

Assignment 1

Wednesday, January 17, 2024 4:32 PM

Part A) Statements in Logic Notation

Do Questions 3.7, 3.8, and 3.9 in the textbook (page 93)

(6 points)

3.7 $x * y$ is a list if $x * y$ is valid Python and x and y are not both numeric values.

3.8 if $x + y$ is a list, then $x * y$ is not a list.

3.9 $x + y$ and $x ** y$ are both valid Python only if x is not a list.

3.7. Let

p :	$x + y$ is valid Python	u :	x is a numeric value
q :	$x * y$ is valid Python	v :	y is a numeric value
r :	$x ** y$ is valid Python	w :	x is a list
s :	$x * y$ is a list	z :	y is a list
t :	$x + y$ is a list		

Then, the compound prop is as follows:

$$q \wedge \neg(u \wedge v) \Rightarrow s$$

3.8 Let

p :	$x + y$ is valid Python	u :	x is a numeric value
q :	$x * y$ is valid Python	v :	y is a numeric value
r :	$x ** y$ is valid Python	w :	x is a list
s :	$x * y$ is a list	z :	y is a list
t :	$x + y$ is a list		

Then, the compound prop is as follows:

$$t \Rightarrow \neg s$$

3.9. Let

p :	$x + y$ is valid Python	u :	x is a numeric value
q :	$x * y$ is valid Python	v :	y is a numeric value
r :	$x ** y$ is valid Python	w :	x is a list
s :	$x * y$ is a list	z :	y is a list
t :	$x + y$ is a list		

Then, the compound prop is as follows:

$$(p \wedge r) \Leftrightarrow \neg w$$

Part B) Truth Tables, Logic Equivalence, and Logic Laws

B.1) Show using truth tables that the following expression is a tautology

(8 points)

$$(p \Rightarrow q) \wedge \neg q \Rightarrow \neg p$$

p	q	$\neg p$	$\neg q$	$p \Rightarrow q$	$(p \Rightarrow q) \wedge \neg q$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	T	T	F

1	1	1	1	1	1
F	T	T	F	T	F
F	F	T	T	T	T

$$(P \Rightarrow q) \wedge \neg q \Rightarrow \neg P$$

T

T

T

T

B.2) Prove, using the logic equivalences discussed in class, that a conditional proposition is logically equivalent to its contrapositive. (8 points)

Proof. Let p and q be some statement,

Then we want to show that:

$$P \Rightarrow q \equiv \neg q \Rightarrow \neg P$$

$$\text{WKT: } P \Rightarrow q \equiv \neg P \vee q \quad \text{by def.}$$

$$\equiv \neg(P \wedge \neg q) \quad \text{by De Morgan's}$$

$$\equiv \neg(\neg q \wedge P) \quad \text{by Commutative}$$

$$\equiv \neg(\neg q) \vee \neg P \quad \text{by De Morgan's}$$

$$\equiv \neg q \Rightarrow \neg P \quad \text{by def.}$$

□

∴ By using demorg's, commutative and def of implication, we learned that $P \Rightarrow q \equiv \neg q \Rightarrow \neg P$.

Part C) Logic and Programming

3.56 Simplify the code in Figure 3.17a as much as possible. (For example, if $p \Rightarrow q$, it's a waste of time to test whether q holds in a block where p is known to be true.)

(a)

1	if $x > 20$ or $(x \leq 20$ and $y < 0)$ then
2	$foo(x, y)$
3	else
4	$bar(x, y)$

Let $p: x > 20$

$q: x \leq 20$

$r: y < 0$

then, the compound prop is,

$p \vee (q \wedge r) \Rightarrow foo(x, y)$

$\neg(p \vee (q \wedge r)) \Rightarrow bar(x, y)$