Simulation of Population Models by R - Programming

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Student's Declaration

I, Ringan Majumdar, hereby declare that the project report titled, "Population Simulation" submitted to St. Xavier's College (Kolkata) is partial fulfilments of the requirements for the award of the degree of Bachelors of Science (Honours), is a record of original and independent research work done by me and it has not formed on the basis for the award of any Degree/Diploma/Fellowship or other similar title of recognition to any candidate of any university.

I affirm that all my sources and no part of my dissertation paper uses unacknowledged materials.

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Introduction

Why do things exist? Now, that’s a pretty tough question of philosophy. Although, you can answer the question in two ways in our case. First, they have started existing or they have been born. Second, they haven't stopped existing yet or they haven't died.

For example, raindrops. Why do raindrops exist? Well, they're good at this first part, they're good at being born. They don't last very long but they form pretty often, often enough where it's not unusual to see them. A different example is planets or stars. Unlike raindrops, they don't form very often at all, but they more than make up for it by lasting a long time. When we look up in the sky or just at the ground, we see planets and stars. They're also common, even though they do it in a very different way.

Anything that exists follow this same age old pattern. This is observed in every living organism too. Now, for living organisms there comes more parameters of existence. Living organisms can reproduce. They need to sustain on food or space. All these factors together can shape or rather have shaped how the world around us looks today.

This project will investigate how nature of populations change based on these elementary factors. And to look into it will use simulations and see how simulations follow the different mathematical models of population. And how they mimic the real life data.

What is a Population?

In simple words, a population refers to a collection of individuals. For example, a population of rabbits or the population of the stars in the Milky way galaxy.

But, in demography Population refers to the totality of all human beings living at a certain time within a territory demarcated by natural, cultural or political boundaries.

A population is conceptually a statistical whole. Hence, its characteristic features are distinct from the characteristics of its own individual members. Although, the nature of the very population is determined by its individual members.

Population is a process of continuous change. The size and the structure of the population undergo changes as parameters often vary. Hence, the populations separated over space and through time tend to be unique in themselves. Again different populations in different conditions might have the same underlying nature. For example, human population growth has been exponential everywhere overtime irrespective of the fact whether there was any link between civilizations in the past. On the other hand, stray cats are thriving although panthers are going extinct both belonging to the very same cat family.

This provides us ample scope for scientific analysis of the size, structure and the distribution of the population. The chances of death, birth or reproduction or even movements of population helps us to describe and measure population growth and changes in their composition.

What is Population Simulation?

A simulation is a programme generated by a computer based on a few parameters which mimics real life processes. A population simulation mimics real life population data. This project shows how few elementary parameters can successfully produce a simulation that can be so close to mathematical models without taking minimum mathematics into consideration.

Aim

The aim of this paper is to complement the science on the model building phase for population models by clarifying the fundamental constituents of a numerical model of populations under study, and to show how a unified theory of population models can be based on these constituents.

What is a Stationary Population?

Suppose a population is closed under migration, this means no other creature comes or goes out the region under consideration. Then, the population experiences constant birth rate and growth rate. However, the population growth remains constant. This is called stable population.

Now, for such a population if the birth rate and death rate becomes equal, then the population growth becomes zero. Thus the population undergoes no growth at all. Such a population is called a stationary population.

Simulation of Stationary Population

Suppose there is a block of land and we are going to simulate the population growth of a creature *Lob* (say) there. Let the birth chance and the death chance of the creature Lob be 100% and 5% respectively in this run. This means at each iteration in the simulation, one of these lob creatures will form and each living lobs will have a one in 20 chance of dying. Let us run this simulation for 250 iterations with one lob in the first iteration.

Algorithm:

In the first step, we are defining variables t and N which are is population sizes of the 1^{st} and nth iteration respectively. Next, the birth chance(b) and death chance(d) are set at the desired level. Along with them the number of iterations(n) are set. Then, an array k is defined, in which the value of the simulated population size would be stored. In the next step, a for loop is initiated. The loop will be executed *n* (no. of iterations) times. Inside the loop, one more variable is defined, Z. Z is defined to control the births. Since, creatures are formed independently but, dies depending upon the population size. Now, *Z* is defined as a random sample of size one from a binomial distribution of size *t* with success probability *b.* The value stored in *Z* is the number of creatures formed in that iteration. Now, we define *N* as the random sample of size one from a binomial distribution of size $N+Z$ with success probability $1-d$. This gives us the population size of that iteration. This is run n times which gives us the simulated population data. Then after plotting the data we get,

(This and every next simulation with the corresponding graphs are generated in R programming language)

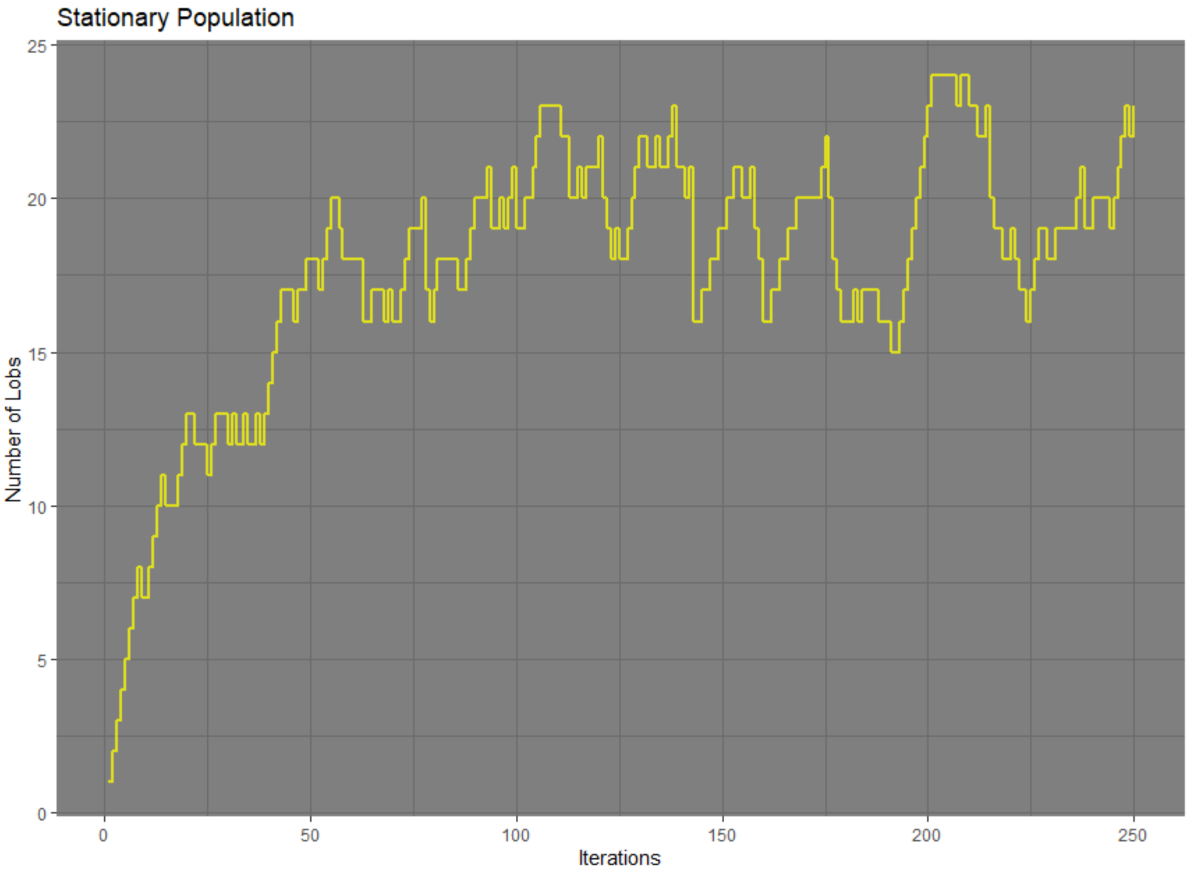

Fig: 1

One can observe (Fig: 1) how the creatures have attained an equilibrium after a few iterations. Although, the number of lobs doesn't get fixed, it fluctuates. The moment population size decreases or increases and gets beyond the state of equilibrium the chance factors of birth and death comes into play and the population returns to equilibrium.

Now, let us have another simulation with another creature *Rob* (say). Robs have a birth chance of 10% and a death chance of 0.5% with respect to each iteration. Therefore, these robs don't form much often although they don't die much often. Now starting with one rob in the first simulation, a simulation of 250 iterations is run. We get,

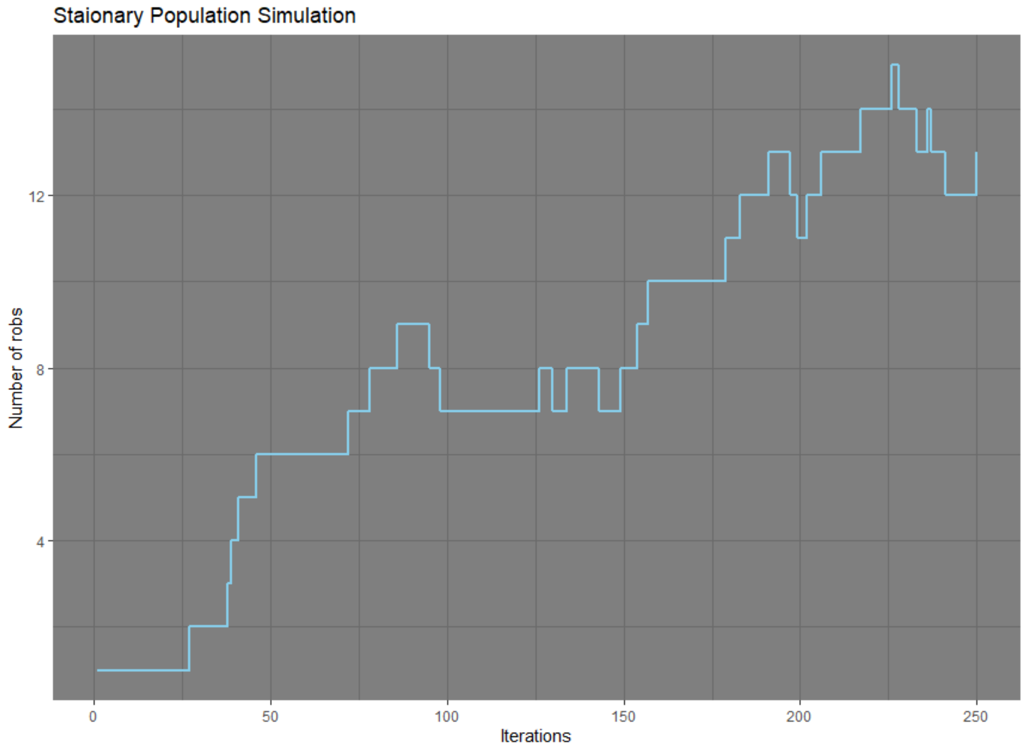

Fig: 2

Now, this seems incomplete (Fig: 3). So, the simulation is again run with 1000 iterations.

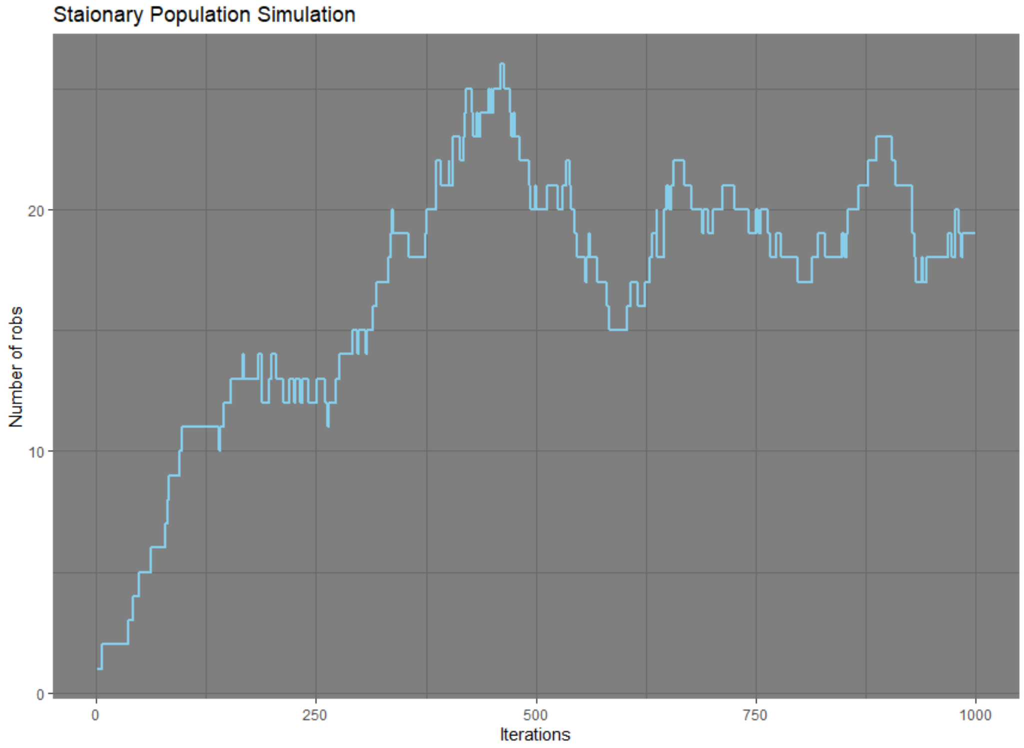

Fig: 3

After 1000 iterations, the number of rob creatures have also attained equilibrium (Fig: 3). Observe that lobs and robs have stabilized over the same region, of around 20(Fig: 4). However, they don't have the same parameter values of birth and death. For a comparative study, a simulation is done of both lobs and robs and then, the plot is imposed on the same graph.

