UNIT→I

Discrete Mathematics :-

(a) used in :-
DBMS, DS, CN, algo., Digital cir

(b) Goals of Subject to develop ability :-
i) Design mathematical arguments
ii) writing mathematical proofs of various state.
iii) Analysis of Complexity of algo.
iv) Solving counting problems using combinational analysis
v) utilizing various graph theory.

FORMAL LOGICS :-

Key points :-
→ A preposition is a sentence which is either true or false but cannot be both

ex :- (i) The sum of five and eight is four F
but preposition ✓

(ii) $4+9=13$ T ✓

(iii) $x=5$ ✗

(iv) This statement is True ⇒ either true or False ✓

(v) This statement is False ⇒ ✗
Here Both conditions applied
→ Statement is Liar paradox.

Note :-
Connectives used with preposition is known as compounds + it can also be represented in T+F.

Lecture 2.

② Logical Operators (connectives).

prepositional variables: P, q, r, P, Q, R.

Compound preposition

P	Q	$\sim P$	$P \cup Q$	$P \wedge Q$	$P \oplus Q$	$P \uparrow Q$	$P \downarrow Q$	$P \to Q$	$P \leftrightarrow Q$
T	T	F	T	T	F	F	F	T	T
T	F	F	T	F	T	T	F	F	F
F	T	T	T	F	T	T	F	T	F
F	F	T	F	F	F	T	T	T	T

Operators:-

i) Negation = $\sim P$
ii) Disjunction = $P \cup Q \Rightarrow P$ or Q.
iii) Conjunction = $P \cap Q \Rightarrow P$ AND Q.
iv) Exclusive OR : $P \oplus R \Rightarrow \sim(PQ) \cup \sim(QP)$
$\Downarrow$
Both same $\Rightarrow$ false, otherwise true.
v) NAND : $P \uparrow Q = \sim(P \wedge Q)$
vi) NOR : $P \downarrow Q = \sim(P \cup Q)$
viii) Conditional : $P \to Q$ (if P then Q) agar Q.f hokki false.
P is called Hypothesis
or Antecendent
or premise.

Q is called conclusion.
or consequence.

$P \to Q$ (implication)
$P \leftrightarrow Q$ (bimplication)

Example :
P : I am a boy.
$\sim P$: I am not a boy
It is not the case that i am a boy

Q) P : Five is greater than two
Q : Sum of five + three is eight.

$P \cup Q$: P or Q five is greater than two or sum of five + three is eight.

$P \cap Q$: P And Q ——— And ———

Q) P : I attend class regularly
Q : I get 1st div.

$P \to Q$ if I attend class regularly then i get 1st div.

Lecture 3.

$$\boxed{P \to Q = \sim P \cup Q}$$

1) Cond^nal / Implication ($P \to Q$)
If P then Q.

ⓐ P is sufficient for Q
ⓑ Q if P $\to$ Q whenever P
ⓓ Q when P $\to$ Q is necessary for P.

(d) Q unless ~P

Ex: P: Maria learns english.
Q: Maria will find a good job

P → Q.
using (d) = Maria will find a good job unless she doesnot learn english.

(2) for conditional statement P → Q. P → Q
• (Only if) is also conditional. || same truth table

Converse	Inverse	Contrapositive.
$q \to p$	$\sim p \to \sim Q$	$\sim Q \to \sim P$.

Example. The Home team wins whenever it is raining.
Q, P, using (b).

Contrapositive: if the home team doesnot win then it is not raining
$\sim Q \to \sim P$

Converse: if the home team wins then it is raining.
$Q \to P$

inverse: if it is not raining the home team does not win.
$\sim P \to \sim Q$.

Biconditional
$P \leftrightarrow Q$. P iff Q.
if P then Q and if Q then P.
$(P \to Q) \wedge (Q \to P) \equiv P \leftrightarrow Q$.

Q) P: you can take flight
Q: you buy a ticket

Biconditional: you can take flight if and only if you buy a ticket.

lecture-4: 02/12/20

#1. well formed formula (WFF) (to find correct answer we use WFF).

i) Every atomic statement is a WFF. and if it can be divided into its sub parts/smaller statements
ex: He is a boy (atomic statement.)

ii) If P is WFF, then ~P is also WFF.
ex: He is not a boy

iii) If P and Q are WFF then $P \cup Q$, $P \cap Q$, $P \to Q$ is also WFF.

iv) Nothing else is WFF.
$P \cup Q \cup R$ ⇒ not WFF.
$((P \cup Q) \cup R)$ ⇒ WFF.

#2. Rules of precedence :-

	N	Con.	Disj, Excl.	impli.	Bi impli.
	$\sim$	$\wedge$	$\vee, \oplus$	$\to$	$\leftrightarrow$
Rank.	1	2	3	4	5

$((\sim P) \wedge Q) \to (R \cup Q)$ (WFF)
(1) (2) (4) (3) (operations sequence)

$\sim P \wedge Q \to R \cup Q$ (not WFF because it is not clearify)

FORMULA
- → Tautology (always True)
- → Contradiction (always False in all statement).
- → Contigency (both true + False occur).

P	Q	$P \wedge Q$	$(P \wedge Q) \to Q$	$P \to Q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	T	T

$P \wedge Q$: Contigency; $(P \wedge Q) \to Q$: Tautology

logical Equivalence (truth table of both condn is same).

Equivalence.

1) $P \wedge T \equiv P$
$P \vee F \equiv P$

P	T	$P \wedge T$	$P \wedge F$	$P \vee F$
T	T	T	F	T
F	T	F	F	F

idempotent law

2) $P \vee P \equiv P$
$P \wedge P \equiv P$ Idempotent law

3) $\sim (P \wedge Q) \equiv \sim P \vee \sim Q$
$\sim (P \vee Q) \equiv \sim P \wedge \sim Q$. Demorgans law.

4) $P \vee (P \wedge Q) \equiv P$
$P \wedge (P \vee Q) \equiv P$. Absorption law.

5) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ Distributive law

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$		$\sim P \vee \sim Q$
T	T	T	F		F
T	F	F	T	= DEMORGAN =	T
F	T	F	T		T
F	F	F	T		T

lecture - 5

Q1) Show that $\sim (P \to Q)$ and $(P \wedge \sim Q)$ are logically equivalent

Soln $\sim (P \to Q)$

$\because (P \to Q) = \sim P \vee Q$.

$\therefore \sim (P \to Q) = \sim (\sim P \vee Q) = P \wedge \sim Q$ (using demorgans law)

Q2) Show that $P \to (Q \to R) \equiv (P \wedge Q) \to R$.

$\because A \to B = \sim A \vee B$. —— (1)

$\therefore P \to (Q \to R) = \sim P \vee (Q \to R)$
$= \sim P \vee (\sim Q \vee R)$
$= (\sim P \vee \sim Q) \vee R$ (using associativity)
$= \sim (P \wedge Q) \vee R$ (using demorgans).
$= (P \wedge Q) \to R$ (using eqn (1))

Q3) Show that $(\sim P \wedge (P \vee Q)) \to Q$ is a tautology without constructing truth table.

$(\sim P \wedge (P \vee Q)) \to Q$.

$\because A \to B \equiv \sim A \cup B$.

$\therefore \sim(\sim P \wedge (P \vee Q)) \vee Q$.

$= (P \vee \sim(P \vee Q)) \vee Q$ (Demorgans law)

$= (P \vee (\sim P \wedge \sim Q)) \vee Q$. (Demorgan's law)

$= (P \vee \sim P) \wedge (P \vee \sim Q) \vee Q$ (using distributive law)

$= T \wedge (P \vee \sim Q) \vee Q$.

$=$ ~~T∧~~ $P \vee \sim Q \vee Q$ (using idempotent law)

T deals with any variable for depends on variable + variable).

$\therefore P \wedge T = P$.

$= P \vee (\sim Q \vee Q)$

$= P \cup T$

$= T$.

<u>lecture 6</u>

Tautological implications (logical implication)

A statement P tautologically implies a statement Q iff $P \to Q$ is a tautology.

$S_1 \Rightarrow S_2$ iff $S_1 \to S_2$ (Tautology)

$T \to F$ (X)

Exy $P \wedge Q \Rightarrow P$

$P \wedge Q \to P$ (tautology)

$\underset{\sim}{F \wedge Q}$ F

$F \to F$ (always tautology)

$P \wedge Q \Rightarrow Q$.

$P \wedge F$ F (Tautology)

$Q \Rightarrow P \vee Q$.

both false then false.

~~F~~ $F \vee F$ ~~(Tautology)~~

$F \to F \cup F$ (tauto—)

$\underbrace{P \wedge (P \to Q)}_{F_1} \Rightarrow \underbrace{Q}_{F_2}$. (Modus ponens)

$T \to F$ F.

