Chapter 1

Eigenvalues, Eigenvectors, and Invariant Subspaces

In chapter 3, we studied linear maps from one vector space to another. We now turn our attention to maps from finite-dimensional vector spaces to themselves.

§1.1 Invariant Subspaces

We start by developing tools to help us understand the structure of operators. Recall that an **operator** is a linear map from a vector space to itself, and we denote the set of operators on V by $\mathcal{L}(V)$; in other words, $\mathcal{L}(V) = \mathcal{L}(V, V)$.
To better understand what an operator looks like, suppose $T \in \mathcal{L}(V)$. If we have a direct sum decomposition

$$V = U_1 \oplus \ldots \oplus U_m,$$

where each U_j is a proper subspace of V, then to understand the behavior of T, we need only understand its behavior on each individual subspace, $T|_{U_j}$.