Stat 230 Introductory Statistics Course Introduction & Descriptive Statistics

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Terminology

Probability is a way to quantify the uncertainty surrounding the outcome of an experiment.

An *experiment* is a process by which we observe something random.

- eg. coin toss or the roll of a die
- eg. apply insecticide to a colony of ants

An *outcome* is the result of an experiment.

- eg. experiment = rolling a die,
  outcome = number of dots facing up when it lands.
- eg. insecticide applied to an ant,
  possible outcomes = the ant lives, the ant dies

What is Probability?

We call the collection of all possible outcomes of an experiment the *sample space*, often denoted by S.

- eg. roll of a die, this is $S = \{1, 2, 3, 4, 5, 6\}$.
- The sample space is comprised of subsets called events.

An *event* is any subset of the sample space, often denoted by the first letters of the alphabet, eg. $A, B, C, \ldots$

- For the roll of a die, the event that an odd number of dots appears is $E = \{1, 3, 5\}$.
- For the ant exposed to insecticide, the event that it dies is $E = \{\text{ant dies}\}$.

Why Do We Need the Probability Concept?

The classical probability concept was first stimulated by the needs of gamblers. *What is my chance to win?*

Now, probability is used to understand the risks associated with different events, such as being struck by lightning, or having a car accident.

An understanding of probability helps us to make predictions that have the least amount of uncertainty.

What is Probability?

We denote the probability that the event E occurs by $P(E)$.

For an experiment with possible outcomes $E_1, E_2, \ldots, E_n$, the probability $P(E_i)$ must obey the following rules:

1. probabilities must be non-negative real numbers
2. the probability of the entire sample space must be equal to 1
3. If two events are disjoint (i.e. they cannot happen at the same time), the probability that either of the events happens is the sum of the probabilities that each happens

Kolmogorov axioms (*Axioms of Probability*)

Let S be the sample space comprised of all possible outcomes $\{E_1, E_2, \ldots, E_n\}$.

Axiom 1 (Non-negative probability)

$P(E_i) \geq 0$ for all E_i

Axiom 2

$P(S) = P(E_1) + P(E_2) + \ldots + P(E_n) = 1$

Axiom 3

$$P(\cup_{i=1}^{m} E_i) = P(E_1) + P(E_2) + \ldots P(E_m)$$

Probability for Equally Likely Outcomes

- The process randomly selects one outcome from a fixed set of equally likely possible outcomes
- If all possible outcomes of a random process are equally likely, the probability of any of the outcomes is $\frac{1}{\text{numbers of all possibilities}}$
- That is, the probability of each possible outcome when all outcomes in the sample space $S = \{o_1, o_2, \ldots, o_n\}$ are equally likely is

$$P(o_1) = P(o_2) = \cdots = P(o_n) = \frac{1}{n}$$

Probability for Equally Likely Outcomes

Example 2.1

A fair die is rolled (the experiment). What is the probability of it landing with 5 dots facing up?

Probability for Equally Likely Outcomes

Theorem 2.1 (Equally likely events)

Suppose an experiment has n equally possible outcomes. If event E comprises r of these outcomes, then

$$P(E) = \frac{r}{n}$$

Example 2.2 (Events with multiple outcomes)

A fair die is rolled (the experiment). What is the probability of it showing an even number?