$T \wedge F$

$F \to F = T$ (tautology)

Satisfiability + validity

if a formulae is true for atleast one case or interpretation then it is satisfiable.

A formula is called valid if it is true in all cases. i.e. tautology.

$P \cup Q \to P \wedge Q$.

$T \to T$ (T)

$\underbrace{P \vee Q}_{F_1} \to \underbrace{P \wedge Q}_{F_2}$.

$F \to F \wedge F$. = True.

$T \cup Q$ $T \wedge F$

$T \to F = F$ (Not tautology)

lecture-7.

03/12/20

Rules of Inferences :-

- Set of premises $P_1 \wedge P_2 \wedge P_3 \wedge \dots P_n$ is called inference systems

set of statements ↳ all but final proposition is the argument are called premises.

Conclusion → final proposition.

- whenever premises are true conclusion must be true.

Ex If you have a password (P) then (→) you can log in to system (Q).

you have password (premises)
therefore
you can login to system. (conclusion).

$$\frac{\begin{array}{l} P \to Q \\ \wedge P \end{array}}{Q}$$

i.e. $\boxed{\cancel{(P \to Q) \wedge P \Rightarrow Q}}$

Ex if meena attend the class (P) then (→) teena attend the class (Q)

Meena attend the class (premises)
therefore
teena attend the class (conclusion).

$\boxed{\cancel{(P \to Q) \wedge P \Rightarrow Q}}$

MMHDASC

	Rule Name	Rule of Inference (True).	Tautology.
1)	Modus ponen	P $\wedge\ P \to Q$ $\therefore Q$.	$(P \wedge (P \to Q)) \to Q$ (on solving we get tautology)
2)	Modus Tollens	$\sim Q$ $\wedge\ P \to Q$ $\therefore \sim P$	$(\sim Q \wedge (P \to Q)) \to \sim P$
3)	Hypothetical sylogism	$P \to Q$ $\wedge\ Q \to R$. $P \to R$.	$(P \to Q) \wedge (Q \to R) \to (P \to R)$
4)	Disjunctive Sylogism	$P \cup Q$ $\wedge \sim P$ Q.	$(P \cup Q) \wedge (\sim P) \to Q$.

lecture 8.

~~Rule of~~

	Rule Name	Rule of Inference	Tautology
5)	Addition	P $\therefore P \cup Q$.	$P \to (P \cup Q)$
6)	Simplification	$P \cap Q$ $\therefore P$	$(P \cap Q) \to P$.
7)	Conjunction	P $\therefore Q$ $P \cap Q$	$P \cap Q$

Ex: Show that premises "It is not sunny this afternoon and it is colder than yesterday." "We will go swimming only if it is sunny". "If we do not go swimming, then we will take agra trip" and if we take agra trip then we will be home by sunset" lead to conclusion" we will be home by sunset".

Solution

Step 1: ~~$\sim P \leftrightarrow Q$~~ $\sim P \wedge Q \Rightarrow \sim P$ using simplification

~~step 2: $R \to P$~~

~~step 3: $\sim R \leftrightarrow S$~~

$\therefore \sim R$.

Step

1. $\sim P \wedge Q$
2. $\sim P$ (simplification.
3. $R \to P$] $\Rightarrow \sim R$. (Modus tollance)
4. $\sim R$.
5. $\sim R \to S$.] S (using modus ponen) (premise)
6. $\therefore S$.
7. $S \to T$ (premise)] $\Rightarrow T$ (using Modus ponen)

$\therefore T$ = we will be Home by sunset

Quantifiers are words that refer to Quantities such as "some or all" and tell for how many elements a given predicate is true.

Lecture 9 (A)

Predicate & Quantifiers (Predicate logic)

A predicate is a sentence that contains finite no. of variable and become a preposition/statement after specific values are substituted for the variables.

P: Every student is Brilliant
Q: Shefali is a student. } proposition.

R: Shefali is Brilliant

Mohan is a student
subject — predicate
⇓
Student(Mohan)

Student (x) { generalised i.e., x is a student ∈ the universe of discourse

P, Q, R — P(x) — Mohan, Shefali, ..., Rohan.

Domain/ Universe of discourse
{ Student (Mohan) ∧ Student (Shefali) ∧ Student (Rohan) }

for ∞ ⟹ $\forall x$, Student (x)
Every x is a student.
⇓
Quantifying

P: Some students are Brilliant.

if any one of them is true then P is true.

Rohan is Brilliant
∪
Mohan is Brillian
∪
Shefali ———

Quantifying :-

$\exists x$ (brilliant)

Quantifier
- Universal Quantifier $\forall x$.
- Essential Quantifier. $\exists x$.

we can't represent everything with propositional logic that's why we study predicate logic.

Ex: P(x,y) : x is greater than y.
predicates — Predicate (∵ we can't say that it is true or False).

& if x = 6 & y = 8 then x is greater than y is proposition ∵ it is true.

lecture 9 (B)

	when true	when false
$\forall x, P(x)$	P(x) is true for all x	There is an x for which P(x) is false
$\exists x, P(x)$	There is an x for which P(x) is true	P(x) is false for every x only.

$\forall x$, — x is Bound variable

$\forall x$ student (x)

$\exists x$, P(x,y) { x is greater than y : P(x,y)
Bound — free variable.

Q4 Ex write symbolic form of
(A) All students are clever $= \forall x, P(x)$
(B) for some students are clever $= \exists x, Q(x)$

P(x): x is clever.
Q(x): x is clever

Q
P(x): x is student.
Q(x): x is clever

for every x, if x is a student then x is clever.
$\forall x, P(x) \rightarrow Q(x)$

There exist x such that x is student and x is clever.
$\exists x (P(x) \land Q(x))$

lecture 10. 04/12/20.

First order logic / Predicate logic

Q4 Every integer is either positive or negative. Symbolise it in predicate logic.

(U.O.D) integers
+ - - +
+ +
+ - - +

Sol (i) P(x) : x be either positive or negative
$\forall x\, P(x)$.

or

(ii) P(x) : x is positive
Q(x) : x is negative
$\forall x\, (P(x) \cup Q(x))$

OR.

(iii) let $p(x)$: x be an integer
$Q(x)$: x is either positive or negative

$\forall x, (p(x) \rightarrow Q(x))$

Negation of Quantifiers:-

$\boxed{\sim \forall x\, P(x) \equiv \exists x \sim P(x)}$

$\boxed{\sim \exists x\, P(x) \equiv \forall x \sim P(x)}$

Q. Q : Every politician is clever.

$P(x)$: x is clever. x, x_2 → politician

$\forall x\, P(x)$

$\sim \forall P(x)$ => There is a politician who is not clever

$\exists x \sim P(x)$

Q. All states in india are Highly populated.

$P(x)$: x is highly populates states of india → x.

$\forall x\, P(x)$.

$\sim \forall P(x)$

There ~~are~~ is a state in ~~a~~ india which is not highly populated

$\exists x \sim P(x)$

Note:- $\forall x\, P(x) = P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots$ → True when all true. x_1, x_2, x_3

$\exists P(x) = P(x_1) \cup P(x_2) \cup P(x_3)$

$\sim \forall x\, P(x) = \sim (P(x_1) \wedge P(x_2) \wedge P(x_3) \ldots)$

$= \sim P(x_1) \cup \sim P(x_2) \cup \sim P(x_3)$

↳ false when all false.

lecture 11:-

Nested Quantifiers:- (ek Quantifier ke upar Quantifiers which occurs dusra Quantifier lagate hai) within the scope of other Quantifier. (order of Quantifier Matters)

$P(x,y)$: $x^2 = y$ x →, y → (+ve)

$\exists y\, P(x,y)$ = There is a ~~some~~ +ve integer y Such that $x^2 = y$ (Here y bound & x free)

$\forall x \exists y\, P(x,y)$ = for every +ve integer x there exist a +ve integer y such that $x^2 = y$

$\forall x \underbrace{\exists y\, P(x,y)}_{Q(x)}$ => $\forall x\, Q(x)$

Q. Let Universe of discourse (U.O.D) for x is the Set $A = \{1,2,3,4\}$ and y is the set $B = \{5,6,7,8\}$ and $P(x,y)$ defined as $P(x,y)$: x is less than y.

Sol: $\forall x \forall y\, P(x,y)$: for every element of x (of set A) for every element of y of set B x is a less than y.

OR

$\forall y \forall x\, P(x,y)$ – True.

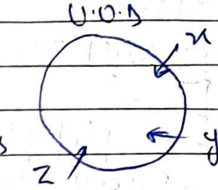
Quantification of of two variable.

