Journal of Environmental Management (1997) **51**, 229–240

Productivity and Undesirable Outputs: A Directional Distance Function Approach

Y. H. Chung, R. Färe and S. Grosskopf

Department of Economics, Southern Illinois University, Carbondale, IL 62901-4515, U.S.A.

Received 26 *April* 1996; *accepted* 16 *June* 1997

Undesirable outputs are often produced together with desirable outputs. This joint production of good and bad outputs is typically ignored in traditional measures of productivity since "prices" are typically unavailable for bad outputs. Here we introduce a directional distance function and use it as a component in a new productivity index that readily models joint production of goods and bads, credits firms for reductions in bads and increases in goods, and does not require shadow prices of bad outputs. This index, as an empirical example shows, solves the problem caused by the joint production of good and bad outputs, and provides a practical managerial tool.

Keywords: productivity, directional distance function, Malmquist–Luenberger productivity index.

1. Introduction

The measurement of productivity has traditionally focused on measuring marketable outputs of firms or industries relative to paid factors of production. This approach, which typically ignores the production of by-products such as pollution, can yield biased measures of productivity growth. Firms in industries that face environmental regulations would typically find that their productivity is adversely affected since the costs of abatement capital would typically be included on the input side, but no account would be made of the reduction in effluents on the output side.*

The major stumbling block in including the output effects of pollution control is the fact that traditional productivity indexes (such as the Törnqvist and Fisher indexes) require prices of all inputs and outputs in order to aggregate to form a total factor productivity index. In the case of pollution, computation of a (shadow) price is not straightforward. One could use emissions trading prices, or following Pittman (1983)

* Several other studies address the issue of performance measurement in the presence of regulation of bads, including Färe *et al.* (1986, 1989b), Färe *et al.* (1989), Haynes *et al.* (1993), Ball *et al.* (1994) and Tyteca (1995). These studies do not, however, explicitly compute a productivity index.

0301–4797/97/110229 + 12 $25.00/0/ev970146

or Färe *et al.* (1993) one could estimate a shadow price. One possible solution is to use a productivity index that does not require information on prices of effluents, for example, the Malmquist index; see Färe and Grosskopf (1996). However, in the presence of undesirable outputs, this index may not be computable.

Here we propose a new index, which we call the Malmquist–Luenberger productivity index which overcomes the shortcomings of the original Malmquist index. This index readily allows for inclusion of undesirable outputs without requiring information on shadow prices. It also explicitly credits firms or industries for reductions in undesirable outputs, providing a measure of productivity which will tell managers whether their "true" productivity has improved over time. This index also tells managers if there has been technical progress (a shift in the best practice frontier) and whether they are catching up to the frontier. Since the index is computed using a data envelopment analysis type approach, information concerning benchmark firms and technical efficiency is also generated for individual firms.

In order to illustrate the applicability of this index, we compute productivity for data from the Swedish paper and pulp industry. We begin with a discussion of the way in which we model technology, and then turn to our measure of productivity based on this model. Section 4 includes a discussion of our data and results. Section 5 provides a brief conclusion.

2. Modelling technology with good and bad outputs

The basic pollution problem is that production of "good" outputs, such as paper or electricity, is typically accompanied by the joint production of undesirable by-products, such as suspended solids or SO_2. The fact that goods and bads are jointly produced means that reduction of bads will be "costly": either resources must be diverted to "clean-up" (e.g. scrubbers), production must be cut back, or fines must be paid.