Probability for Equally Likely Outcomes

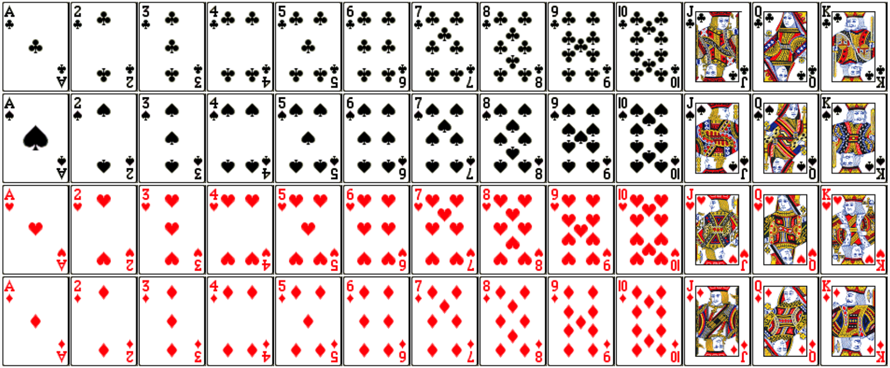
Example 2.3

A card is selected from a well shuffled pack (the experiment). What is the probability of it being a heart?

Example 2.4

A card is selected from a well shuffled pack (the experiment). What is the probability of it being black or a jack?

A Deck of Ordinary Playing Cards



Probability for Equally Likely Outcomes

Example 2.5

Toss a quarter and a loonie in the air. What is the probability that both land heads facing up?

Probability for Equally Likely Outcomes

Example 2.6

What is the probability that one coin lands tails up and the other lands heads up?

Exercises to try on your own

Exercise 2.1 A card is selected from a well shuffled pack (the experiment). What is the probability of it being red?

Exercise 2.2 A ball is chosen (at random) from a bag containing 3 red and 4 blue balls. What is the probability that the chosen ball is blue?

Exercise 2.3 Two fair dice are rolled.

2.3.1 What is the probability that both numbers are odd? [Hint: write out all of the possible outcomes; there are 36.]

2.3.2 What is the probability that both numbers are the same?

Counting

The list of total possible outcomes by rolling two dice (sample space)

	1	2	3	4	5	6
1	$(1,1)$	$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(1,6)$
2	$(2,1)$	$(2,2)$	$(2,3)$	$(2,4)$	$(2,5)$	$(2,6)$
3	$(3,1)$	$(3,2)$	$(3,3)$	$(3,4)$	$(3,5)$	$(3,6)$
4	$(4,1)$	$(4,2)$	$(4,3)$	$(4,4)$	$(4,5)$	$(4,6)$
5	$(5,1)$	$(5,2)$	$(5,3)$	$(5,4)$	$(5,5)$	$(5,6)$
6	$(6,1)$	$(6,2)$	$(6,3)$	$(6,4)$	$(6,5)$	$(6,6)$

Table: Possible combinations of two dice.

Introduction to Counting

- Without understanding the principals of counting, it is not possible to completely answer even some the most rudimentary probability problems.
- We are often required to count different permutations in order to solve problems.
- Let's look at some efficient ways of counting.

General Counting Rule

Theorem 2.2 (Fundamental Principle of Counting)

If a particular task may be accomplished n_1 ways and then a second task may be accomplished in n_2 ways, then the first task followed by the second task may the accomplished in $n_1 n_2$ different ways.

Example 2.7

When buying a new car from a particular dealership, you're given the option of four exterior colours (blue, black, white, red) and two interior materials (leather, cloth). How many unique versions of your desired car are there?

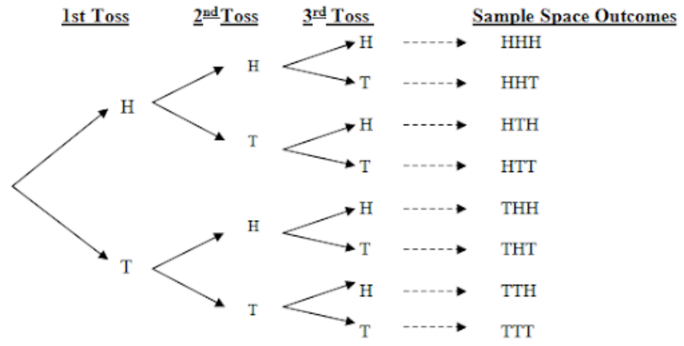
Counting Rule Extension

Theorem 2.3

If there is a job comprised of k tasks, the first of which may be accomplished n_1 ways, the second n_2, ..., and the k-th n_k then the job may be accomplished in $n_1 n_2 \cdots n_k$ unique ways.

