Final examination DSE 210

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Be clear and concise. Write your answers in the space provided. Use the backs of pages for scratchwork.

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TOTAL POINTS: 80

1. **(10 points)** You are dealt two cards at random from a standard deck. What is the probability that:

(a) The first card is an ace?

$4/52$

(b) The first and second cards are both aces?

$$\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{13} \cdot \frac{1}{17} = \frac{1}{221}$$

(c) The second card is an ace?

$$\left(\frac{48}{52}\right)\left(\frac{4}{51}\right) + \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \boxed{\frac{1}{13}}$$

(d) The first card is an ace, given that it is a heart?

$$Pr(A|H) = \frac{Pr(A \cap H)}{Pr(H)} = \frac{\left(\frac{1}{52}\right)}{\left(\frac{1}{13}\right)} = \frac{13}{52} = \boxed{\frac{1}{4}}$$

(e) The second card is an ace, given that the first card is an ace?

$$Pr(A_2|A_1) = \frac{Pr(A_2 \text{ and } A_1)}{Pr(A_1)} = \frac{\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)}{\left(\frac{4}{52}\right)} = \frac{3}{51}$$

2. **(3 points)** Ten cards are chosen at random from a standard deck. Which of the following pairs of events A, B are independent? Circle them.

- A: first card is a ten, B: tenth card is a nine
- A: first card is a ten, B: second card is a heart
- A: second card is a heart, B: fifth card is a club

3. **(10 points)** Short answer questions.

(a) The letters G, H, I, R, T are randomly permuted. What is the probability that the result is the word R, I, G, H, T?

$|\Omega| = 5! = 120 \quad |E_1| = 1 \quad \frac{|E_1|}{|\Omega|} = Pr(E_1)$

$\boxed{\frac{1}{120}}$

(b) Three fair dice are rolled. What is the probability that they all have the same value?

$\frac{|E|}{|\Omega|} = \frac{6}{6^3} = \boxed{\frac{1}{36}}$

(c) Each time you go to the gym, you have a 20% chance of running into your worst enemy. What is the expected number of trips to the gym before you meet this person?

$E(x) = \frac{1}{p} = \frac{1}{0.2} = \boxed{5}$

(d) A certain population consists of 40% men and 60% women. Of the men, 20% are left-handed, and of the women, 10% are left-handed. A person is picked at random from this population and is found to be left-handed. What is the probability that this person is female?

$Pr(F|L) = \frac{Pr(L|F) \cdot Pr(F)}{Pr(L)} = \frac{Pr(L|F) \cdot Pr(F)}{Pr(L|F) \cdot Pr(F) + Pr(L|M) \, Pr(M}$

$= \frac{(0.1)(0.6)}{(0.1)(0.6) + (0.2)(0.4)} = 0.43 \quad \boxed{43\%}$

(e) A man has a bottle containing ten identical-looking pills. Two of them contain medicine while the other 8 are placebos. Upon taking a pill, the man feels either good or not good, with the following probabilities:

$$\Pr(\text{feel good} \mid \text{medicine}) = \frac{3}{4}$$

$$\Pr(\text{feel good} \mid \text{placebo}) = \frac{1}{2}$$

Today, the man picks a pill at random and finds that he feels good. What is the probability that the pill contained medicine?

$Pr(M|G) = \frac{Pr(G|M) \cdot Pr(M)}{Pr(G)} = \frac{Pr(G|M) \, Pr(M)}{Pr(G|M) \, Pr(M) + Pr(G|P) \cdot Pr(P)}$

$= \frac{(0.75)(0.2)}{(0.75)(0.2) + (0.5)(0.8)} = .27 \quad \boxed{27\%}$

4. **(8 points)** A die has six sides that come up with different probabilities.

$$\Pr(1) = \Pr(2) = \Pr(3) = \frac{1}{12}, \quad \Pr(4) = \Pr(5) = \Pr(6) = \frac{1}{4}.$$

(a) You roll the die; let X denote the outcome. What is $\mathbb{E}(X)$?

$E(x) = (1)(\frac{1}{12}) + (2)(\frac{1}{12}) + 3(\frac{1}{12}) + 4(\frac{1}{4}) + 5(\frac{1}{4}) + 6(\frac{1}{4})$

$E(x) = \frac{17}{4} = \boxed{4.25}$

(b) What is $\text{var}(X)$?

$Var(x) = E(x^2) - M^2$, $E(x) = 4.25$ $E(x^2) = 20.42$

$Var(x) = 20.42 - (4.25)^2$

$var(x) = \boxed{2.3575}$

(c) Now you roll this die a hundred times, and let Z be the sum of all the rolls. What is $\mathbb{E}(Z)$?