More formally, if we denote good outputs by $y \in \mathbb{R}^M_+$, bad outputs by $b \in \mathbb{R}^I_+$, and inputs by $x \in \mathbb{R}^N_+$, then we can describe technology in a very general way via the output sets

$$P(x) = \{(y, b): x \text{ can produce } (y, b)\}. \tag{2.1}$$

We model the idea that reduction of bads is costly (what we call weak disposability of undesirable outputs) as:

$$(y, b) \in P(x) \text{ and } 0 \leq \theta \leq 1 \text{ imply } (\theta y, \theta b) \in P(x). \tag{2.2}$$

In words, this states that a reduction in bads is feasible only if goods are simultaneously reduced, given a fixed level of inputs. In addition, we assume that the good or desirable outputs are freely disposable, i.e.

$$(y, b) \in P(x) \text{ and } y' \leq y \text{ imply } (y', b) \in P(x). \tag{2.3}$$

The notion that the good inputs are jointly produced with the bads is modelled by

$$\text{if } (y, b) \in P(x) \text{ and } b = 0 \text{ then } y = 0. \tag{2.4}$$

In words, (2.4) says that the good outputs are "null-joint" with the bad outputs if the only way to produce no bads is by producing zero good outputs. Alternatively, this

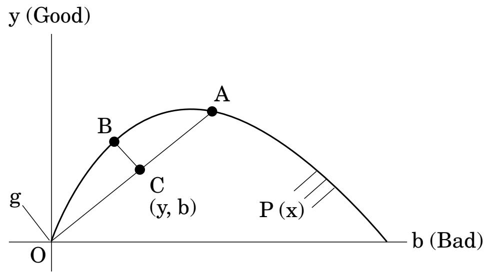

Figure 1. Distance functions.

means that if a good output is produced in a positive amount some bad output must also be produced. Conditions (2.2)–(2.4) will be incorporated into our computational model discussed in Section 3.

The original Malmquist index uses Shephard output distance functions to represent technology (Shephard, 1970). These are defined as

$$D_o(x, y, b) = \inf\{\theta : ((y, b)/\theta) \in P(x)\}. \tag{2.5}$$

This function expands the good and bad outputs (y,b) proportionally as much as is feasible.* It does not credit reduction of bads, since both types of outputs are expanded at the same rate, which is one of the reasons we wish to modify the original Malmquist index.

The reciprocal of the distance function is known as the Farrell output measure of technical efficiency (Farrell, 1957); see Färe *et al.* (1994) for details. In order to allow for the possibility of crediting firms for the reduction of bad outputs, we use a directional output distance function instead of the Shephard output distance function to represent technology. In contrast to the Shephard output distance function which seeks to increase goods and bads simultaneously, the directional distance function seeks to increase the good outputs while simultaneously decreasing the bad outputs. Formally, it is defined as†

$$\vec{D}_o(x, y, b; g) = \sup\{\beta : (y, b) + \beta g \in P(x)\}, \tag{2.6}$$

where "g" is the vector of "directions" in which outputs are scaled. In our case, $g = (y, -b)$, i.e. good outputs are increased and bad outputs are decreased.

To illustrate the directional output distance function and to compare it to Shephard's output distance function, let us again represent the technology by an output set. If we impose conditions (2.2)–(2.4) on this set, it may take the form shown in Figure 1.

* The output distance function is a complete characterization of the technology, and it was shown by Färe and Primont (1995) that under weak disposability of outputs,

$$(y, b) \in P(x) <=> D_o(x, y, b) \leq 1.$$

The distance function is also homogeneous of degree $+1$ and concave in outputs; see Färe and Primont (1995). We do not impose separability of inputs or outputs.

† Briec (1995) develops a distance function for the growth of the technology similar to Luenberger's shortage function. See Luenberger (1992a, 1992b, 1994a, 1994b, 1995a, 1995b).

The output set is denoted by $P(x)$, good output by y and the bad by b. The outputs (y, b) are weakly disposable and y by itself is strongly disposable. Moreover, the good output y is null-joint with b, since if $b=0$, then the only y with $(y, b) \in P(x)$ is $y=0$.

