REVIEW SHEET 10, Math 540, Summer 2021, Melody Chan

Due Fri June 18 at 11:59pm Eastern Time

Submit all of the following on Gradescope, and don't forget to tag each answer to its page. We have implemented a course policy whereby failing to tag results in half credit.

I put a copy of this review sheet in the Overleaf folder.

(1) Compute each of the following matrix products, or explain why they are not defined:

(a)
$$\begin{pmatrix} 2 & 5 \\ 3 & 7 \\ 11 & 13 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 0 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & -2 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 3 & 7 \\ 11 & 13 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1+i \\ 0 \\ i \end{pmatrix} \begin{pmatrix} -i & 0 & i \end{pmatrix}$$

(d)
$$\begin{pmatrix} -i & 0 & i \end{pmatrix} \begin{pmatrix} 1+i \\ 0 \\ i \end{pmatrix}$$

(2) Let $T_\theta\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by counterclockwise rotation by angle θ about the origin. (Convince yourself that T is indeed linear.)

(a) Let A be the matrix of T_θ with respect to the standard basis of $\mathbb{R}^2$. Compute A, by using what you know about sine and cosine.

(b) Compute A^2 and A^3. Then interpret these products geometrically to deduce formulas for $\cos(2\theta), \sin(2\theta), \cos(3\theta)$, and $\sin(3\theta)$.

(3) Suppose $T \in \mathcal{L}(V, V)$ is a linear operator on V. Does $T^2 - T = 0$ imply that $T = 0$ or $T = 1$? Here 1 denotes the identity map $V \to V$.

Either prove the original statement, or prove the negation by giving a counterexample.