$E(Z) = 100 \cdot E(x) = 100 \cdot 4.25 = \boxed{425}$

(d) What is $\text{var}(Z)$?

since each roll independent

$Var(Z) = 100 \cdot 2.3575 = \boxed{235.75}$

5. **(3 points)** A pair of random variables X_1 and X_2 have the following properties:

- They both take values in $\{-1, 1\}$
- X_1 has mean 0 while X_2 has mean 0.5
- The correlation between X_1 and X_2 is 0.25

Suppose we fit a (bivariate) Gaussian to (X_1, X_2). Give the mean and covariance matrix of this Gaussian.

$Var(X_1) = 1$ $var(X_2) = ?$ since $M_2 = 0.5$ we can say $0.5 = (1)(\frac{3}{4}) + (-1)(\frac{1}{4})$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Pr(X_2=1) \qquad \Pr(X_2=-1)$

so $Var(X_2) = E(X_2^2) - M_2^2$

$E(X_2^2) = (1^2)(\frac{3}{4}) + (-1^2)(\frac{1}{4}) = 1$

so $var(X_2) = 1 - (0.5)^2 = 0.75$

$corr(X_1, X_2) \cdot std(X_1) \cdot std(X_2) = cov(X_1, X_2)$

$(0.25)(\sqrt{1})(\sqrt{0.75}) = cov(X_1, X_2)$

$0.2165 = cov(X_1, X_2)$

cov matrix

$M = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} \quad \Sigma = \begin{pmatrix} 1, & 0.2165 \\ 0.2165, & 0.75 \end{pmatrix}$

6. **(10 points)** A certain random variable $X \in \mathbb{R}^3$ has mean and covariance as follows:

$$\mathbb{E}X = \begin{pmatrix}1\\2\\3\end{pmatrix}, \quad \mathrm{cov}(X) = \begin{pmatrix}5 & -3 & 0\\ -3 & 5 & 0\\ 0 & 0 & 4\end{pmatrix}$$

(a) The eigenvectors of $\mathrm{cov}(X)$ can be found in the following list:

$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$$

Circle them.

(b) Find the eigenvalues corresponding to each of the eigenvectors in part (a). Make it clear which eigenvalue belongs to which eigenvector.

$$\boxed{u_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} \quad \lambda_1 = 8} \quad \boxed{u_2 = \begin{pmatrix}0\\0\\1\end{pmatrix} \quad \lambda_2 = 4} \quad \boxed{u_3 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} \quad \lambda_3 = 2}$$

(c) Suppose we used principal component analysis (PCA) to project points X into *two* dimensions. Which directions would it project onto?

use u_1 and u_2

$$U^T = \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}x_1 - \frac{1}{\sqrt{2}}x_2\\ x_3\end{pmatrix} \leftarrow U^Tx$$

$\leftarrow$ project to $\mathbb{R}^2$

(d) Continuing from part (c), what would be the resulting two-dimensional projection of the point $x = (4, 0, 2)$?

Plug x into U^Tx

$$U^Tx = \begin{pmatrix}\frac{1}{\sqrt{2}}(4) - \frac{1}{\sqrt{2}}(0)\\ 2\end{pmatrix} = \begin{pmatrix}\frac{4}{\sqrt{2}}\\ 2\end{pmatrix} \rightarrow \begin{pmatrix}2.83\\ 2\end{pmatrix}$$

(e) Continuing from part (d), suppose that starting from the 2-d projection, we tried to reconstruct the original x. What would the three-dimensional reconstruction be, exactly?

$x \mapsto UU^Tx$ same as $x \to (u_1 \cdot x)u_1 + (u_2 \cdot x)u_2$

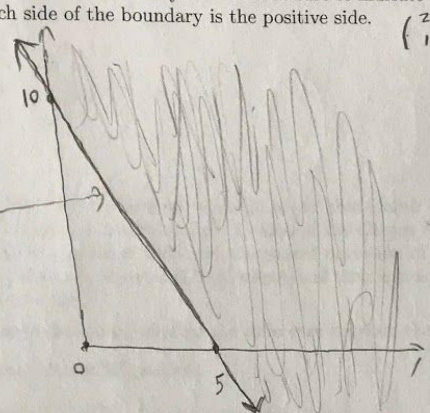
$$U = \begin{pmatrix}\frac{1}{\sqrt{2}} & 0\\ -\frac{1}{\sqrt{2}} & 0\\ 0 & 1\end{pmatrix}\underbrace{\begin{pmatrix}\frac{4}{\sqrt{2}}\\ 2\end{pmatrix}}_{U^Tx} = \begin{pmatrix}\frac{4}{\sqrt{2}\sqrt{2}}\\ -\frac{4}{\sqrt{2}\sqrt{2}}\\ 2\end{pmatrix} = \begin{pmatrix}2\\-2\\2\end{pmatrix} = UU^Tx$$