Counting Rule Extension

For example we toss a coin three times. Suppose the first toss is task 1, the second toss task 2 and the third toss task 3, then the total number of unique outcomes from the three tasks taken together is $2 \times 2 \times 2$. We can use a tree diagram to see how many outcomes for the three coin tosses are possible.



[1]

[1] http://web.mnstate.edu/peil/MDEV102/U3/S25/S25_print.html

Counting Rule Extension

Example 2.8

Example 2: When buying a new car from a particular dealership, you're given the option of four exterior colours (blue, black, white, red) and two interior materials (leather, cloth). The dealer then offers you a free upgrade of your choice: a moon roof, a spoiler, or a subwoofer for your sound system. How many unique versions of your desired car are there?

Permutation

 [2] 

[3]

[2] https://www.hokeypokeyshop.ca/color-factory-paint-palette-713x337-plastic-rectangular-9-well

[3] https://www.alamy.com/stock-photo-top-view-closeup-of-water-color-palette-in-white-\plastic-containers-137829692.html

Permutation

Theorem 2.4

The number of ways of arranging n distinct objects in a line is

$$n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$$

$n!$ *is called* **n *factorial***.

Definition 2.5 (permutation)

Each of $n!$ arrangement of n distinct objects is called a **permutation** of the n objects.

Exercises

Exercise 2.4 How many ways can the letters PAUL be arranged to give a four letter 'word'?

Exercise 2.5 What if the letters could be repeated?

Exercise 2.6 How many ways can 8 horses finish a race (assuming all finish and there are no ties)?

Exercises

Exercise 2.7 How many 7 letter 'words' can be formed from the letters KELOWNA?

Exercise 2.8 There are 14 teams in a soccer league. In how many different orders can the teams finish? You may assume that two teams cannot be tied.

Permutation with Repeats

Theorem 2.6

The number of ways of arranging n objects, of which m are identical is

$$\frac{n!}{m!}$$

Exercise 2.9 How many ways can the colors in a 7-color palette be arranged if we have a total of 7 packs of pigment where 3 of the packs are the same color?

Theorem 2.7

The number of ways of arranging n objects of k different kinds of which n_1 are of one kind, n_2 are of the second kind, $\dots$, n_k are of the kth kind, and $n_1 + n_2 + \cdots + n_k = n$ is of which m are identical is

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

Exercise 2.10 How many ways can the letters OKANAGAN be arranged if all must be used?

Exercises

Exercise 2.11 How many ways can the letters STATISTICS be arranged to make a 10 letter ‘word’?

Exercise 2.12 How many ways can the letters SCIENCE be arranged if all letters must be used?

Exercise 2.13 How many ways can the letters ONTARIO be arranged if all letters must be used?

Combination

Definition 2.8

The number of ways of choosing r items from n distinct items (in any order) is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Definition 2.9 (combination)

We call this type of selection, i.e. one that does not consider the order or the arrangement important, a **combination**.

Exercises

Exercise 2.14 How many ways can the colors in a 7-color palette be arranged if we have a total 14 distinct color pigments.

Exercise 2.15 How many ways can a committee of 6 people be chosen from 10 people?

Exercise 2.16 How many different poker hands (of 5 cards) are there? i.e. how many different choices are there of drawing 5 cards from a standard deck of 52 playing cards?

Exercises

As you will find throughout this course, there will often be multiple ways of arriving at the same conclusion

Exercise 2.17 In how many different ways can six tosses of a coin yield two heads and four tails?
Direct enumeration

Using Definition 2.8

Using Theorem 2.7

Probability & Counting

Exercise 2.18 In a lottery, 6 numbers are drawn from a drum with 49 numbers. How many different winning combinations are possible?