Shephard's distance function applied to the output vector (y, b) places it on the boundary of $P(x)$ at A, and yields a value of OC/OA, i.e. if goods and bads were both increased by a factor of OA/OC, the firm would be judged efficient. Of course, if this firm faces regulations concerning the bad, we would not want increases in the bad to give the firm a better performance score. In contrast, the directional distance function starts at C and scales in the direction of increased goods and decreased bads and projects C on the boundary at B. In Figure 1 this amounts to the ratio of the distances (BC/Og). This means that if the firm moved from C to B (i.e. reduced bads and increased goods) it would be judged efficient based on the directional distance function. We note that the values of the directional distance function for point B is zero, and at C it is positive-valued given our g vector.

In order to relate the two distance functions to each other, let $g=(y, b)$, then through (2.6), we get

$$\begin{aligned}\vec{D}_o(x, y, b; y, b) &= \sup\{\beta : D_o(x, (y, b)+\beta(y, b)) \leq 1\} \\ &= \sup\{\beta : (1+\beta) D_o(x, y, b) \leq 1\} \\ &= \sup\{\beta : \beta \leq \frac{1}{D_o(x, y, b)} - 1\} \\ &= 1/D_o(x, y, b) - 1.\end{aligned} \tag{2.7}$$

This expression shows that Shephard's output distance function is a special case of the directional distance function. The relation between the two can be written as

$$\vec{D}_o(x, y, b; y, b) = (1/D_o(x, y, b)) - 1 \tag{2.8}$$

or equivalently

$$D_o(x, y, b) = 1/(1 + \vec{D}_o(x, y, b; y, b)) \tag{2.9}$$

3. Productivity measurement

Färe *et al.* (1989a) defined a productivity index based on Shephard's output distance function. Their index is the geometric mean of two Malmquist productivity indexes, which were introduced by Caves (Caves *et al.*, 1982). They named their index after the Swedish statistician Sten Malmquist, who used the distance function in 1953 in defining input quantity indexes.

In this section we give a short presentation of the output-oriented Malmquist productivity index and we introduce our new productivity index based on the directional distance function. We call this the Malmquist–Luenberger productivity index.

Suppose there are $t=1, \ldots, T$ time periods, then the Färe, Grosskopf, Lindgren and Roos (FGLR) output-oriented Malmquist productivity index is defined by

$$\mathrm{M}_t^{t+1}=\left[\frac{D_{\mathrm{o}}^t(x^{t+1},y^{t+1},b^{t+1})}{D_{\mathrm{o}}^t(x^t,y^t,b^t)}\frac{D_{\mathrm{o}}^{t+1}(x^{t+1},y^{t+1},b^{t+1})}{D_{\mathrm{o}}^{t+1}(x^t,y^t,b^t)}\right]^{1/2}. \tag{3.1}$$

The Malmquist index (3.1) can be decomposed into two component measures, one accounting for efficiency change (MEFFCH), and one measuring technical change (MTECH). These are

$$\mathrm{MEFFCH}_t^{t+1}=\frac{D_{\mathrm{o}}^t(x^{t+1},y^{t+1},b^{t+1})}{D_{\mathrm{o}}^t(x^t,y^t,b^t)} \tag{3.2}$$

and

$$\mathrm{MTECH}_t^{t+1}=\left[\frac{D_{\mathrm{o}}^t(x^{t+1},y^{t+1},b^{t+1})}{D_{\mathrm{o}}^{t+1}(x^{t+1},y^{t+1},b^{t+1})}\frac{D_{\mathrm{o}}^t(x^t,y^t,b^t)}{D_{\mathrm{o}}^{t+1}(x^t,y^t,b^t)}\right]^{1/2}. \tag{3.3}$$

The product of the component measures exhausts the productivity measure so that

$$\mathrm{M}_t^{t+1}=\mathrm{MEFFCH}_t^{t+1}\cdot\mathrm{MTECH}_t^{t+1}. \tag{3.4}$$

This index has several nice features. First of all, it is a total factor productivity index. Perhaps most importantly, it only requires information on input and output quantities in contrast to the Fisher and Törnqvist indexes which in addition to the quantity data, requires information on input and output prices, which makes these other indexes inappropriate for measuring productivity in the public sector or in the presence of non-marketable goods like pollution. Although the Malmquist index can in principle "deal with" pollution since it requires no prices, as shown above, the distance functions on which it is based do not allow us to credit firms for reductions in pollution. To do so, we substitute directional distance functions for the output distance functions in the Malmquist index and rename it the Malmquist–Luenberger productivity index.

