

# STAT230 Assignment 1

Rin Meng

Student ID: 51940633

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1. Let  $S$  be the sample of the given student, then we have

$$S = \{m, m, m, m, m, m, w, w, w, w\}$$

where  $m$  is men, and  $w$  is women.

- (a) Let  $n = \text{sample size}$ . Then there are a number of

$$n! = 10! = 3628800$$

different ways to rank the students by Theorem 2.4.

- (b) Let  $n = \text{sample size}$ ,  $w = \text{women in } n$  and  $m = \text{men in } n$ . Then there are a number of

$$\frac{n!}{m!w!} = \frac{10!}{6! \times 4!} = 210$$

different ways of ranking them with regards to men and women by Theorem 2.7.

2. Let  $S$  be the sample of the given flags, then we have

$$S = \{w, w, w, w, r, r, r, b, b\}$$

where  $w$  is white flag,  $r$  is red flag, and  $b$  is blue flag. Then there are a number of

$$\frac{n!}{w! \times r! \times b!} = \frac{9!}{4! \times 3! \times 2!} = 1260$$

different ways to arrange the flags if all flag in the same color is identical.

3. Let  $S$  be the set of all possible rolls of a two fair dice, and let  $n$  be the number of all possible rolls which is  $6 \times 6 = 36$ .

- (a) Let  $S_6$  be set of two fair dice getting a total number of 6, then we have

$$S_6 = \{(1, 5), (5, 1), (4, 2), (2, 4), (3, 3), \}$$

Let  $P(S_6)$  be the probability of getting the number 6 by rolling two fair dice, then we have

$$P(S_6) = \frac{\text{Size of } S_6}{n} = \frac{5}{36}$$

- (b) Let  $S_{3 \vee 10}$  be the set of two fair dice getting a total number of  $3 \vee 10$ , then we have

$$S_{3 \vee 10} = \{(1, 2), (2, 1), (5, 5), (4, 6), (6, 4)\}$$

Let  $P(S_{3 \vee 10})$  be the probability of getting the number  $3 \vee 10$  by rolling two fair dice, then we have

$$P(S_{3 \vee 10}) = \frac{\text{Size of } S_{3 \vee 10}}{n} = \frac{5}{36}$$

- (c) Let  $S_{\text{odd} \vee \text{prime}}$  be the set of two fair dice getting a total number that is  $\text{odd} \vee \text{prime}$ , then we have

$$S_{\text{odd} \vee \text{prime}} =$$

$$\{(1, 1), (1, 2), (2, 1), (2, 3), (3, 2),$$

$$(4, 1), (1, 4), (3, 4), (4, 3), (2, 5),$$

$$(5, 2), (1, 6), (6, 1), (5, 6), (6, 5)\}$$

Let  $P(S_{\text{odd} \vee \text{prime}})$  be the probability of getting a number that is  $\text{odd} \vee \text{prime}$  by rolling two fair dice, then we have

$$P(S_{\text{odd} \vee \text{prime}}) = \frac{\text{Size of } S_{\text{odd} \vee \text{prime}}}{n} = \frac{15}{36}$$

4. Let us denote the given data by  $\phi$ , the size of  $\phi$  as  $n$  which is 20 data points.

(a) Then the mean of  $\phi$  is

$$\bar{\phi} = \frac{\sum \phi}{n} = \frac{74.4}{20} = 3.72$$

(b) Since we know that  $\sum \phi^2 = 278.4196$ , the standard deviation of  $\phi$  is

$$s_{\phi} = \sqrt{\frac{\sum \phi^2 - \frac{(74.4)^2}{20}}{n-1}} = \sqrt{\frac{278.4196 - 276.768}{20-1}} = 0.295$$

(c) The median value of  $\phi$  is

$$\phi_{sorted} =$$

$$\{3.55, 3.55, 3.56, 3.56, 3.57, 3.57, 3.59, 3.59, 3.59, 3.60, \\ 3.61, 3.63, 3.65, 3.66, 3.71, 3.73, 3.75, 3.99, 4.15, 4.79\}$$