Experimental Probability

Definition 2.10

Experimental probability refers to the probability of an event occurring when an experiment was conducted.

$$\text{Probability} = \frac{\text{Number of event occurrences}}{\text{Total number of trials}}$$

Definition 2.11

The law of large numbers states that as more observations are collected, the proportion of occurrences with a particular outcome, $\hat{p}_n$, converges to the probability of that outcome, p

Experimental Probability

Example 2.9

Canada's population is approximately $36,000,000$ and in any given year, approximately 150 Canadians are hit by lightning. What is the probability that a Canadian would be hit by lightning in any one year period?

Experimental Probability

Example 2.10

What is the probability that a randomly sampled student thinks marijuana should be legalized <u>or</u> they agree with their parents' political views

	Share Parents' Politics		
Legalize MJ	No	Yes	Total
No	11	40	51
Yes	36	78	114
Total	47	118	165

Probability Rules - Multiple Events

Earlier, we computed probabilities of events occurring when tossing 1 die.
What would happen with more than 2 dice? Would we have to write out all of the possible outcomes?

	1	2	3	4	5	6
1	$(1,1)$	$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(1,6)$
2	$(2,1)$	$(2,2)$	$(2,3)$	$(2,4)$	$(2,5)$	$(2,6)$
3	$(3,1)$	$(3,2)$	$(3,3)$	$(3,4)$	$(3,5)$	$(3,6)$
4	$(4,1)$	$(4,2)$	$(4,3)$	$(4,4)$	$(4,5)$	$(4,6)$
5	$(5,1)$	$(5,2)$	$(5,3)$	$(5,4)$	$(5,5)$	$(5,6)$
6	$(6,1)$	$(6,2)$	$(6,3)$	$(6,4)$	$(6,5)$	$(6,6)$

Table: Possible combinations of two dice.

Multiple Events

- What if we rolled a die and chose a card — imagine writing out all of those outcomes ($6 \times 52 = 312$ in all)!
- Thankfully, we do not need to write out all of these outcomes — there are rules we can use... but first some definitions.

Set Notation

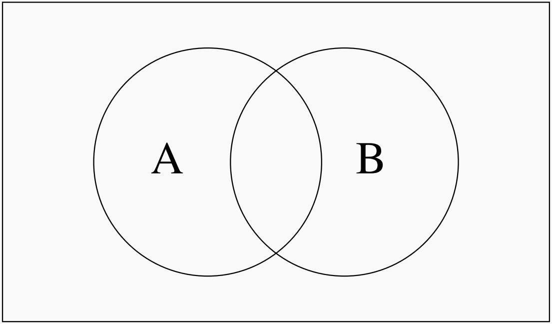
- Union: $A \cup B$
  - either A or B or both
- Intersection: $A \cap B$
  - A and B (both)
- Complement: A' (you may also run across A^c)
  - not A
- Two events are said to be disjoint (or mutually exclusive) if $A \cap B = \emptyset$

Multiple Events

Example 2.11

Consider rolling a die, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$
Let A be the event of rolling an even number: $A = \{2, 4, 6\}$
Let B be the event of rolling a value larger than 3: $B = \{4, 5, 6\}$

We can use *Venn diagrams* to visualize these events.



Useful Terminology for Multiple Events

Often events are combinations of two or more events formed by taking *unions, intersections,* and *complements*

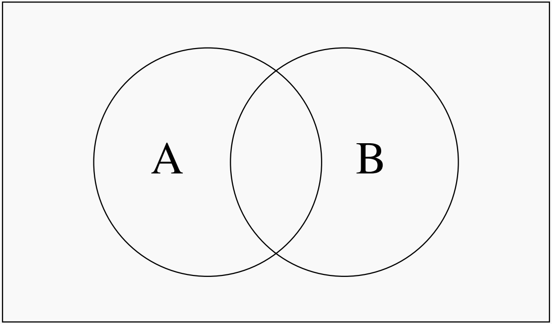
- You can interchange “union” for "or",
- “intersection” for "and" and
- “complement” (opposite) for "not"