Statement	When true	When false
i) $\forall x \forall y\, P(x,y)$ $\forall y \forall x\, P(x,y)$	$P(x,y)$ is true for every pair x,y	There is a pair x,y for which $P(x,y)$ is false.
ii) $\exists x \exists y\, P(x,y)$ $\exists y \exists x\, P(x,y)$	There is a pair x,y for which $P(x,y)$ is true	$P(x,y)$ is false for every pair of x,y.
iii) $\forall x \exists y\, P(x,y)$	for every x there is a y $P(x,y)$ is true.	there is an x such that $P(x,y)$ is false for every y.
iv) $\exists x \forall y\, P(x,y)$	there is an x for which $P(x,y)$ is true for every y	for every x there is a y for which $P(x,y)$ is false.

Q) $\exists x \forall y\, P(x,y)$

There is exist x for every y $P(x,y)$
There is an element x of set A for every element of set B such that x is less than y.
True. (Last Question)

Q. $\forall x \forall y \forall z\, P(x,y,z)$



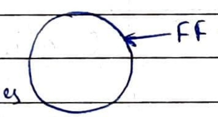
for every element x that belongs to U.O.D for every y that belongs to U.O.D for every z that belongs to U.O.D $P(x,y,z)$, and then check True or false according to given Question.

lecture 12.

Rules of Inference for Quantified Statements

(i) Universal instantiation / Specification

(1) – $\forall x\, P(x)$ → if this true. Every flip flop stores 1 bit
U.O.D
$\therefore P(c)$ → then this true.
c is any instance → S-R flip flop stores 1 Bit



Q. Ex: $A = \{3, 6, 9\}$ $P(x)$: x be a multiple of 3

$\forall (x)\, P(x) \Rightarrow P(3)$

$\forall (x)\, P(x) \Rightarrow P(y)$ – (ii)

(i) + (ii) are same.

ii) Universal generalisation:-

Premise(s)

$P(c)$ for any arbitrary c

Conclusion: $\forall x\, P(x)$

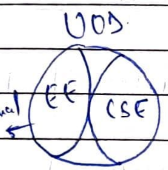
A flip flop stores 1 Bit

Every flip flop stores 1 Bit.

iii) Existential instantiation:-

$\exists x\, P(x)$

$\therefore P(c)$ for some element c. Vimal



Some students has done Electrical engg.

Vimal has done elect. engg.

c always must be new for every next time.

$\exists x\, Q(x)$

$Q(c)$

$$\boxed{\exists x\, P(x) \land \exists x\, Q(x) \Rightarrow P(y) \land Q(z)}$$

iv) Existential generalisation:-

Premise

$P(c)$ for some element c.

$\therefore \exists\, P(x)$

Conclusion

Vimal has done EE

Some students has done EE

Q) $\forall x\, (P(x) \rightarrow Q(x)) \land \underset{\sim\sim}{\exists x\, P(x)} \Rightarrow \exists x\, Q(x)$

$P(y)$ or $P(c)$ (using existential inst.)

$\forall x\, (P(x) \rightarrow Q(x)) \land P(y)$

$P(y) \rightarrow Q(y) \land P(y)$ (using Universal inst.)

$Q(y)$ (using Modus Ponens)

$Q(y) = \exists x\, Q(x)$ (using Existantial gen.)

Lecture 13. 05/12/20.

Method of proof :-

1) Direct proof :-

Steps

(i) In this method we rephrase the theorem/statement as a conditional statement $P \rightarrow Q$.

(ii) Start with assumption that P is true and then use the rules of inferences with given ~~option~~ axioms and already proved theorems and defitions to show that Q is also true.

Q) Show that square of even no. is an even no.

Soln: if n is an even no. then n^2 is also an even no.

P: n is an even no.

Q: n^2 is an even no.

$P \to Q$ k: integer i.e. $k \in Z$.

$n = 2k$

Now

$n^2 = (2k)^2 = 4k^2 = 2\cdot(2k^2)$

$\therefore n^2$ is also an even no. Ans

Q4 Show that the sum of two odd integers is an even no.

Soln:

let a and b be two odd integers

P: a is odd and b is odd.

Q: a+b is even.

$a, b \to$ odd.

We can write $a = 2l+1$; $l \in Z$.

$b = 2m+1$; $m \in Z$.

$\therefore a+b = 2l+1+2m+1$

$= 2\underbrace{(l+m+1)}_{\text{integer}}$

= even (∵ any integer multiplied by 2 is even)

$\therefore P \to Q$. Ans

lecture 14

#2. Proof by contraposition. (indirect proof)

Conditional statement $P \to Q$ can be proved by showing that its contrapositive, $\sim Q \to \sim P$ is true.

∵ Truth table of $P \to Q$ & $\sim Q \to \sim P$ are same,

Q4 Proove that n^2 is odd then n is odd

P: n^2 is odd

Q: n is odd.

using proof by contraposition.

$\therefore \sim Q \to \sim P$.

$\sim Q$ is true (say)

$\therefore \sim Q$: n is even.

$\sim P$: n^2 is even.

$n = 2k$; $k \in Z$ $\Rightarrow n^2 = 2(2k^2)$

$\Rightarrow n^2$ = even.

$\sim P$ is also true.

$\therefore \sim Q \to \sim P$

Hence the equivalent statement of this $P \to Q$ is true that is n^2 is odd, then n is odd.

lecture 15

3) Proof by contradiction:-

To proove that statement P is true, we assume that $\sim P$ is true and taking as premise we draw a contradiction of F as the conclusion.

Steps

(i) Assume that P is false

(ii) using this assumption show a contradiction.

Q) Show that $\sqrt{2}$ is an irrational number.

Soln. P: $\sqrt{2}$ is an irrational no.

let ~P: $\sqrt{2}$ is not an irrational no. True.

for rational no ∵ $\frac{p}{q}$ ($q \neq 0$) where P and Q have no common factor.

let $\sqrt{2} = \frac{p}{q}$ $\Rightarrow 2 = \frac{p^2}{q^2}$

$\Rightarrow 2q^2 = p^2$ — (I)

$\Rightarrow p^2$ is an even no. (using (I))

∴ p is also an even no.

∴ $P = 2k$ ($k \in Z$)

$p^2 = 4k^2 = 2(2k^2)$ — (II)

On Comparing (I)+(II)

$q^2 = 2k^2$ $\Rightarrow q^2$ = even no.

∴ q is even no.

∵ P+q are even no. ∴ they have a common factor 2 which does not satisfy the condition of rational no. which contradict our statement.

Hence $\sqrt{2}$ is an irrational no.

lecture-16. 06/12/20

UNIT-II

SET OVERVIEW:-

A set is a well-defined collection of elements

small letters elements a, b, c, d, e } elements

set represented by capital letter Set → X, A, B

A = {1, 2, 3}

Two common methods to denote sets:-

i) Roster Notation:-

It is the complete listing of all the elements of set.

B = {2, 4, 6 - - - 20} A = {a, b, c, d}

ii) Set builder Notation:-

It is used when the roaster method is cumbersome. i.e. when so many elements are present in a set then we use it.

B : {x : 2 ≤ x ≤ 20 & x is an even no.}.

imp.

Some set Sets and symbols:-

i) Set of all natural no. - N = {1, 2, 3 - - -}

ii) Set of integers Z = {- - - - -2, -1, 0, 1, 2, - - - - -}

iii) Set of real no. Q.

R, C, R^+, Z^+

Q. Set of first 5 natural nos. in Roster and Set builder Notation.

Roster Notation - A = {1, 2, 3, 4, 5}

Set Builder - A = {x : x is a natural no. < 6}

Cardinality of sets:-
No. of distinct elements in any set X.
It is denoted by $|X|$ or $n(X)$

$|A| = 5$, $n(A) = 5$

(i) **Empty set:-**
Cardinality = 0
denoted by ϕ, $\{\}$
$A = \phi$, $A = \{\}$ But $A = \{\phi\}$ ✗ (Because it represent an element).

(ii) **Singleton set:** (Only 1 element)
Cardinality = 1
$A = \{3\}$
$|A| = 1$

lecture-17

(iii) **Finite set and Infinite set:-**
Finite set = Finite no. of elements
ex: $A\{1,2\}$.

Infinite set: $A\{0 \dots 2\}$ no. of real nos. b/w 0 + 2.

(iv) **Countable and Uncountable set:-**

Countable → A set of +ve even nos. is a Countable and infinite set.

Uncountable → set of real nos. b/w 0 + 2.

Note:- $A\{1,2,3,4\}$. $\{1,3\} \subset A$
But $\{3\} \subset A$ (False ∵ 3 + $\{3\}$ are diff ...).

