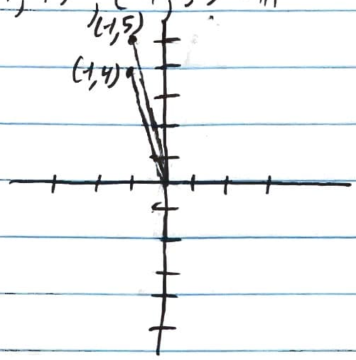
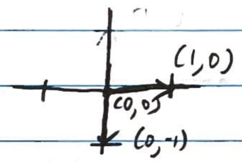
Review Sheet 6

1) a. $(-1,4),(-1,5)\in\mathbb{R}^2$



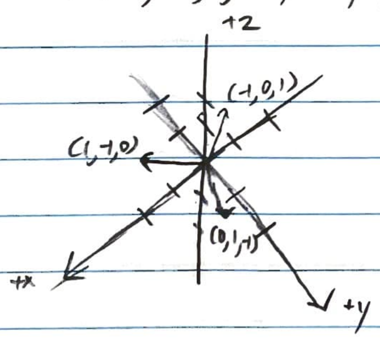
Linearly independent.

b. $(1,0),(0,-1),(0,0)\in\mathbb{R}^2$



Linearly dependent; any $a\in\mathbb{R}$ satisfies $a(0,0)=0$ [i.e. $a=3\neq 0$]

c. $(1,-1,0),(0,1,-1),(-1,0,1)\in\mathbb{R}^3$



Linearly dependent: $(1,-1,0)+(0,1,-1)+(-1,0,1)=(1-1,-1+1,-1+1)=(0,0,0)$

Any $a_1,a_2,a_3\in\mathbb{R}$ where $a_1=a_2=a_3$ satisfies $a_1v_1+a_2v_2+a_3v_3=0$.

2) (2.A.10) Suppose $v_1,\dots,v_m$ is linearly independent in V and $w\in V$. Prove that if $v_1+w,\dots,v_m+w$ is linearly dependent, then $w\in\operatorname{span}(v_1,\dots,v_m)$.

Let $v_1,\dots,v_m$ be linearly independent. Suppose $w\in W$ such that $v_1+w,\dots,v_m+w$ is linearly dependent. Then

$$a_1(v_1+w)+\dots+a_m(v_m+w)=a_1v_1+\dots+a_mv_m+(a_1+\dots+a_m)w=0,$$

where $a_i\in\mathbb{R}$, $i\in\{1,\dots,m\}$ not all 0. Let $\alpha=a_1+\dots+a_m$. Then

$$w=\frac{a_1}{\alpha}v_1+\dots+\frac{a_m}{\alpha}v_m,$$

where $a_i\in\mathbb{R}$ not all zero and so $\alpha\neq 0$. Thus $w\in\operatorname{span}(v_1,\dots,v_m)$. □