Linear ~~reg.~~ Assumption

(a) Ans: The bias of an estimator measures how far its average value is from the true parameter. Mathematically $Bias(\hat{\beta}) = E[\hat{\beta}] - \beta$ if it is equals 0, the estimator is unbiased.

(b) for OLS unbiased meaning $E[\hat{\beta}] = \beta$

$$\hat{\beta} = (X'X)^{-1}X'y \qquad y = X\beta + \varepsilon$$

$$\hat{\beta} = (X'X)^{-1}X'X\beta + (X'X)^{-1}X'\varepsilon \Rightarrow$$

$$\hat{\beta} = \beta + (X'X)^{-1}X'\varepsilon$$

$$E[\hat{\beta}|X] = E[\beta + (X'X)^{-1}X'\varepsilon|X] =$$

$$= \beta + (X'X)^{-1}X'E[\varepsilon|X]$$

OLS assumes the errors have zero ~~meaning~~ mean

given X $E[\varepsilon|X] = 0 \Rightarrow$ $\boxed{E[\hat{\beta}|X] = \beta}$

c) Homoskedasticity

Mathematical form: $Var(\varepsilon_i | X) = \sigma^2 \ \forall i$

That means every error term ε_i has the same variance - the spread of the error is constant across all levels of X.

d) $E[\varepsilon | X] = 0$

$Var(\varepsilon | X) = \sigma^2 I_n$

X has full rank (so $X'X$ is invertible)

OLS

$$\hat{\beta} = (X'X)^{-1} X'y \qquad y = X\beta + \varepsilon$$

$$\hat{\beta} = (X'X)^{-1} X'(X\beta + \varepsilon) = \beta + (X'X)^{-1} X'\varepsilon$$

* $Var(A\varepsilon | X) = A \, Var(\varepsilon | X) A'$

$$Var(\hat{\beta} | X) = Var((X'X)^{-1} X'\varepsilon | X) = (X'X)^{-1} X' Var(\varepsilon | X) X (X'X)^{-1}$$

$$Var(\hat{\beta} | X) = (X'X)^{-1} X' (\sigma^2 I_n) X (X'X)^{-1} =$$

$$= \sigma^2 \underbrace{(X'X)^{-1} X'X}_{I} (X'X)^{-1} = \sigma^2 (X'X)^{-1}$$

e) $E[\varepsilon|X]=0$ $\quad Var(\varepsilon|X)=\sigma^2 I_n$

The avg res. is 0
they're centered around the regression line

The spread of res is the same for all obs

$Var(\hat{\varepsilon}|X)=\sigma^2(I-H)$

$H=X(X'X)^{-1}X'$

If $\varepsilon \sim N(0,\sigma^2 I)$ then the res are normally distributed as well: $\hat{\varepsilon} \sim N(0,\sigma^2(I-H))$

$$\hat{y}=X\hat{\beta}=X(X'X)^{-1}X'y=Hy$$

$$\hat{\varepsilon}=y-\hat{y}=y-Hy=(I-H)y$$

$$\hat{\varepsilon}=(I-H)(X\beta+\varepsilon)$$

$(I-H)(X\beta)=0$ (a key prop of the hat matrix)

$$(I-X(X'X)^{-1}X')X=$$
$$=X-X\underbrace{(X'X)^{-1}X'X}_{I}=X-X=0.$$

so $\hat{\varepsilon}=(I-H)\varepsilon$

1 D
2 B
3 B
4 D
5 D

6 C
7 D
8 A
9 B
10 A

11 D
12 C
13 A
14 B

15 A
16 d
17 b
18 C

15 b