(v) **Universal set:-**
A set that contains all the elements of the universe is called a universal set.
'U' donated by
ex:- R, letters + each + every thing

(vi) **Subset and Proper subset:-**

Subset: $X \subseteq Y$ iff $\forall x (x \in X \Rightarrow x \in Y)$

X

proper subset: $X \subset Y$ iff $\forall x (x \in X \Rightarrow x \in Y)$
+ $|X| < |Y|$

ex:- $Y = \{1,2,3,4\}$.
$X \{1,2,3\}$ proper subset.
∵ $|X| = 3$ + $|Y| = 4$.
∴ $|X| < |Y|$.

ex: $Y \{a,b,c\}$.
$X \{a,b,c\}$ $|X| = 3$, $|Y| = 3$ not a proper Subset ∵ $|X| = |Y|$.

(vii) **Power set: $P(x)$**

Power set of X is the set that consists of all subsets of the set X and It is denoted by $P(X)$
Cardinality of the $P(X)$ of set of n elements is 2^n.

$X = \{1,2,3\}$.
$P(X) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}\}$
all are subset of X.
$|P(X)| = 8 = 2^3$

(viii) Equality Sets:-

$X = Y$ iff $X \subseteq Y$ & $Y \subseteq X$

Order and repetition has no effect.

ex: $\{1,2,3\} = \{3,2,1\}$.

$\{1,1,2,3,3,3\} = \{3,2,2,1,1,3\}$

lectures.

Operations on set:-

VENN Diagram (John Venn)

→ Universal set represented by rectangle □

→ Other sets are represented by circle O.

$A = \{1,2,5\}$ 

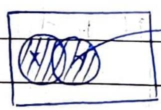
Operations on set:-

ⓐ Union:-

$X \cup Y = \{x : x \in X \text{ or } x \in Y\}$

ex: $X = \{1,2,3\}$, $Y = \{2,3,4,5\}$

$\therefore X \cup Y = \{1,2,3,4,5\}$.

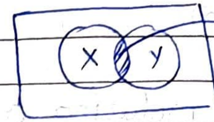


→ $X \cup Y$.

ⓑ intersection:-

$X \cap Y = \{x : x \in X \text{ and } x \in Y\}$

$X \cap Y = \{2,3\}$



→ $X \cap Y$.

ⓒ Difference of two sets:-

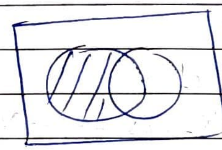
$X - Y = \{x : x \in X \text{ and } x \notin Y\}$.

$X = \{1,2,3,4\}$ $Y = \{3,4,5\}$

$X - Y$ $\{1,2\}$.

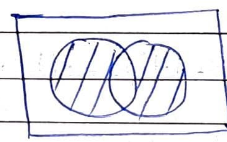
↑

Only belongs to x.



ⓓ Symmetric difference of two sets

$X \oplus Y = (X - Y) \cup (Y - X)$



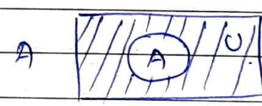
$X \oplus Y$.

$X = \{1,2,3,4\}$ $Y = \{3,4,5\}$

$X \oplus Y = \{1,2,5\}$.

ⓔ Compliment of a set:-

$\overline{X} = \{x : x \in U - X\}$.



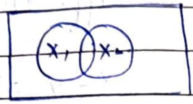
$\overline{A} = U - A$

lecture 19 (A)

principle of inclusion and exclusion:- (Counting Technique)

$X_1 = \{1,2,3\}$ $X_2 = \{2,3,4,5\}$

$|X_1| = 3$ $|X_2| = 4$.



$\boxed{|X_1 \cup X_2| = |X_1| + |X_2| - |X_1 \cap X_2|}$ = principle of inclusion and exclusion

for n set

$X_1, X_2, X_3 \text{-----} X_n$

$|X_1 \cup X_2 \cup X_3 \text{----} X_n| = \Sigma|x_i| - \Sigma|x_i \cap x_j|$
$+ \Sigma|x_i \cap x_j \cap x_k| \text{----}$
$\text{---} (-1)^{n+1} \Sigma |X_1 \cap X_2 \text{---} X_n|$

for 3 set.

$|X_1 \cup X_2 \cup X_3| = |X_1| + |X_2| + |X_3| - (|X_1 \cap X_2| + |X_2 \cap X_3| + |X_3 \cap X_1|)$
$+ |X_1 \cap X_2 \cap X_3|$.

Q) In a survey of on group of 80 people. It is found that 60 like egg and 30 like fish find the percentage of people like both egg and fish.

Sol. $|E| = 60$; $|F| = 30$; $|E \cap F| = ?$

$|E \cup F| = 80$ (given).

$\because |E \cup F| = |E| + |F| - |E \cap F|$

$\Rightarrow 80 = 60 + 30 - |E \cap F|$

$\therefore |E \cap F| = 10$ Ans

Q) In a survey of the usage of three tooth pastes A, B, C. It is found that 60 people like A, 55 like B, 40 like C, 20 like A and B, 35 like B and C, 15 like A + C and 10 like all find no. of person included in survey.

$|A| = 60$, $|B| = 55$, $|C| = 40$ $|A \cap B| = 20$

$|B \cap C| = 35$ $|A \cap C| = 15$ $|A \cap B \cap C| = 10$.

$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |C \cap A|$
$+ |A \cap B \cap C|$

$= 60 + 55 + 40 - (20 + 35 + 15) + 10$
$= 155 - 70 + 10$
$= 95$ Ans

lectur 19 B.

(f) Disjoint sets:-

if $X \cap Y = \phi$

ex: $X = \{1, 8\}$

$Y = \{1, 2, 5, 9\}$ $X \cap Y = \phi \Rightarrow$ disjoint set.

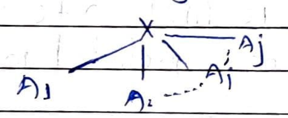
(g) Partition set:-

let x be a set and $S = \{A_i ; A_i \subseteq X, i \in N\}$ be the set of subset x.

S is said to be partion of x if

(i) Union of all A_i's is the set X i.e. $\cup A_i$'s $= X$

(ii) All A_i's are disjoint. i.e. $A_i \cap A_j = \phi$



Ex: $X = \{1, 2, 3, 4, 5, 6, 7\}$

$S = \{\{1, 2, 3\}, \{4, 5\}, \{6, 7\}\}$.

A_1 A_2 A_3

$A_1, A_2, A_3 \subseteq X$.

$A_1 \cap A_2 = 3$ which is not null set

∴ not a partition set.

(h) Order set:

Ordered collection of distinct elements

week {Sun, Mon, Tue, Thu, Fri, Sat}

wed,

(i) Cartesian Product of sets:-

Cartesian product of two sets X and Y is a set of ordered pair in which the first element from X and second from Y.

$X \times Y = \{(x, y) : x \in X \text{ and } y \in Y\}$

$X = \{1, 2\}$ $Y = \{3, 4\}$

$X \times Y = \{(1,3), (1,4), (2,3), (2,4)\}$

$Y \times X = \{(3,1), (3,2), (4,1), (4,2)\}$.

Algebra of set:-

(i) Idempotent law:-

~~X∩~~ $X \cup X = X$, $X \cap X = X$

(ii) distributive law:-

$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

(iii) Demorgan's law:-

$(\overline{X \cup Y}) = \bar{X} \cap \bar{Y}$

$(\overline{X \cap Y}) = \bar{X} \cup \bar{Y}$

(iv) Compliment law.

$\bar{\bar{A}} = A$, $A \cup \bar{A} = U$ (universal set)

$\because \bar{A} = U - A$, $X \cap \phi = \phi$

$\bar{U} = \phi$, $\bar{\phi} = U$, $X \cup \phi = X$

$X \cup U = U$, $X \cap U = X$

lecture 20

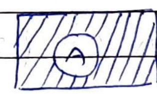
Demorgan's laws proof using SET:-

(i) prove $\overline{X\cap Y} = \bar{X}\cup\bar{Y}$

∵ for equality sets
$A=B$
$A\subseteq B$ and $B\subseteq A$.