Let’s return to Example 2.11 to see what I mean

Example 2.11 (cont'd)

Consider rolling a die, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$
Let A be the event of rolling an even number: $A = \{2, 4, 6\}$
Let B be the event of rolling a value larger than 3: $B = \{4, 5, 6\}$

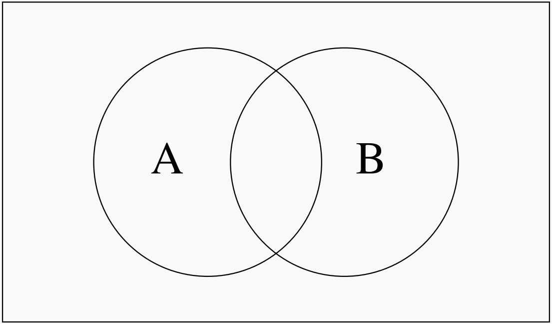
The *union of A and B*, denoted $A \cup B$ and read "A or B", is the set of all elements from A and B.



Example 2.11 (cont'd)

Consider rolling a die, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$
Let A be the event of rolling an even number: $A = \{2, 4, 6\}$
Let B be the event of rolling a value larger than 3: $B = \{4, 5, 6\}$

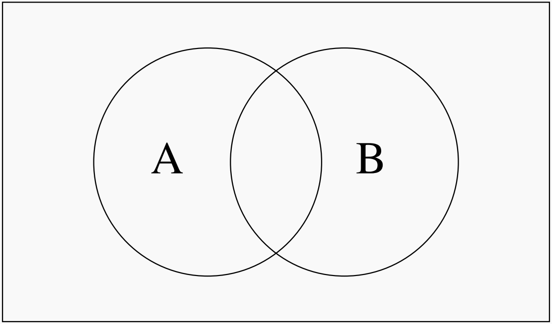
The *intersection of A and B* denoted by $A \cap B$ and read "A and B', is the set of all elements that A and B have in common



Example 2.11 (cont'd)

Consider rolling a die, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$
Let A be the event of rolling an even number: $A = \{2, 4, 6\}$
Let B be the event of rolling a value larger than 3: $B = \{4, 5, 6\}$

The *complement of A*, denote $\overline{A}$ *or* A^c and read "A complement" or "not A', is all elements in S that are not in A



Venn Diagrams

Example 2.12 (Venn Diagrams for Multiple Events)

Consider rolling a fair die. The sample space is $\{1, 2, 3, 4, 5, 6\}$
Let A be the event of rolling a one, two, or three: $A = \{1, 2, 3\}$
Let B be the event of rolling an even number: $B = \{2, 4, 6\}$
Let C be the event of rolling a 3: $C = \{3\}$. Draw the corresponding Venn Diagram

Venn Diagrams for Multiple Events

Definition 2.12 (mutually exclusive)

Two events are *mutually exclusive if they have no elements in common i.e. the events cannot occur at the same time.*

- For example, $B = \{2, 4, 6\}$ and $C = \{3\}$ have no elements in common. Therefore they are mutually exclusive. (We cannot roll a number that is both even and a 3)
- B and C are mutually exclusive if $B \cap C = \emptyset$.
- On a Venn Diagram, two events that have no overlap are mutually exclusive

Example 2.12 (Venn Diagrams for Multiple Events)

Consider rolling a fair die. The sample space is $\{1, 2, 3, 4, 5, 6\}$
Let A be the event of rolling a one, two, or three: $A = \{1, 2, 3\}$
Let B be the event of rolling an even number: $B = \{2, 4, 6\}$
Let C be the event of rolling a 3: $C = \{3\}$. Draw the corresponding Venn Diagram

C is *contained in A*, denoted $C \subset A$ (C is a subset of A), if all elements in C are elements in A.

Addition Rule - Probability of a Union

Theorem 2.13 (Addition Rule or the "OR" Rule)

*For any two events E and F, the probability of E **or** F occurring is given by*

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

In reference to the rolling a die experiment in Example 2.12, if A $= \{1, 2, 3\}$ and B $= \{2, 4, 6\}$, what is the probability of A union B?

