REVIEW SHEET 3, Math 540, Summer 2021, Melody Chan

Due Fri May 21 at 11:59pm Eastern Time

Submit all of the following on Gradescope, and don't forget to tag each answer to its page. We have implemented a course policy whereby failing to tag results in half credit.

I put a copy of this review sheet in the Overleaf folder located here

`https://www.overleaf.com/read/hffhqdrqxbjf`

for those of you who are typing your homework. I'll try to keep doing this for future homeworks.

(1) Recall that $\mathbb{C}^2$ is a vector space over $\mathbb{C}$. Let's practice scalar multiplication in $\mathbb{C}^2$. Let v be the vector

$$v = (2 + 2i, -1 + i).$$

(a) Compute $\frac{1}{2} \cdot v$.

(b) Compute $(3i) \cdot v$.

(c) Compute $(-1 + i) \cdot v$.

(d) (Optional, ungraded, but maybe helpful for intuition)
It's hard to draw the vector space $\mathbb{C}^2$—I can't see in four real dimensions, can you? But you might try to "draw" a point $(z_1, z_2) \in \mathbb{C}^2$ as a pair of points z_1 and z_2 in the complex plane.[1] Try this for the vector v and its three scalar multiples that you computed. How are the pictures related?

(2) For each of the following statements, write down its negation, then assert whether the original statement or the negation is true. Finally, prove your assertion.[2]

Example: In any field $\mathbb{F}$, for all $a, b, c \in \mathbb{F}$, $ab = ac$ implies $b = c$.

Answer:

The negation is: There exists a field $\mathbb{F}$ and $a, b, c \in \mathbb{F}$, such that $ab = ac$ but $b \neq c$.

The negation is true. Proof: Let $\mathbb{F} = \mathbb{R}$, and let $a = 0, b = 2$, and $c = 3$. Then $ab = 0$ and $ac = 0$, but $b \neq c$.

[1]To draw points in the *complex plane*, draw $a + bi$ at location (a, b).

[2]Remember in a *direct proof,* try to respond to a universal quantifier with "given," and try to respond to an existential quantifier, i.e., a claim that something exists, by actually producing something with the desired properties.

(a) There is some $\alpha \in \mathbb{C}$ such that
$$\alpha \cdot (1+i, 1-i) = (1, i).$$

(b) For all $s \in \mathbb{C}$, there exist $t, u \in \mathbb{C}$ such that
$$(1, s) = t \cdot (u, -i).$$

(c) In any field $\mathbb{F}$ and given any $a, b \in \mathbb{F}$ such that $a \neq 0$, there is some $x \in \mathbb{F}$ such that
$$ax = b.$$

(d) For every $a, b \in \mathbb{Q}$, not both 0, there exist $c, d \in \mathbb{Q}$ such that the following equation holds in $\mathbb{R}$:
$$(a + b\sqrt{3})(c + d\sqrt{3}) = 1.$$