∴ $\overline{X\cap Y} \subseteq \bar{X}\cup\bar{Y}$ and $\bar{X}\cup\bar{Y} \subseteq \overline{X\cap Y}$

let $x\in(\overline{X\cap Y}) \Rightarrow x\notin X\cap Y$



$\bar{A}$

$\Rightarrow x\notin X$ or $x\notin Y$
$\Rightarrow x\in\bar{X}$ or $x\in\bar{Y}$
∴ $x\in\bar{X}\cup\bar{Y}$

Thus $\overline{X\cap Y} \subseteq \bar{X}\cup\bar{Y}$ —(I)

$x\in(\bar{X}\cup\bar{Y}) \Rightarrow x\in\bar{X}$ or $x\in\bar{Y}$
$\Rightarrow x\notin X$ or $x\notin Y$
$\Rightarrow x\notin(X\cap Y)$
$\Rightarrow x\in(\overline{X\cap Y})$

∴ $\bar{X}\cup\bar{Y} \subseteq \overline{X\cap Y}$ —(II)

using (I) + (II)
$\overline{X\cap Y} = \bar{X}\cup\bar{Y}$

(ii) prove $\overline{X\cup Y} = \bar{X}\cap\bar{Y}$

we will show

$\overline{X\cup Y} \subseteq \bar{X}\cap\bar{Y}$ and $\bar{X}\cap\bar{Y} \subseteq \overline{X\cup Y}$

let $x\in\overline{X\cup Y} \Rightarrow x\notin(X\cup Y)$
$\Rightarrow x\notin X$ and $x\notin Y$
$\Rightarrow x\in\bar{X}$ and $x\in\bar{Y}$
∴ $x\in(\bar{X}\cap\bar{Y})$

Thus $\overline{X\cup Y} \subseteq \bar{X}\cap\bar{Y}$ —(I)

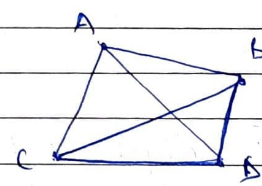
let $x\in(\bar{X}\cap\bar{Y}) \Rightarrow x\in\bar{X}$ and $x\in\bar{Y}$
$\Rightarrow x\notin X$ and $x\notin Y$
$\Rightarrow x\notin X\cup Y$
∴ $x\in(\overline{X\cup Y})$

Thus $\bar{X}\cap\bar{Y} \subseteq \overline{X\cup Y}$ —(II)

using (I) + (II) $\overline{X\cup Y} \subseteq \bar{X}\cap\bar{Y}$

lecture 21.

permutation and Combination



for triangle.
$\Delta ABC = \Delta BAC$.
Order does not matter (comb

$4C_3 = \frac{4!}{3!\,1!} = 4$.

comb^n.

Q. 1 2 3 no. digits
4 3 123 order matters.
321 ⇒ permutation
213 $^4P_3 = \frac{4!}{(4-3)!} = 4! = 24$.

permutation :-
Each of different arrangements / selection / group which can be made by taking some or all of the given things or objects at a time is called permutation.

$^nP_r = r! \; ^nC_r$.

$^nC_r = \frac{n!}{r!(n-r)!}$

$^nC_r = {^nC_p} \Rightarrow r = p$ or $r + p = n$.

Q 42 men hand shake. $A \rightleftarrows B$ (same) ⇒ Combⁿ.

$^{42}C_2 = \frac{42!}{2!40!} = \frac{42 \times 41 \times \cancel{40!}}{\cancel{40!} \times 2!} = 21 \times 41$ Ans

lecture 22 :

Q. How many no. of five digits can be form with the digits 1, 3, 5, 7 and 9 no digit being repeated.

$^5P_5 = 5!$

$\boxed{5|4|3|2|1} = 120$

for repetition :-
$\boxed{5|5|5|5|5}$
$= 5^5 = 3125$

Q. 0, 2, 4, 6, 8.

$\boxed{4|4|3|2|1} = 96$.

zero not possible.

for repetion: $\boxed{4|5|5|5|5} = 2500$ Ans

Q. Find the no. of triangles formed by 11 pts. (out of which 5 are collinear).

$^{11}C_3 - {^5C_3} = \frac{11!}{8!3!} - \frac{5!}{2!2!}$

lecture 23 :-

Pigeonhole principle :-

If n pigeonholes are occupied by n+1 or more pigeons then atleast one pigeonhole is occupied by more than one pigeon.

let pigeonholes = □ □ □
pigeons = P1, P2, P3, P4.

(Here atleast 1 box contain more than one pen or it might contain all pens.

Q) Find the min no. of students in a class so that 2 students were born in same month.

1 year = 12 month. => 12+1 = 13 students min. such that 2 students born in same month.

for same date.
1 year = 365 (say) => 365+1 = 366 students.

Generalized pigeonhole principle.

If n pigeonholes are occupied by kn+1 or more pigeons, where k is positive integer, then atleast one pigeonhole is occupied by k+1 or more pigeons.

□ □ □ $\quad kn+1 = 7$

n = 3 $\quad k = 2.$

$\therefore k+1 = 3.$ => atleast one box has atleast 3 items.

Q. Find min no. of students in a class to be sure that 4 of them were born in same month.

Here $k+1 = 4$, $n = 12$.
$k = 3$

$\therefore$ min no. of student:
$kn+1 = 12 \times 3 + 1 = 37$ Ans

Lecture 25 (24 Missing) $\quad$ 08/12/20

Type of Relation :-

1) Reflexive Relation :-
A Relation is said to be reflexive relation if xRx or $(x,x) \in R$ for all $x \in X$.

extra set doesn't bother.

$A \{1,2,3,4\}$, $R = \{(1,1),(2,2),(3,3),(2,3),(4,4)\}$

Q. Let $A = \{1,2,3,4\}$ determine whether the following is reflexive or not.

(a) $R_1 = \{(1,1),(2,2),(3,3),(2,4)\}$ × ($\because$ (4,4) not present)

(b) $R_2 = \{(1,1),(1,2),(2,2),(3,3),(4,4)\}$ ✓ possibility

2) Symmetric Relation :-

If xRy then yRx for all $x, y \in X$
or $(x,y) \in R \Rightarrow (y,x) \in R$ for all $x,y \in X$.

$A = \{1,2,3,4\}$ $\quad R = \{(1,2),(2,1),(2,4),(4,2)\}$ ✓
does not require all elements

$R_1 = \{(1,3),(3,1),(3,4),(4,4)\}$ × $\because$ (4,3) doesn't incl.

3) Transitive Relation :-
if xRy and yRz then xRz $\forall x,y,z \in X$
$(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$.

(a) $A = \{1,2,3,4\}$ $\quad R_1 = \{(1,2),(2,3),(1,3),(3,2)\}$ × $\because$ (2,2) nhi h.

$R_2 = \{(2,3),(3,4),(2,4),(3,1),(2,1)\}$

4) Equivalence Relation:-
R is said to be equivalence relation if it is Reflexive, Symmetric and Transitive

5) Compitable Relation:-
Reflexive and Symmetric.

Lecture 26.

6) Irreflexive Relation:-
if $\forall x \in X$, $x \not R x$ or $(x,x) \notin R$
then R is irreflexive.

Ex: A{1,2,3} + R = {(1,1) (2,2) (1,3)} X
$\because$ (1,1) (2,2) $\in$ R.

R1 {(1,2) (2,3) (1,3)} ✓.

7) Assymmetric Relation:-
let R be a relation define on set X
if $x R y$ then $y \not R x$
$(x,y) \in R \Rightarrow (y,x) \notin R$

A{1,2,3}, R = {(1,1) (2,2) (1,3) (3,1)} X
$\because$ (3,1) exist.
R1 = {(1,1) (1,3), (2,3)} ✓.

8) Antisymmetric Relation:-
If $x R y$ and ~~y R x~~ $y R x$ then $x = y$ $\forall x,y \in X$

Ex {1,2,3}, R = {(1,1) (2,2) (2,1) (1,2) (2,3)} X
$\because 2 \neq 1$

R' {(1,1) (2,2) (3,3)}.

Symmetric. ✓ ($\because$ Reverse (1,1) = (1,1))
Asymmetric X (—"—"—)
Antisymmetric. ✓ ($\because$ 1 = 1)

9) Partial Order Relation:-
Reflexive, antisymmetric, and transitive all at a same time is partial order rel^n.

R1 = {(1,1), (2,3), (3,3)} A = {1,2,3}

Reflexive = X
Symmetric = X ($\because$ (3,2) not exis).
Assymmetric = X
Antisymmetric = ✓

Lecture 27.

Q) Let R be a rel^n define on a set of +ve integers such that for all $x,y \in Z^+$ $x R y$ iff $x+y$ is an even no. Proove that R is an equivalence relation.

(i) Sol^n Reflexive:- $\forall x \in Z^{+}$, $x + x = 2x$ where x is even/odd
$\therefore \forall x \in Z^{+}$ $(x,x) \in R$. $\Rightarrow$ Reflexive.

(ii) Symmetric:- $y R x$.
let $x,y \in Z^{+}$ and $(x,y) \in R$.
$x R y \Rightarrow x+y$ is an even no.
$\Rightarrow y+x$ is an even no.
$\Rightarrow y R x$ Symmetric

iii) Transitive : $(x,y) \in R$ and $(y,z) \in R$ then $(z,x) \in R$.

let $x, y, z \in Z$
xRy and yRz
$\Rightarrow$ $x+y$ is even and $y+z$ is even.

$\therefore (x+y+y+z) =$ even.
$x+2y+z = 2k$ $\quad k \in Z$.
$\Rightarrow x+z = 2k-2y$
$\Rightarrow x+z = 2(k-y)$
$\therefore x+z =$ even. $\quad$ Transitive.