(d) The *IQR* of  $\phi$  is

$$Q1 = \frac{3.57 + 3.57}{2} = 3.57$$

$$Q3 = \frac{3.71 + 3.73}{2} = 3.72$$

$$IQR = Q3 - Q1 = 3.72 - 3.57 = 0.15$$

5. Since  $P(\{1\}) = \frac{1}{2}$ , we can also rewrite as  $P(\{1\}) = \frac{3}{6}$  and since  $P(\{1, 2\}) = \frac{2}{3}$ , we can rewrite as  $P(\{1, 2\}) = \frac{4}{6}$

Then we can calculate  $P(\{2\})$  as follow:

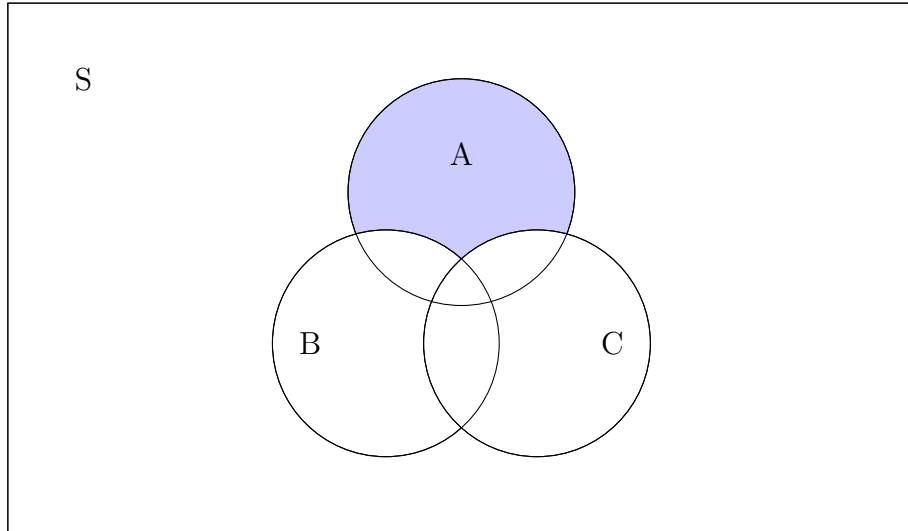
$$P(\{2\}) = P(\{1, 2\}) - P(\{1\}) = \frac{4}{6} - \frac{3}{6}$$

$$P(\{2\}) = \frac{1}{6}$$

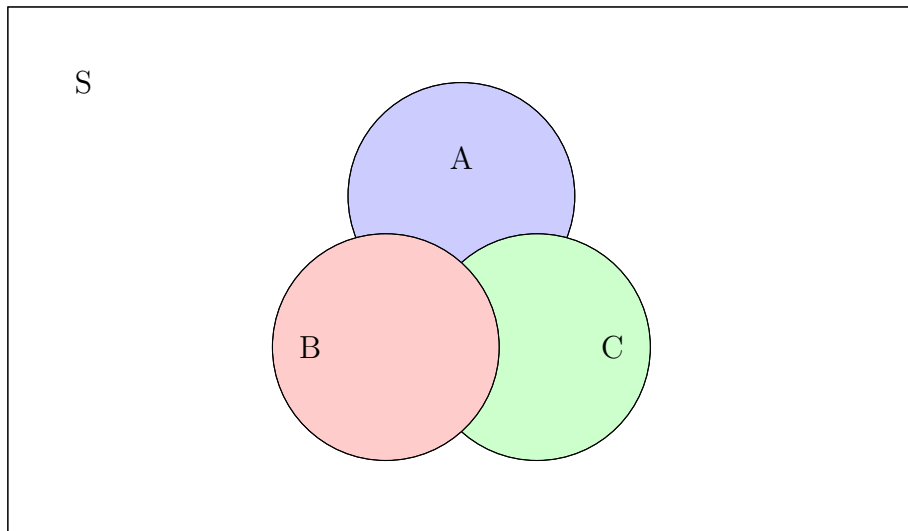
Then from this, we can conclude that  $P(\{3\})$  must be equal to  $\frac{2}{6}$  because  $P(\{1, 2, 3\})$  cannot exceed past  $\frac{6}{6}$  namely 1, by *Kolmogorov Axiom 2*.