Addition Rule - Probability of a Union

Note that if E and F are mutually exclusive then $P(E \cap F) = 0$ and we have the special case formula previously discussed: $P(E \cup F) = P(E) + P(F)$.

The **"OR Rule" (or Addition Rule) for mutually exclusive events**: if E and F are mutually exclusive events then

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F)$$

Returning to Example 2.12, if B $= \{2, 4, 6\}$ and C $= \{3\}$ are mutually exclusive, then what is the $P(B \cup C)$?

Addition Rule - Probability of a Union

Example 2.13

A coin is flipped 3 times, what is the probability that it lands heads at least twice?

Let "2" be the event that the coin lands heads (exactly) twice
Let "3" be the event that the coin lands heads (exactly) three times
$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

Addition Rule - Probability of a Union

Example 2.14

A card is selected from a well shuffled pack, what is the probability that it is a jack or a black card?

Example 2.15

A card is selected from a well shuffled pack. What is the probability of it being a jack or a 5?

Conditional Probability

Conditional probability aims at finding the probability of an event, given that a certain event has already occurred.

Definition 2.14

The conditional probability of A given B is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)},$$

provided $P(B) > 0$.

Conditional Probability

Example 2.16

Suppose a fair die is rolled. (i) What is the probability that the number of dots landing face up is odd? (ii) What is the probability that the number of dots is odd given that the number is greater than 3?

Conditional Probability

Example 2.17

Two cards are selected from a well shuffled pack. If the first card drawn was a jack, what is the probability that the second card drawn is also a jack?

Probability of the Intersection of 2 Events

Definition 2.15 (Multiplication Rule or "AND Rule")

If E and F are two events then

$$\begin{aligned} P(E \cap F) &= P(E)P(F \mid E) \\ &= P(F)P(E \mid F) \end{aligned}$$

where $F \mid E$ means the occurrence of an event F given that an event E has already occurred and $E \mid F$ means the occurrence of an event E given that an event F has already occurred.

Probability of the Intersection of 2 Events

Example 2.18

Two cards are selected from a well shuffled pack. What is the probability that they are both jacks?

Let $J1$ be the event that the first card is a jack and
Let $J2$ be the event that the second card is a jack, then

Probability of the Intersection of 2 Events

Example 2.19

There are 3 red and 4 green balls in a bag. Two balls are selected consecutively, at random without replacement. What is the probability that the first ball is red and the second is green?

Generalizations to more than 2 events

The AND and OR rules can be extended to more than two events.

Theorem 2.16 (Generalization of Theorem 2.13)

If sample space S is comprised of k events: $A_1, A_2, \ldots, A_k$ such that $P(A_i \cap A_j) \neq 0$ for all $i \neq j$, then

$$P(\cap_{i=1}^{k} A_k) = P(A_1) \cdot P(A_2|A_1) \cdots P(A_k|A \cap_{i=1}^{k-1} A_k).$$

For instance, if $k = 3$:
If A, B, and C are any three events in a sample space S such that $P(A \cap B) \neq 0$, then

$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B).$$

Probability of the Intersection of More than 2 Events

Example 2.20

There are 3 red, 4 green and 7 blue balls in a bag. Three balls are selected consecutively, at random. What is the probability that the first is a red, the second green and the third blue?

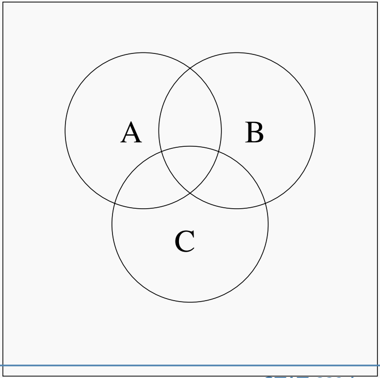
Let $R1$ be the event that the first ball is red
Let $G2$ be the event that the second ball is green
Let $B3$ be the event that the third ball is blue

Probability of the Union of More than 2 Events

Theorem 2.17 (Generalization of Theorem 2.15)

If A, B and C are three events in a sample space S, then

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) \\ &\quad - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C). \end{aligned}$$



Statistical Independence

There is an important special case of the AND rule — the case where the events are independent. In fact, statistical independence is often defined in terms of the AND rule.