As the Relation R is Reflexive, Symmetric and Transitive $\therefore$ R is an equivalence relation.

Lecture 28

POSET (Partially ordered set)

A Relation R on a set P is called partially ordered relation if it is reflexive.

$a \in P$, $(a,a) \in R$ or aRa → Reflexive.
Antisymmetric → if $(a,b) \in R$ & $(b,a) \in R$. then $a=b$.
Transitive if $(a,b) \in R$, $(b,c) \in R$, $(c,a) \in R$.

The set P is called together with a partial order relation $\leq$ is called partial order relation.

$\boxed{(P, \leq) - \text{POSET}}$

$a \leq b$ $\quad$ b succeeds a / a precedes b
if $a \neq b$ and $a \leq b$ then a strictly precedes b.

Q) Check POSET for following set with given relatn.

ⓐ $(Z, \geq)$ integers greater than equal to
$x \geq y$
$y \geq x$
$R = \{x, y\}$ $\quad x, y \in Z$
only possible when $x = y$
Antisymmetric.

→ $x \geq x \Rightarrow$ Reflexive.

→ $x \geq y$ and $y \geq z$. $\therefore x \geq z$
for $R = \{(x,y), (y,z), (x,z)\}$ $\therefore$ transitive.

$\therefore$ POSET. $\quad \because Z$ was a set and after applying operation on it, it becomes POSET.

ⓑ $(Z, >)$
$x > y$ $\quad x > x$ not possible. (No Reflexive)
Not a POSET.

ⓒ $(P, \subseteq)$ subset. where P is collection of sets
Rule.
subset/set
→ Reflexive
$(x,y) \subseteq (x,y)$
Z $\quad$ Z.

→ Transitive $Z_1 \subseteq Z_2$, $Z_2 \subseteq Z_3$ $\therefore Z_1 \subseteq Z_3$

Antisymmetric

$z_1 \in R$ $\quad (x, y)$ $\quad z_2 \in R$ $\quad (y, x)$ $\quad \therefore z_1 = z_2$ antisymmetric.

Here we deal with
Set: $(x, y) = (y, x)$
$\therefore$ set $(2,1)$ = set $(1,2)$

$\therefore$ POSET.

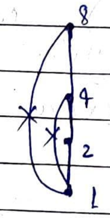
lecture 29:- 09/12/20

HASSE DIAGRAM:-
It is used for the representation of POSET.

Steps :-
i) Create vertex of element
ii) if aRb then draw edge from a to b.
iii) Remove self loop and transitive edges

Let $(P, \prec)$ be a poset and $x, y \in P$. The element y is called the cover of x if $x \prec y$.
y is called cover of x if y is immediate successor of x.

Ex) $A = \{1, 2, 4, 8\}$.
↑
Partial Order set
$(A, \prec)$ $\quad$ x divides y.
$x \prec y$.

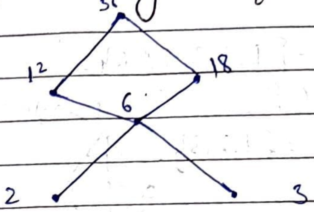


Hasse diagram

Here 1 to 4 transitive edge

Q) Set $A = \{2, 3, 6, 12, 18, 36\}$ and partial order relation $\prec$ is defined as $\forall x, y \in X$ $x \prec y$ iff x divides y.
Draw Hasse diagram of POSET $(A, \prec)$

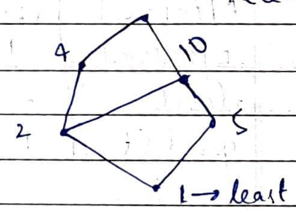
Sol^n.



$(A, \prec) = \{(2,6), (3,6), (6,12), (6,18), (12,36), (18,36)\}$

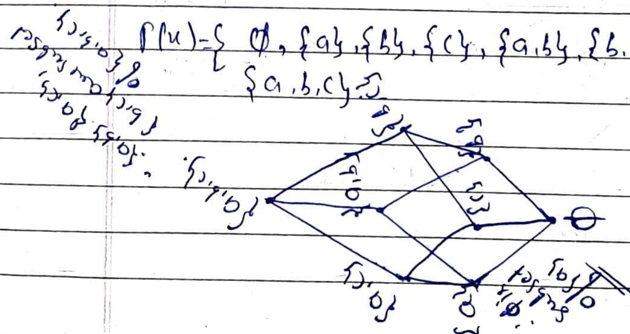
lecture 30 :-

Q) Draw Hasse diagram of poset $(D_{20}, \prec)$ and find its least and greatest elements, where D_{20} is the set of +ve divisions of 20.
→ which can divide 20
$D_{20} = \{1, 2, 4, 5, 10, 20\}$



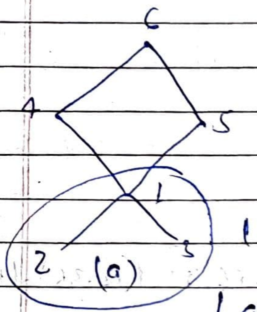
Q) Let $X = \{a, b, c\}$ draw Hasse diagram of $(P(x), \subseteq)$

$P(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$ Powerset of x.

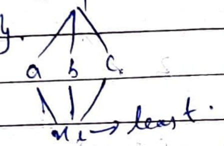


lecture 30.2

POSET Elements



P = {1,2,3,4,5,6}
S ⊆ P
S = {1,2,3}.



only 1 value.

	a	
kuch nahi hona chahiye unke/uske niche = Minimal	{2,3}	leaf side (subordinate value)
Maximal	1	
least	–	(lowest value which is connected to its)
Greatest	1	

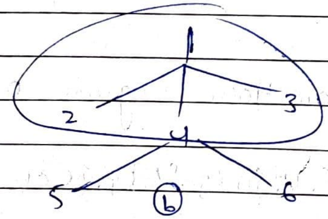
us ke upar wale sb connected hote hai use → least

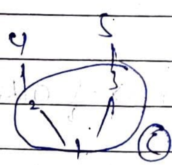
Greatest → apne sb niche waalo se connected.

all upper elements + subordinates

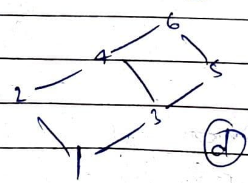
2 3 not connected

	(b)	(c)	(d)
Minimal	{2,3}	1	1
Maximal	1	{2,3}	4
least	–	1	1
greatest	1	–	4





S = {1,2,3}.

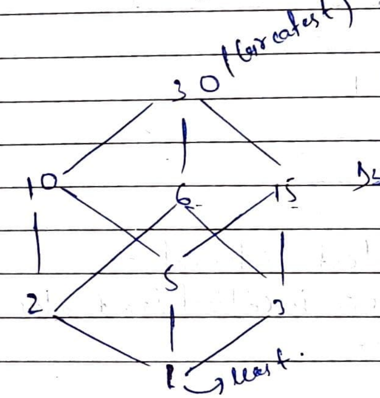


S = {1,2,3,4}.

lecture 30.3

greatest lower Bound.

	upper Bound	lower Bound	LUB	GLB
10,15	30	{5,1}	30	5



30

(a) 10 + 15 (lower Bound) = {5,1} (and operation)
→ 10 L.B → 2,5,1 (10 se jo niche hai)
→ 15 L.B → 5,3,1 (lower subordinates)

(b) G.L.B 10+15 (10+15 connected to 5 + 5 connected to 1 + 5>1)

∴ 5 GLB

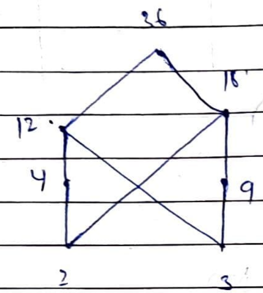
(c) upper Bound = 30

(d) LUB 30

Lecture 30.4

Elements in POSET

Q4 Draw a Hasse diagram of {2, 3, 4, 9, 12, 18, 36}.



least and Greatest Element:-
An element $a \in A$ is called least element of A if $a \leq x$ $\forall x \in A$.

An element $a \in A$ is called greatest element of A if $a \geq x$ $\forall x \in A$.

$A_1 = \{12, 18\}$
Can't find least and greatest element as there is no such edge.

Minimal and Maximal element :-
An element $a \in A$ is called minimal element of A if no $x \in A$ exists such that $x < a$.

$A_1 : \{12, 18\}$ 12,18 minimal) / ~~lower bow~~
Maximal {12, 18}

lower and upper Bound
An element $x \in X$ is called a lower bound of A if $x \leq a$ $\forall a \in A$ - (x is successor of A)

lower Bound = {2, 3}
Upper Bound = {36}.

Greatest lower Bound and least upper Bound

If the set of lower bound of A has greatest element then this element is called GLB or infimum of A.

GLB = X
LUB = 36.

