REVIEW SHEET 9, Math 540, Summer 2021, Melody Chan

Due Weds June 16 at 11:59pm Eastern Time

Submit all of the following on Gradescope, and don't forget to tag each answer to its page. We have implemented a course policy whereby failing to tag results in half credit.

I put a copy of this review sheet in the Overleaf folder.

(1) Let $S, T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be the linear maps uniquely determined by the following information:

$$S(1,1) = (2,-1,3), \quad S(1,-1) = (2,-2,4),$$
$$T(1,0) = (4,0,0), \quad T(0,1) = (-4,-1,0).$$

Compute the following; no proofs needed. For matrices associated to linear maps, please write them with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively.

(a) $(-S)(4,0)$

(b) $(-S+T)(4,0)$

(c) $(-S+T)(4,0) + (-S+T)(-1,3) + (-S+T)(-2,-2) + (-S+T)(-1,-1)$

(d) $\mathcal{M}(-S)$

(e) $\mathcal{M}(T)$

(f) $-\mathcal{M}(S) + \mathcal{M}(T)$

(2) Let $\mathcal{P}_3(\mathbb{R})$ denote the vector space of real polynomials in x of degree at most 3.

Write[1] the linear map

$$T\colon \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$$

given by

$$T(f) = f + f' + f''$$

as a matrix with respect to the basis

$$1, \quad x-2, \quad (x-2)^2, \quad (x-2)^3$$

for both the domain and codomain.

[1] All you need to know about differentiation is that it is linear and that $((x-2)^d)' = d(x-2)^{d-1}$ for each d.