Definition 2.18 (independence)

Two events E and F are said to be *statistically independent* if

$$P(E \cap F) = P(E)P(F)$$

Some Notes

- In essence: two events are independent if the outcome of one has no effect on the outcome of the other.
- Thus, the following rules also apply if events E and F are statistically independent:
  - $P(F \mid E) = P(F)$
  - $P(E \mid F) = P(E)$
- Note that this is very different from *mutually exclusive* - many students mix the two definitions up!
- In fact, if two events E and F are mutually exclusive, then the following is true:
  - $P(F \mid E) = 0$
  - $P(E \mid F) = 0$

Some Examples

Example 2.21 (Ex:1)

A card is selected from a well shuffled pack and a die is rolled. What's the probability of obtaining a red card and an even number?

Example 2.22

A card is selected from a well shuffled pack and a coin is tossed. What is the probability of obtaining a queen and a tail?

The Complement Rule

Theorem 2.19 (Complement Rule)

For any event E, $P(E) = 1 - P(\overline{E})$.

Note that basic probability rules discussed previously also hold conditionally, i.e.

$$P(A \mid B) = 1 - P(\overline{A} \mid B)$$
$$P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$$

The Complement Rule

Example 2.23

A coin is flipped 20 times, what is the probability that it lands heads at least once?

Let H_i be the event that i coins land head facing up
Let E be the event that the coin lands heads at least once

$S =$

$E =$

$\overline{E} = E^c =$

$P(\overline{E}) =$

$P(E) = 1 - P(\overline{E})$

Some Food for thought

In Example 2.23, what would happen if we had to work out the probability of it landing heads up 11 times?

- We need to study counting and some further probability to work this out.

Exercises

Exercise 2.19 Two cards are chosen at random from a well shuffled pack. What is the probability that:

2.19.1 Both are red?

2.19.2 None are red?

2.19.3 One is a club and one is a heart? [Be careful!]

Exercise 2.20 A die is rolled 10 times. What is the probability that:

2.20.1 The number 6 does not show at all?

2.20.2 No odd numbers show?

2.20.3 An odd number shows at least once?

More Exercises

Exercise 2.21 Lotto rules: Choose 6 numbers from 1–49, each number is only selected once (in other words, the combination {1,30,32,32,5,8} is impossible because the number 32 cannot arise twice). Order of selection does not matter.

2.21.1 What is the probability of winning the Lotto?

2.21.2 What is the probability of matching at least one number in the Lotto?

Exercise 2.22 Suppose that a class consists of eleven females (including the professor) and four males. If two students are chosen at random, what is the probability that:

2.22.1 Both are female?

2.22.2 Both are male?

Exercise 2.23 A card is chosen from a well shuffled pack. What is the probability that it is:

2.23.1 A queen or a heart?

2.23.2 A 3 or a black card?

2.23.3 An odd numbered card or a diamond?

Exercise 2.24 A bag contains nine balls numbered 1 – 9; balls 1 – 3 are red, balls 4 – 6 are blue and balls 7 – 9 are green. A ball is selected at random. What is the probability that:

2.24.1 It is blue?

2.24.2 It has an odd number on it?

2.24.3 It is blue or it has an odd number on it?

Fact

Let A and B be events. We may express A as

$$A = \{A \cap B\} \cup \{A \cap \bar{B}\}$$

where $\bar{B}$ is the complement of event B.

