STAT230 Assignment 2

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March 3, 2024

1. Given: $\sigma = 1.4$ cm, $\mu = 6$ cm, $x = 8$ cm.
   $Z = \frac{x-\mu}{\sigma} = \frac{8-6}{1.4} \simeq 1.43$

   (a) $P(Z \geq 1.43) = 1 - P(z \leq 1.43) = 1 - 0.9236 = 0.0764$

   (b) $P(Z \leq 1.43) = 0.9236$

   (c) In normal distribution, the Q1 and Q3 is given by:

   $$Q1 = \mu - 0.675\sigma = 6 - 0.675(1.4) = 5.055$$

   $$Q3 = \mu + 0.675\sigma = 6 + 0.675(1.4) = 6.945$$

   Therefore,

   $$IQR = Q3 - Q1 = 6.945 - 5.055 = 1.89$$

2. Given:

   - Acceptance rate $= 1 - 0.04 - 0.10 = 0.86$
   - Infimum (lower bound): $1 - 0.04 = 0.96 = P(Z \leq 1.75) \Rightarrow Z_{3.6mm} = -1.75 = \frac{3.6-\mu}{\sigma}$
   - Supremum (upper bound): $1 - 0.10 = 0.90 = P(Z \leq 1.28) \Rightarrow Z_{4.8mm} = 1.28 = \frac{4.8-\mu}{\sigma}$

Solving for σ:

$$\mu - 1.75\sigma = 3.6$$

$$\mu + 1.28\sigma = 4.8$$

$$3.6 + 1.75\sigma = \mu = 4.8 - 1.28\sigma$$

$$1.75\sigma + 1.28\sigma = 4.8 - 3.6$$

$$3.03\sigma = 1.2$$

$$\sigma = \frac{1.2}{3.03} \simeq 0.40$$

Solving for μ:

$$\mu = 3.6 + 1.75(0.40) \simeq 4.3$$

3. Given: $f(t) = Ce^{-t/3}$

   (a) $F(t) =$

$$\int_0^\infty Ce^{-t/3}dt = 1$$

$$C\int_0^\infty e^{-t/3}dt = 1$$

Use u-substitution: $u = -\frac{t}{3} \Rightarrow du = -\frac{1}{3}dt \Rightarrow dt = -3du$

$$C\int_0^\infty e^u(-3du) = 1$$

$$-3C(e^u)|_0^\infty = 1$$

Substitute $-\frac{t}{3} = u$

$$-3C(e^{-t/3})|_0^\infty = 1$$

$$-3C(\lim_{t\to\infty} e^{-\frac{t}{3}} - e^0) = 1$$

$$-3C(\lim_{t\to\infty} \frac{1}{e^{t/3}} - e^0) = 1$$

$$-3C(0 - 1) = 1$$

$$-3C(-1) = 1$$

$$C = \frac{1}{3}$$

(b) Now that $C = \frac{1}{3}$,

$$P(T > 5) = 1 - P(T < 5) = 1 - \frac{1}{3}\int_0^5 e^{-t/3}dt$$

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$$P(T > 5) = 1 - \frac{1}{3}\int_0^5 e^{-t/3}dt$$

Using the recent integrated integral, we can say that:

$$P(T > 5) = 1 - \frac{1}{3}(-3e^{-t/3})|_0^5 dt$$

$$P(T > 5) = 1 + (e^{-5/3} - e^{0/3})$$

$$P(T > 5) = 1 + (\frac{1}{e^{5/3}} - 1)$$

$$P(T > 5) = 1 - 0.811 = 0.19$$

(c) Given that $C = \frac{1}{3}$ and $f(t) = Ce^{-t/3}$ then,

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} \frac{1}{3}e^{-x/3}xdx$$

$$E[X] = \int_0^1 \frac{1}{3}e^{-x/3}xdx$$

$$= \frac{1}{3}\int_0^1 xe^{-x/3}dx$$

Now solving for:

$$\int xe^{-x/3}dx$$

Substitute $u = -\frac{x}{3}, x = -3u \Rightarrow du = -\frac{1}{3}dx \Rightarrow dx = -3du$

$$\int (-3u)e^u(-3du)$$

$$= 9\int ue^u du$$

Using integration by parts, $\int fg' = fg - \int f'g$
$f = u,\ g' = e^u du,\ f' = du,\ g = e^u$

$$\int ue^u du = ue^u - \int e^u du$$

$$\int ue^u du = ue^u - e^u$$

$$9\int ue^u du = 9ue^u - 9e^u$$

Substitute $-\frac{x}{3} = u$

$$\frac{1}{3}\int_0^1 e^{-x/3}xdx = \frac{1}{3}(-3xe^{-x/3} - 9e^{-x/3})|_0^1$$

$$= \frac{1}{3}(-3xe^{-x/3} - 9e^{-x/3})|_0^1$$

$$= -e^{-x/3}(x+3)|_0^1$$

$$= -e^{-1/3}(1+3) - (-e^{-0/3}(0+3))$$

$$= -e^{-1/3}(4) - (-1(0+3))$$

$$= -4e^{-1/3} - (-3)$$

$$= -4e^{-1/3} + 3$$

$$E[X] \simeq 0.134$$

4. Given: Sample size: 500, population in favor of incumbent: 0.46

(a) Since this is a Binomial Distribution, we have
$P(X \geq 250) = 1 - P(X < 250)$
$\mu = np = 500 \times 0.46 = 230$
$\sigma = \sqrt{np(1-p)} = \sqrt{230(1-0.46)} \simeq 11.14$

$$Z = \frac{x-\mu}{\sigma} = \frac{249.5 - 230}{11.14} = 1.75$$

where x is adjusted for continuity correction, then we have:

$$P(Z \geq 1.75) = 1 - P(Z < 1.75) = 1 - 0.9599 \simeq 0.04$$

(b) Given: sample $n = 50$. Using Poisson Approximation, the mean is $\lambda = np = 50(0.46) = 23$. So, it is true that

$$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!} = e^{-23} \cdot \frac{23^{25}}{25!} \simeq 0.07$$

5. Given that: $f(x) = 2x$, $Y = g(x) = \frac{1}{1+x}$

   Then it is true that:

$$E[Y] = E[g(x)] = \int_{-\infty}^{\infty} g(x)f(x)dx$$

$$\int_0^1 \frac{2x}{1+x}dx$$

   Solving by u-substitution: $u = 1 + x \Rightarrow x = u - 1 \Rightarrow du = dx$

   Now finding the new supremum and infimum for u:

   Supremum $= u = 1 + 1$, infimum $= u = 1 + 0$

$$= \int_1^2 \frac{2(u-1)}{u}du$$

$$= \int_1^2 \frac{2u-2}{u}du$$

$$= \int_1^2 2 - \frac{2}{u}du$$

$$= 2(u - ln(|u|)|_1^2 du$$

$$= 2(2 - ln(2)) - 2(1 - ln(1) = 2(1.31) - 2(1 - 0) = 2.62 - 2$$

$$E[Y] \simeq 0.62$$

End of Assignment 2.