

University of British Columbia
Sauder School of Business

Introductory Statistics
STAT 230

Assignment 3
Due March 18, 2024

Full Name: _____

Out of 25 _____

Instructions:

- **You must show significant steps to get full marks!**
- This assignment is out of 25 points.

1. **Formally** prove the following:

- (a) (2 points) Let X be a continuous random variable that is normally distributed with mean μ and standard deviation σ . Show that approximately 95% of the area under the normal density curve is within 2 standard deviations of the mean.
- (b) (2 points) Let Y_1 and Y_2 be uncorrelated (independent) random variables and let $U_1 = Y_1 + Y_2$ and $U_2 = Y_1 - Y_2$. Find $\text{Cov}(Y_1, Y_2)$ in terms of the variance of Y_1 and Y_2 .

2. (4 points) Given the following probability density function,

$$f(x, y) = \begin{cases} 2x, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the covariance of X and Y .

3. (4 points) Let X and Y be discrete random variables with joint probability distribution shown below.

	x		
y	-1	0	1
-1	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{16}$
0	$\frac{3}{16}$	0	$\frac{3}{16}$
1	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

Show that X and Y are dependent, but have zero covariance.

4. A chemical process has produced, on the average, 600 tons of chemical per day. The daily yields for the past week are 585, 604, 590, 593, 602, 598 and 593 tons.
- (a) (6 points) Do these data indicate that the average yield is less than 600 tons and hence that something is wrong with the process? Test at the 5% level of significance.
 - (b) (1 point) What assumptions are required for the valid use of the procedure you used to analyze these data.
 - (c) (1 point) Would your decision change if the level of significance was $\alpha = 0.01$? Explain.
 - (d) (1 point) Would your decision change if the alternative hypothesis was two-tailed with $\alpha = 0.05$? Explain.
 - (e) (3 points) Construct a 95% confidence interval for the true mean weekly yield and interpret the interval.
 - (f) (1 point) Is the result consistent with your conclusion in part (a)? Explain.