STAT230 Assignment 1

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1. Let S be the sample of the given student, then we have

$$S = \{m, m, m, m, m, m, w, w, w, w\}$$

where m is men, and w is women.

(a) Let $n = sample\, size$. Then there are a number of

$$n! = 10! = 3628800$$

different ways to rank the students by Theorem 2.4.

(b) Let $n = sample\, size$, $w = women\, in\, n$ and $m = men\, in\, n$. Then there are a number of

$$\frac{n!}{m!w!} = \frac{10!}{6! \times 4!} = 210$$

different ways of ranking them with regards to men and women by Theorem 2.7.

2. Let S be the sample of the given flags, then we have

$$S = \{w, w, w, w, r, r, r, b, b\}$$

where w is white flag, r is red flag, and b is blue flag. Then there are a number of

$$\frac{n!}{w! \times r! \times b!} = \frac{9!}{4! \times 3! \times 2!} = 1260$$

different ways to arrange the flags if all flag in the same color is identical.

3. Let S be the set of all possible rolls of a two fair dice, and let n be the number of all possible rolls which is $6 \times 6 = 36$.

   (a) Let S_6 be set of two fair dice getting a total number of 6, then we have
   $$S_6 = \{(1,5),(5,1),(4,2),(2,4),(3,3),\}$$
   Let $P(S_6)$ be the probability of getting the number 6 by rolling two fair dice, then we have
   $$P(S_6) = \frac{Size\,of\,S_6}{n} = \frac{5}{36}$$

   (b) Let $S_{3 \vee 10}$ be the set of two fair dice getting a total number of $3 \vee 10$, then we have
   $$S_{3 \vee 10} = \{(1,2),(2,1),(5,5),(4,6),(6,4)\}$$
   Let $P(S_{3 \vee 10})$ be the probability of getting the number $3 \vee 10$ by rolling two fair dice, then we have
   $$P(S_{3 \vee 10}) = \frac{Size\,of\,S_{3 \vee 10}}{n} = \frac{5}{36}$$

   (c) Let $S_{odd \vee prime}$ be the set of two fair dice getting a total number that is $odd \vee prime$, then we have
   $$S_{odd \vee prime} =$$
   $$\{(1,1),(1,2),(2,1),(2,3),(3,2),$$
   $$(4,1),(1,4),(3,4),(4,3),(2,5),$$
   $$(5,2),(1,6),(6,1),(5,6),(6,5)$$
   $$(3,6),(6,3),(4,5),(5,4)\}$$
   Let $P(S_{odd \vee prime})$ be the probability of getting a number that is $odd \vee prime$ by rolling two fair dice, then we have
   $$P(S_{odd \vee prime}) = \frac{Size\,of\,S_{odd \vee prime}}{n} = \frac{19}{36}$$

4. Let us denote the given data by ϕ, the size of ϕ as n which is 20 data points.

   (a) Then the mean of ϕ is

   $$\overline{\phi} = \frac{\sum \phi}{n} = \frac{74.4}{20} = 3.72$$

   (b) Since we know that $\sum \phi^2 = 278.4196$, the standard deviation of ϕ is

   $$s_\phi = \sqrt{\frac{\sum \phi^2 - \frac{(74.4)^2}{20}}{n-1}} = \sqrt{\frac{278.4196 - 276.768}{20-1}} = 0.295$$

   (c) The median value of ϕ is

   $$\phi_{sorted} =$$

   $$\{3.55, 3.55, 3.56, 3.56, 3.57,$$

   $$3.57, 3.59, 3.59, 3.59, 3.60,$$

   $$3.61, 3.63, 3.65, 3.66, 3.71,$$

   $$3.73, 3.75, 3.99, 4.15, 4.79\}$$

   $$\tilde{\phi} = \frac{3.60 + 3.61}{2} = 3.605$$

   (d) The IQR of ϕ is

   $$Q1 = \frac{3.57 + 3.57}{2} = 3.57$$

   $$Q3 = \frac{3.71 + 3.73}{2} = 3.72$$

   $$IQR = Q3 - Q1 = 3.72 - 3.57 = 0.15$$

5. Since $P(\{1\}) = \frac{1}{2}$, we can also rewrite as $P(\{1\}) = \frac{3}{6}$ and since $P(\{1,2\}) = \frac{2}{3}$, we can rewrite as $P(\{1,2\}) = \frac{4}{6}$