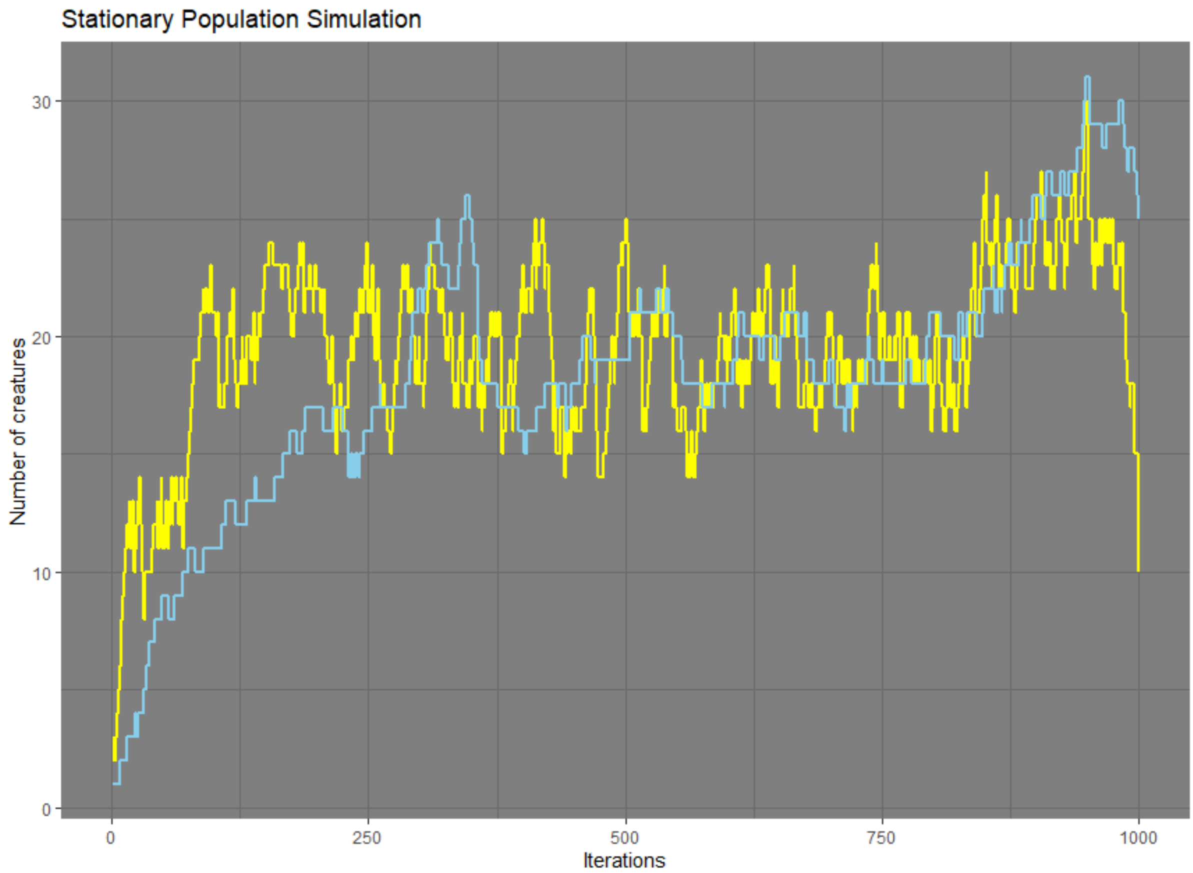

Fig: 4

One can see the lobs are attaining the equilibrium conditions in less number of iterations than that of robs. But, the lobs fluctuate much more from the equilibrium than that of the robs. This is where comes our first story, the raindrops and planets. The lobs form more often and die more often but, on the other hand robs form less often and die less often too.

Now, comes the question of why they are stabilizing around the same region. For that the means of both the runs are computed after both of them have attained equilibrium *(judged by naked eye)*. Computing means of the population number values from after 250 iterations we get,

Mean Population value of lobs = 20.02929

Mean Population value of robs = 20.60985

Therefore, they are very close to each other. What is the reason? What is the reason of the equilibrium number? Why are the populations stabilizing around the ratio of birth chance to death chance?

Suppose, we have equal birth and death chances. The chance of a creature being formed in an iteration is equivalent to the chance of it dying. Then, intuitively you can understand the total population will fluctuate around of a constant size because, the existence of a creature depends upon both the chances simultaneously. The moment one creature forms the death chance factor will try to kill it in the next iteration or the other since the creatures form and die at the same rate. Now, if the birth chance is greater than the death chance, say 75% and 5% respectively. Then, a creature forms more or less 75 times out 100 iterations. Now, out of those existing creatures a creature will die with a chance of 5%. The population size will fluctuate at some higher value. And since each creature individually has a chance of dying, the total death rate depends on the current number of live creatures. Therefore, we can formulate out a relationship basing on the simulation data,

$$\boldsymbol{Birth\ Chance\ (B) = Number\ of\ creatures * \ Death\ Chance\ (D)}$$

Using numbers from our first simulation, where one lob was formed each iteration, and each creature had a one in 20 chance of dying each iteration, we can see that the expected birth rate and death rates should be equal when there are 20 creatures and that's what we saw in the simulation. But we also saw in the simulation that the number of creatures fluctuated all over the place. It didn't just stick at 20. It's possible for all of the creatures to get lucky and not die, in which case, the number of creatures rises to 21 in the next frame but then the expected death rate will be higher than the birth rate and then on average, we'd expect more than one creature to die in the next frame, which would push the number back toward 20. This means that under these conditions the population reaches point where the death factor cannot kill significant part of the population and the birth factor cannot form them since, they will tend to die. As a result, they balance out each other, hence having equilibrium. This makes us to rewrite the previous equation as below, *(replacing number of creatures with equilibrium number since eventually the number of creatures stabilizes at some given number)*

$$\boldsymbol{Equilibrium\ Number}\ (\boldsymbol{E}) = \frac{\boldsymbol{Birth\ Chance}\ (\boldsymbol{B})}{\boldsymbol{Death\ Chance}\ (\boldsymbol{D})}$$

Equilibrium number can be defined as the population size around which the population size is stabilizing.

Mathematical basis

Till now we've tried to build simulation graphs based on natural understanding. But, graphs change for every other simulation even with same parameter values. Now, let us try to build a crude mathematical relation based on the parameters. These relations will act as the skeletons of the simulations. The formulae will help us with what these population number-over-iteration graphs have in common and even predict what we'd expect the graph to look like before we run a simulation.

For this type, instead of plotting data from a simulation, we're going to graph a function where one quantity depends directly on another without any randomness.

The starting point is to set the Birth rate (b) = Death rate (d). When these are equal, we expect some creatures to die each iteration, but we expect the same number to appear to replace them. The overall expected change in the population would be zero, a stationary population.

Now for making a function equation, instead of assuming the birth and death rate are the same, we can subtract them to get the overall expected change. For this, we'll use the symbol delta. (Δ) and replace the rates by chances,

$$\Delta = B - D$$

(Note that, birth and death chances and birth and death rates are not same. The former ones are set by the simulator and the latter ones are computed one the basis of the simulation data. The chances are denoted by capital letters whereas the rates by small letters. In other words, the expectation of the rates are chances.)

The total expected change (Δ) will still be zero some of the time. But, simulation data are never fixed. The number of creatures change constantly. So, it will take non zero values some time too. Therefore, we can try to formulate a function equation where the total expected change (Δ) is a function of the population number (N). The equation is given by,

$$\Delta = \mathrm{B} - \mathrm{D} * \mathrm{N}$$

Now, let’s plot the function for the lobs and the mobs.

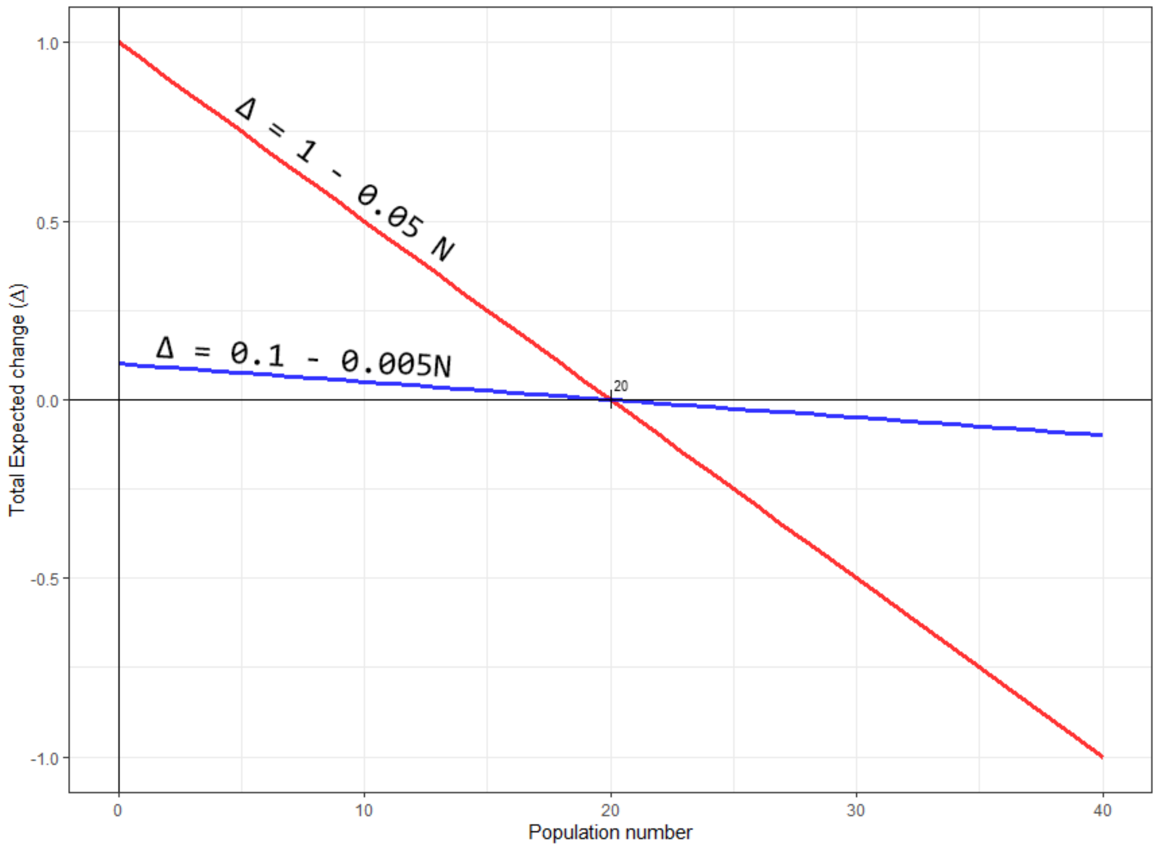

Fig: 5

Now, obviously that will be a straight line (Fig: 5). It cuts the Δ axis at value B value. But one can see both the straight lines intersect the Population number–axis at the same point, 20. And that was the equilibrium number for both type of creatures. But, what is the reason behind this? Now, for a population to be stationary the total expected change must be zero. Thus the point where the line cuts the axis becomes the equilibrium number.

Although, they do have different slopes and the value of slope determines how fast the population number is going to stabilize. That is why lobs have stabilized much faster than that of the robs. This equation can be applied to any simulation with parameters B and D. In latter parts of the project the equation will be modified as new parameters would come.