7. **(4 points)** Consider the linear classifier $w \cdot x \geq \theta$, where

$$w = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \theta = 10.$$

Sketch the decision boundary in $\mathbb{R}^2$. Make sure to indicate where the boundary intersects the two axes, and which side of the boundary is the positive side.

$\begin{pmatrix} 2 \\ 1 \end{pmatrix}(x_1, x_2) \geq 10 \rightarrow 2x_1 + x_2 \geq 10$



decision boundary $2x_1 + x_2 \geq 10$

(shaded area is positive side of boundary)

8. **(4 points)** A survey is taken to determine what fraction of freshman computer science majors have prior programming experience. Call this unknown fraction p. Out of the nationwide pool of computer science freshmen, 100 are chosen at random. Of them, 40% had prior programming experience.

(a) The natural estimate of p is 0.4. Give a 95% confidence interval for the estimate.

$\hat{\theta} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.4)(0.6)}{100}} = \sqrt{\frac{0.24}{100}} = 0.049 = \hat{\theta}$

95% confidence interval

$0.4 \pm 2(.049)$

$\boxed{0.4 \pm 0.098}$

(b) Suppose we now want to estimate p more accurately, to within a 95% confidence interval of ± 0.01. What sample size should we use?

want $2 \cdot \theta \leq 0.01$

so $\theta \leq 0.005$

$\sqrt{\frac{(0.4)(0.6)}{n}} = 0.005$

$\frac{(0.4)(0.6)}{n} = (0.005)^2$

$.24 = 0.000025n$

$9600 = n$

sample size should be at least 9600

9. **(2 points)** A school wants to determine the average number of hours that the students spend on homework; call this unknown number μ. 100 students are chosen at random, and each of them is asked to report the typical number of hours per week that he or she spends on homework. The reported numbers have a mean of 12.2 and a standard deviation of 5.4. Give a 95% confidence interval for μ.

$M + 2 \cdot \sigma$

$\sigma = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{5.4}{\sqrt{100}} = \frac{5.4}{10}$

$12.2 \pm 2(0.54)$

$\sigma = .54$

$\boxed{12.2 \pm 1.08}$

10. **(6 points)** Genius Academy is a high school that claims to prepare its students exceptionally well for the SAT exam. A random sample is taken of 100 Genius Academy students, and their SAT scores turn out to have a mean of 1930 and a standard deviation of 150. A random sample is also taken of 100 students from the other local high school, and their scores have a lower mean, of 1860, with a standard deviation of 200.

We wish to determine whether the difference between these observed averages is significant.

(a) State the null hypothesis.

the mean test scores of the two distributions (academy and other school) are the same.

(b) Compute a suitable z-statistic for this situation.

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{(150)^2 + (200)^2} = 250$

$z\text{-stat} = \frac{1930 - 1860}{250}$

$z\text{-stat} = \boxed{0,28}$

(c) What is the p value, and what conclusion would you draw?

p-value is $\approx \boxed{0.39}$

not significant at $p < 0.05$ level (95 % confidence interval)

conclusion: accept the null, the mean test scores of the two schools are the $\boxed{\text{same}}$

Scratch Work

(4) die 6 sided $Pr(1) = Pr(2) = Pr(3) = \frac{1}{12}$ $Pr(4) = Pr(5) = Pr(6) = \frac{1}{4}$

(a) $E(X) = \sum_x x Pr(X=x) = (1)(\frac{1}{12}) + 2(\frac{1}{12}) + 3(\frac{1}{12}) + 4(\frac{1}{4}) + 5(\frac{1}{4}) + 6(\frac{1}{4})$

$E(X) = \frac{1}{12} + \frac{2}{12} + \frac{3}{12} + \frac{4}{4} + \frac{5}{4} + \frac{6}{4}$

$E(X) = \frac{2}{4} + \frac{4}{4} + \frac{5}{4} + \frac{6}{4}$

$\boxed{E(X) = \frac{17}{4} = 4.25}$

$4\frac{1}{4}$

(b) $Var(X) = E(X^2) - \mu^2$ where $\mu = E(X) = 4.25$

$E(X)^2 = (1)^2(\frac{1}{12}) + (2)^2(\frac{1}{12}) + (3)^2(\frac{1}{12}) + 4^2(\frac{1}{4}) + (5)^2(\frac{1}{4}) + (6)^2(\frac{1}{4})$

