Basic definitions

An **alphabet** is a finite set whose elements are called **letters**.

A **string** is a sequence of letters of an alphabet. A **text** and a **word** are other names for strings, depending on the context. The empty string is noted ϵ.

The **length** of a string x is the length of the associated sequence and is noted $|x|$.

Basic definitions (cont)

The sequence of letters is written simply by enumerating in order the letters, like `abraca`.

For $i = 1, 2, \ldots, |x|$, we note $x[i]$ the letter at **index** (or position) i in x. For example, if $x = \mathtt{abba}$ then $x[1] = x[4] = \mathtt{a}$.

From the previous definitions, we have $x = x[1]x[2]\ldots x[|x|]$ for all non-empty string x.

Basic definitions (cont)

We can define the equality of two strings using the equality of two letters: by definition $x = y$ if $x = y = \epsilon$ or $|x| = |y|$ and then for all i such that $1 \leqslant i \leqslant |x|$, we have $x[i] = y[i]$.

The **product** or **concatenation** of two strings x and y is the string composed of the letters of x followed by the letters of y. It is noted xy or $x \cdot y$.

Property $x \cdot \epsilon = \epsilon \cdot x = x$ holds for all strings x.

Basic definitions (cont)

A word x is a **factor** of a word y if there exists two words u and v such that $y = uxv$.

Then there exists a position i such that $x = y[i]y[i+1] \dots y[i+|x|-1]$, noted more simply as $x = y[i \dots i+|x|-1]$. One says that x **occurs** in y.

Also, when $y = xv$, then x is a **prefix** of y, noted $x \preccurlyeq y$.

When $y = ux$, then x is a **suffix** of y. A factor x of y such that $x \neq y$ is a **proper factor**. We note $x \prec y$ if x is a proper prefix of y.

Basic definitions (cont)

The **n-th power** of word x is defined as $x^0 = \epsilon$ and $x^{n+1} = x^n x$, for all $n \geqslant 0$. This notation denotes the repetition of a word. So, for example, if $x = \mathtt{abb}$, then

$$\begin{aligned}
x^0 &= \epsilon \\
x^1 &= x^0 x = \varepsilon x = x = \mathtt{abb} \\
x^2 &= x^1 x = (x^0 x)x = xx = (\mathtt{abb})(\mathtt{abb}) = \mathtt{abbabb} \\
x^3 &= x^2 x = (x^1 x)x = ((x^0 x)x)x = xxx \\
&= (\mathtt{abb})(\mathtt{abb})(\mathtt{abb}) = \mathtt{abbabbabb}
\end{aligned}$$

It is good to remember a similar concept (power) and notation for functions. If f is a function from a set onto itself, then, for all $n \geqslant 0$

$$f^0(x) = x \qquad f^{n+1}(x) = f^n(f(x))$$

Basic definitions (cont)

So, for example, if $f(x) = x + 1$, then

$$
\begin{aligned}
f^0(x) &= x \\
f^1(x) &= f^0(f(x)) = f(x) = x + 1 \\
f^2(x) &= f^1(f(x)) = f^0(f(f(x))) = f(f(x)) = f(x+1) = (x+1)+1 \\
&= x + 2
\end{aligned}
$$

Note that it was possible to define the power of a function f as

$$f^0(x) = x \qquad f^{n+1}(x) = f(f^n(x))$$

It is possible to compose two different functions too. If f and g are two composable functions, then we define the composed function $f \circ g$ as

$$(f \circ g)(x) = f(g(x))$$