Chapter 1

Set Theory

Set theory forms a basis for all of higher mathematics. We begin with a brief introduction.

§1.1 Sets

Definition 1.1.1: Sets

A **set** is a (possibly empty) collection of elements. If S' is a set and a is some object, then a is either an element of S or not. We write:

- $a \in S$ if a is an element of S.
- $a \notin S$ if a is not an element of S.

Definition 1.1.2: Natural Numbers

The **natural numbers** are the set

$$\mathbb{N} = \{1, 2, \ldots\}.$$

Formally, we define $\mathbb{N}$ as follows:

1. $\mathbb{N}$ contains an initial element 1.
2. $\forall n \in \mathbb{N}$, there is an incremental rule that creates the next element $n + 1$.
3. We can reach every element of $\mathbb{N}$ by starting with 1 and repeatedly adding 1.

Remark 1. *$\mathbb{N}$ is totally ordered. We say m is less than n if n appears before n when we start from 1 and add repeatedly.*
In this case we write $m < n$ and $m \leq n$.

Example 1. *Let*

$$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$$

denote the set of integers, and

$$Q = \{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\}.$$

the set of rationals.

Definition 1.1.3: Set Operations

Let S, T be sets.

[label=()]S is a **subset** of T if every element of S is an element of T, i.e. $a \in S \to a \in T$. We write

$$S \subset T.$$

The union of S and T is the set of elements that belong to S or belong to T, denoted

$$S \cup T = \{a \mid a \in S \text{ or} a \in T\}.$$