University of British Columbia

Sauder School of Business

Introductory Statistics
STAT 230

Assignment 2
Due February 03, 2024

Full Name:_____ Out of 30 _____

Instructions:

- **You must show significant steps to get full marks!**
- This assignment is out of 30 points.

1. The length of eggs from a species of chicken are known to be normally distributed with a mean of 6 cm and a standard deviation of 1.4 cm.
   (a) (2 points) What is the probability of finding an egg bigger than 8 cm in length?
   (b) (2 points) What is the probability of finding an egg smaller than 8 cm in length?
   (c) (3 points) What is the interquartile range for the length of eggs?

2. (5 points) A machinist doesn't know the mean and standard deviation of the diameters of ball bearings he is producing. However, a sorting system rejects all bearings larger than 4.8 mm and those under 3.6 mm. Out of 1000 ball bearings, 4% are rejected as too small and 10% as too big.
   (a) What is the mean and standard deviation of the ball bearings produced (to two decimal places)?

3. Let $T \geq 0$ be the random variable of some life time (in years) of a standard light bulb. Suppose that

$$f(t) = Ce^{-t/3}$$

   for some constant C.
   (a) (2 points) Find C such that the $f(t)$ is a probability distribution.
   (b) (2 points) What is the probability that a light bulb lasts longer than 5 years, given your answer in part (a)?
   (c) (3 points) What is the expected lifetime of a light bulb, given your answer in part (a)?

4. In a local city, 46 percent of the city's population favor the incumbent for mayor.
   (a) (4 points) A simple random sample of 500 is taken. Use the normal distribution (with continuity correction) to find the probability that at least 250 favor the incumbent for mayor.
   (b) (3 points) If only 50 people are randomly sampled, use the Poisson approximation to find the probability that exactly half the sample favor the incumbent for mayor.

5. (4 points) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Suppose $Y = g(x) = \frac{1}{1+x}$. Find the expected value of Y. (Hint: See definition 4.5 in the notes and use the substitution $u = 1 + x$ in the integral.