Similar Simulations

For lobs the birth chance was *100%* and death chance was *5%* and for robs the birth and death chance was respectively *10%* and *0.5%*, hence, in both the cases the equilibrium number is *20.* This is observed for any value of birth and death chances. Here are four examples,

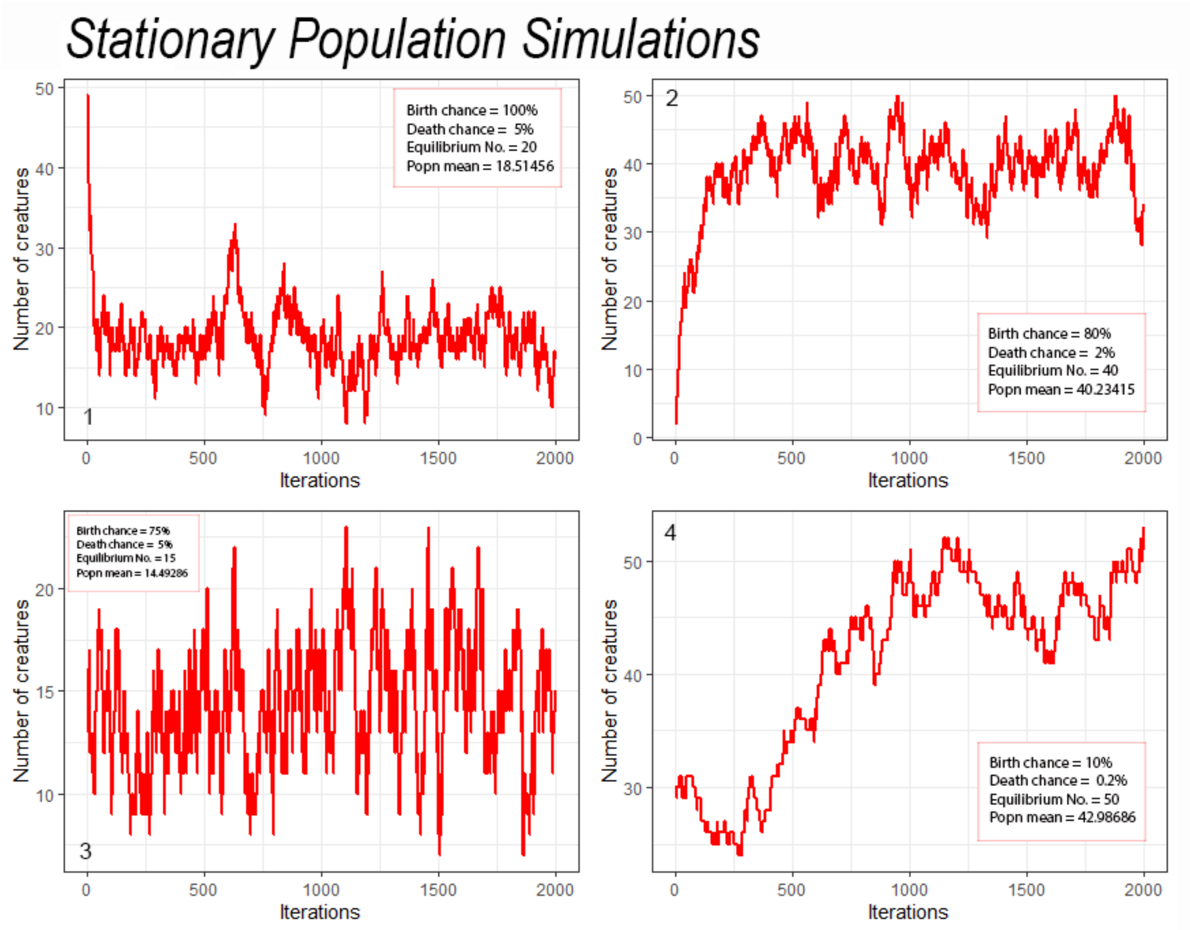

Fig: 6

In these 4 simulations (Fig: 6), the population of 3 of them stabilizes although the 4th case doesn't seem to attain equilibrium even after 2000 iterations. This is where, the probability factor comes into play and something unusual occurs. Moreover, no matter what the number of creatures is in the 1st iteration, the population will always stabilize to the equilibrium number *(in most of the cases)*. In the 1st simulation the starting population number was 50 but, it stabilized at the desired equilibrium number. In case of simulation 3, the population number was already set at the equilibrium number 15.

The mathematical equations of the four graphs is given by

1. $\Delta = \mathbf{1 - 0.05N}$ cuts the population number axis at 20.
2. $\Delta = \mathbf{0.8 - 0.02N}$ cuts the population number axis at 40.
3. $\Delta = \mathbf{0.75 - 0.05N}$ cuts the population number axis at 15.
4. $\Delta = \mathbf{0.1 - 0.002N}$ cuts the population number axis at 50.

All the straight lines cut the population number axis which was predicted equilibrium number.

Although, the stationary populations cannot be seen in real life examples of living organisms. The next part of the project will describe and discuss the reasons and try to simulate why two types of population models and have a comparative analysis with real life population data.

Populations of Living Organisms

So far we’ve seen how for a given chance birth and death chance a simulation can act upon. But, what does this have to do with living things? From past simulations it kind of seems like we shouldn't exist. We're too complex to form spontaneously the way raindrops do. Imagine all the right atoms and molecules just happening to come together to form a rabbit. It's pretty unlikely and we also don't live all that long, but somehow, living organisms are still pretty common, so what's going on?

Living things have a special feature of reproduction unlike a planet or a raindrop. We can make more of ourselves. Therefore, now comes an extra parameter in our simulation, a chance of replication.

Now, living organisms need food to sustain or a place to live. If the place under consideration of the creatures does not have enough food or to nourish the whole population the population will start to decrease. As even if they reproduce with same probability they will die too soon to reproduce again. Hence, there comes out another parameter of food and nourishment.

Why Build Population Models?

Scientists build models to investigate processes that are too difficult or impossible to study directly through experiments or actual measurements. They use different types of models (such as physical models, diagrams, or mathematical models) depending on the problem they are investigating. Mathematical models typically simplify reality to help scientists understand how key components of a complex system interact and can be used to make predictions about future events.

Ecologists use population-based mathematical models to study how population sizes change over time. These models can help identify factors that regulate population dynamics and promote a better understanding of how ecosystems work. The knowledge gained can be applied to guide conservation and wildlife management decisions.

This project mainly explores simulations based on two simple models of population dynamics: the exponential growth model and the logistic growth model.

Exponential Model:

The exponential model is one of the simplest population growth models. Its defining characteristic is that population change is proportional to population size.

Charles Darwin, in developing his theory of natural selection, was influenced by the English clergyman Thomas Malthus. Malthus published his book in 1798 stating that populations with abundant natural resources grow very rapidly; however, they limit further growth by depleting their resources. The early pattern of accelerating population size is called exponential growth.

Malthusian law

Suppose, a population has a size P, at time t and size $P + \Delta P$ at time $t + \Delta t$. The rate of increase of the population at time t is given by,

$$\frac{dP}{dt} = \lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t}$$

Now, the population change is proportional to the population size the relative growth rate, r $(r > 0)$ is defined as,

$$\frac{dP}{dt} = rP$$

$$\Rightarrow \frac{1}{P} dp = rdt$$

$$\Rightarrow \int \frac{1}{P} dP = r \int dt + c$$

$\Rightarrow \ln P = rt + c$ ………... (c is the constant of integration)

$\Rightarrow \boldsymbol{P(t) = P_0\, e^{rt}}$ ……….. $(P_0 = \ e^c > 0; a\ constant\ )$

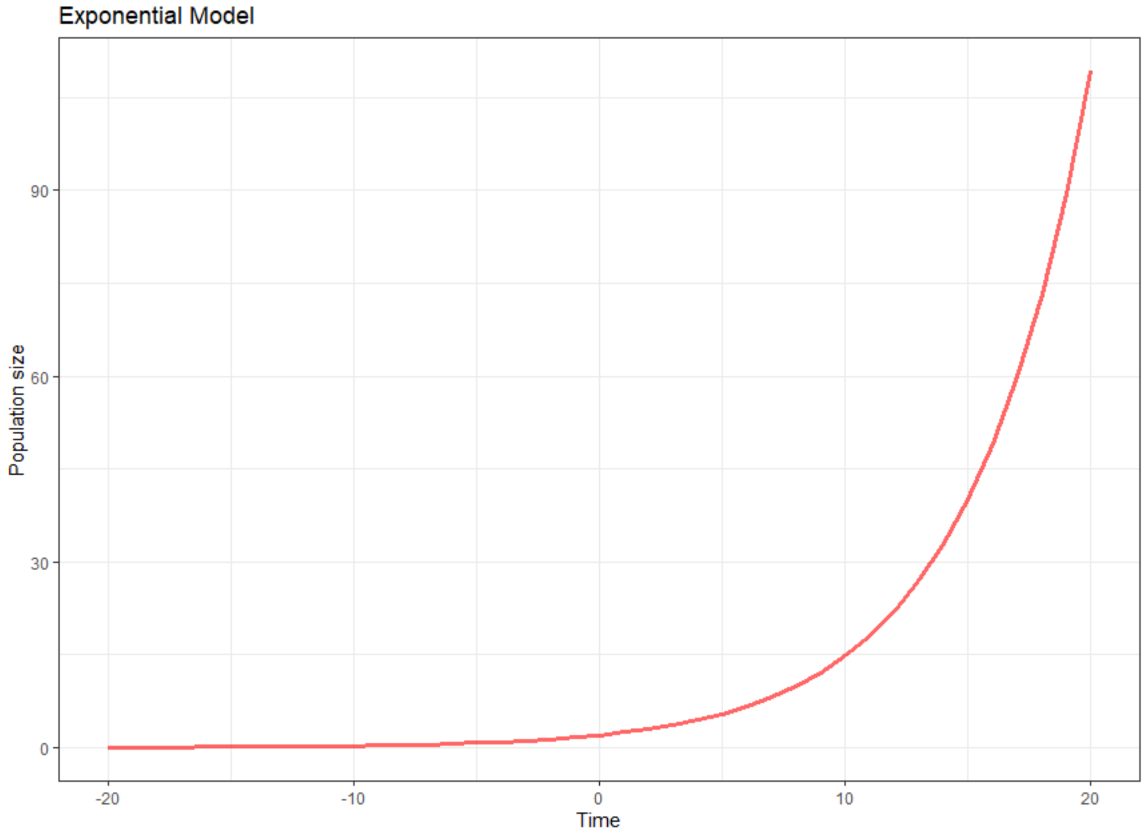

Fig: 7

Thus, if a population grows exponentially it gives a J-shaped curve (Fig: 7). The population growth is called Malthusian law of growth.

Note that, as $t \rightarrow -\infty, P \rightarrow 0$, again, $t \rightarrow -\infty, P \rightarrow \infty$ which is quite unrealistic, because a population with limited resources *(food, space or competition)* cannot go on increase indefinitely and if it does it leads to a crisis situation. This is called Malthusian crisis and this leads the next population model.

Logistic Model:

The logistic growth model expands the exponential growth model by making population growth **density dependent**. As the density (size within a certain area) of a population increases, the growth rate decreases.

Density dependence is represented by the **carrying capacity** (*k*); the maximum number of individuals a particular location can support. The value of *k* depends on variables such as food, space, and competition.

Logistic law

Here suppose, the population has a size *P;* like an exponential model the growth rate is supposed to be changing with time but, in this case there is an extra parameter of crowding. A very plausible assumption for this population that is growing in an area of fixed limits. Now, once the population has become enough dense, the relative growth rate gradually decreases (with P and t). One of the simplest form of decreasing function of P is given by,

$$r(1-kP)\ , where\ r = relative\ growth\ rate; k = carrying\ capacity, r > 0, k > 0$$

Therefore, the differential equation is given by,

$$\frac{dP}{dt} = rP(1 - kP)$$

$$\Rightarrow \frac{1}{P(1-kP)} dp = rdt$$

$$\Rightarrow \frac{k}{kP(1-kP)} dp = rdt$$

$$\Rightarrow k\{\frac{1}{kP} + \frac{1}{1-kP}\}dp = rdt$$

$\Rightarrow \int \frac{1}{P} dp + \int \frac{k}{1-kP} dp = r\int dt + c$ ………. (c is the constant of integration)

$$\Rightarrow \ln P - \ln(1 - kp) = rt + c$$

$$\Rightarrow \ln \frac{P}{1-kP} = rt + c$$

$\Rightarrow \frac{P}{1-kP} = P_0\, e^{rt}$ ……….. $(P_0 = e^c > 0; a\ constant\ )$

$$\Rightarrow P = \frac{P_0\, e^{rt}}{1+kP_0\, e^{rt}}$$

$$\Rightarrow \boldsymbol{P} = \frac{\mathbf{1}}{\frac{\mathbf{1}}{\boldsymbol{P_0}} \boldsymbol{e^{-rt}} + \boldsymbol{k}}$$

This is the characteristic expression of a Logistic population growth.

Note that, as $t \rightarrow -\infty$, $P \rightarrow 0$ but, $t \rightarrow -\infty$, $P \rightarrow 1/k$. Here 1/k is the maximum size of the population.

