

Problem §1

Translate from set-builder notation into English.

Solution:

- (a) $\{x \in \mathbb{R} \mid x > 0\} \cup \{x \in \mathbb{R} \mid x < 0\}$ is the set of all non-zero real numbers.
- (b) $\mathbb{R} \setminus \mathbb{Q}$ is the set of all irrational numbers.
- (c) $\{\alpha \in \mathbb{C} \mid \alpha^2 \in \mathbb{Q}\}$ is the set of all complex numbers whose squares are rational.
- (d) $\{(x, y) \in \mathbb{C}^2 \mid y = -x\}$ is the set of ordered pairs whose sum is zero.
- (e) $\{(x, 2x) \mid x \in \mathbb{R}\}$ is the set of ordered pairs whose second number is double the first number.
- (f) $\{(a, b, 0) \mid a, b \in \mathbb{R}\} \cap \{(x, 0, z) \mid x, z \in \mathbb{R}\}$ is the x axis.

Problem §2

Translate from English to set-builder notation.

Solution:

- (a) The set of real numbers whose cube is an integer is

$$\{x \in \mathbb{R} \mid x^3 \in \mathbb{Z}\}.$$

- (b) The set of real numbers whose square is greater than 2 is

$$\{x \in \mathbb{R} \mid x^2 > 2\}.$$

- (c) The set of rational numbers strictly between 0 and 1 is

$$\{x \in \mathbb{Q} \mid 0 < x < 1\}.$$

- (d) The set of ordered pairs of complex numbers in which the second is i times the first one is

$$\{(\alpha, \beta) \in \mathbb{C}^2 \mid \alpha i = \beta\}.$$

- (e) The set of ordered triples of real numbers in which the third one is zero is

$$\{(a, b, c) \in \mathbb{R}^3 \mid c = 0\}.$$

(Alternatively, $\{(a, b, 0) \mid a \in \mathbb{R}\}$).

- (f) The set of ordered triples of real numbers in which at least one of the three is zero is

$$\{(a, b, c) \in \mathbb{R}^3 \mid a = 0 \vee b = 0 \vee c = 0\}.$$

- (g) The set of nonzero complex solutions to the equation $x^5 + 17x^3 - x^2 = 0$ is

$$\{\alpha \in \mathbb{C} \setminus \{0\} \mid \alpha^5 + 17\alpha^3 - \alpha^2 = 0\}.$$