   Then we can calculate $P(\{2\})$ as follow:

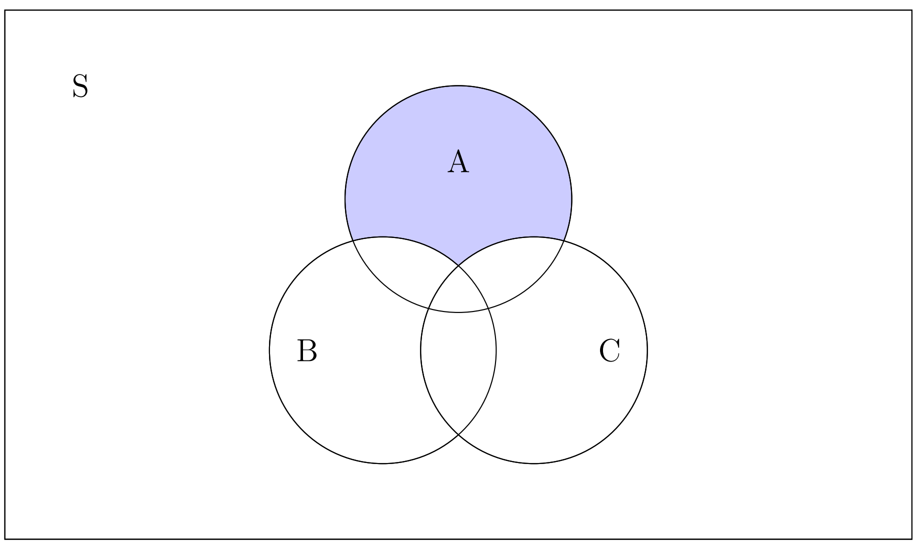
$$P(\{2\}) = P(\{1,2\}) - P(\{1\}) = \frac{4}{6} - \frac{3}{6}$$

$$P(\{2\}) = \frac{1}{6}$$

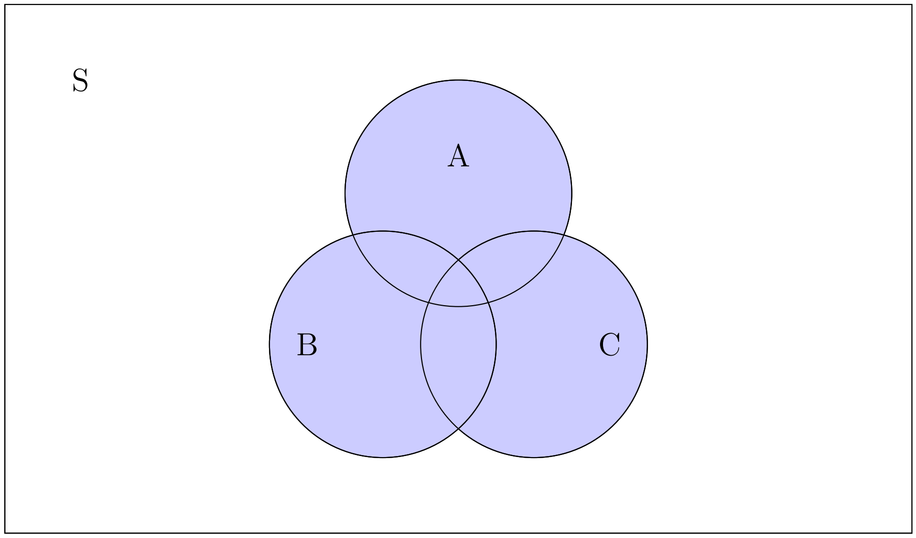
   Then from this, we can conclude that $P(\{3\})$ must be equal to $\frac{2}{6}$ because $P(\{1,2,3\})$ cannot exceed past $\frac{6}{6}$ namely 1, by $Kolmovgorov\ Axiom$ 2.