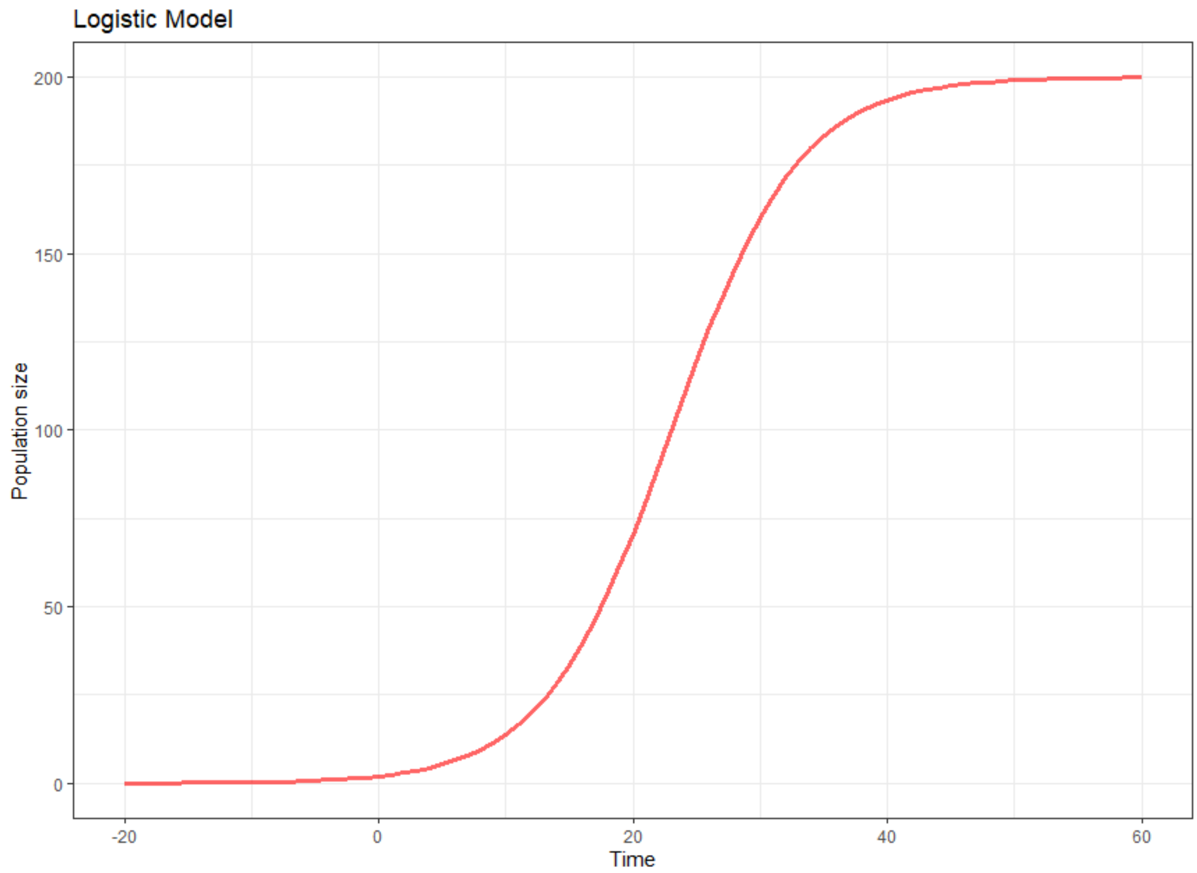

Fig: 8

Logistic population model gives us an S-shaped curve (Fig: 8). Now, the logistic curve can be divided into three different phases.

- ***Pre-transitional phase***: This phase is characterized by very low growth rate although increasing due to both and similar high birth rate and death rates.
- ***Transitional phase***: This is the steep region of the curve. This is the deciding phase whether the population will go on increasing exponentially or will it stabilize around some population size to behave as a logistic population. Here, the birth remains high or declines very slowly whereas, death rate declines very rapidly.
- ***Post-transitional phase***: In this region, both the birth rate and death rate has declined. They are low and more or less equal. The curve becomes decreasing in this phase.

The next part will discuss about simulations of these two types of population models.

Simulation of the Population Models

In the previous simulations we have simulated creatures like *lobs* and *robs* based on only two parameters, birth chance and death chance. But, that is quite unrealistic. Since, the probability of living organism forming on its own is very tiny. So, as stated earlier, a new parameter of replication chance is introduced.

Model Simulation of Populations with replication chance:

Suppose there is a block of land and we are going to simulate the population growth of a creature *mobbit* (say) there. Let the spontaneous birth chance and the death chance of the creature Lob be 80% and 5% respectively in this run. Now, unlike *lobs* or *robs* this creature has a replication chance of 2%. This means more or less 3 in every 100 *mobbits* will make another mobbit in the next iteration. Here, they don't have a problem of resources. They have ample amount of food or space. Therefore, no competition.

One more thing, the population models are continuous in nature but we are simulating in a discrete setup. Here, the iteration variable is analogous to time.

Algorithm:

The algorithm for simulation of population with the extra parameter of replication rate is very similar to the previous simulations.

Similar to the stable population simulations, variables *t, N,* are *n* are defined. The birth(b), death(d) and replication(r) chances are set to their desired values. An array k is defined, in which the value of the simulated population size would be stored with respect to each iteration. Next, a for loop is initiated. Two variables *Z* and *N* are defined similar to the previous simulation. Now, *Z* is defined as a random sample of size one from a binomial distribution of size t with success probability b. The value stored in Z is the number of creatures formed in that iteration. N is defined as the random sample of size one from a binomial distribution of size $N+Z$ with success probability $1-d$. But, here another variable M is defined. M is defined as the random sample of size one from a binomial distribution of size $N+M$ with success probability r. This is the one of the only two differences from the previous code. This gives us the population size of that iteration. This is run n times which gives us the simulated population data. Then after plotting the data we get,

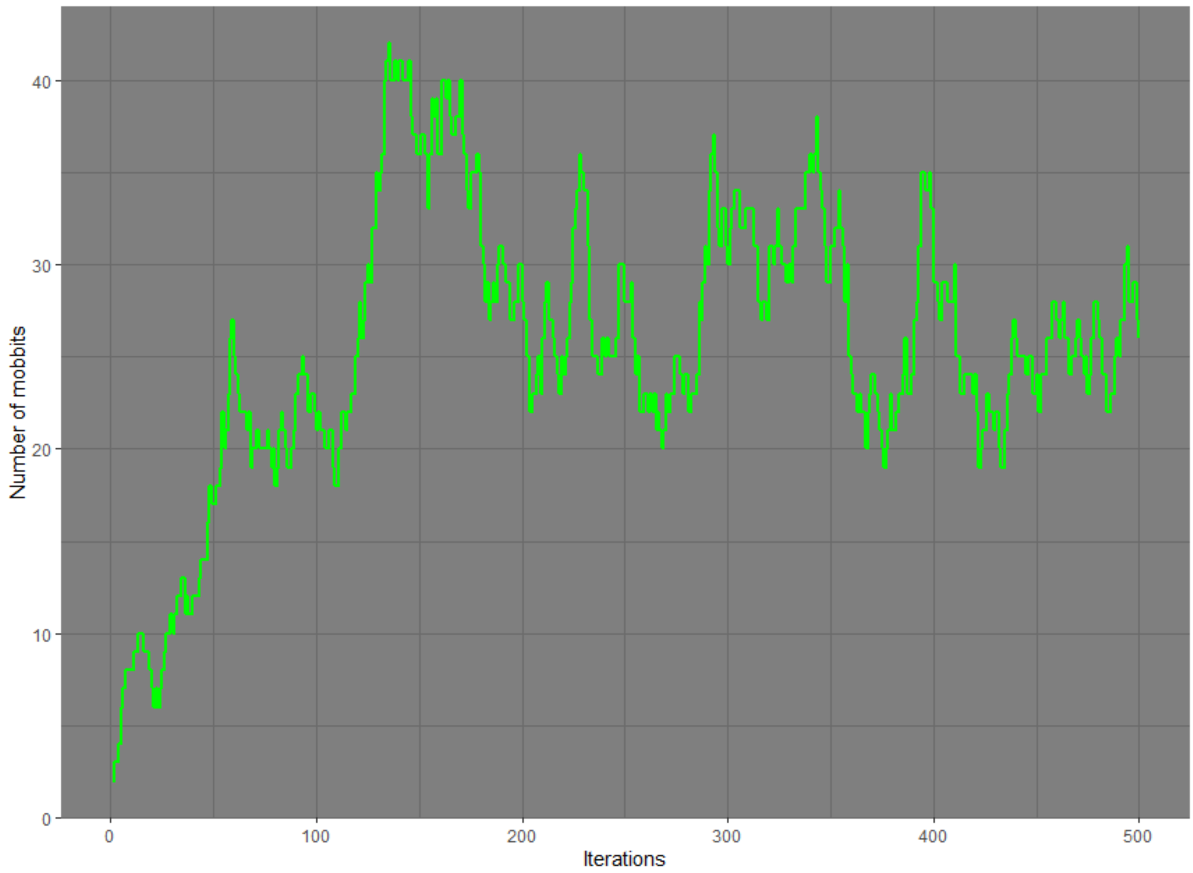

Fig: 9

Here, the simulation data is behaving like a stationary population with no growth at all (Fig: 9). Unlike the first case where the replication chance was greater than death chance and outweighing it, here the death chance is outweighing the replication chance, which results to have replication chance no effect in the simulation whatsoever.

Now, let us run another simulation with equal death chance and replication chance. Let the spontaneous birth chance be 80%, and the death and replication chance be 5%.

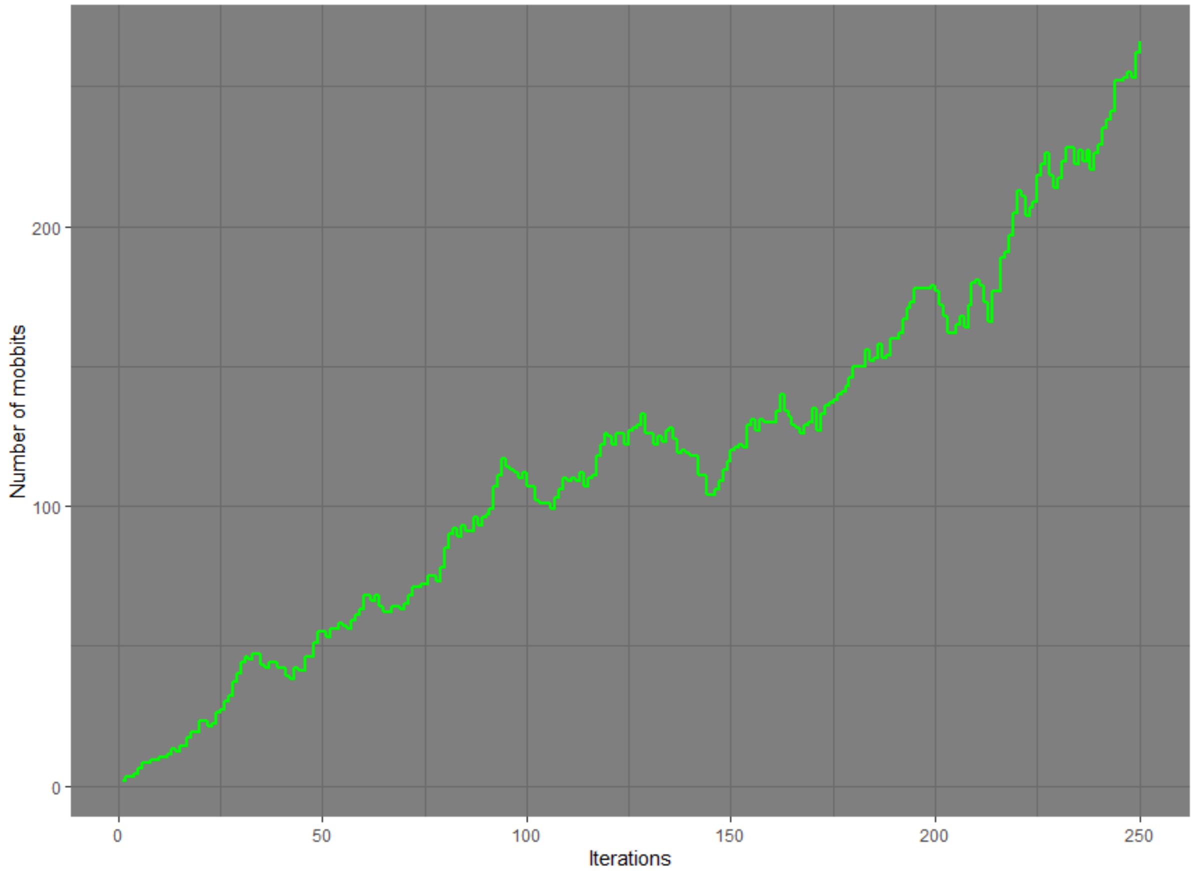

Fig: 10

This curve does not mimic an exponential curve. Rather, it is very much similar to a straight line (Fig: 10). The cause being the equal death and replication rates which makes them cancel out each other in a long run and the only main reason of the population growth being the spontaneous birth chance itself. But, this type of population cannot be seen in nature because, it is very unlikely to have same death and birth chance for an organism.

Now, what if the replication chance becomes greater than that of death chance keeping the spontaneous birth chance same.

Running another simulation with death chance greater than that of replication chance. Let the spontaneous birth chance be 80%, and the death chance be 5% and replication chance be 10%.