To define an output-oriented Malmquist–Luenberger (ML) productivity index that is comparable to the Malmquist index we choose the direction to be $g=(y,-b)$ and define it as:

Definition (3.5): The output-oriented Malmquist–Luenberger productivity index, with undesirable output, is

$$\mathrm{ML}_t^{t+1}=\left[\frac{(1+\vec{D}_{\mathrm{o}}{}^{t}(x^t,y^t,b^t,-b^t))}{(1+\vec{D}_{\mathrm{o}}{}^{t}(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}))}\frac{(1+\vec{D}_{\mathrm{o}}{}^{t+1}(x^t,y^t,b^t;y^t,-b^t))}{(1+\vec{D}_{\mathrm{o}}{}^{t+1}(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}))}\right]^{1/2}. \tag{3.5}$$

Our definition is such that when the direction g is (y,b) rather than $(y,-b)$, the Malmquist–Luenberger index coincides with the Malmquist index. As in the case of the Malmquist index, the new index can also be decomposed into two components, namely

$$\text{MLEFFCH}_t^{t+1} = \frac{1 + \vec{D}_o{}^t(x^t, y^t, b^t, -b^t)}{1 + \vec{D}_o{}^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})}. \tag{3.6}$$

$$\text{MLTECH}_t^{t+1} = \left[\frac{\{1 + \vec{D}_o{}^{t+1}(x^t, y^t, b^t, -b^t)\}}{\{1 + \vec{D}_o{}^t(x^t, y^t, b^t, -b^t)\}} \frac{\{1 + \vec{D}_o{}^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})}{\{1 + \vec{D}_o{}^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})\}} \right]^{1/2} \tag{3.7}$$

and their product equals ML_t^{t+1}. The Malmquist–Luenberger measure and the Malmquist measure indicate productivity improvements if their values are greater than one and decreases in productivity if the values are less than one.

Our next task is to develop a procedure for calculating the two indexes and their decompositions. This requires the computation of four distance functions for each index. We assume that at each time $t = 1, \ldots, T$, there are $k = 1, \ldots, K$ observations of inputs and outputs,

$$(x^{t,k}, y^{t,k}, b^{t,k}), k = 1, \ldots, K, t = 1, \ldots, T. \tag{3.8}$$

In our example k is a paper and pulp mill.

Following Färe *et al.* (1994) the output set that meets conditions (2.2)–(2.4) and is derived from the data (3.8) is

$$P(x) = \Bigg\{ (y, b) \colon \sum_{k=1}^{K} z_k y_{km}^t \geq y_m^t, m = 1, \ldots, \text{M}, \tag{3.9}$$

$$\sum_{k=1}^{K} z_k b_{ki}^t = b_i^t, i = 1, \ldots, I,$$

$$\sum_{k=1}^{K} z_k x_{kn}^t \leq x_n^t, n = 1, \ldots, N,$$

$$z_k \geq 0, k = 1, \ldots, K \Bigg\}.$$

This activity analysis model (3.9) also satisfies constant returns to scale, i.e.

$$P(\lambda x) = \lambda P(x), \lambda > 0 \tag{3.10}$$

and strong disposability of inputs,

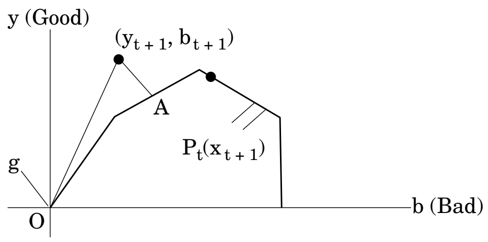

Figure 2. A mixed period distance function.