6. Let S be the sample space, where events A, B, C can occur in S, then

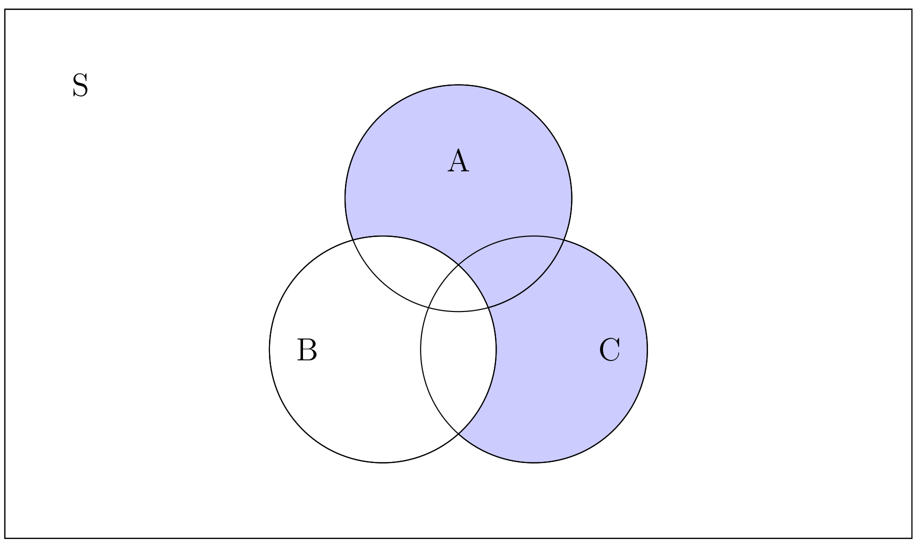
   (a) This is the Venn Diagram among A, B, and C, only A occurs.



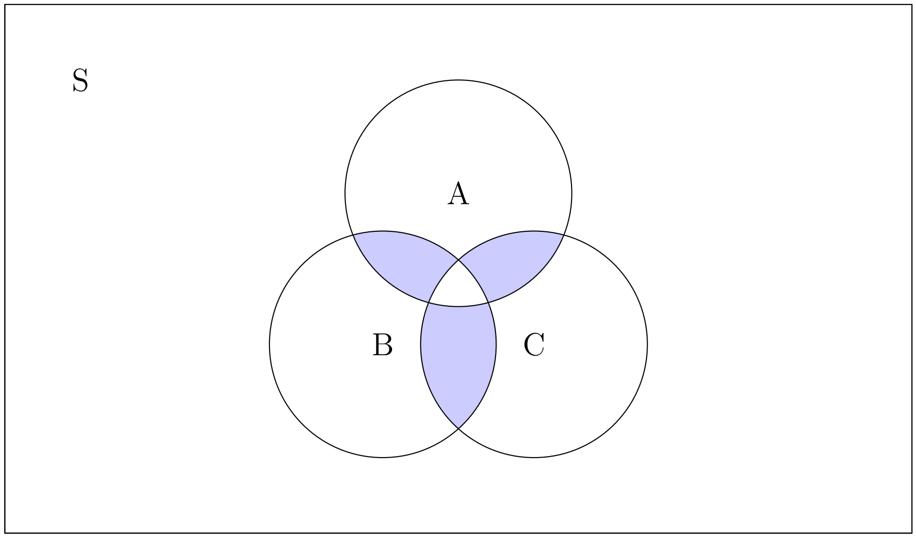
   (b) This is the Venn Diagram where at least one of the events A, B, or C occurs.