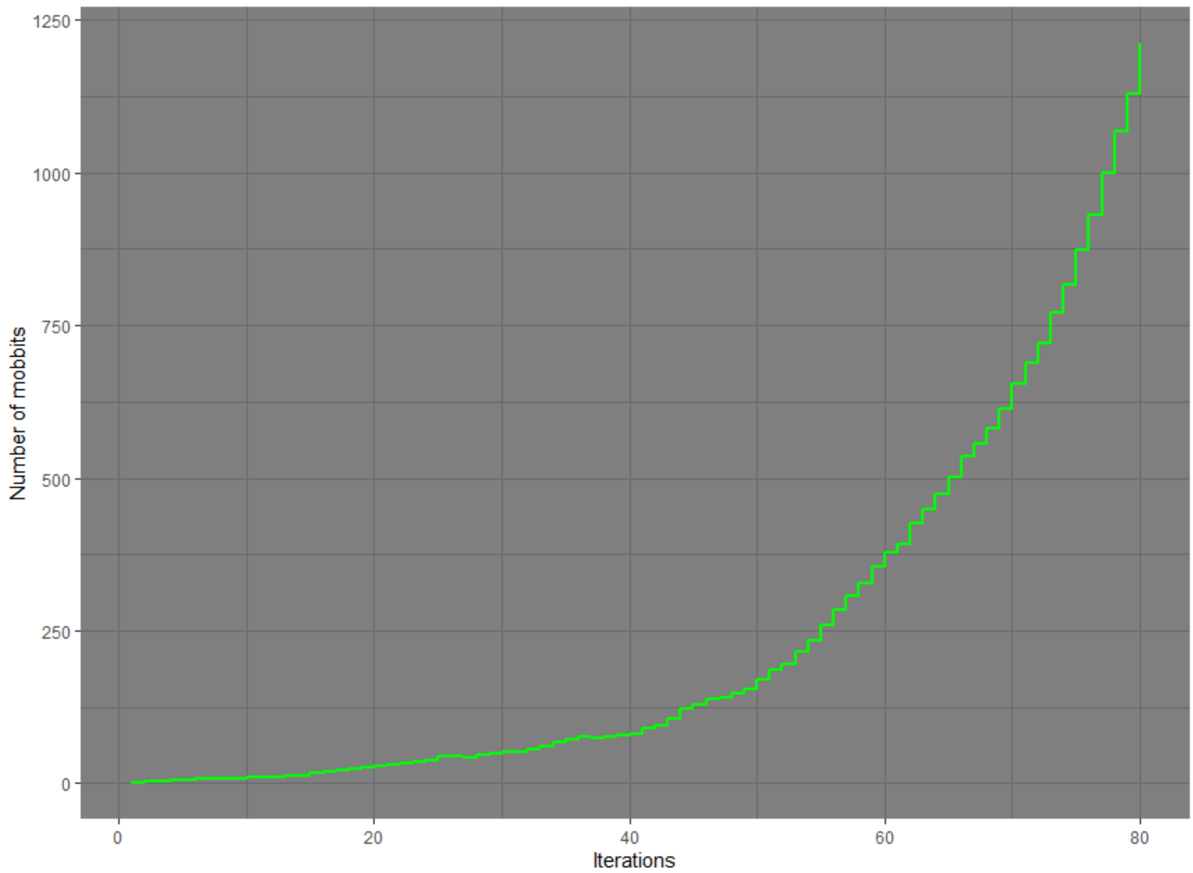

Fig: 11

Clearly, this simulation data follows an exponential curve (Fig: 11). Intuitively you can understand that since the replication rate is greater than that of the death chance, so it is outweighing the effect of the death chance as a whole, making the graph multiplicative in nature. Hence, the graph does not stabilize anywhere.

Mathematical Basis

Recall, in the first section of this project we derived a functional relation of Total Expected change (Δ) and population number (N). The spontaneous birth and death chance was parameters. The expression was given by,

$$\Delta = B - D * N$$

Now, here the new parameter replication chance is introduced. Hence, the above equation is modified as,

$$\Delta = B - (D - R) * N$$

Plotting this function equation with the parameter values of above three cases.

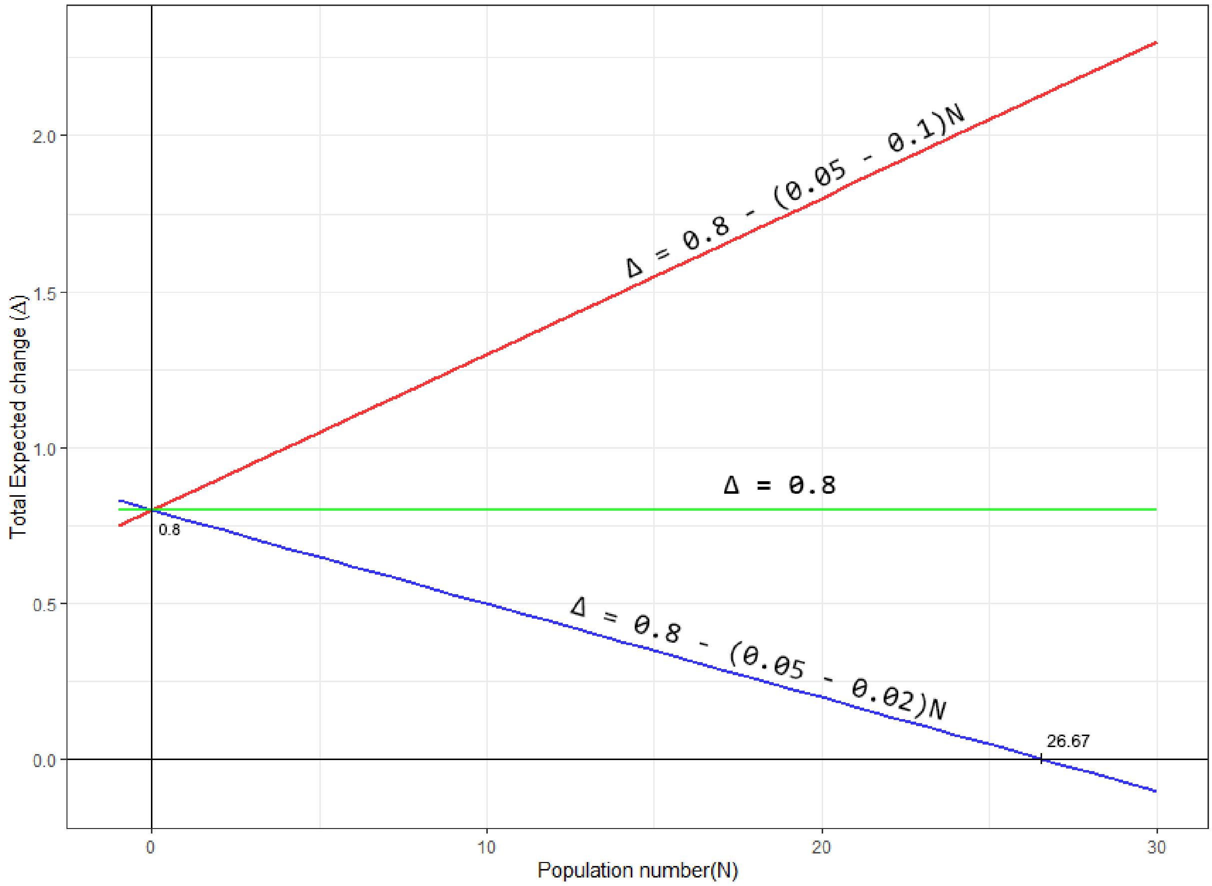

Fig: 12

Clearly, for equation with the replication chance lesser than that of death chance (*blue straight line*) acts similar to the equations of the first section. This straight line has a negative slope and cuts the Population number axis at a point (*for this case it is 26.67*). On the other hand, the population mean comes out to be 28.03 (Fig: 9) which is pretty close to that cut point value. Therefore, until the replication chance is less than that of the death chance, the straight line will always cut the population number axis at some point, hence, will act as a stationary population. Since, here we get a population size value where the total expected change to be zero. Hence, here the expected population curve is straight line parallel to iteration – axis.

In the second case, the replication rate equals to that of the death rate. This makes our function curve (*green straight line*) a straight line parallel to the Population number axis hence, it will never cut it. Therefore, there will be no equilibrium number in this cases because for population value we get the total expected change to be zero. Rather we get a population with a stable or constant increase in size. This can be seen in the second simulation Fig: 10. here, the expected population curve is a straight line with a positive slope.

In the third case, the replication chance exceeds the birth chance. The straight line (*red*) for this case has a positive slope. Therefore, the total expected change increases with increase in the population number. Hence, the greater the population size, more will it increase. This is the basis of exponential population model, the rate of increase of population size is directly proportional to the population size. The expected population curve is an exponential curve. Hence not with much surprise we get simulation data which follows exponential growth. (Fig: 11).

Exponential Population simulation

So far this project has shown how mere introduction of a replication chance can affect the nature of the simulation so vastly. But, in real life we don't really see much of a stationary or stable population with constant growth. Since, the evolution of living organisms has made it very unlikely where more creatures will die than born. Moreover, equal death and replication chances are ever more unlikely. That is why exponential model of population growth is so common in nature.

Now, let us try to simulate some exponential populations based on given parameter values super imposed on the same graph. The algorithm of running R – code for obtaining the graphs remains same.

The graph is provided below,

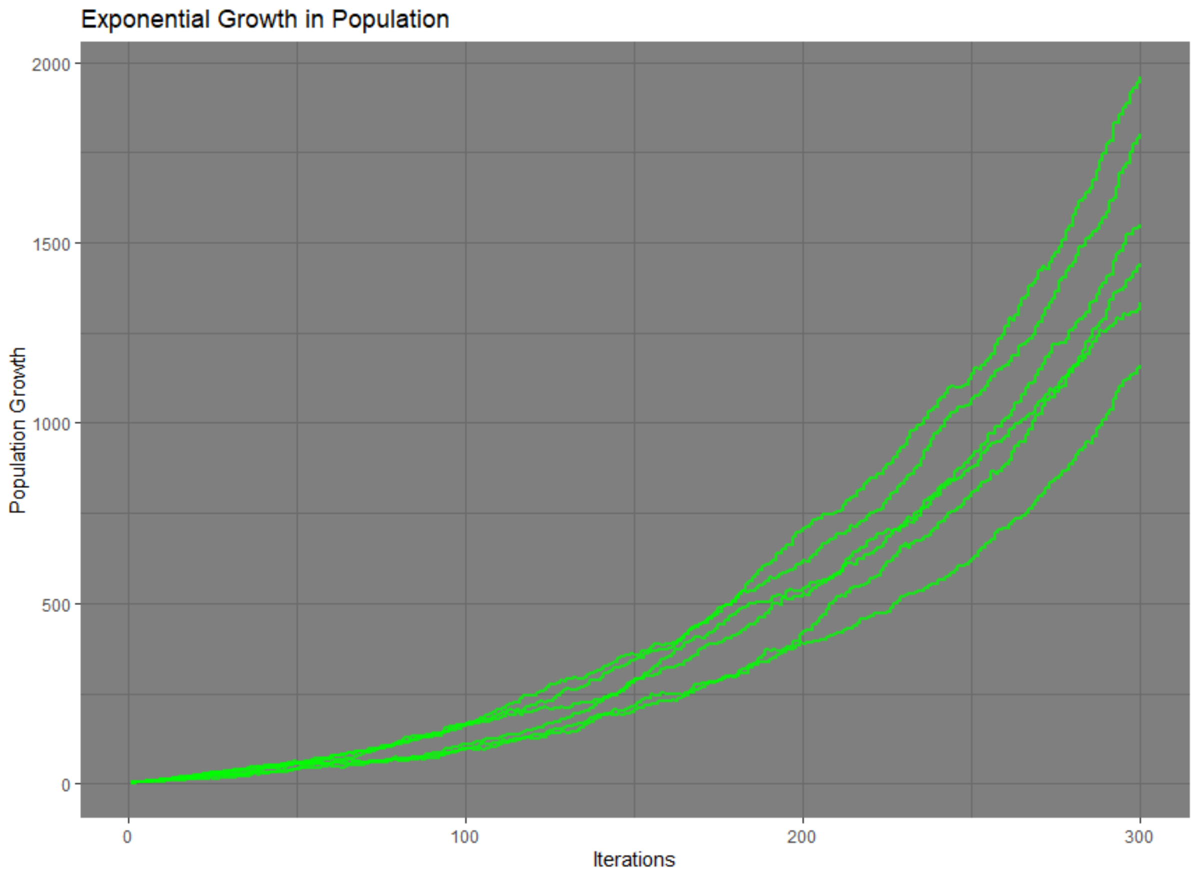

Fig: 13

Here are six runs of simulation which are based on the same parameter values. Spontaneous birth chance 80%, death chance 20% and the replication chance 30%. Although all the graphs have not follow the same path. Some populations have grown faster than the others but, all have them have the same underlying parameters. This is the beauty of probability factor. But, one can clearly see how the nature of all the graphs remain same.

But, living organisms don’t form spontaneously in the real world. The only way they form is by reproduction? So, the spontaneous birth chance parameter is futile in case of living organisms. Therefore, let us try simulate a population where the spontaneous birth chance is zero but the replication rate, 10% and let the death chance be 5% remain same. We are starting with 5 creatures for 50 iterations (Fig: 14).

