Problem §1
Indicate if each statement is true or false.

(a) A statement and its negation may be both false.

(b) $3 \leq 5$ and 10 is odd.

(c) $x > 0$ or $x^3 \leq 0$.

(d) If 2 is odd, then 10 is even.

(e) 2 is odd $\iff$ 4 is odd.

(f) $xy > 0$ whenever $x > 0$ and $y > 0$.

Solution:

(a) false; a statement must be either true or false, so its negation must be the opposite.

(b) false; 10 is not odd.

(c) true; any x is either positive, or its cube is non-positive (the domain of x^3 is $\mathbb{R}$, so $x \leq 0$ is contained in x^3).

(d) true; since 2 is not odd, the statement is always true regardless of the conclusion.

(e) true; both statements are false, so the biconditional is true.

(f) true; if x and y are both positive, then xy must also be positive.

Problem §2 Construct truth tables to show that:

1. $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$
2. $\neg(p \to q) \equiv p \wedge (\neg q)$

Solution:

§2.1

p	q	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$	$\neg(p \vee q)$
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

§2.2

p	q	$p \wedge (\neg q)$	$\neg(p \to q)$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

Problem §3 Write the negation of each statement:

(a) A circle of radius 1 has smaller area than a square of side 1.

(b) It is sunny and windy.

(c) I can finish the homework provided that I attend the class.

(d) I can speak Spanish or Japanese.

Solution:

(a) A circle of radius 1 does not have a smaller area than a square of side 1.

(b) It is not sunny or not windy.

(c) I attend the class and I cannot finish the homework (D:).

(d) I cannot speak Spanish and I cannot speak Japanese.

Problem §4 Rewrite the following statements using quantifiers, then write the negation first using quantifiers and then in common English:

(a) If $x > 3$, then there exists $y > 0$ such that $x^2 > 9 + y$.

(b) For every $x \in A$ and $y \in B$, $|x - y| \geq 1$.

(c) For every $M > 0$, there exists $A > 0$ such that if $|x| > A$ then $|f(x)| > M$.

Solution:

(a) Rewrite using quantifiers: $x > 3, \exists y > 0, x^2 > 9 + y$.
Negation using quantifiers: $x > 3, \forall y > 0, x^2 \leq 9 + y$.
Negation in common English: $x > 3$, and for every positive y, $x^2 \leq 9 + y$.

(b) Rewrite using quantifiers: $\forall x \in A, y \in B, |x - y| \geq 1$.
Negation using quantifiers: $\exists x \in A, y \in B$ $s.t.$ $|x - y| < 1$.
Negation in common English: There exists an $x \in A$ and $y \in B$ where $|x - y| < 1$.

(c) Rewrite using quantifiers: $\forall M > 0, \exists A > 0$ $s.t.$ $|x| > A \rightarrow |f(x)| > M$.
Negation using quantifiers: $\exists M > 0, \forall A > 0$ $s.t.$ $|x| > A \wedge |f(x)| \leq M$.
Negation in common English: There exists an $M > 0$ where for all $A > 0$, $|x| > A$ and $|f(x)| \leq M$.

Problem §5 A function f is *periodic* if there exists a $T > 0$ such that for every x, $f(x + T) = f(x)$.

(a) Write the above definition using quantifiers.

(b) By taking the negation, give the definition of *non* periodic functions.

Solution:

(a) $\exists T > 0$ s.t. $\forall x, f(x + T) = f(x) \rightarrow f$ is periodic.

(b) A function is *non* periodic if for all $T > 0$, there exists an x such that $f(x + T) \neq f(x)$.