6. Let  $S$  be the sample space, where events  $A, B, C$  can occur in  $S$ , then

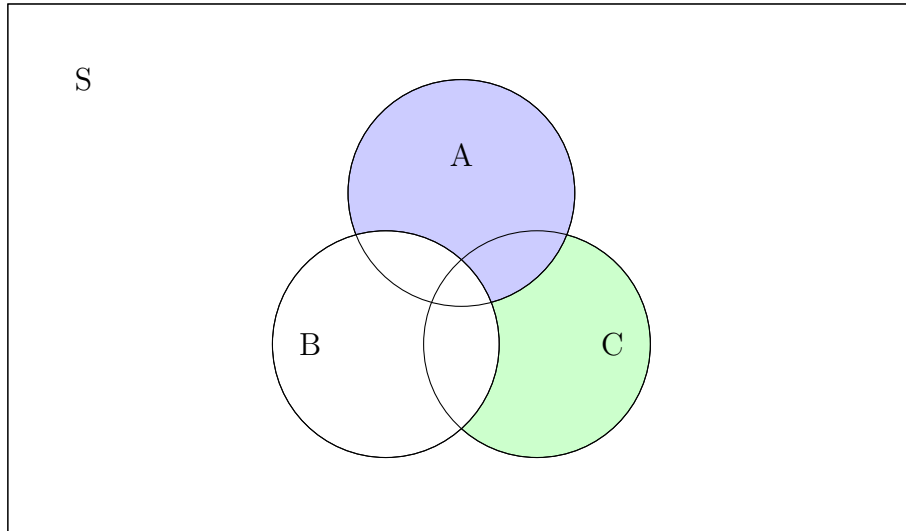
- (a) This is the Venn Diagram among  $A$ ,  $B$ , and  $C$ , only  $A$  occurs.



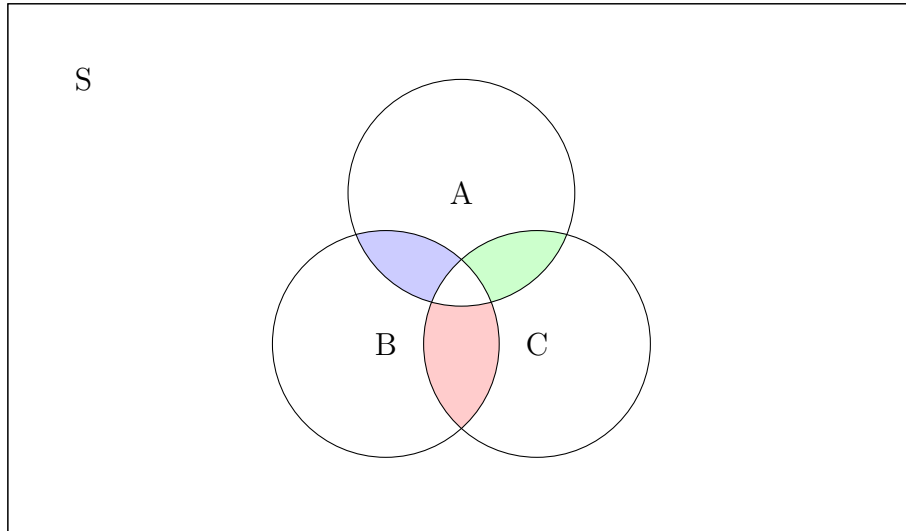
- (b) This is the Venn Diagram where at least one of the events  $A$ ,  $B$ , or  $C$  occurs.



- (c) This is the Venn Diagram where one of  $A$  or  $C$  occurs, but not  $B$ .



