Rightarrow q$	(implications)

Propositional Logic

- Propositions
- Connectives

Boolean expression/formula over p, q. Truth value depends on

- truth value of p
- truth value of q
- the logic connectives

Propositions and Their Truth Value

- Atomic propositions
- Compound propositions

Logic Connectives

$\neg p$	(negation)
$p \wedge q$	(and, conjunction)
$p \vee q$	(or, disjunction)
$p \Rightarrow q$	(implications)

Additional Notes

Logic is the study of the principles of reasoning, including the mathematical foundation of well-defined languages and valid inference methods.

Propositional Logic (Calculus)

CORE CONCEPT

"Formal" system for reasoning with propositions

- ▷ Symbols — abstract representation
- ▷ Syntax — well-defined expressions
- ▷ Rules — to work with expressions

- Atomic Propositions
- Compound Propositions

How to decide the truth value
of a compound statement?

Propositional Logic

Truth Values and Truth Tables

- Atomic Propositions
- Compound Propositions

Not defined by propositional logic itself!
Depends on the "'world" state.

p = "It is snowing"

- p = F today, but
- p = T on a snowing day

How to decide the truth value
of a compound statement?

- Atomic Propositions
- Compound Propositions

Need a **table** to map **truth assignments** (for the boolean variables) to **truth values** of the Boolean expression

- Atomic Propositions
- Compound Propositions

Need a **table** to map **truth assignments** (for the boolean variables) to **truth values** of the Boolean expression

Truth Table for $\wedge$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- Atomic Propositions
- Compound Propositions

Need a **table** to map **truth assignments** (for the boolean variables) to **truth values** of the Boolean expression

Boolean variables

Truth Table for $\wedge$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- Atomic Propositions
- Compound Propositions

Need a **table** to map **truth assignments** (for the boolean variables) to **truth values** of the Boolean expression

Boolean variables

Truth Table for $\wedge$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Assignment:
$(p = F, q = T)$

- Atomic Propositions
- Compound Propositions

Example 2. Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

- Expression over p and q
- and of two or's

- Atomic Propositions
- Compound Propositions

Example 3. Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

- Expression over p and q
- and of two or's

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	?

- Atomic Propositions
- Compound Propositions

Q.3) The value is

A) T B) F

C) Unknown D) X

Example 4. Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

- Expression over p and q
- and of two or's

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	?

- Atomic Propositions
- Compound Propositions

New Connective: $p \oplus q$

"Exclusive OR" (abbreviated as XOR)

Example 5. Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

- Expression over p and q
- and of two or's

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	?

Memorize/Understand the Truth Tables

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$p \oplus q$
		$p \times q$	?	$1 - p$	(p + q) modulo 2
T(1)	T(1)	1	1	0	0
T(1)	F(0)	0	1	0	1
F(0)	T(1)	0	1	1	1
F(0)	F(0)	0	0	1	0

Announcement: Assignment 1 Online

- ▷ Individual Written Assignment
- ▷ Due Feb 2 (Friday), 11:59pm
- ▷ Hand in a digital copy on Canvas.

iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Q.4) The truth value of $p \oplus p$ is

(A) always T

(B) always F

(C) T if p is T

(D) T if p is F

iClicker Quiz: Truth Values and Truth Tables

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p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Q.5) The truth value of $(p \oplus q) \oplus r$ is

(A) always T

(B) always F

(C) T if p, q, r are all T

(D) T if p, q are both T

iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
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Q.5) The truth value of $(p \oplus q) \oplus r$ is

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True iff one or three T's

iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

Parity Function

$f(x_1, x_2, \cdots, x_n) = 1$ iff

$$x_1 \oplus x_2 \oplus \cdots \oplus x_n$$

is true.

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Q.5) The truth value of $(p \oplus q) \oplus r$ is

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iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

Parity Function

$f(x_1, x_2, \cdots, x_n) = 1$ iff

$$x_1 \oplus x_2 \oplus \cdots \oplus x_n$$

is true.

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Error-Correcting Codes (Section 4.2.4: Hamming Codes)

Message	Codeword	Received	Error Position
abcd			
0000	0000 000	1000 000	bit a

iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

Parity Function

$f(x_1, x_2, \cdots, x_n) = 1$ iff

$$x_1 \oplus x_2 \oplus \cdots \oplus x_n$$

is true.

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Error-Correcting Codes (Section 4.2.4: Hamming Codes)

Message	Codeword	Received		Error Position
abcd				
0000	0000	000	1000	000 bit a

Sender calculates three parity bits

$b \oplus c \oplus d$

$a \oplus c \oplus d$

$a \oplus b \oplus d$

iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Error-Correcting Codes (Section 4.2.4: Hamr

Message	Codeword	Received		Error
abcd 0000	0000	000	1000	000 bit a

Fig. 4.10

x		x	x			x		parity bit #1: $b \oplus c \oplus d$
x	x		x		x			parity bit #2: $a \oplus c \oplus d$
x	x	x		x				parity bit #3: $a \oplus b \oplus d$
bit d	bit a	bit b	bit c	parity #3	parity #2	parity #1	no error!	location of error

Sender calculates three parity bits

$b \oplus c \oplus d$

$a \oplus c \oplus d$

$a \oplus b \oplus d$

Receiver calculate three parity bits

$b \oplus c \oplus d$

$a \oplus c \oplus d$

$a \oplus b \oplus d$

iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
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T	F	F	T	T

Q.6) Is this truth table complete?

(A) Yes

(B) No

iClicker Quiz: Truth Values and Truth Tables

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p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Q.7) If not, how many rows are missing?

(A) 1

(B) 2

(C) 3

(D) 4

iClicker Quiz: Truth Values and Truth Tables

New Connective: $p \oplus q$ (XOR)

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T

Q.7) If not, how many rows are missing?

