DSE 210: Probability and statistics **Winter 2018**

Worksheet 6 — Generative models 2

1. Would you expect the following pairs of random variables to be uncorrelated, positively correlated, or negatively correlated?
   (a) The weight of a new car and its price.
   (b) The weight of a car and the number of seats in it.
   (c) The age in years of a second-hand car and its current market value.
2. Consider a population of married couples in which every wife is exactly 0.9 of her husband's age. What is the correlation between husband's age and wife's age?
3. Each of the following scenarios describes a joint distribution (x, y). In each case, give the parameters of the (unique) bivariate Gaussian that satisfies these properties.
   (a) x has mean 2 and standard deviation 1, y has mean 2 and standard deviation 0.5, and the correlation between x and y is -0.5.
   (b) x has mean 1 and standard deviation 1, and y is equal to x.
4. Roughly sketch the shapes of the following Gaussians $N(\mu, \Sigma)$. For each, you only need to show a representative contour line which is qualitatively accurate (has approximately the right orientation, for instance).
   (a) $\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix}$
   (b) $\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & -0.75 \\ -0.75 & 1 \end{pmatrix}$
5. For each of the two Gaussians in the previous problem, check your answer using Python: draw 100 random samples from that Gaussian and plot it.