# Math 1010–Midterm 1

You can take the exam on Gradescope during any 90-minute session that starts after 6:00pm ET on Fri June 18 and ends before noon on Sat June 19. This exam has **4 problems**.

**You are only allowed to spend 75 minutes answering the exam questions.** After 75 minutes, you must immediately stop working and begin scanning and uploading to Gradescope. We strongly recommend that you scan and upload in PDF format.

If you have a technical issue that prevents you from uploading (to Gradescope) on time, communicate with Huy Nguyen (huy_quang_nguyen@brown.edu ) as early as possible. Your priority should be to get me documentation that your exam was complete on time; **send your exam first, explain afterward**. A PDF is strongly preferred, but send images if you must. If it takes more than 90 minutes from the time you start the exam on Gradescope to the time **I receive** your exam by email, your exam is considered a late submission and will receive a penalty. We will evaluate late submissions on a case-by-case basis.

Exam Rules

You are ALLOWED to use:

- The textbook and handouts for this course
- Notes you have taken yourself during this course
- Solutions you have created yourself when solving problems in this course

You are NOT ALLOWED to use:

- Calculators or other calculating devices
- Websites, apps, or other electronic/internet references
- Communication with other people
- Books or publications other than the course textbook
- Notes or solutions from other courses
- Notes or solutions created by other people

For every question, unless the question specifically says otherwise, you are meant to submit your answer and your work that led to that answer. It is extremely important that you write down and submit all of your reasoning, so that we can see where your answers came from. To avoid confusion, please do not write on separate "scrap paper". Put all of your attempts on the paper you are submitting, and if you write something that turns out not to be relevant to your answer, erase or cross it out. If you forget to submit part of your work, it cannot be used as justification after the fact. Partially correct work leading to an incorrect answer may receive partial credit. Problems in which logical steps are skipped may receive no credit at all (even if part of the problem or the final answer is correct), and if the work appears to have come from a prohibited source, we will report it to Brown as a potential Academic Code violation.

**We wish you luck!**

For each of the following problems, if an statement is true, give a **detailed proof**; on the other hand, if a statement is false, give a **counterexample**.

**Problem 1 (10 points).**

(a) Write the definition of non Cauchy sequences using quantifiers and then in common English.

(b) Prove that $\sqrt[3]{2+\sqrt{2}}$ is not a rational number.

**Problem 2 (10 points).** For any nonempty subsets $A$ and $B$ of $\mathbb{R}$, define $AB = \{ab : a \in A,\ b \in B\}$. All the sets considered in this problem are *nonempty*.

(a) Let $A$ and $B$ be subsets of $(0, \infty)$ such that they are bounded above. Prove that $\sup A$ and $\sup B$ are positive, $AB$ is bounded above, and $\sup(AB) = (\sup A)(\sup B)$.

(b) If $A$ and $B$ are subsets of $\mathbb{R}$ and are bounded above, is $AB$ bounded above? If $A$ and $B$ are subsets of $\mathbb{R}$ and are bounded, is the identity $\sup(AB) = (\sup A)(\sup B)$ always true?

**Problem 3 (10 points).** Let $(s_n)_{n\in\mathbb{N}}$ converge to $s$ and $(t_n)_{n\in\mathbb{N}}$ converge to $t$.

(a) If $s_n < t_n$ for all $n \in \mathbb{N}$, can one conclude that $s < t$?

(b) Assume that $\lim s_n = s \neq 0$, and $\lim t_n = 0$ where $t_n \neq 0$ for all $n$. What is $\lim \frac{|s_n|}{|t_n|}$? Does $\lim \frac{s_n}{t_n}$ always exist (as a real number, or $\infty$, or $-\infty$)?

For part (b) you can use without proof the fact that if $\lim r_n = r \in \mathbb{R}$ then $\lim |r_n| = |r|$.

**Problem 4 (10 points).** Define a sequence recursively by $s_1 = 2$ and

$$s_{n+1} = \frac{1}{2}(s_n + \frac{2}{s_n}) \quad \forall n \geq 1.$$

(a) Prove that $s_n^2 \geq 2$ for all $n \in \mathbb{N}$.

(b) Prove that $(s_n)$ converges and find $\lim s_n$.