MATH 540 HONORS LINEAR ALGEBRA SUMMER 2021
EXAM 1

You can use your notes, the course notes, Canvas and Gradescope course pages, and the course textbooks. Other than that, you may use the Internet only at the level of generality of Wikipedia, though honestly I think you're better off sticking with class resources. You're welcome to use a calculator, but I don't see how it would be useful.

Under no circumstances may you communicate with anyone, either in the class or outside of the class, about any aspect of the exam, until the entire exam period is over. If you have any uncertainty about what is or is not allowed, it is on you to ask me for clarification.

Some partial credit will be provided for well-written scaffolding and setting up of proofs, even in the absence of a full proof.

Feel free to send me an email at melody_chan+linear@brown.edu if you think something on the exam needs to be clarified.

Best of luck! –Melody

4 questions, 2 hours and 15 minutes, 12+12+16+20 points

1. Negate the following statement. Then prove either the original statement or the negation, whichever one is true. (Please indicate clearly which one is true.)

   For any linearly dependent vectors $v, w \in \mathbb{R}^2$, there is some $c \in \mathbb{R}$ such that $cv = w$.

2. Let V be the real vector space of functions from $\mathbb{R}$ to $\mathbb{R}$. Let W be the set of functions from $\mathbb{R}$ to $\mathbb{R}$ that are bounded from above. (We say a function $f\colon \mathbb{R} \to \mathbb{R}$ is *bounded from above* if there exists some $M \in \mathbb{R}$ such that $f(x) \leq M$ for all $x \in \mathbb{R}$.)

   Is W a subspace of V? Prove your answer either way.

3. Let a be a real number. Prove that the linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

   $$T(x, y) = (y, ax - y)$$

   is injective if and only if $a \neq 0$.

4. Let V and W be vector spaces over a field $\mathbb{F}$, and let $T\colon V \to W$ be a linear map. Suppose $v_1, \ldots, v_m$ are vectors in V such that $T(v_1), \ldots, T(v_m)$ are linearly independent. Prove that

   $$\operatorname{null} T \cap \operatorname{span}(v_1, \ldots, v_m) = \{\mathbf{0}\}.$$