$$x' \geq x \Rightarrow P(x') \supseteq P(x). \tag{3.11}$$

The inequalities for inputs in (3.9) makes them freely disposable, and the same holds for the good outputs. The bad outputs are modelled with equalities; this makes them not freely disposable. Finally, the non-negativity constraints on the intensity variables z_k allow the model to exhibit constant returns to scale.*

For each observation the distance functions in the Malmquist index are computed as the solutions to a linear programming problem. For example, for k',

$$\begin{aligned}
&(D_o^t(x^{t,k'}, y^{t,k'}, b^{t,k'}))^{-1} = \max\theta \qquad (3.12)\\
&\text{s.t.} \sum_{k=1}^{K} z_k y_{km}^t \geq \theta y_{k'm}^t, m=1,\ldots,\mathrm{M},\\
&\sum_{k=1}^{K} z_k b_{k'i}^t = \theta b_{k'i}^t, i=1,\ldots,I,\\
&\sum_{k=1}^{K} z_k x_{k'n}^t \leq x_{k'n}^t, n=1,\ldots,N,\\
&z_k \geq 0, k=1,\ldots,N,
\end{aligned}$$

To use the original Malmquist productivity index when undesirable outputs are present may create a problem, which we illustrate in Figure 2.

The technology is based on t period data as indicated by P^t. The observation being evaluated is from the following period $t+1$, $(x^{t+1}, y^{t+1}, b^{t+1})$. Note that (y^{t+1}, b^{t+1}) lies outside the technology from the previous period, t, i.e. it is not feasible in period t, presumably because technical progress has occurred allowing production of more good with less bad than was possible in period t. If we try to compute the mixed period distance function

$$D_o^t(x^{t+1}, y^{t+1}, b^{t+1}), \tag{3.13}$$

its value will be $+\infty$, in which case the Malmquist index is not well-defined. In our empirical section we encounter this problem for a third of our observations.

* We use constant returns to scale because that is a necessary condition for the resulting productivity indexes to be true total factor productivity indexes; see Färe and Grosskopf (1996).

The directional distance functions may also be calculated as solutions to linear programming problems. Again as an example,

$$\vec{D}^{t}_{o}(x^{t,k'}, y^{t,k'}, b^{t,k'}; y^{t,k'}, -b^{t,k'}) = \max \beta \tag{3.14}$$

$$\text{s.t.} \sum_{k=1}^{K} z_k y^t_{k'm} \geq (1+\beta) y^t_{k'm}, m = 1, \ldots, \mathrm{M},$$

$$\sum_{k=1}^{K} z_k b^t_{ki} = (1-\beta) b^t_{k'i}, i = 1, \ldots, I,$$

$$\sum_{k=1}^{K} z_k x^t_{kn} \leq (1\text{-}\beta) x^t_{k'n}, n = 1, \ldots, N,$$

$$z_k \geq 0, k = 1, \ldots, K$$

This shows that the directional distance function may also be computed using linear programming. We note that in the mixed period problem illustrated in Figure 2, the directional distance function places (y^{t+1}, b^{t+1}) on the output set $P^{t}(x^{t+1})$ at A. Thus, in this case we no not encounter any computational problems with the Malmquist–Luenberger index. The same is true in our empirical example.

4. Data and results

We measured productivity changes in the Swedish pulp and paper industry. We use the same panel data as in Brännlund *et al.* (1995a) and Brännlund *et al.* (1995b). The data sources are primary data for the pulp and paper industry gathered by Statistics Sweden and the Swedish Environmental Protection Board. The part of the data used here is annual data on quantities of outputs and inputs from 39 paper and pulp mills for the period 1986–1990.* To produce desirable output pulp, y, we observe four inputs, namely: labour (x_1), wood fibre (x_2), energy (x_3) and capital (x_4). The desirable output y is jointly produced with several bads, which include biological oxygen demand (BOD), chemical oxygen demand (COD) and suspended solids (SS).