- $\{A \cap B\}$ and $\{A \cap \bar{B}\}$ are disjoint events
- The probability of event A is

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap \bar{B}) \\ &= P(A|B)P(B) + P(A|\bar{B})P(\bar{B}) \end{aligned}$$

Fact

More generally if D, E, and F are disjoint events which contain all of the outcomes of another event A, then

$$\begin{aligned} P(A) &= P(A \cap D) + P(A \cap E) + P(A \cap F) \\ &= P(A|D)P(D) + P(A|E)P(E) + P(A|F)P(F) \end{aligned}$$

We can extend this in analogous ways to more than three events.

The Partition Theorem

What we actually used in the last example is The Partition Theorem. This approach can be generalized to get a formula for $P(E)$ in terms of conditional probabilities.

Theorem 2.20 (Rule of total probability)

The Partition Theorem: *If $F_1, F_2, \ldots, F_n$ are mutually exclusive events of which one must occur, then*

$$P(E) = \sum_{i=1}^{n} P(E \mid F_i) P(F_i)$$

The events $\{F_1, F_2, \ldots, F_n\}$ make up a **partition** of the sample space of an experiment.

Partition Theorem Example

Example 2.24

In a region, 31% of people are smokers, 19% of smokers develop lung cancer and 2% of non-smokers develop lung cancer. What is the probability that a person chosen at random will develop lung cancer; i.e. what percentage of this region will suffer from lung cancer?

Partition Theorem Example

Example 2.25

Consider the population of people who drive cars. We let Y, M, and O represent age groups (young, middle-aged, and old) and A as the event that a randomly selected driver has an accident. Suppose the proportion of the population that is the young group is $P(Y) = 0.132$, the proportion that is the middle-aged age group is $P(M) = 0.356$, and the proportion that is the old group is $P(O) = 0.512$. Suppose also that we know 16% in the young group had an accident, 8% of the middle-aged group had an accident, and 4% in the old group. Now we can find the probability that a driver has an accident.

Conditional Probability Example

Exercise 2.25 Suppose two cards are dealt from a well-shuffled deck of 52 cards. What is the probability that the second card is black?

Conditional Probability II

- Using the fact that $P(E \cap F) = P(F \cap E)$ and noting that $P(F \cap E) = P(E \mid F)P(F)$, we can write the expression for $P(F \mid E)$ as follows.

$$P(F \mid E) = \frac{P(E \mid F)P(F)}{P(E)} \tag{1}$$

- Now, we can also rewrite the term $P(E)$ using partition theorem in this equation.

Bayes' Theorem

Theorem 2.21 (Bayes' Theorem)

If $F_1, F_2, \ldots, F_n$ are mutually exclusive events of which one must occur, nd $P(F_i) \neq 0$ for $i = 1, 2, ..., n$ then for any event E

$$P(F_j \mid E) = \frac{P(E \mid F_j)P(F_j)}{\sum_{i=1}^{n} P(E \mid F_i)P(F_i)}, \quad j = 1, 2, \ldots, n.$$

Bayes' Theorem Example

Example 2.26

Returning to the lung cancer example, work out the probability that someone smokes given that they have lung cancer.

Exercise 2.26 The members of a firm rent cars from three rental agencies: 60% from agency 1, 30% from agency 2, and 10% from agency 3. If 9% of cars from agency 1 need a tune-up, 20% of the cars from agency 2 need a tune-up, and 6% of cars from agency 3 need a tune-up, what is the probability that a rental car delivered to the firm needs a tune-up?

Exercise 2.27 Referring back to Example 2.26, if we discover that a rental car does in fact need a tune-up, what is the probability the delivered car is from agency 2?

Exercise

Exercise 2.28 Personal computers are assembled on two production lines, 60% are assembled on Line 1 and 40% on Line 2. QC records show that both lines are not equally reliable: 95% of units assembled by Line 1 require no rework, while the figure for Line 2 is only 88%.

2.28.1 What percentage of all computers require rework?

2.28.2 If a computer is found to require rework, what is the probability that it came from Line 1?