$E(X)^2 = \frac{1}{12} + \frac{4}{12} + \frac{9}{12} + \frac{16}{4} + \frac{25}{4} + \frac{36}{4} = \frac{14}{12} + \frac{77}{4} \approx 20.42$

so $Var(X) = 20.42 - (4.25)^2 =$

$\boxed{var(X) = 2.3575}$

(c) roll die hundred times Z sum of all roles, what is $E(Z)$?

simply, $100 \cdot E(X) = 100 \cdot 4.25 = 425$

(d) $var(Z) = \sum_1^{100} var(X_i)$ since each dice roll independent

$100 \cdot var(Z) = 100 \cdot 2.3575$

$\boxed{var(Z) = 235.75}$

⑤ X_1 and X_2 both take values in $\{-1, 1\}$
X_1 has mean 0, while X_2 has mean 0.5

$\mu_1 = 0$ $\mu_2 = 0.5$ $\text{corr}(X_1, X_2) = 0.25$

$\mu = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$ cov_matrix $= \begin{pmatrix} & \\ & \end{pmatrix}$ $0.25 = \dfrac{\text{cov}(X,Y)}{\text{std}(X)\,\text{std}(Y)}$

need to calculate variance of X_1 and X_2

$E(x_i^2) - \mu^2$ $E(x_i) = 0$ $\text{var}(x_i) = 1$

$(1)\left(\frac{3}{4}\right)$ $(-1)\left(\frac{1}{4}\right)$

$\{^{-1}_{1}$ $\mu_2 = 0.5$

$1 \cdot p_1 -$ $p_1 - p_2 = 0.5$

$\text{var}(X_2) = E(X_2^2) - \mu_2^2$ $p_1 + p_2 = 1$

$E(X_2^2) = (1^2)\frac{3}{4} + (-1)^2\left(\frac{1}{4}\right)$ $p_1 = 0.5 + p_2$

$= 1$ $2p_2 = 0.5$

$1 - (0.5)^2 = 0.75$

(6) $EX = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\quad \hat{cov}(X) = \begin{pmatrix} 5 & -3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(b) $Mu = \lambda u$

$$\begin{pmatrix} 5 & -3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = 4u_1$$

eigenvect $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ has eigenvalue $\lambda = 4$

$$\begin{pmatrix} 5 & -3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}} \\ -\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} \\ 0 \end{pmatrix} = 2u_2$$

eigenvector $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ has eigenvalue $\lambda = 2$

$$\begin{pmatrix} 5 & -3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{2}} + \frac{3}{\sqrt{2}} \\ -\frac{3}{\sqrt{2}} - \frac{5}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{8}{\sqrt{2}} \\ -\frac{8}{\sqrt{2}} \\ 0 \end{pmatrix} = 8u_3$$

eigenvect $\frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ has eigenvalue $\lambda = 8$

(c) PCA project X to 2 dimensions, which directions would it project to

Σ is 3×3

$u^T \Sigma u$ maximized by setting u to first eigenvector
max value is corr. eigenvalue

$X = \mathbb{R}^3$ project to $\mathbb{R}^2$

$U^T x$ projects to $\mathbb{R}^2$ same as saying $x \to (u_1 \cdot x, u_2 x \cdot)$

u_1 and u_2 are the 2 eigenvectors w/ max eigenvalues

first eigenvector is $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix} - u_1$ since max eigenvalue of 8

second eigenvector is $\begin{pmatrix}0\\0\\1\end{pmatrix} - u_2$ since 2^{nd} highest eigenvalue of 4

rewrite u_1 as $\begin{pmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\\0\end{pmatrix}$ $u_2 = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $X = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$

$$U^T = \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \boxed{\begin{pmatrix}\frac{1}{\sqrt{2}}x_1 - \frac{1}{\sqrt{2}}x_2\\ x_3\end{pmatrix}}$$ $\leftarrow$ projection to $\mathbb{R}^2$

2×3 $\quad 3 \times 1$

direction $\frac{1}{\sqrt{2}}x_1 \quad \frac{1}{\sqrt{2}}x_2$

(d) $$\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}4\\0\\2\end{pmatrix} = \boxed{\begin{pmatrix}\frac{4}{\sqrt{2}}\\2\end{pmatrix}}$$

