REVIEW SHEET 5 "PICTURE DAY," Math 540, Summer 2021, Melody Chan

Due Fri May 28 at 11:59pm Eastern Time

Submit all of the following on Gradescope, and don't forget to tag each answer to its page. We have implemented a course policy whereby failing to tag results in half credit.

I put a copy of this review sheet in the Overleaf folder.

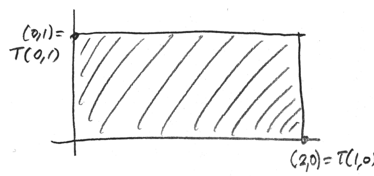
(1) Let

$$Q = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$$

be the unit square in the first quadrant. Draw pictures of the image of Q under the following linear transformations T. In other words, draw the set $\{Tv : v \in Q\}$. Please also show where $T(1,0)$ and $T(0,1)$ are in your picture.

Example: The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (2x, y)$.

Answer:



(a) The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (y, x + y)$.

(b) The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (0, x + 2y)$.

(c) The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (y, x)$.

(d) The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(x, y) = (y, y, x)$.

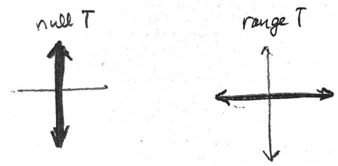
Study the definition of the *nullspace* of a linear map T (see Axler pp. 59), as follows: The *nullspace* of a linear map $T\colon V \to W$, denoted $\operatorname{null} T$, is the set of vectors in V that are sent to 0:

$$\operatorname{null} T = \{v \in V : T(v) = 0\}.$$

(2) Draw pictures of $\operatorname{null} T$ and $\operatorname{range} T$ for the following linear maps T.

Example: The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (-x, 0)$.

Answer:



(a) The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (0, 0)$.

(b) The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (3x - 3y, y - x)$.

(c) The linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (y, x)$.

(d) The linear map $T\colon \mathbb{R}^3 \to \mathbb{R}$ given by $T(x, y, z) = x - y$.

It sure looks like the nullspace is always a subspace of V and the range is always a subspace of W. Prove this! This is optional, but is recommended for practicing proofs. Afterwards, compare with the textbook pages 60 and 62 respectively.

(3) (Optional, ungraded) Let $T\colon V \to W$ be a linear map.

(a) Prove $\operatorname{null} T$ is a subspace of V.

(b) Prove $\operatorname{range} T$ is a subspace of W.