Descriptive statistics for the data are presented in Table 1. The data shows that the amount of inputs and outputs increased between 1986 and 1988 and decreased thereafter.

The average values for the Malmquist–Luenberger productivity indexes which are based on the directional distance functions as specified in (3.14) and its components are reported in Table 2.

This result shows that the productivity in this industry has improved on average over the entire time period. The main source of the productivity improvements is technological advance rather than efficiency improvement. In fact, technical efficiency fell during every period except the 1987/1988 period.

We also computed the traditional Malmquist based on the maximization problems in (3.12) and (3.13) which are solved as linear programming problems. The solutions for each observation are then used to obtain the original Malmquist productivity indexes as well as their components. The solutions for the maximization problems for the mixed period distance functions did not exist for almost one third of the observations.

* Only the data of the firms which operated during the whole five years are included here.

TABLE 1. Descriptive statistics

T	Variable	Mean	SD	Min.	Max.
1986	y (tons)	233 851·0	174 073·6	2718·0	824 250·0
($K=39$)	b_{BOD} (tons)	3013·6	2640·3	110·7	10 740·0
	b_{COD} (tons)	12 996·2	11 976·3	254·5	47 256·0
	b_{ss} (tons)	1309·1	1209·7	51·9	4400·0
	x_1 (hrs)	828 897·4	129 386·8	107 000·0	2 383 000·0
	x_2 (m^3)	1·01E+06	7·17E+05	7000·0	2·5E+06
	x_3 (kwh)	3·54E+08	3·19E+08	1·61E+07	1·26E+09
	x_4 (mil)	454·7	330·2	6·5	1585·9
1987	y	249 792·0	186 907·4	2974·0	853 520·0
($K=39$)	b_{BOD}	3098·8	2631·8	121·8	11 993·0
	b_{COD}	13 019·8	11 826·1	316·7	47 256·0
	b_{ss}	1346·4	1481·8	69·6	7318·5
	x_1	826 435·9	524 013·3	102 000·0	2 332 000·0
	x_2	1·05E+06	7·25E+05	8000·0	2·52E+06
	x_3	3·81E+08	3·63E+08	1·37E+07	1·46E+09
	x_4	435·0	315·4	6·4	1515·1
1988	y	257 866·3	191 670·6	2346·0	878 000·0
($K=39$)	b_{BOD}	3197·5	3167·3	115·8	14 040·0
	b_{COD}	12 703·3	11 664·8	247·1	47 060·0
	b_{SS}	1756·1	2704·0	49·4	14 480·0
	x_1	829 717·9	528 028·4	102 000·0	2 345 000·0
	x_2	1·27E+06	1·75E+06	7000·0	1·11E+07
	x_3	4·00E+08	3·83E+08	1·40E+07	1·50E+09
	x_4	419·4	303·9	6·6	1434·9
1989	y	257 977·9	194 005·7	2077·0	872 610·0
($K=39$)	b_{BOD}	2825·8	2714·6	120·0	11 067·0
	b_{COD}	11 788·2	11 773·4	257·7	49 980·0
	b_{ss}	1775·4	2735·7	42·4	14 994·0
	x_1	827 846·2	526 682·6	107 000·0	2 268 000·0
	x_2	1·06E+06	7·31E+05	6000·0	2 543 000·0
	x_3	4·17E+08	4·23E+08	1·50E+07	1·58E+09
	x_4	408·6	299·7	6·3	1368·5
1990	y	251 887·3	180 568·9	1964·0	855 080·0
($K=39$)	b_{BOD}	2383·8	2196·4	110·7	9632·0
	b_{COD}	10 028·6	9606·3	252·6	38 880·0
	b_{SS}	1657·9	2538·1	4·9	12 384·0
	x_1	809 846·2	509 687·0	96 000·0	2 202 000·0
	x_2	1·01E+06	6·94E+05	6000·0	2·25E+06
	x_3	4·05E+08	4·15E+08	1·52E+07	1·51E+09
	x_4	403·1	292·3	6·25	1325·6

TABLE 2. The Malmquist–Luenberger productivity indexes and components*

$K=39$	ML_t^{t+1}	$MLEFFCH_t^{t+1}$	$MLTECH_t^{t+1}$
1986/1987	1·027	0·984	1·043
1987/1988	1·087	1·011	1·075
1988/1989	1·031	0·995	1·035
1989/1990	1·059	0·882	1·200

*The geometric mean of individual indexes.

