REVIEW SHEET 7, Math 540, Summer 2021, Melody Chan

Due Wed June 9 at 11:59pm Eastern Time

Submit all of the following on Gradescope, and don't forget to tag each answer to its page. We have implemented a course policy whereby failing to tag results in half credit.

I put a copy of this review sheet in the Overleaf folder.

(1) (Ungraded, 0 points) Go over your answers to Review Sheet 5 Problem 2, and find agreement with the Rank-Nullity theorem.

(2) (2 points) Consider the following list of vectors in $\mathbb{R}^3$. Working from left to right, remove each vector if it is in the span of the vectors to the left of it in the list. What list of vectors remains? Just the answer is fine; no proofs needed.

$(1,1,0)$, $(-2,-2,0)$, $(0,0,0)$, $(0,0,4)$, $(1,1,1)$, $(4,0,0)$, $(0,1,1)$, $(4,-5,3)$, $(7,2,2)$

(3) (1.5 points each) Let V be a finite-dimensional vector space over a field $\mathbb{F}$. Prove the following statements **briefly**. Please do invoke already-proved statements from class. If you are writing more than a couple sentences, try to find a shorter proof.

Some of these are proved in the textbook. It's fine for you to peek in the textbook if you're stuck, but then please close the book before writing down your solutions.

(a) Prove that if $\dim V > 0$ then V has a nonzero vector.

(b) Conversely, prove that if $\dim V = 0$ then $V = \{\mathbf{0}\}$. That is, prove that V consists only of the zero vector.

(c) Prove that $\dim U \leq \dim V$ if U is a subspace of V.

(d) Prove that if $T\colon V \to W$ is injective then $\dim V = \dim \operatorname{range} T$. Then, conclude that $\dim V \leq \dim W$ if W is finite-dimensional.

(e) Prove[1] that if $T\colon V \to W$ is surjective then $\dim V \geq \dim W$.

(f) Prove that if $T\colon V \to W$ is bijective then $\dim V = \dim W$. (Hint: don't think too hard.)

[1] You already showed, in a previous problem, that W must also be finite-dimensional if these hypotheses hold.