REVIEW SHEET 2, Math 540, Summer 2021, Melody Chan

Due Weds May 19 at 11:59pm Eastern Time

Submit all of the following on Gradescope, and don't forget to tag each answer to its page. We have implemented a course policy whereby failing to tag results in half credit.

I put a copy of this review sheet in the Overleaf folder located here

`https://www.overleaf.com/read/hffhqdrqxbjf`

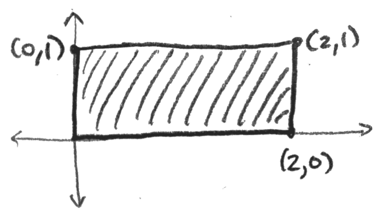
for those of you who are typing your homework. I'll try to keep doing this for future homeworks.

(1) Draw pictures of the following subsets of $\mathbb{R}^2$.

Example:

$$\{\lambda_1 \cdot (2,0) + \lambda_2 \cdot (0,1) : \lambda_1, \lambda_2 \in \mathbb{R},\ 0 \leq \lambda_1, \lambda_2 \leq 1\}$$

Answer:



(a) $\{\lambda_1 \cdot (1,1) + \lambda_2 \cdot (0,1) : \lambda_1, \lambda_2 \in \mathbb{R},\ 0 \leq \lambda_1, \lambda_2 \leq 1\}$

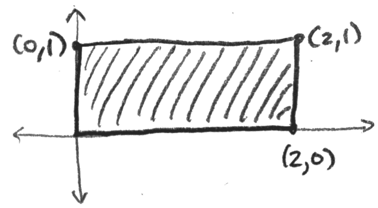
(b) $\{\lambda_1 \cdot (0,2) + \lambda_2 \cdot (0,1) : \lambda_1, \lambda_2 \in \mathbb{R},\ 0 \leq \lambda_1, \lambda_2 \leq 1\}$

(c) $\{\lambda_1 \cdot (1,1) + \lambda_2 \cdot (0,1) : \lambda_1, \lambda_2 \in \mathbb{R}_{\geq 0}\}$

(d) $\{\lambda_1 \cdot (1,1) + \lambda_2 \cdot (0,1) : \lambda_1, \lambda_2 \in \mathbb{R}\}$

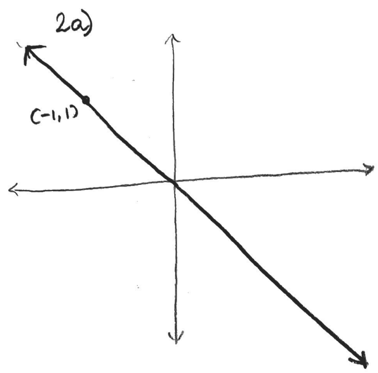
(2) Describe the following subsets of $\mathbb{R}^2$, drawn below, using set-builder notation. Many different answers are certainly possible.

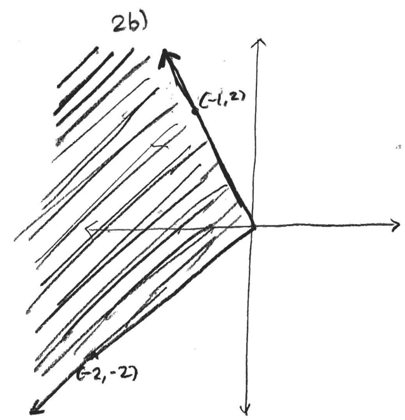
Example:

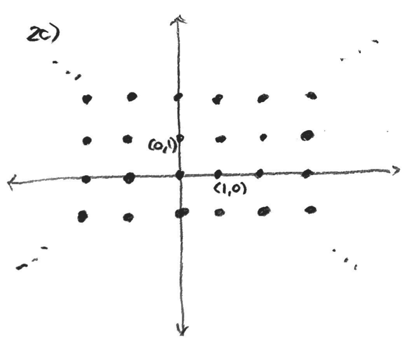


is the set

$$\{\lambda_1 \cdot (2,0) + \lambda_2 \cdot (0,1) : \lambda_1, \lambda_2 \in \mathbb{R},\ 0 \leq \lambda_1, \lambda_2 \leq 1\}.$$







(3) (Starting to write mathematics!) Define the functions below using $f(\cdot)$ or $\mapsto$ notation. Don't be afraid to give things names! Then, assert whether they are injective, surjective, both (bijective), or neither.

Example: The absolute value function with domain and codomain $\mathbb{R}$.

Answer 1: Let $f\colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x|$. I claim that f is neither injective nor surjective.

Another answer: Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $x \mapsto |x|$. Then I assert that f is neither injective nor surjective.

Notice that both answers are composed of two sentences, each beginning with an English. I really do want you to write "Let" and "I assert"/"I claim." The latter two phrases signal that you are claiming something is true but that you haven't yet justified it.

(a) The natural logarithm function from $\mathbb{R}_{>0}$ to $\mathbb{R}$.

(b) The binary operation of multiplication in the field of rational numbers. Regarded as a function, it has domain $\mathbb{Q} \times \mathbb{Q}$ and codomain $\mathbb{Q}$.

(c) The function from $\mathbb{R}^2$ to $\mathbb{R}^3$ sending (x, y) to $(x, x + y, y)$.

(d) The function from $\mathbb{R}^2$ to $\mathbb{R}^2$ sending (x, y) to $(x + y, -x - y)$.