(e) $x \mapsto UU^T x$ same as $x \mapsto (u_1 \cdot x)u_1 + (u_2 \cdot x)u_2$

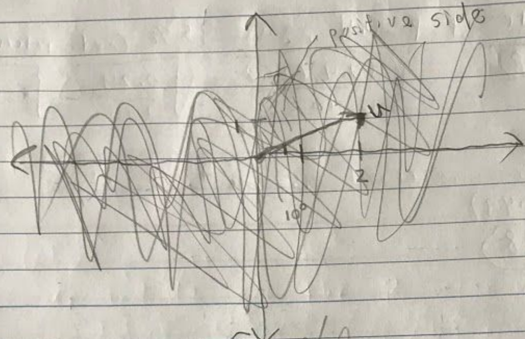
we have $U^T x = \begin{pmatrix}\frac{4}{\sqrt{2}}\\2\end{pmatrix}$ (projection)

matrix associative

$$\begin{pmatrix}\frac{1}{\sqrt{2}} & 0\\ -\frac{1}{\sqrt{2}} & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{4}{\sqrt{2}}\\2\end{pmatrix} = \begin{pmatrix}\frac{4}{\sqrt{2}\cdot\sqrt{2}} + 0\\ \frac{-4}{\sqrt{2}\sqrt{2}}\\ 2\end{pmatrix} = \begin{pmatrix}\frac{4}{2}\\ \frac{-4}{2}\\ 2\end{pmatrix} = \boxed{\begin{pmatrix}2\\-2\\2\end{pmatrix}}$$

$3 \times 2 \quad 2 \times 1$

7 $w \cdot x \geq \theta$ $w = \binom{2}{1}$ $\theta = 10$

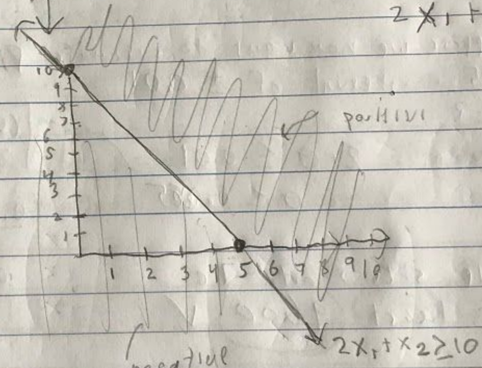


$\binom{2}{1}(x_1, x_2) \geq 10$

$2x_1 + x_2 \geq 10$

$[2, 1]\binom{x_1}{x_2}$

$2x_1 + x_2 \geq 10$



(8) unknown p $n=100$ $\hat{p}=40$

(a) natural estimate of $p=0.4$, give 95% Conf. interval for estimate

std. $\sigma = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.4)(0.6)}{100}} = \sqrt{\frac{0.24}{100}} = \sqrt{.0024} = .049$

$\sigma = .049$

95 confidence interval is $\mu \pm 2 \cdot$ std

$0.4 \pm 2(.049)$

0.4 ± 0.098

(b) Suppose we now want to estimate p more accurately.

95% interval of ± 0.01

$n=?$

want $2 \cdot \sigma \leq 0.01$

so $\sigma \leq 0.005$

$\left(\sqrt{\frac{(0.4)(0.6)}{n}}\right)^2 = (0.005)^2$

$\frac{(0.4)(0.6)}{n} = .000025$

$0.24 = 0.000025n$

$9600 = n$

sample size should be at least 9600 ✓

Final

(9) school wants to determine average $M = ?$

$n = 100$ $\hat{M} = 12.2$ $\hat{std} = 5.4$

95% confidence interval

$$\sigma = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{5.4}{\sqrt{100}} = \frac{5.4}{10}$$

$$\sigma = .54$$

$\boxed{12.2 \pm 2(0.54)}$

95%

when to divide std by $\sqrt{n}$

(10) $n = 100$ from academy $\hat{M}_1 = 1930$ $\hat{\sigma}_1 = 150$

$n = 10$ from local higher $\hat{M}_1 = 1860$ $\hat{\sigma}_2 = 200$

(a) <u>null hypothesis</u>: The mean test scores of the two distributions (academy and other highschool) are the same

(b) Z-stat $\frac{1930 - 1860}{250}$

Z-stat $= 0.28$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$$

$$\sigma = \sqrt{(150)^2 + (200)^2}$$

$$\sigma = \sqrt{22{,}500 + 40{,}000}$$

$$\sigma = 250$$

(c) p-value is 0.3897

the result is <u>not</u> significant at $p < 0.05$ level (95% confidence)

<u>conclusion</u>: accept the null, the mean test scores of the two schools <u>are the same</u>