COSC 221 - Introduction to Discrete Structures

Lecture - Logic-02

Readings

- Propositional Logic: Sections 3.1, 3.2, 3.3
- Predicate Logic: Sections 3.4
- Computer Science Connections
  1. Computational Complexity (Section 3.3)
  2. Modern Compilers (Section 3.3)
  3. Game Trees (Section 3.4)

Propositional Logic

- New Connective: Implication
- Truth Value of Implication
- Variants of Implication

Logic Connectives

CORE CONCEPT

- $\neg$, $\wedge$, $\vee$, $\oplus$
- Implication: $p \Rightarrow q$
- Biconditional: $p \Leftrightarrow q$

- New Connective: Implication
- Truth Value of Implication
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A.k.a Conditional Statement

p : Hypothesis (antecedent)

q : Conclusion (consequence)

- ▷ New Connective: Implication
- ▷ Truth Value of Implication
- ▷ Variants of Implication

read

- ▷ "p implies q"
- ▷ "if p, then q"
- ▷ "p only if q"
- ▷ "q if p"

Logic Connectives

Core Concept

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- "p implies q"
- "if p, then q"
- "p only if q"
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Example:
if $0 = 100$, then 100 is odd.

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Example:
if $0 = 100$, then 100 is odd.

Q.1) p: "$0 = 100$", q: "100 is odd"

The truth value of $p \Rightarrow q$ is

A) T

B) F

C) Unknown

D) Undefined

Logic Connectives

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- Truth Value of Implication

Truth Table

p	q	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

Q.1) p: "$0 = 100$", q: "100 is odd"

The truth value of $p \Rightarrow q$ is

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T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

Q.2) $p \Rightarrow q$ is True on

A) Feb 28, 2018 B) Feb 28, 2024

C) Feb 27, 2018 D) Both B) and C)

p = "Today is Feb 28"
q = "Tomorrow is Feb 29"

Logic Connectives

CORE CONCEPT

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- New Connective: Implication
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Truth Table

p	q	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

Q.3) The truth value of

"If $1+1=3$, then $2+1=4$" is

A) T

B) F

Logic Connectives CORE CONCEPT

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A.k.a Conditional Statement

p : Hypothesis (antecedent)

q : Conclusion (consequence)

Propositional Logic

- New Connective: Implication
- Truth Value of Implication
- Variants of Implication

Variants of $p \Rightarrow q$

- Contrapositive
- Biconditional
- Converse: $q \Rightarrow p$ (p, if q)
- Inverse: $\neg p \Rightarrow \neg q$.

- ▷ New Connective: Implication
- ▷ Truth Value of Implication
- ▷ Variants of Implication

Variants of $p \Rightarrow q$

- ▷ Contrapositive
- ▷ Biconditional
- ▷ Converse: $q \Rightarrow p$ (p, if q)
- ▷ Inverse: $\neg p \Rightarrow \neg q$.

$$\neg q \Rightarrow \neg p$$

- New Connective: Implication
- Truth Value of Implication
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Are these two logically identical?

$\varphi =$ "if n is even, then n^2 is even"

$\psi =$ "If n^2 is not even, then n is not even"

Variants of $p \Rightarrow q$

- Contrapositive
- Biconditional
- Converse: $q \Rightarrow p$ (p, if q)
- Inverse: $\neg p \Rightarrow \neg q$.

$\neg q \Rightarrow \neg p$

- New Connective: Implication
- Truth Value of Implication

Truth Table

p	q	$\neg q$	$\neg p$	φ	ψ
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

Are these two logically identical?

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- Converse: $q \Rightarrow p$ (p, if q)
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- New Connective: Implication
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Truth Table

p	q	$\neg q$	$\neg p$	φ	ψ
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
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- Biconditional
- Converse: $q \Rightarrow p$ (p, if q)
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$\neg q \Rightarrow \neg p$

Propositional Logic

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Logic Equivlence — next topic

$$(p \Rightarrow q) \equiv (\neg q \Rightarrow \neg p)$$

(Identical Truth Tables)

Variants of $p \Rightarrow q$

- Contrapositive
- Biconditional
- Converse: $q \Rightarrow p$ (p, if q)
- Inverse: $\neg p \Rightarrow \neg q$.

$\neg q \Rightarrow \neg p$

- ▷ New Connective: Implication
- ▷ Truth Value of Implication
- ▷ Variants of Implication

Q.4) "If n^2 is even, then n is even"

Its contrapositive is

A) If n^2 is not even, then n is not even

B) If n is even, then n^2 is even

C) If n is not even, then n^2 is not even

D) n^2 is even, but q is not even

Variants of $p \Rightarrow q$

- ▷ Contrapositive
- ▷ Biconditional
- ▷ Converse: $q \Rightarrow p$ (p, if q)
- ▷ Inverse: $\neg p \Rightarrow \neg q$.

$$\boxed{\neg q \Rightarrow \neg p}$$

- New Co
- Truth V
- Variants

Q.5) "A graph is a forest if there is no cycle in it"

Its contrapositive is

A) If a graph is a forest, then there is no cycle in it

B) If there is a cycle in a graph, then it is not a forest

C) If a graph is not a forest, then there exists at least one cycle in it

D) A graph is a forest or there is a cycle in it

Variants of $p \Rightarrow q$

- Contrapositive
- Biconditional
- Converse: $q \Rightarrow p$ (p, if q)
- Inverse: $\neg p \Rightarrow \neg q$.

$\neg q \Rightarrow \neg p$

- New Connective: Implication
- Truth Value of Implication
- Variants of Implication

Variants of $p \Rightarrow q$

- Contrapositive
- Biconditional
- Converse: $q \Rightarrow p$ (p, if q)
- Inverse: $\neg p \Rightarrow \neg q$.

$$p \iff q$$

$$(p \Rightarrow q) \wedge (q \Rightarrow p)$$

Example: $p =$ "Today is Easter", $q =$ "Tomorrow is Monday"

- If today is Easter, then tomorrow is Monday
- If tomorrow is Monday, then today is Easter
- If today is not Easter, then tomorrow is not Monday
- If tomorrow is not Monday, then today is not Easter
- Today is Easter, but tomorrow is not Monday

References I