Answers to the mid-term examination on Logic Circuit Design

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1 Binary numbers

Question. If $N = b_{n-1}b_{n-2}\dots b_0$ is an n-bit binary number, what is $2N$ and $\lfloor N/2 \rfloor$?

Answer. By definition

$$\begin{aligned}
N &= b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \cdots + b_1 2 + b_0\\
&= 2(b_{n-1}2^{n-2} + b_{n-2}2^{n-3} + \cdots + b_1) + b_0\\
\lfloor N/2 \rfloor &= b_{n-1}2^{n-2} + b_{n-2}2^{n-3} + \cdots + b_1\\
&= 0 \times 2^{n-1} + b_{n-1}2^{n-2} + b_{n-2}2^{n-3} + \cdots + b_1\\
&= 0b_{n-1}b_{n-2}\dots b_1
\end{aligned}$$

And

$$\begin{aligned}
2N &= 2(b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \cdots + b_1 2 + b_0)\\
&= b_{n-1}2^{n} + b_{n-2}2^{n-1} + \cdots + b_1 2^2 + b_0 2^1 + 0 \times 2^0\\
&= b_{n-1}b_{n-2}\dots b_1 b_0 0
\end{aligned}$$

2 Boolean algebra

Question. Using truth tables, prove the distributive laws

$$\begin{aligned}
A + BC &\stackrel{1}{=} (A+B)(A+C)\\
A(B+C) &\stackrel{2}{=} AB + AC
\end{aligned}$$

Answer. Using truth tables, prove the distributive laws

$$A + BC \overset{1}{=} (A+B)(A+C)$$
$$A(B+C) \overset{2}{=} AB + AC$$

A	B	C	BC	$\mathbf{A+BC}$	$A+B$	$A+C$	$\mathbf{(A+B)(A+C)}$
0	0	0	0	**0**	0	0	**0**
0	0	1	0	**0**	0	1	**0**
0	1	0	0	**0**	1	0	**0**
0	1	1	1	**1**	1	1	**1**
1	0	0	0	**1**	1	1	**1**
1	0	1	0	**1**	1	1	**1**
1	1	0	0	**1**	1	1	**1**
1	1	1	1	**1**	1	1	**1**

A	B	C	$B+C$	$\mathbf{A(B+C)}$	AB	AC	$\mathbf{AB+AC}$
0	0	0	0	**0**	0	0	**0**
0	0	1	1	**0**	0	0	**0**
0	1	0	1	**0**	0	0	**0**
0	1	1	1	**0**	0	0	**0**
1	0	0	0	**0**	0	0	**0**
1	0	1	1	**1**	0	1	**1**
1	1	0	1	**1**	1	0	**1**
1	1	1	1	**1**	1	1	**1**

Given the distributive laws and the following

$$1 + A \overset{3}{=} 1 \qquad 0 \cdot A \overset{4}{=} 0 \qquad 0 + A \overset{5}{=} A \qquad 1 \cdot A \overset{6}{=} A$$
$$A + A \overset{7}{=} A \qquad A \cdot A \overset{8}{=} A \qquad A + \overline{A} \overset{9}{=} 1 \qquad A \cdot \overline{A} \overset{10}{=} 0 \qquad \overline{\overline{A}} \overset{11}{=} A$$

prove the absorption laws

$$A + AB \overset{12}{=} A$$
$$A(A+B) \overset{13}{=} A$$

without truth tables.

Question. Given the distributive laws and the following

$$1 + A \overset{3}{=} 1 \qquad 0 \cdot A \overset{4}{=} 0 \qquad 0 + A \overset{5}{=} A \qquad 1 \cdot A \overset{6}{=} A$$
$$A + A \overset{7}{=} A \qquad A \cdot A \overset{8}{=} A \qquad A + \overline{A} \overset{9}{=} 1 \qquad A \cdot \overline{A} \overset{10}{=} 0 \qquad \overline{\overline{A}} \overset{11}{=} A$$

prove the absorption laws

$$A + AB = A \tag{1}$$
$$A(A + B) = A \tag{2}$$

without truth tables.

Answer. For the sake of simplicity, we assume $AB = BA$ and $A + B = B + A$. There are several possible answers.

$$A + AB \overset{6}{=} A \cdot 1 + AB \overset{2}{=} A(1 + B) \overset{3}{=} A \cdot 1 \overset{6}{=} A$$

Note: equation 2 is used after replacing C by 1.

$$A(A + B) \overset{5}{=} (A + 0)(A + B) \overset{1}{=} A + 0 \cdot B \overset{4}{=} A + 0 \overset{5}{=} A$$

Note: equation 3 is used after replacing C by 0. Or, assuming $\overset{12}{=}$:

$$A(A + B) \overset{2}{=} A \cdot A + AB \overset{8}{=} A + AB \overset{12}{=} A$$

Note: equation 2 is used after replacing C by A.

3 Addition of BCD numbers

Question. State any overflow or underflow.

$$\begin{array}{lrrrr} \text{(a)} & 0111 & 0101 & 1001 & 0000 \\ & +\,0010 & 1001 & 0011 & 0101 \\ \hline \end{array}$$

$$\begin{array}{lrrrr} \text{(b)} & 0011 & 0110 & 0001 & 0111 \\ & +\,0001 & 0100 & 0110 & 1000 \\ \hline \end{array}$$

$$\begin{array}{lrrrr} \text{(c)} & 1001 & 0101 & 1000 & 0111 \\ & +\,0011 & 0101 & 0110 & 0010 \\ \hline \end{array}$$

Answer. Note that there cannot be underflows since we are adding positive numbers. There cannot be overflows because the number of digits of the result is not specified.

```
(a)           1 1          1      1 1
              0 1 1 1   0 1 0 1   1 0 0 1   0 0 0 0
              0 0 1 0   1 0 0 1   0 0 1 1   0 1 0 1
    -------------------------------------------------
                  1 1   1 1   1   1
              1 0 0 1   1 1 1 0   1 1 0 0   0 1 0 1
                        0 1 1 0   0 1 1 0
    -------------------------------------------------
            1 1 1
              1 0 1 0   0 1 0 1   0 0 1 0   0 1 0 1
              0 1 1 0
    -------------------------------------------------
    0 0 0 1   0 0 0 0   0 1 0 1   0 0 1 0   0 1 0 1

(b)             1 1     1
              0 0 1 1   0 1 1 0   0 0 0 1   0 1 1 1
              0 0 0 1   0 1 0 0   0 1 1 0   1 0 0 0
    -------------------------------------------------
                    1   1 1       1 1 1 1   1 1
              0 1 0 0   1 0 1 0   0 1 1 1   1 1 1 1
                        0 1 1 0             0 1 1 0
    -------------------------------------------------
              0 1 0 1   0 0 0 0   1 0 0 0   0 1 0 1

(c)             1 1     1   1               1 1
              1 0 0 1   0 1 0 1   1 0 0 0   0 1 1 1
              0 0 1 1   0 1 0 1   0 1 1 0   0 0 1 0
    -------------------------------------------------
            1 1     1   1 1   1   1 1
              1 1 0 0   1 0 1 0   1 1 1 0   1 0 0 1
              0 1 1 0   0 1 1 0   0 1 1 0
    -------------------------------------------------
    0 0 0 1   0 0 1 1   0 0 0 1   0 1 0 0   1 0 0 1
```