TABLE 3. Comparison of the Malmquist and Malmquist–Luenberger indexes*

Period	M_o (ML_o)	MEFFCH (MLEFFCH)	MTECH (MLTECH)
1986/1987 ($K=26$)	1·052	0·964	1·091
	(1·024)	(0·977)	(1·049)
1987/1988 ($K=23$)	1·053	1·028	1·024
	(1·060)	(1·010)	(1·050)
1988/1989 ($K=28$)	0·940	0·995	0·945
	(1·024)	(0·992)	(1·032)
1989/1990 ($K=27$)	0·959	0·929	1·032
	(1·049)	(0·855)	(1·227)
Total ($K=104$)	0·997	0·977	1·020
	(1·039)	(0·955)	(1·088)

*The geometric mean of individual scores.

Thus, the Malmquist productivity indexes in (3.1) and technical change (3.3) for those observations is undefined. This is in contrast to the case of the directional distance functions where no computational problems arise.

The Malmquist productivity index is not defined for all observations (see previous section), yet we still want to see how it compares to the Malmquist–Luenberger index. We limit the comparison to those observations for which the Malmquist index is well-defined. The geometric means for the observations that were well-defined for the traditional Malmquist index, as well as the means for those same observations for the Malmquist–Luenberger index, are included in Table 3. Looking first at the grand means over all time periods, we see that the Malmquist–Luenberger (ML) index suggests greater improvements in productivity and technical change on average than the traditional Malmquist (M) index. The mean values of the ML index are in fact frequently greater than the mean values of the M index (except for efficiency change) in the individual years.

We tested for differences in the indexes for the observations that had well-defined M indexes. Based on a battery of non-parametric tests of location using the SAS NPARIWAY procedure, we reject the hypothesis that the two indexes are "the same" at the 0·01 significance level. However, based on a simple t-test with the null hypothesis that $ML^k - M^k = 0$, we could not reject the null at conventional significance levels.

5. Summary

This paper introduces a performance measure that credits the reduction of undesirable outputs like pollution while simultaneously crediting increases in desirable outputs. This function is used in the construction of a Malmquist type productivity measure. The new index, termed the Malmquist–Luenberger index, which also accounts for reduction of bads, can be decomposed into two parts: efficiency change and technology change. We show how to compute these indexes using simple linear programming problems, and provide an empirical example for the case of Swedish paper and pulp mills operating over the 1986–1990 period. This technique provides individual plant-specific information on productivity and its sources without requiring data on prices of inputs and outputs. This is especially useful where firms face environmental regulations, since the regulated effluents are typically non-marketed.

TABLE 4. Non-parametric tests of comparison of ML and M indexes (for well-defined M)

ANOVA F (prob>F)	Wilcoxon Z (prob>Z)	Kruskal–Wallis χ^2 (prob>χ^2)	Median Z (prob>Z)	Van-der Werden Z (prob>Z)	Savage Z (prob>Z)	Kolmogorov–Smirnov KSa (prob>KSa)
19·90 (0·0001)	−4·08 (0·0001)	16·69 (0·0001)	−2·97 (0·0030)	−4·32 (0·0001)	−2·66 (0·0077)	2·08 (0·0003)

In modelling the joint production of goods and bads, and providing a performance measure that credits firms for reduction of bads, managers have a reliable measure of their "true" productivity and its sources.

This research has been partially funded by USEPA CR823009010. We are grateful to Finn Førsund, three referees and the participants at the Atlantic Economic Meetings in Williamsburg for their comments.

