MATH 540 HONORS LINEAR ALGEBRA SUMMER 2021, Melody Chan
PROBLEM SET G

Due Wednesday July 7 at 11:59pm Eastern

Submit all of the following on Gradescope, unless otherwise indicated, and don't forget to tag each answer to its page. 0+0+6+6 points.

1. (ungraded) Look through your classmates' comments on "Should there be a Mathematicians' Ethical Code?" linked here.
   (a) Please add some "+1"s for ideas that you find striking (anonymously is fine).
   (b) Optionally, please leave some responses. Write your name if responding, and please of course be respectful.
2. (ungraded) Convince yourself that if $V = U_1 \oplus U_2$ then a basis of U_1 together with a basis of U_2 forms a basis of V. You may want to use this fact later.
3. (All about projections) Let V be a finite-dimensional vector space over a field $\mathbb{F}$ and let $T \in \mathcal{L}(V)$ be an operator. Prove that the following are equivalent:
   (a) $T^2 = T$.
   (b) There exist subspaces U_1, U_2 of V such that $V = U_1 \oplus U_2$ and
   $$T(u_1 + u_2) = u_1$$
   for all $u_1 \in U_1$, $u_2 \in U_2$.
   (c) T is diagonalizable and its set of eigenvalues is a subset of $\{0, 1\}$.

   Note: such linear operators are called **projections**. *You have seen many examples, e.g., (5) and (6) on Review Sheet 12.*
4. Axler 5.B.11 on page 153. Possible hints below.[1]

[1] For the forward direction, you could consider the polynomial $q(z) = p(z) - \alpha$, and recall that it must factor into linear factors. The backwards direction is spelled out in 5.B.10.