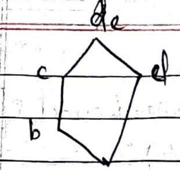
$A_2 = \{4, 9\}$
No least and greatest since they are not connected to each other
Minimal - {4, 9} = Maximal

lecture 31.1 10/12/20

LATTICES:-

A POSET $(L, \leq)$ is called lattice if every pair of elements in L has L.U.B and G.L.B

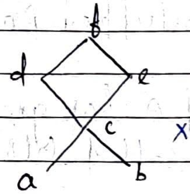
GLB of x, y is called meet of x and y $(x \wedge y)$
L.U.B of x, y is called joint of x and y $(x \vee y)$

Q4 

$L = \{a, b, c, d, e\}$

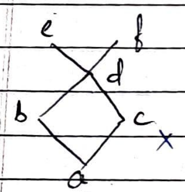
(i) G.L.B of c + d
$c = \{b, a\}$ and $d\{a\}$
$c \wedge d = a$ = G.L.B

(ii) L.U.B of c + d = e.
∵ b and a are comparable and a is lower of a + b + e is LUB.

(b) 

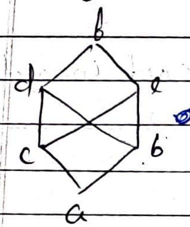
∵ we can't find lower Bound of $a \wedge b$ ∴ it can't be a lattice

X

(c) 

∵ we can't find L.U.B of e + f.

X

(d) 

G.L.B of a + b = a.
L.U.B of a + b = f.

G.L.B of b + d = a
L.U.B of b + d = f.

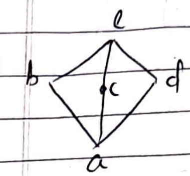
X

(d, e)

d : L(L.B) = {c, a, b}
e (L.B) = {b, c, a}.
∴ L.B = {b, c, a}.
∵ b + c are non comparable ∴ we can't find G.L.B of d + e.

(b, c) ∴ it is not a lattice.
G.U.B = {d, e} non parable.

two terms are non-comparable if they are not connected th' any direct path.

(c) 

✓

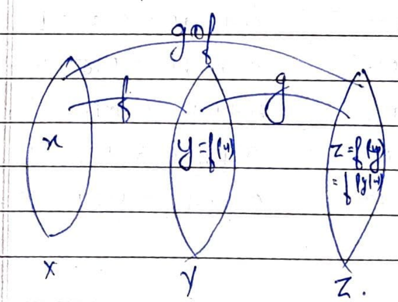
if we can't compare any two elements then it is not a lattice

lecture 22.

Composition of function:-

let $f : x \to y$ and $g : y \to z$ be two f^n(s) when co-domain of f is the domain of g. then composition of f and g, written as gof is a function from x to y defined as $gof(x) = g(f(x))$ $\forall x \in X$.

$y = x + 3$ and $z = y + 10$
$z = x + 3 + 10$



gof.

Q. let $f: R \to R$ and $g: R \to R$ be two function defined as $f(x) = x^2$ and $g(x) = 3x+1$. find $gof(x)$ and $fog(x)$

$gof(x) = g(f(x)) = g(x^2) = 3x^2+1$.

$fog(x) = f(g(x)) = f(3x+1) = (3x+1)^2$.

$fogoh(x) = fog(h(x))$
$= f(g(h(x)))$

lecture 30.5

M-Notations : Minterms and Maxterms

Variable A	B	C	Minterms m_i	Maxterms M_i	f
0	0	0	$\bar A\bar B\bar C = m_0$	$\bar A+\bar B+\bar C = M_0$	0
0	0	1	$\bar A\bar B C = m_1$	$\bar A+\bar B+C = M_1$	0
0	1	0	$\bar A B\bar C = m_2$	$\bar A+B+\bar C = M_2$	1
0	1	1	$\bar A BC = m_3$	$\bar A+B+C = M_3$	1
1	0	0	$A\bar B\bar C = m_4$	$A+\bar B+\bar C = M_4$	0
1	0	1	$A\bar B C = m_5$	$A+\bar B+C = M_5$	0
1	1	0	$AB\bar C = m_6$	$A+B+\bar C = M_6$	1
1	1	1	$ABC = m_7$	$A+B+C = M_7$	0

$n=3 \quad 2^n = 2^3 = 8$

$f(A,B,C) = \Sigma m(2,3,6)$ (SOP)

$f = \Pi M(0,1,4,5,7) = (A+B+C)(A+B+\bar C)\ldots$ (POS)

K-Map (Karnaugh map) or Veith diagram

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

(A, B)

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	15	15	14
10	8	9	11	10

Q	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

A \ B	0	1
0	0	1
1	1	0

Q $Y = AB\bar C + ABC + \bar A\bar B\bar C$

A \ BC	00	01	11	10
0	1	0	0	0
1	0	0	1	1

lecture 31.2

Rule of Karnaugh Maps (K-Maps)

1. No zero allowed
2. No diagonal
3. only power of 2 no. of cells
4. Group should be as large as possible
5. Every one must be in at least one group
6. overlapping allowed.
7. wrap around allowed.

Inverse of a function:-

Q1 $f(x) = \frac{x-3}{2}$ ⇒ put $y = f(x)$

⇒ $y = \frac{x-3}{2}$ ⇒ swap x with y

⇒ $x = \frac{y-3}{2}$ ⇒ solove for y { y in one hand (side) }

⇒ $2x + 3 = y$

$\therefore y = 2x + 3$.

$\because y = f^{-1}(x)$ ($\because$ we swap x with y)

$f^{-1}(x) = 2x + 3$

checking

$x = 5$

$f(5) = \frac{5-3}{2} = 1$. $f^{-1}(1) = 2(1) + 3$

$= 5$ correct.

Q2 $f(x) = \frac{3x+2}{4x-1}$

$y = \frac{3x+2}{4x-1}$ ⇒ $x = \frac{3y+2}{4y-1}$

⇒ $4xy - x = 3y + 2$.

⇒ $4xy - 3y = x + 2$ ⇒ $y = \frac{x+2}{4x-3}$

$\because y = f^{-1}(x)$

⇒ $f^{-1}(x) = \frac{x+2}{4x-3}$

Binary operations:-

A Binary operation $*$ (star / sterics) on set A is a function

$*: A \to A$ we denote $*(a,b)$ by $a*b$

Bi = 2. (only two operands.

operands = a, b (used in operation)

operations = +, −, ÷, ×

Closure property:-

Real No. a, b.

① Add^n = a+b (we get a real no.) $\in \mathbb{R}$

∴ it is closed on R.

② Division is not closed on Natural no.

$a, b \in N$

$a = 4, b = 2, a/b = 2 \in N$

$a = 2, b = 4, a/b = 1/2 \notin N$

$A*B*C = D$ → n-ary (operation b/w more than 2 element)

$*(2,4)$ = it means here we select the no. which is smaller. this is the work of sterics

UNIT-III

11/12/20

lecture-50

principal of Mathematical Induction:-

let $P(n)$ be a statement defined on positive integers $n \in N$ such that it has following properties

(i) $P(1)$ is true (Basis of induction)

we take the 1st value for which our Question is true. and then we assume our statement is true for k and we prove our statement for k+1.

(ii) $P(k+1)$ is true whenever $P(k)$ is true for $k \geq 1$

Q1 prove for every natural no. $1+2+3+\dots+n = \frac{n(n+1)}{2}$. $n = \{1,2,3,\dots n\}$

Sol^n let $p(n)$
$n=1$. L.H.S $= 1$, R.H.S $= \frac{1(1+1)}{2} = 1$.

L.H.S = R.H.S.

$P(1)$ true.
let the statement $p(n)$ is true for $n=k$ then
$1+2+3+\dots k = \frac{k(k+1)}{2}$.

Now for $n=k+1$

L.H.S: $1+2+3+\dots+k+k+1$.

$= \frac{k(k+1)}{2} + k+1 = \frac{k(k+1)+2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2} = $ R.H.S.

$\therefore P(n)$ is true for $n=k+1$

Q2 prove that for every positive integer $n \geq 4$. $2^n < n! \Rightarrow p(n)$

Sol^n for $n=4$ L.H.S $2^4 = 16$,
R.H.S $= 4! = 4\times3\times2\times1 = 24$.
$16 < 24$ - True.

let $k \Rightarrow 2^k < k!$
for $k+1 \Rightarrow 2^{(k+1)} < (k+1)!$
$\Rightarrow 2^k \cdot 2 < (k+1)\,k!$

$\because 2^k < k!$
$2\cdot 2^k < 2k! < (k+1)k!$ $(\because k \geq 4)$
$\Rightarrow 2\cdot 2^k < (k+1)\,k!$

$\therefore P(n)$ is true for $n=k+1$.

lecture 51

principal of complete induction:-

let $P(n)$ be a statement define on +ve integers $n \in N$ such that

(1) $P(m)$ is true of some $m \in N$.

instead of only 1 value we check for some more value.

