REVIEW SHEET 4, Math 540, Summer 2021, Melody Chan

Due Weds May 26 at 11:59pm Eastern Time

Submit all of the following on Gradescope, and don't forget to tag each answer to its page. We have implemented a course policy whereby failing to tag results in half credit.

I put a copy of this review sheet in the Overleaf folder.

Are the following subsets actually subspaces? If the answer is yes, just write yes. If the answer is no, show briefly how the subset fails to be a subspace.

Example: Let W be the closed first quadrant of $\mathbb{R}^2$; that is,

$$W = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0\}.$$

Is W a subspace of $\mathbb{R}^2$?

Answer: No. For instance, $(1, 0) \in W$ but $-1 \cdot (1, 0) = (-1, 0) \notin W$.

(1) Let V be any vector space. Is $\emptyset$ a subspace of V?

(2) Let V be any vector space. Is V a subspace of V?

(3) Is $W = \{(\alpha, i\alpha) : \alpha \in \mathbb{C}\}$ a subspace of the complex vector space $\mathbb{C}^2$?

(4) Is $W = \{(x, y, z) \in \mathbb{R}^3 : x = 0 \text{ or } y = 0 \text{ or } z = 0\}$ a subspace of $\mathbb{R}^3$?

(5) Let V be a vector space over $\mathbb{F}$, and let $v \in V$ be any vector in V. Is

$$W = \{av : a \in \mathbb{F}\}$$

a subspace of V?

(6) Let V be a vector space over $\mathbb{F}$, and let $v, w \in V$ be any vectors in V. Is

$$W = \{av + bw : a, b \in \mathbb{F}\}$$

a subspace of V?

(7) Let V be a vector space over $\mathbb{F}$ and let U, W be subspaces of V. Is
$$X = \{u + w : u \in U,\ w \in W\}$$
a subspace of V?

For all of the remaining questions, let V be the real vector space of functions $\mathbb{R} \to \mathbb{R}$.

(8) Let W be the set of functions $\mathbb{R} \to \mathbb{R}$ that are nondecreasing. (We say a function $f : \mathbb{R} \to \mathbb{R}$ is *nondecreasing*[1] if $f(b) \geq f(a)$ whenever $b > a$.) Is W a subspace of V?

(9) Let W be the set of functions $\mathbb{R} \to \mathbb{R}$ that have bounded support. (We say a function $f : \mathbb{R} \to \mathbb{R}$ has *bounded support*[2] if there is some $N \in \mathbb{R}_{\geq 0}$ such that $|x| > N$ implies $f(x) = 0$.) Is W a subspace of V?

(10) Let W be the set of functions $f : \mathbb{R} \to \mathbb{R}$ such that the range of f has exactly one element. Is W a subspace of V?

(11) Let W be the set of functions $f : \mathbb{R} \to \mathbb{R}$ such that the range of f has at most two elements. Is W a subspace of V?

(12) Let W be the set of functions $f : \mathbb{R} \to \mathbb{R}$ such that the range of f has finitely many elements. Is W a subspace of V?

[1] Roughly, the graph never goes down as you move to the right.

[2] Roughly, the function is zero outside some interval $[-N, N]$.