Review Sheet 14

1) $T:\mathbb{R}^2 \to \mathbb{R}^2$ reflection across $y=x$.

With $\lambda_1 = 1$, we have $v_1 = (1,1)$; and $\lambda_2 = -1$, $v_2 = (-1,1)$.

$B = \{v_1, v_2\}$.

$M(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2) $T:\mathbb{R}^2 \to \mathbb{R}^2$ $180°$ rotation about origin.

$\lambda_1 = -1$, $v_1 = (1,0)$; $v_2 = (0,1)$

$B = \{v_1, v_2\}$

$M(T) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

3) $T:\mathbb{R}^2 \to \mathbb{R}^2$ $60°$ CC rotation

No such basis exists

4) $T:\mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (x+y, x+y)$

$\lambda_1 = 2$, $v_1 = (1,1)$; $\lambda_2 = 0$, $v_2 = (-1,1)$ $B = \{v_1, v_2\}$

$M(T) = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$

5) $T:\mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (x,x)$

$\lambda_1 = 1$, $v_1 = (1,1)$; $\lambda_2 = 0$, $v_2 = (0,1)$ $B = \{v_1, v_2\}$

$M(T) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

6) $T:\mathbb{R}^3 \to \mathbb{R}^3$, $T(x,y,z) = (x,y,0)$

$\lambda_1 = 0$, $v_1 = (0,0,1)$; $\lambda_2 = 1$, $v_2 = (1,0,0)$, $v_3 = (0,1,0)$ $M(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

7) $T:\mathbb{R}^3 \to \mathbb{R}^3$ $180°$ rotation about x-axis

$\lambda_1 = -1$, $v_1 = (0,1,0)$, $v_2 = (0,0,1)$; $\lambda_2 = 1$, $v_3 = (1,0,0)$ $M(T) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

8) $T:\mathbb{R}^3 \to \mathbb{R}^3$ $T(x,y,z) = (x, 2y, 3z)$

$\lambda_1 = 1$, $v_1 = (1,0,0)$; $\lambda_2 = 2$, $v_2 = (0,1,0)$; $\lambda_3 = 3$, $v_3 = (0,0,1)$ $B = \{v_1, v_2, v_3\}$

$M(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

9)
- -1: Swapping column 1&2 flips sign, & det. of diagonal matrix is $1 \cdot \ldots \cdot 1 = 1$.
- -1: We need to swap 5 rows, so $(-1)^5 \cdot 1 = -1$.
- -720: We need to swap 3 columns; then we multiply along the diagonal: $(-1)^3(1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6) = -720$.
- 0: By adding rows (specifically, $1+4+5+6 = (0\ 0\ 0\ 0\ -1\ -1)$, and $2+3 = (0\ 0\ 0\ 0\ 1\ 1)$), we see that one row is linearly dependent on the others. Thus $\det A = 0$.