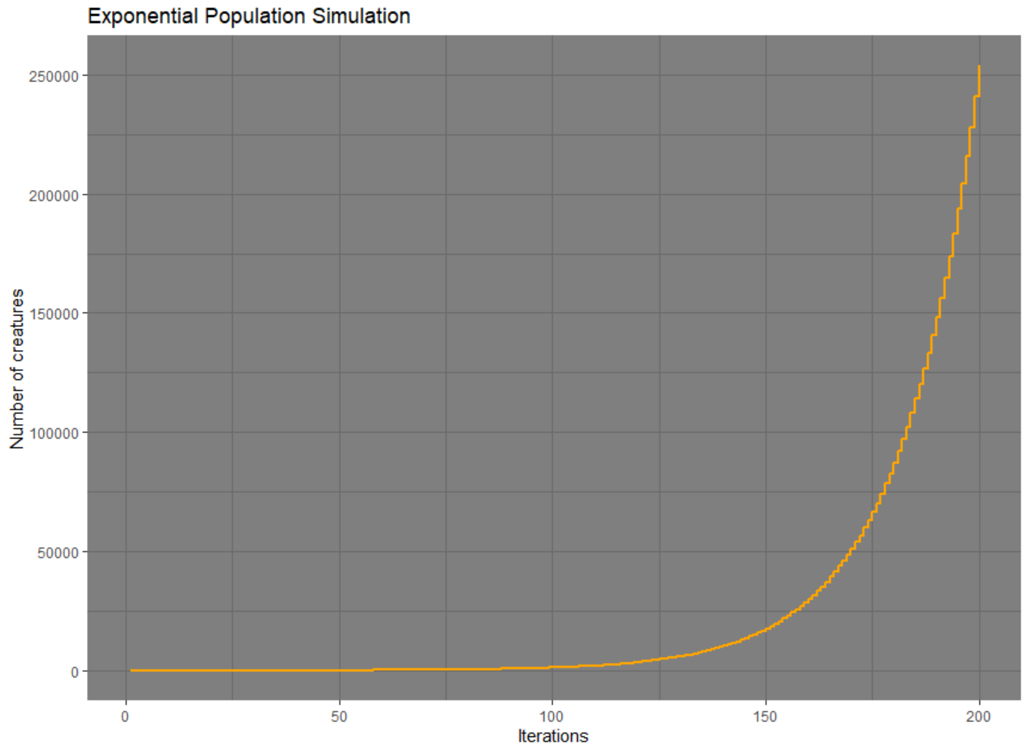

Fig: 14

Observe that, this population too does follow an exponential curve, despite zero birth chance.

This can make you understand how powerful the replication chance is.

Although, sometimes this can occur (Fig: 15)

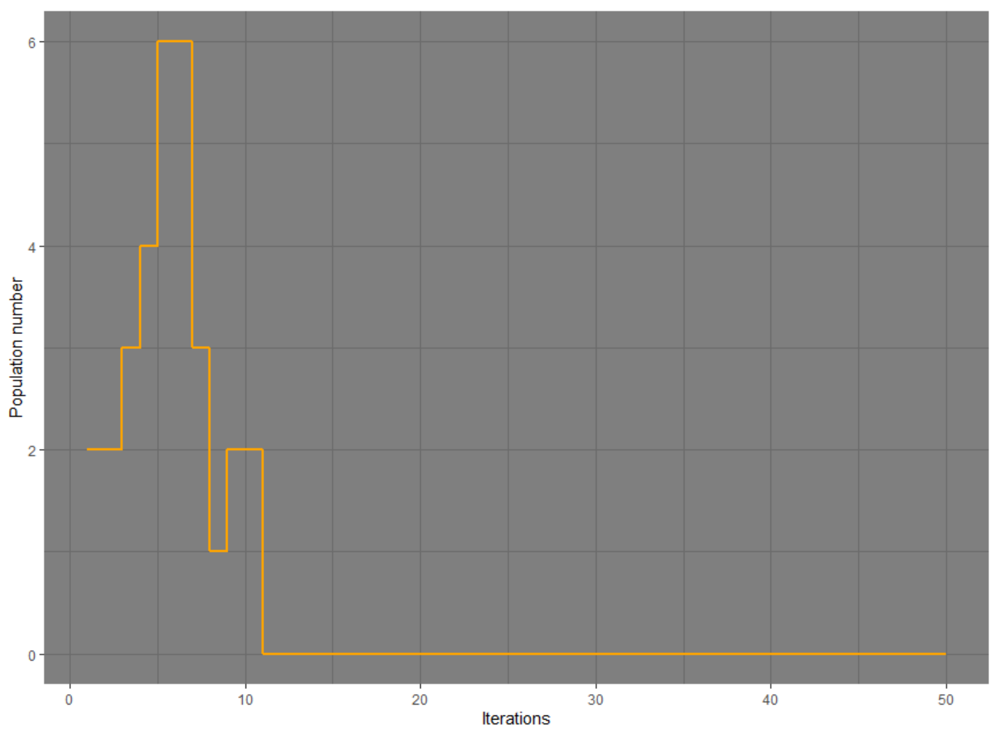

Fig: 15

The creatures can be unlucky enough to go extinct while having the same parameter values.

This makes you think on your existence. Life is so unlikely to exist but, we do exist. This is evident since, scientists are yet to discover any signs of life outside our mother Earth.

Examples of Exponential Population

There are numerous examples of naturally occurring exponential populations.

One of the examples are **bacterial growth**. Microbes grow at a fast rate when they are provided with unlimited resources and a suitable environment. It makes the study of the organism in question relatively easy and, hence, the disease/disorder is easier to detect.

Human population growth is also a major example of exponential growth. The human population is increasing exponentially. As of February 2019, the total population of the world exceeded 7.71 billion, and the numbers are amplifying day-by-day. However, in some areas, growth is slow or the population is on the verge of decline. China is the most populous country and India ranks second. It is, however, estimated that India will lead the world by 2030.

The size of the world population over the last 12.000 years

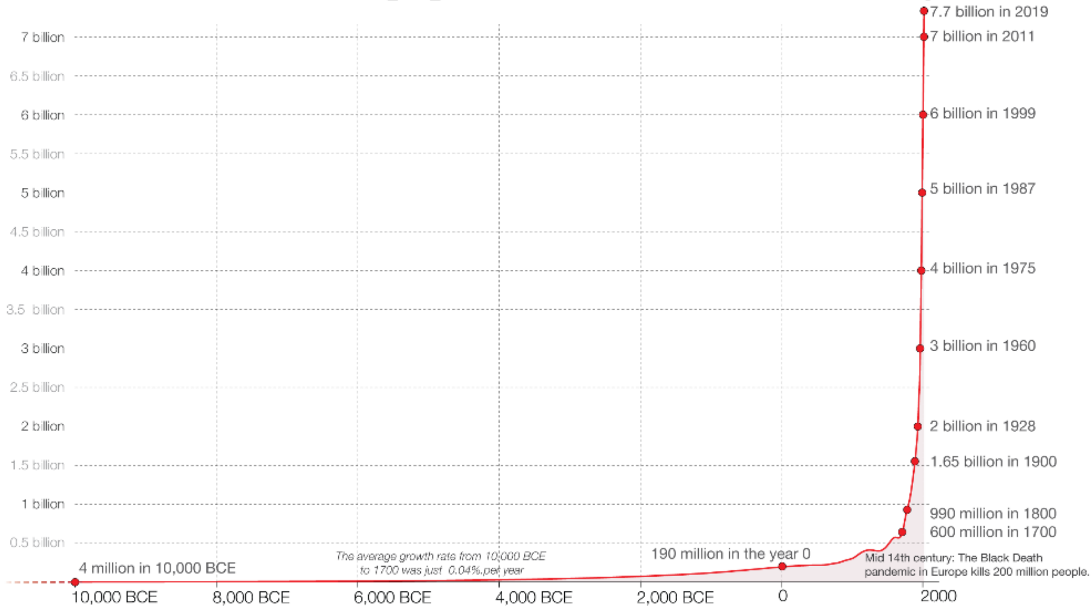

Fig: 16

Even the **COVID-19 pandemic caused by SARS CoV2** worldwide has also shown an exponential growth.

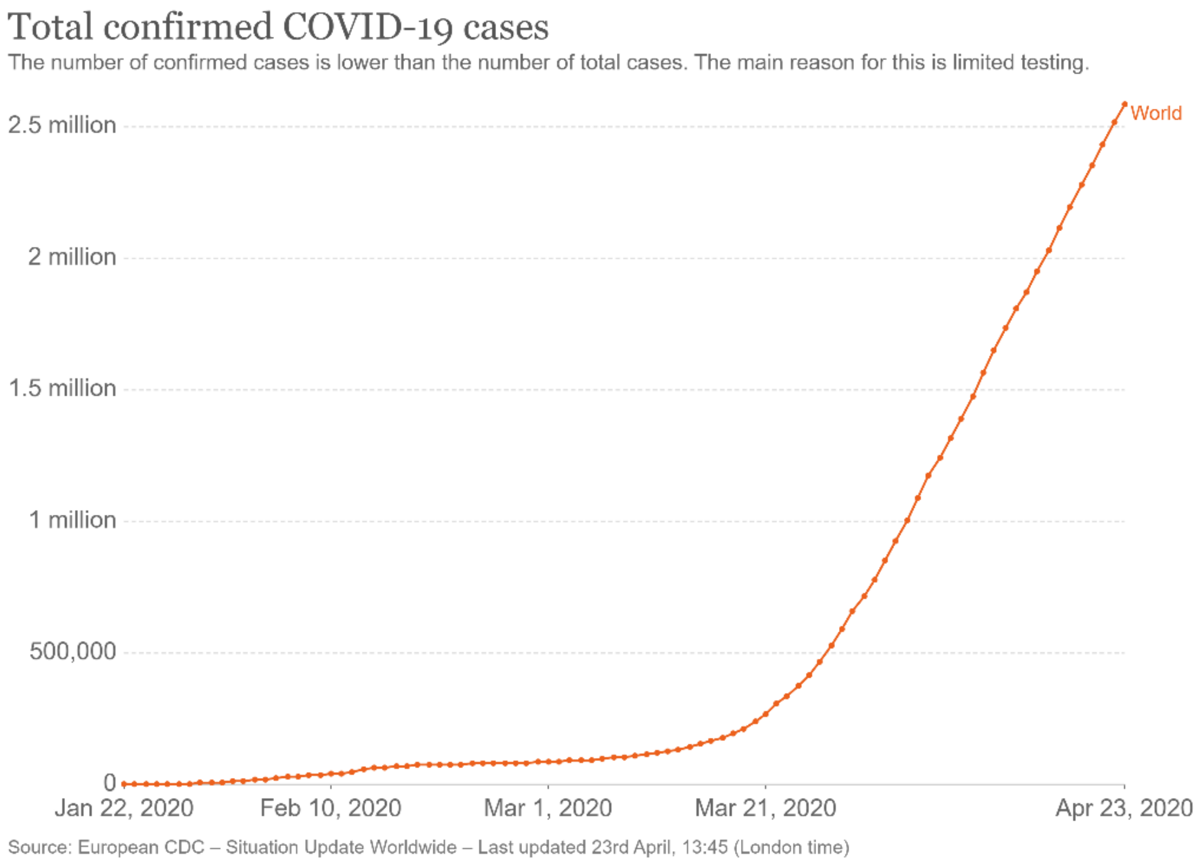

Fig: 17

Exponential populations need unlimited resources. Then why are exponential populations so frequently observed in nature. This is because, our mother earth has ample amount of resources, which seems to be unlimited which makes life to thrive here so good. Although, recent studies have shown many resources are diminishing day by day.

This gives birth to a new parameter based on resources, based on which the logistic model is formed. Simulations of population model will be discussed in the next section.

Logistic Population Simulation:

Suppose for another block of land and we are going to simulate the population growth of another creature *gobbit* (say) there. Let the birth chance and the death chance of the creature Lob be 100% and 5% respectively in this run. Now, like *mobbits* this creature has also a replication chance of 10%. This means more or less 1 in every 10 *gobbits* will make another *gobbit* in the next iteration. But, the previous setup had no constraint of resources. In this setup a new parameter crowding constant *(c)* is introduced. Let c = 0.0001. This is analogous to the carrying capacity term of the model.

Algorithm:

This is almost similar to the algorithm of the exponential population.

