CS 663: Digital Image Processing
Assignment – 5

Question – 5

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For correcting the translation between the two images, this paper refers and uses the method of “Phase-Correlation”. Hence, that problem is already solved and that algorithm is usable. Now, to talk about correction of rotation between two images, I will ignore the possible scaling between images for sake of clarity and simplicity.

The author uses a very smart technique to correct for rotation. He first explains the shift theorem of Fourier transform as follows:

$$f_2(x, y) = f_1(x - x_0, y - y_0)$$

$$F_2(u, v) = e^{-j2\pi \cancel{(u+v)}\,(ux_0 + vy_0)} \times F_1(u, v)$$

Hence, it is straight forward to correct for translation of an image. Translational variation is captured by phase change of the Fourier transform as we see in the above equations. For Images that differ both in terms of translation and rotation, we observe the following relations:

$$f_2(x, y) = f_1(x\cos\theta + y\sin\theta - x_0, \; -x\sin\theta_0 + y\cos\theta_0 - y_0)$$

$$F_2(u, v) = e^{-j2\pi(ux_0 + vy_0)} \times F_1(u\cos\theta_0 + v\sin\theta_0, \; -u\sin\theta_0 + v\cos\theta_0)$$

If M denotes the magnitude of the Fourier transform, then the following relation follows:

$$M_2(u,v) = M_1(u\cos\theta_0 + v\sin\theta, -u\sin\theta_0 + v\cos\theta_0)$$

Hence, we see the strategy adopted here. In the fourier domain the two intermixed variations of translation and rotation get seperated in the fourier domain. Translation can be corrected easily just by correcting the phase. And the entire of effect of rotation of image affects only the magnitude response as we see in the above equation. Now, the problem of correcting for rotation of (rotated+translated) image boils down to correcting the rotation of the magnitude of the fourier transform of the image.

Now, if we express the function in polar coordinates, then we can convert the problem of rotation in the (u,v) domain into a problem of translation in the (r,theta) domain as follows:

$$M_1(\rho,\theta) = M_2(\rho, \theta - \theta_0)$$

This problem of translation can be solved simply by using the method of phase correlation as mentioned in the section 1 of the paper.