CS 663: Digital Image Processing
Assignment – 5

Question – 1,2

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Question – 1)

Qn: 1)

$$g_1 = f_1 + h_2 * f_2$$
$$g_2 = h_1 * f_1 + f_2$$

Taking Discrete Fourier Transform, we get

$$G_1 = F_1 + H_2 * F_2$$
$$G_2 = H_1 F_1 + F_2$$

Upon solving the two linear equations, we get

$$F_1 = \frac{G_1 - H_2 G_2}{1 - H_1 H_2} \quad ; f_1 = iDFT(F_1)$$

$$F_2 = \frac{G_2 - H_1 G_1}{1 - H_1 H_2} \quad ; f_2 = iDFT(F_2)$$

We will face a computational issue whenever $H_1 . H_2 = 1$.

Fact : $\int_{-\infty}^{\infty} h_1(x)\,dx = \int_{-\infty}^{\infty} h_2(x)\,dx = 1$ (For Blur kernels)

for a function $f(x)$, and its DFT $F(x)$, we know that

$$F(0) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi(0)x}\,dx = \int f(x)\,dx = \text{average of fn. } f$$

$\therefore$ We know that $H_1(u=0) = H_2(u=0) = 1$.

$$\Rightarrow H_1(0).H_2(0) = 1.$$

$\therefore$ We can't evaluate $F_1(0)$ and $F_2(0)$,

$\therefore$ we can't compute the expectation/mean of the original images f_1 and f_2.

Question – 2)

2) The problem we are trying to solve here is to estimate a function given its gradient. The issue faced here is not specific to only the discrete case. It is also an issue in the continuous case

Given the gradient / derivative of a function, we can estimate the function upto an error of a constant.

That is because the Indefinite Integral of a derivative gives the output as

$$\int f'(x)\,dx = f(x) + c.$$

The constant 'c' can be eliminated if we had the limits on the integral that would convert it to a Definite Integral.

To know the limits of the integral, we must have information about the boundary conditions of the function.

Hence, without the knowledge of appropriate boundary conditions, we cannot evaluate the exact function if we are only given its gradient.

1D case : $h[n] = [1 \quad -1]$

$$\Rightarrow g[n] = f[n] - f[n-1]$$

$$\Rightarrow G(u) = F(u)\left(1 - e^{-\frac{2\pi u}{N} j}\right)$$

$$\Rightarrow F(u) = \frac{G(u)}{1 - e^{-\frac{2\pi u j}{N}}}$$

We observe the same issue in fourier domain as well. For 'u = 0', the denominator becomes zero. Hence, we can't retrieve the DC value of the signal from F(u).
We need to have proper boundary conditions for evaluating that. ($u = 0, N, \ldots$)

2D case : $f_x(x,y)$, $f_y(x,y)$ are x & y partial derivatives

$$\therefore F_x(u,v) = F(u,v)\left(1 - e^{-\frac{2\pi u j}{N}}\right)$$

$$F_y(u,v) = F(u,v)\left(1 - e^{-\frac{2\pi v j}{N}}\right).$$

{Image size N×N}

Again in the 2D case, for values of $u = 0, N, \ldots$ and $v = 0, N, \ldots$ we can't evaluate $F(u,v)$ from either of the partial derivatives or the components of the gradient vector.

Hence, even for the 2D case, we cannot evaluate the DC component of the image from the gradient information alone. We need to exploit the information from the boundary conditions of the image to completely reconstruct the image.