(c) This is the Venn Diagram where one of A or C occurs, but not B.



(d) This is the Venn Diagram where at most, two of the events A, B, or C occurs.



7. *Proof.* "A and B are independent $\Leftrightarrow$ A^c and B are independent."

"$\Rightarrow$": Since A and B are independent, we have:

$$P(A \cap B) = P(A)P(B) \qquad (\triangle)$$

by Definition 2.18 (independence). Now we want to show that:

$$P(A^c \cap B) = P(A^c)P(B)$$

We know that:

$$P(B) = P(A^c \cap B) + P(A \cap B) \qquad (\theta)$$

because the probability of B, is the same as probability of A and B happening added with probability of $\neg A$ and B. We then also learn that:

$$P(A^c \cap B) = P(B) - P(A \cap B) \qquad (\Theta)$$

by rearranging (θ).

We will now substitute $(\triangle)$ into (Θ) where we will have

$$P(A^c \cap B) = P(B) - P(A)P(B)$$

$$P(A^c \cap B) = P(B)(1 - P(A)) \qquad (|)$$

and by Theorem 2.19 (Complement Rule), we learn that

$$(1 - P(A)) = P(A^c)$$

then this equation,

$$P(A^c \cap B) = P(A^c)P(B)$$

is identical to $(|)$. Hence we have proven that

$$A \text{ and } B \text{ are independent } \Rightarrow A^c \text{ and } B \text{ are independent.}$$

"$\Leftarrow$": Now, since A^c and B are independent, we have:

$$P(A^c \cap B) = P(A^c)P(B) \qquad (\phi)$$

by Definition 2.18 (independence). This proof is trivial as it follows the same structure as the last one I did. Now we want to show that:

$$P(A \cap B) = P(A)P(B)$$

We know that:

$$P(B) = P(A^c \cap B) + P(A \cap B) \qquad (\gamma)$$

because the probability of B, is the same as probability of A and B happening added with probability of $\neg A$ and B. We then also learn that:

$$P(A \cap B) = P(B) - P(A^c \cap B) \qquad (\Gamma)$$

by rearranging (γ).

We will now substitute (ϕ) into (Γ) where we will have

$$P(A \cap B) = P(B) - P(A^c)P(B)$$

$$P(A \cap B) = P(B)(1 - P(A^c)) \qquad (\psi)$$

and by Theorem 2.19 (Complement Rule), we learn that

$$(1 - P(A^c)) = P(A)$$

then this equation,

$$P(A \cap B) = P(A)P(B)$$

is identical to (ψ). Hence we have proven that

$$A^c \text{ and } B \text{ are independent } \Rightarrow A \text{ and } B \text{ are independent.}$$

Therefore, We have proven that A and B are independent $\Leftrightarrow$ A^c and B are independent. $\square$

8. Let S be the event that the silver coin is found and let C be the event that a cabinet was selected.

   We know that

$$P(C_A) = P(C_B) = \frac{1}{2}$$

   because both drawers in cabinet A (C_A) contains the silver coin, we have

$$P(S|C_A) = \frac{1}{2} + \frac{1}{2} = 1$$

   and because only one of the silver coin is in cabinet B (C_B), we have

$$P(S|C_B) = \frac{1}{2}$$

   Using Theorem 2.20 (Rule of Probability), then we learn that

$$\begin{aligned} P(S) &= P(S|C_A)P(C_A) + P(S|C_B)P(C_B) \\ &= (1 \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2}) \\ &= \frac{3}{4} \end{aligned}$$

   and using Theorem 2.21 (Bayes' Theorem), we can conclude that

$$P(C_A|S) = \frac{P(S|C_A)P(C_A)}{P(S)} = \frac{1 \times \frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

   End of Assignment 1.