References

Ball, V., Eden, C., Lovell, A. K., Nehring, R. F. and Somwaru, A. (1994). Incorporating undesirable outputs into models of production: an application to U.S. agriculture. *Cabiers d'Economie et Sociologie Rurales* no. 31, pp. 60–74.

Brännlund, R., Färe, R. and Grosskopf, S. (1995a). Environmental regulation and profitability: an application to Swedish pulp and paper industry. *Environmental and Resource Economics* **6**, 23–36.

Brännlund, R., Chung, Y., Färe, R. and Grosskopf, S. (1995b). Emission Trading and Profitability: The Swedish Pulp and Paper Industry. Discussion Paper No. 95–14, Southern Illinois University at Carbondale.

Briec, W. (1995). A Graph Type Extension of Farrell Technical Efficiency Measure. Memo, GREQAM.

Caves, D. W., Christensen, L. R. and Diewert, E. (1982). The economic theory of index numbers and the measurement of input, output, and productivity. *Econometrica* **501**, 1393–1414.

Färe, R. and Grosskopf, S. (1996). *Intertemporal production frontiers: with dynamic DEA.* Boston, Kluwer Academic Publishers.

Färe, R. and Primont, D. (1995). *Multi-Output Production and Duality: Theory and Applications.* Boston: Kluwer Academic Publishers.

Färe, R., Grosskopf, S. and Pasurka, C. (1986). Effects on relative efficiency in electric power generation due to environmental controls. *Resources and Energy* **8**, 167–184.

Färe, R., Grosskopf, S. and Pasurka, C. (1989). The effects of environmental regulation on the efficiency of electric utilities. 1969 vs. 1925. *Applied Economics* **21**, 225–235.

Färe, R., Grosskopf, S., Lindgren, B. and Roos, P. (1989a). Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach. Discussion Paper No. 89-3. Southern Illinois University at Carbondale.

Färe, R., Grosskopf, S., Lovell, C. A. K. and Pasurka, C. (1989b). Multiple productivity comparisons when some outputs are undesirable: a nonparametric approach. *The Review of Economics and Statistics* **71**, 90–98.

Färe, R., Grosskopf, S. and Lovell, C. A. K. (1994). *Production Frontiers.* Cambridge University Press.

Färe, R., Grosskopf, S., Lovell, C. A. K. and Yaisawarng, S. (1993). Derivation of shadow prices of undesirable outputs: a distance function approach. *Review of Economics and Statistics* **75**, 274–380.

Farrell, M. J. (1957). The measurement of productivity efficiency. *Journal of Royal Statistical Society A* **20**, 283–281.

Haynes, K., Patrik, S., Bowen, W. and Cummings-Saxton, J. (1993). Environmental decision models: U.S. experience and a new approach to pollution management. *Environmental International* **19**, 261–275.

Luenberger, D. G. (1992a). Benefit functions and duality. *Journal of Mathematical Economics* **21**, 461–486.

Luenberger, D. G. (1992b). New optimality principles for economic efficiency and equilibrium. *Journal of Optimization Theory and Applications* **75**, 221–264.

Luenberger, D. G. (1994a). Dual pareto efficiency. *Journal of Economic Theory* **62**, 70–85.

Luenberger, D. G. (1994b). Optimality and the theory of value. *Journal of Economic Theory* **63**, 147–169.

Luenberger, D. G. (1995a). *Microeconomic Theory.* Boston: McGraw-Hill.

Luenberger, D. G. (1995b). Externalities and benefits. *Journal of Mathematical Economics* **24**, 159–177.

Pittman, R. W. (1983). Multilateral productivity comparisons with undesirable outputs. *Economic Journal* **93**, 883–891.

Shephard, R. W. (1970). *Theory of Cost and Production.* Princeton: Princeton University Press.

Tyteca, D. (1995). DEA models for the measurement of environmental performance of firms: concepts and empirical results. Working Paper, IAG, University of Catholique de Louvain.