Answers to the mid-term exam on Introduction to the Internet

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I. Questions. Suppose two hosts A and B, separated by 1,000 km, and connected by a direct link of $R = 1$ Mbps. Propagation speed of the link is 2.5×10^8 m/sec.

1. Calculate the *bandwidth-delay product* $R \times d_{\text{prop}}$.

2. Consider sending a file of 400,000 bits from host A to host B. Suppose the file is sent continuously as one big message. What is the maximum number of bits that we will be in the link at any given time?

3. Provide an interpretation of the bandwith-delay product.

4. What is the width (in meters) of a bit in the link?

Answers.

1. The propagation delay of one bit is $10^6/(2.5 \times 10^8) = 4$ ms. We have $R \times d_{\text{prop}} = 10^6 \times (4 \times 10^{-3}) = 4,000$ bits.

2. During the time equal to the propagation delay, the number of bits that have been pushed on the link, i.e., transmitted, is $R \times d_{\text{prop}} = 10^6 \times (4 \times 10^{-3}) = 4,000$ bits.

3. The bandwidth-delay product is the maximum number of bits on the link.

4. Since there are 4,000 bits on the link at any time, the link, whose length is 1,000 km is shared by intervals of $10^6/(4 \times 10^3) = 0.25 \times 10^3 = 250$ meters.

II. Question. Referring to the previous question, suppose we can modify R. For what value of R is the width of a bit as long as the length of the link?

Answer. We must have the equation $R \times d_{\text{prop}} = 1$, thus, $R = 1/(4 \times 10^{-3}) = 0.25 \times 10^3 = 250$ bps.

III. Question. Consider sending a large file of F bits from host A to host B. There are two links (and one switch) between them and the links are uncongested (that is, no queuing delays). Host A segments the file into segments of S bits each and adds 40 bits of header to each segment, forming packets of $L = 40 + S$ bits. Each link has a transmission rate of R bit/s. Assuming that F/S is an integer, find the value of S that minimises the delay of moving the file from host A to host B. Disregard propagation delay.

Answer. We can generalise the answer to any header size. So let $h = 40$. Then the size of each packet is $L = h + S$. The number of packets is F/S, which is an integer by assumption. One packet takes L/R seconds to cross over one link. So the first packet takes $2L/R$ seconds from source to destination. Then, every L/R seconds another packet, among the $F/S - 1$ remaining, arrives to destination, because of pipelining. So the total time to receive all the packets is

$$2\frac{L}{R} + \Big(\frac{F}{S} - 1\Big)\frac{L}{R} = \Big(1 + \frac{F}{S}\Big)\frac{L}{R} = \Big(1 + \frac{F}{S}\Big)\frac{h + S}{R}.$$

In other words, the delay $\mathcal{D}$ in function of S is expressed as

$$\mathcal{D}(S) = \frac{1}{R}\frac{(S + F)(h + S)}{S} = \frac{1}{R}\Big(h + F + S + \frac{hF}{S}\Big).$$

The minumum of this function is reached at S_m such as

$$\frac{d}{dS}\mathcal{D}(S_m) = 0.$$

Since

$$\frac{d}{dS}\mathcal{D}(S) = \frac{1}{R}\Big(1 - \frac{hF}{S^2}\Big).$$

We deduce finally $S_m = \lfloor\sqrt{hF}\,\rfloor$, where $\lfloor x \rfloor$ is the biggest integer which is smaller than x (in this example, we could choose also the smallest integer which is bigger than x).