In the variables *t, N,* are *n* are defined. The birth(b), death(d) and replication(r) chances are set to their desired values. An array k is defined, in which the value of the simulated population size would be stored with respect to each iteration. Next, a for loop is initiated. Three variables *Z, N* and *M* are defined similar to the previous simulation. Now, *Z* is defined as a random sample of size one from a binomial distribution of size *t* with success probability *b.* The value stored in *Z* is the number of creatures formed in that iteration. *N* is defined as the random sample of size one from a binomial distribution of size *N+Z* with success probability *1-d*. *M* is defined as the random sample of size one from a binomial distribution of size *N+M* with success probability *r-c*N.* This *c*N* act as the factor which will change the growth rate, as when the population N increases *c*N* increases too, which on the other hand decreases *r-c*N.* This is the one of the only two differences from the previous code. This gives us the population size of that iteration. This is run *n* times which gives us the simulated population data. Then after plotting the data we get,

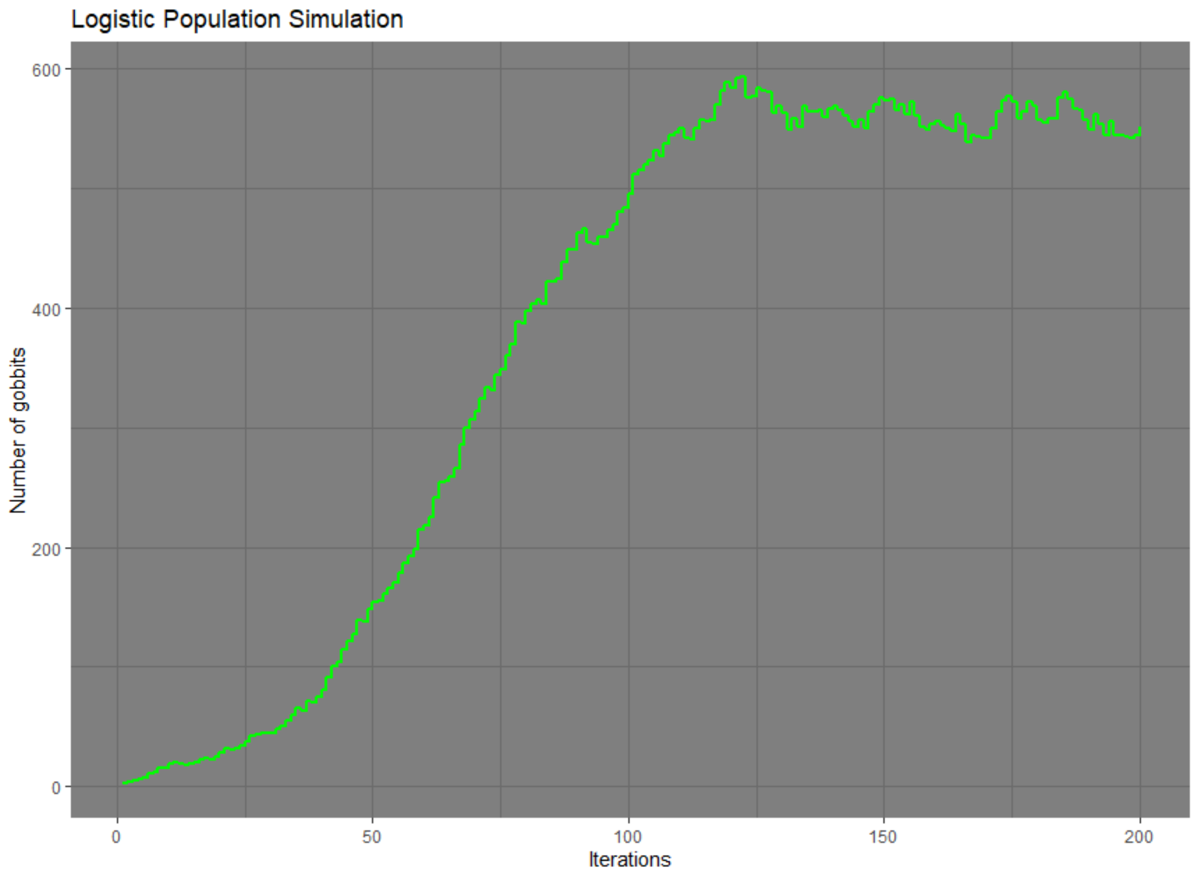

Fig: 18

This graph based on the simulation looks very much similar to the logistic curve (Fig: 8). The first part of the graph mimics an exponential curve, but as soon as the it reaches the limit the curve starts to stabilize. In the beginning the birth rate (*due to both spontaneous birth rate and replication rate*) and death rate are both high. Then, the death rate falls as the population increases although the birth rate remains high. But, when the population reaches its limit which by the way depends upon the crowding constant the birth rate starts to decrease. This whole thing is the foundation of Logistic model. Since, this can be seen in our simulated data, we can infer our simulation to be more or less a success.

This curve stabilizes around a population number of 530. But, this is a single simulation, which is not much trustworthy for a simulation code.

Now, if we run the simulation 6 times and plot all of the data in the same graph we get,

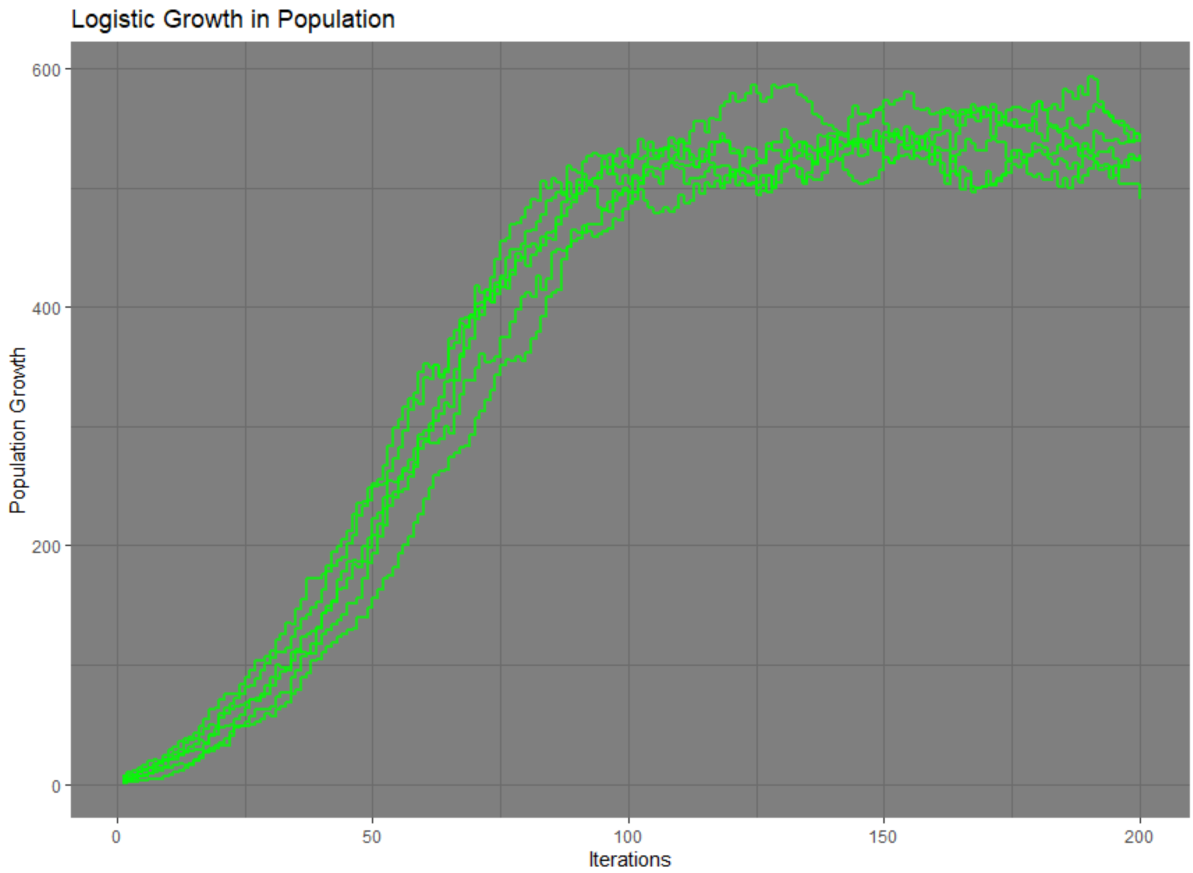

Fig: 19

Obviously not all runs have produced same data but, the overall nature remains same like in Fig: 13. Moreover, all the simulations are getting stabilized at the same region. Therefore, the R-code has successfully simulated a logistic population.

Mathematical Basis

For exponential model, the function equation of the total expected change and the population number based on the parameters of birth, death and replication chance was derived as,

$$\Delta = B - (D - R) * N$$

In the exponential population model, the replication chance (> death chance) remains same throughout. This is the reason why population follows the exponential path since, more the population size more they will replicate. But, here the new parameter of resources has been taken into account. Therefore, the reproduction or the replication rate cannot be constant in this case. In logistic model the first half of the population behaves as the exponential model and grows very fast but, when the population has increased enough the population growth slows down. Hence, the population growth depends on the population number. Hence, we can modify the function equation and write it as follows,

$$\Delta = B - (D - R + (c * N)) * N$$

$$\Rightarrow \Delta = B - (D - R) * N - c * N^2$$

Plotting this function equation with the parameter values.

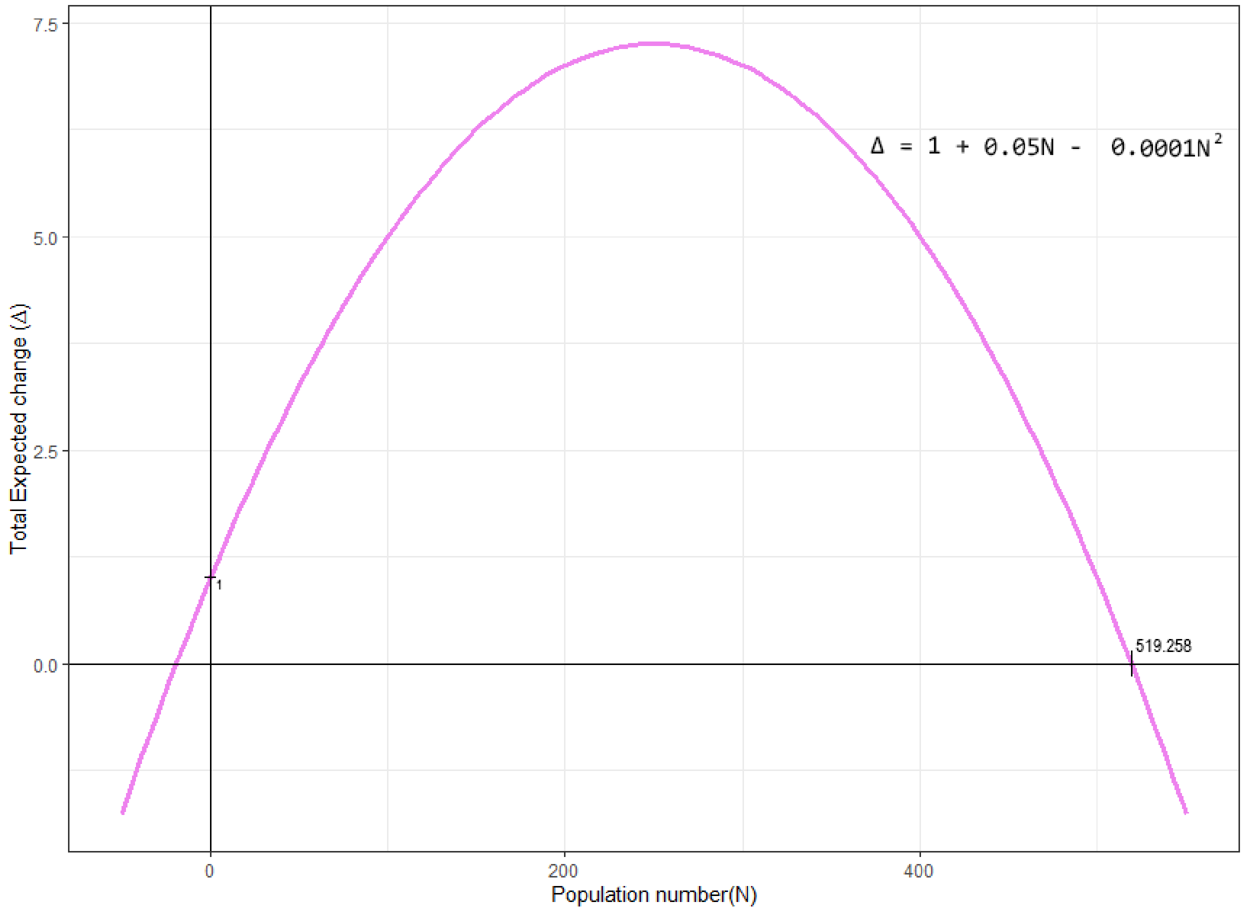

Fig: 20

Not with much surprise this is a parabola as the function was in 2^{nd} order of N. It peaks at 250 population number which means the total expected change is maximum when the number of creatures is around 250. On the other hand, the curve cuts the population number axis at 519.258. It is evident from the Fig: 18 that the population has more or less stabilized there. So it is leveling out at 520 creatures approximately.