(A) 1

(B) 2

(C) 3

(D) 4

Total number of truth assignments: 2^3

Propositional Logic

Modelling Tool

- Propositions
- Truth Values
- Connectives

Propositional Logic

Modelling Tool

- Propositions
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Modelling the Wumpus World

$p_{i,j}$ = "there is a pit in room (i, j)"

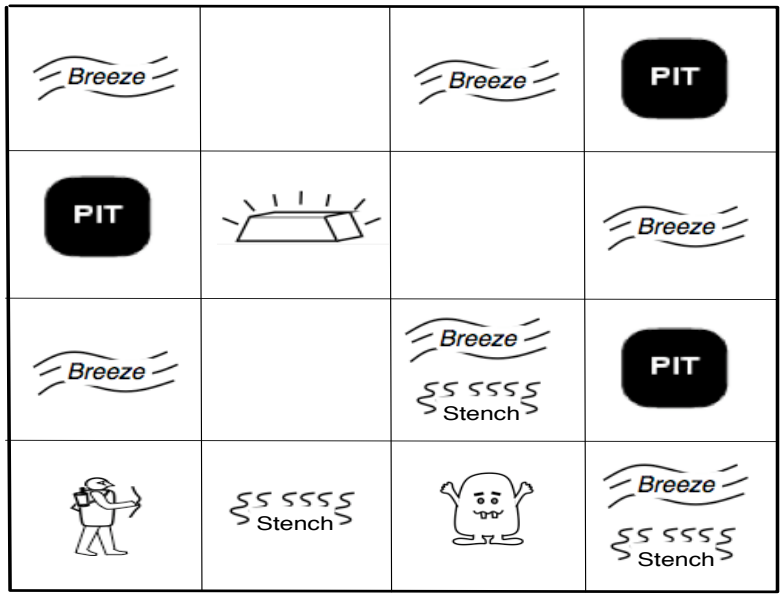
$w_{i,j}$ = "wumpus is in room (i, j)"

$s_{i,j}$ = "room (i, j) is smelly"

$b_{i,j}$ = "room (i, j) is breezy"

$g_{i,j}$ = "gold is in room (i, j)"

Wumpus World (Russell's AI textbook)



Propositional Logic

Modelling Tool

- Propositions
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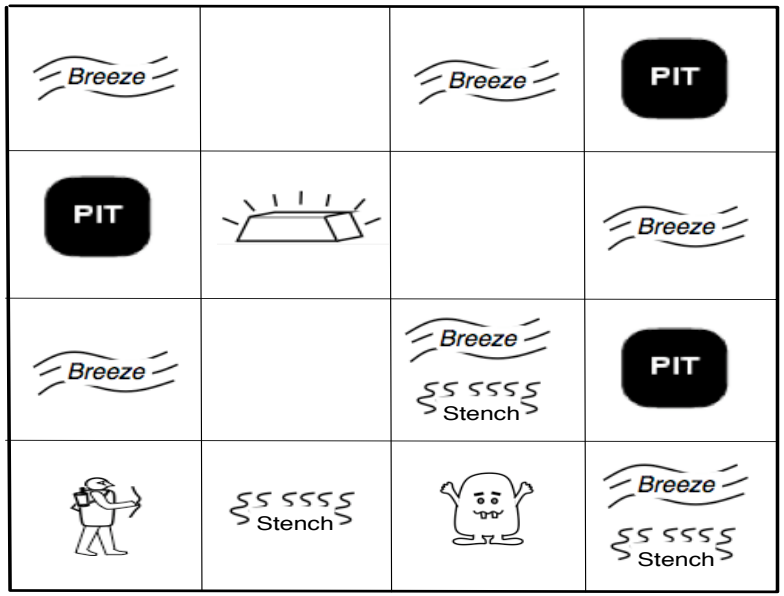

Propositional Logic

Modelling Tool

- Propositions
- Truth Values
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Gold in the third row

- $g_{3,1} \vee g_{3,2} \vee g_{3,3} \vee g_{3,4}$

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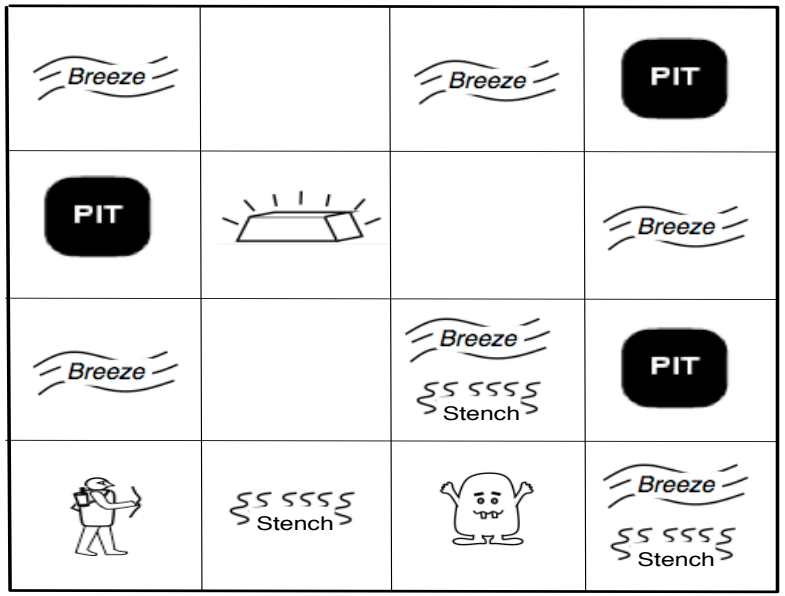

References I