- (d) This is the Venn Diagram where at most, two of the events  $A$ ,  $B$ , or  $C$  occurs.



7. *Proof.* “ $A$  and  $B$  are independent  $\Leftrightarrow A^c$  and  $B$  are independent.”

“ $\Rightarrow$ ”: Since  $A$  and  $B$  are independent, we have:

$$P(A \cap B) = P(A)P(B) \quad (\Delta)$$

by Definition 2.18 (independence). Now we want to show that:

$$P(A^c \cap B) = P(A^c)P(B)$$

We know that:

$$P(B) = P(A^c \cap B) + P(A \cap B) \quad (\theta)$$

because the probability of  $B$ , is the same as probability of  $A$  and  $B$  happening added with probability of  $\neg A$  and  $B$ . We then also learn that:

$$P(A^c \cap B) = P(B) - P(A \cap B) \quad (\Theta)$$

by rearranging  $(\theta)$ .

We will now substitute  $(\Delta)$  into  $(\Theta)$  where we will have

$$P(A^c \cap B) = P(B) - P(A)P(B)$$

$$P(A^c \cap B) = P(B)(1 - P(A)) \quad (|)$$

and by Theorem 2.19 (Complement Rule), we learn that

$$(1 - P(A)) = P(A^c)$$

then this equation,

$$P(A^c \cap B) = P(A^c)P(B)$$

is identical to  $(|)$ . Hence we have proven that

$$A \text{ and } B \text{ are independent} \Rightarrow A^c \text{ and } B \text{ are independent.}$$

“ $\Leftarrow$ ”: Now, since  $A^c$  and  $B$  are independent, we have:

$$P(A^c \cap B) = P(A^c)P(B) \quad (\phi)$$

by Definition 2.18 (independence). This proof is trivial as it follows the same structure as the last one I did. Now we want to show that:

$$P(A \cap B) = P(A)P(B)$$

We know that:

$$P(B) = P(A^c \cap B) + P(A \cap B) \quad (\gamma)$$

because the probability of  $B$ , is the same as probability of  $A$  and  $B$  happening added with probability of  $\neg A$  and  $B$ . We then also learn that:

$$P(A \cap B) = P(B) - P(A^c \cap B) \quad (\Gamma)$$

by rearranging  $(\gamma)$ .

We will now substitute  $(\phi)$  into  $(\gamma)$  where we will have

$$P(A \cap B) = P(B) - P(A^c)P(B)$$

$$P(A \cap B) = P(B)(1 - P(A^c)) \quad (\psi)$$

and by Theorem 2.19 (Complement Rule), we learn that

$$(1 - P(A^c)) = P(A)$$

then this equation,

$$P(A \cap B) = P(A)P(B)$$

is identical to  $(\psi)$ . Hence we have proven that

$$A^c \text{ and } B \text{ are independent} \Rightarrow A \text{ and } B \text{ are independent.}$$

Therefore, We have proven that  $A$  and  $B$  are independent  $\Leftrightarrow A^c$  and  $B$  are independent.  $\square$

8. Let  $S$  be the event that the silver coin is found and let  $C$  be the event that a cabinet was selected.

We know that

$$P(C_A) = P(C_B) = \frac{1}{2}$$

because both drawers in cabinet A ( $C_A$ ) contains the silver coin, we have

$$P(S|C_A) = \frac{1}{2} + \frac{1}{2} = 1$$

and because only one of the silver coin is in cabinet B ( $C_B$ ), we have

$$P(S|C_B) = \frac{1}{2}$$

Using Theorem 2.20 (Rule of Probability), then we learn that

$$\begin{aligned}P(S) &= P(S|C_A)P(C_A) + P(S|C_B)P(C_B) \\&= (1 \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2}) \\&= \frac{3}{4}\end{aligned}$$

and using Theorem 2.21 (Bayes' Theorem), we can conclude that

$$P(C|S) = \frac{P(S|C)P(C)}{P(S)} = \frac{1 \times \frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

End of Assignment 1.