Now, in the previous section the spontaneous birth chance was omitted in a simulation although the graph looked pretty much same like an exponential curve (Fig: 14). Let us try to omit the spontaneous birth chance and run a simulation for logistic population. All the parameter values will remain same except the birth chance will be null and the number of creatures in the first iteration will be 10. We get

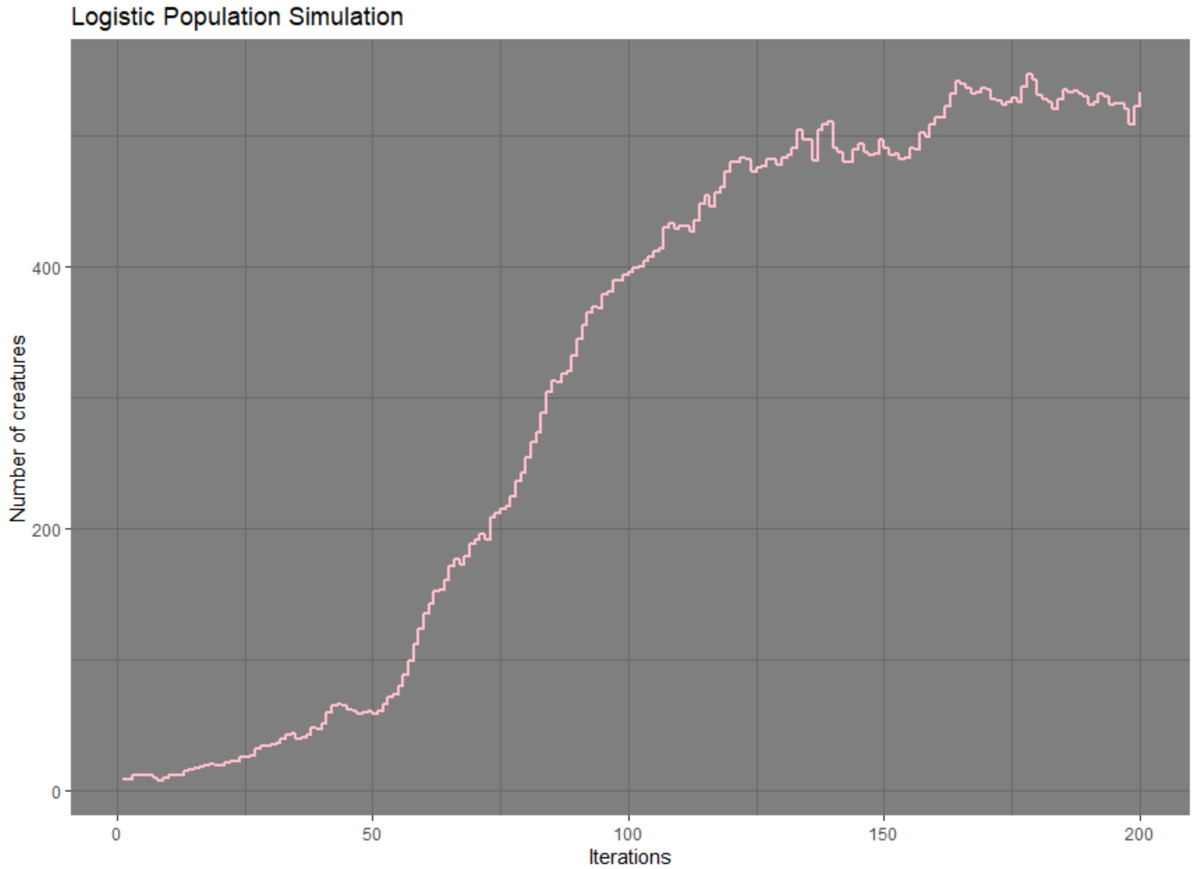

Fig: 21

Nothing much changes and the graph obtained on the basis of simulation data looks same that of a logistic curve and again it gets proven that spontaneous birth chance is not an important simulation parameter for the living organisms. This can be evident from the total expected change – population number function equation too. Plotting the function, we get,

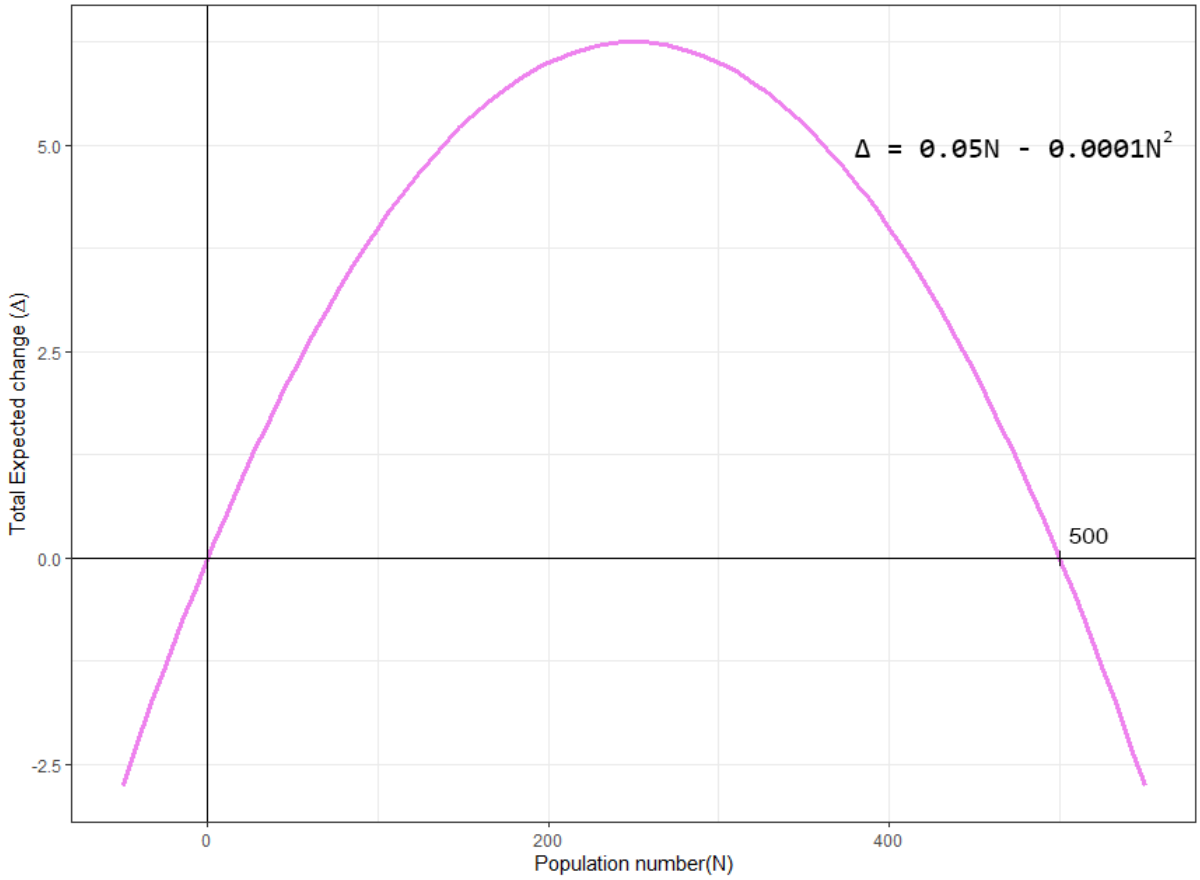

Fig: 22

The curve remains a parabola even when the birth chance is zero and cuts the population number axis at 500. Clearly, in Fig: 21 one can see the population data is stabilizing around 500.

But, the creatures don't turn out lucky every time. This was seen in simulation of Fig: 15 which was done for the exponential population. This same phenomenon can be seen in simulation for logistic population too.

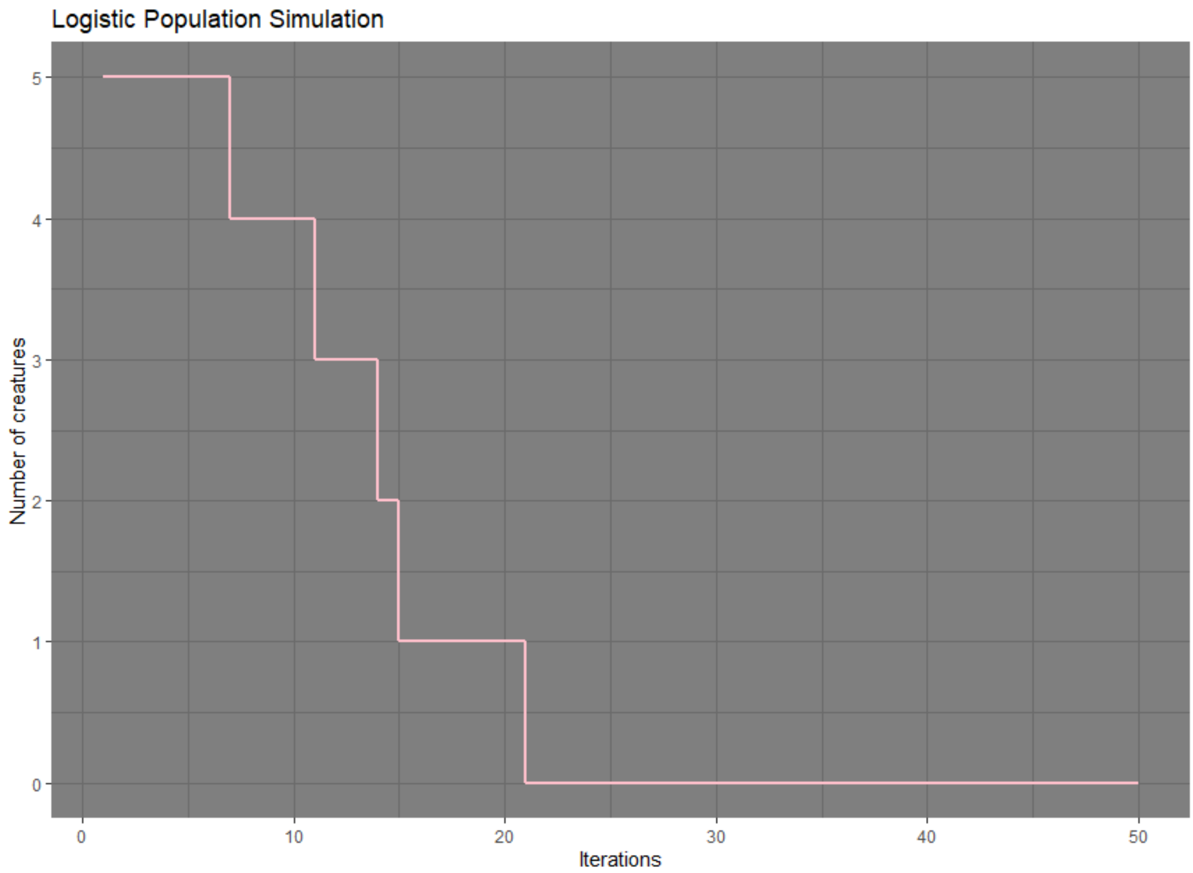

Fig: 23

Therefore, a logistic population can also go extinct. This has a small probability but, this does occur.

Examples of Logistic populations

There are many examples of logistic population. Some of them are as follows.

Yeast, a microscopic fungus used to make bread and alcoholic beverages, exhibits the classical S-shaped curve when grown in a test tube. Its growth levels off as the population depletes the nutrients that are necessary for its growth.

In the real world, however, there are variations to this idealized curve. Examples in wild **populations include sheep and harbour seals**. In both examples, the population size exceeds the carrying capacity for short periods of time and then falls below the carrying capacity afterwards. This fluctuation in population size continues to occur as the population oscillates around its carrying capacity. Still, even with this oscillation, the logistic model is confirmed.

Major **metropolitan cities of first world countries** also show logistic population growth. There may be many reasons for this and mostly they are city specific in nature. Cities like Vancouver, Tokyo or Copenhagen exhibit this nature of population.

And for the most recent example, the places around the world which have taken the outbreak under control shows logistic nature when we plot total confirmed cases affected by coronavirus with time and that is pretty much understandable. The number of total confirmed cases first increases exponentially. The government takes necessary actions action that. If the population obeys that actions the strength of the outbreak diminishes as the number of cases goes down around zero. This can be seen in Japan or South Korea or even in the Indian states like Kerala.

R – Code:

```
library(ggplot2)
t = t0            #total number of creatures at the beginning.
N = N0            #total number of creatures at nth iteration.
b = b0            #Birth chance
d = d0            #Death chance
r = r0            #Replication chance
n = n0            #Number of iterations.
c = c0
y = array(0)
k = array(0)
M = 0
for(i in 1:n)
{
  Z = rbinom(1, t, b)
  N = rbinom(1, N+Z, 1-d)
  M = rbinom(1, N+M, r-c*N)
  N = M+N
  k[i] = N
  y[i] = sum(k)
}
z = k
data = as.data.frame(z)

ggplot(data,aes(1:n)) +
  geom_step(aes(y=z), color = "", size =, alpha = ) +
  theme_dark() +
  xlab("Iterations") +
  ylab("Population Number") +
  ggtitle("Population Simulation")
```

This simple R – code is responsible for all the simulations which have done in the project. Note that, the package "ggplot2" has been load externally for obtaining aesthetic graphs.

Conclusion

Modelling and simulation to describe and analyze a complex system are based on mathematics, statistics, logic and numerical methods. However, in contrast to these fundamental sciences, modelling and simulation have developed into an unstructured multitude of approaches and ideas that are combined in a variety of ways.

This whole project was an exercise on how populations of things be it living or non-living grows and by understanding their nature of growth I have tried to simulate their populations. Starting from the populations of non-living things to the complex human population this journey of simulating their population has been really worthwhile. Lobs, robs or mobbits has helped in this journey a lot. The simulations have shown how close a computer generated data can get to real life population on the basis of only few parameters of birth, death, and replication.

But, this project is wholly based on R-programme and this shows how powerful this statistical software is. Without R none of the graphs could have been obtained.

Population simulation is a very powerful tool for predicting population nature and will be used in far future more massively.

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