(ii) whenever $P(m), P(m+1) \dots P(k)$ are true then $P(k+1)$ is true, where $k \geq m$.

Assumption: Our statement is true for $P(m), P(m+1) \ldots P(k)$ then it is also ~~and prove~~ Hence we

and prove it is also true for $P(k+1)$

Q) P.T any integer $n \geq 2$ is either a prime or product of prime.

Sol^n let $P(n)$ is be a statement that n is prime or product of prime.

$n=2$. $P(2) = 2 = 2 \times 1$
$P(3) = 3 = 3 \times 1$.
$P(4) = 4 = 2 \times 2 \rightarrow$ product of prime.

let $p(n)$ is true of $2 \leq n \leq k$ (assumption)
i.e. $P(2), P(3), P(4), \ldots P(k)$ true.

Now for $n = k+1$.

(a) if $k+1$ is a prime then it is ~~true~~ prime.

(b) if $k+1$ is not a prime.

$k+1 = u \cdot v$ (say) ; $2 \leq u < k$, $2 \leq v < k$.

$\because$ $P(2), P(3) \ldots P(k)$ is true.
$\therefore$ $P(u)$ & $P(v)$ lies in b/w $P(2)$ to $P(k)$
$\therefore$ $P(u)$ and $P(v)$ are product of prime no.

$\therefore$ $k+1$ is product of prime.

lecture 52.1

Recursive definition:-
$\hookrightarrow$ function call itself

1) Basic step: Set of primitive values
2) Recursive step: This step defines the rule(s) to ~~define~~ find a new element from existing elements

Q) $1, 2, 3, \ldots$
$a_1 = 1$.
$a_n = a_{n-1} + 1$
$a_3 = a_2 + 1 = 2+1 = 3$.

Q) $2, 4, 8, 16, \ldots \Rightarrow a_1 = 2$.
$a_n = 2 \cdot a_{n-1}$.

Recursive f^n:-
$f(x) = 2^x$
$x = 0$ $f(0) = 2^0 = 1$.
$f(x+1) = 2^{x+1} = 2^x \cdot 2^1$

$$f(x) = \begin{cases} 1 & ; x = 0 \\ 2 \cdot f(x-1) ; & x \geq 1. \end{cases}$$

$x = 3$.
$f(x) = 2 \cdot 2^2 = \underline{\underline{8}}$.

Note:-
the order of a recurrence relation can be calculated ~~also~~ as the difference b/w the largest and the smallest subscripts of a appearing in the recurrence relation.

ex

(i) $a_r = 2a_{r-1} - a_{r-2}$ Order. $r-(r-2) = 2$. degree $= 1$.
largest subscript Smallest subscript

(iv) $a_r = r \cdot a_{r-1} + a^2_{r-2}$ Order = 2, d = 2.

(v) $a_r = r a_{r-1} + a_{r-2} + r^2$ Order = 2. d = 1.

HOMOGENOUS	Non-Homogenous.
Not dependent on r	dependent on r.
$a_r = a_{r-1} + a_{r-2}$.	$a_r = a_{r-1} + r$.
linear recurrance relation:- degree 1	Quadratic RR. degree 2.

Lecture 52.2.

Solve Recurrence relation using Recursive Method.

Q1 $a_r = a_{r-1} + 3$. for $r \geq 1$.
$a_0 = 1$.

$a_r = a_{r-1} + 3$ for $(r-1)$ $a_{r-1} = a_{r-2} + 3$

$a_r = a_{r-2} + 3 + 3$ $a_{r-2} = a_{r-3} + 3$

$= a_{r-2} + 2 \cdot 3$

$= a_{r-3} + 3 + 2 \cdot 3$

$= a_{r-3} + 3 \cdot 3$

put $r = 1$.

$a_r = a_0 + r \cdot 3$

$\boxed{a_r = 1 + r \cdot 3}$ for $r \geq 0$

Q1 $a_r = a_{r-1} + r$ for $r \geq 1$
$a_0 = 0$

$a_r = a_{r-1} + r$ $a_{r-1} = a_{r-2} + r - 1$

$= a_{r-2} + (r-1) + r$. $a_{r-2} = a_{r-3} + (r-2)$

$= a_{r-3} + (r-2) + (r-1) + r$.

$\vdots$

$a_r = a_0 + 1 + 2 + \cdots (r-2) + (r-1) + r$.

$a_r = 0 + \frac{r(r+1)}{2}$.

$a_r = \frac{r(r+1)}{2}$ for $r \geq 0$ ($\because$ if we take $a_0 = 0$)

Q1 $a_r = 2 \cdot a_{r-1}$ for $r \geq 0$
$a_0 = 1$.

$a_r = 2 \cdot a_{r-1}$ $a_{r-1} = 2 \cdot a_{r-2}$.

$= 2 \cdot (a_{r-1})$ $a_{r-2} = 2(a_{r-3})$

$= 2(2)(a_{r-2})$ $\vdots$

$= 2^{(r)} a_0$

$= 2^r$.

Lecture 53 12/12/20

Linear Recurrence Relation with Constant Coefficients

A linear relation with constant coefficients is a recurrence relation of the form

$a_r = C_1 a_{r-1} + C_2 a_{r-2} + \cdots + C_k a_{r-k} + f(r)$

where, $C_1, C_2, \cdots C_k$ are real numbers and $C_k \neq 0$

if $f(r) = 0 \rightarrow$ Homogenous L.R.R.

Otherwise $\rightarrow$ Non Homogenous LRR.

HOMOGENOUS solution:-

$a_r = \alpha^r$ where α is constant.

$a_r = c_1 a_{r-1} + c_2 a_{r-2} + \dots + c_k a_{r-k}$
$\alpha^r = c_1 \alpha^{r-1} + c_2 \alpha^{r-2} + \dots + c_k \alpha^{r-k}$.

divide by α^{r-k}.

$\alpha^k = c_1 \alpha^{k-1} + c_2 \alpha^{k-2} + c_3 \alpha^{k-3} + c_4 \alpha^{k-4} + \dots c_k$.

$\Rightarrow \underbrace{\alpha^k - c_1 \alpha^{k-1} - c_2 \alpha^{k-2} - \dots - c_k = 0}$
characteristics eq^n.
$\alpha \to$ values
characteristics roots.

Q) Solve the recurrence relation.

$a_r = 6a_{r-1} - 8a_{r-2}$, $d=1$, $O=2$. Here $f(r)=0$
$\alpha^r = 6\alpha^{r-1} - 8\alpha^{r-2}$.
characteristics eq^n. Here $r-k = \underline{r-2}$.

$\therefore \alpha^2 - 6\alpha + 8 = 0$ = c.eq^n.
$(\alpha-4)(\alpha-2) = 0$
$\alpha = 4, 2$. = c/roots.

$\therefore a_r = c_1 2^r + c_2 4^r$ — (a)

$a_0 = 0$, $a_1 = 4$. (if given)

$a_0 = c_1 2^0 + c_2 4^0 \Rightarrow c_1 + c_2 = 0$ — (1)

$4 = a_1 = 2c_1 + 4c_2$ — (2)
$c_1 = -2$, $c_2 = 2$.

$\therefore a_r = -2^{r+1} + 2 \cdot 4^r$ (using eq^n (a))

lecture 54
Particular.
Non-Homogenous linear Recurrence Relation

$a_r = c_1 a_{r-1} + c_2 a_{r-2} + \dots + c_k a_{r-k} + f(r)$
where c_i's are real nos. $(1 \le i \le k)$

and $f(r) = (b_q r^q + b_{q-1} r^{q-1} + \dots + b_1$

and $f(r) = (b_q r^q + b_{q-1} r^{q-1} + \dots b_1 r + b_0)\beta$

→ If β is not root of characteristics equation of associated linear Homogenous Recurrence Relation then there is a particular solution in the form

$a_r^p = (d_q r^q + d_{q-1} r^{q-1} + \dots d_1 r + d_0)\beta^r$

if β is a root with multiplicity m, then solution

$a_r^p = r^m (d_q r^q + d_{q-1} r^{q-1} + \dots d_1 r + d_0)\beta^r$

Q) $a_r = 5a_{r-1} - 8a_{r-2} + 4a_{r-3} + f(r)$

$\therefore \alpha^3 - 5\alpha^2 + 8\alpha - 4 = 0$
$(\alpha-1)(\alpha-2)(\alpha-2) = 0$
$\therefore \alpha = 1, 2, 2$.

$-f(r) = 5 = 5\cdot 1^r$ and $\beta = \beta\cdot 1^r$.

1 is a root of characteristics eqn.

$\therefore\ a_r^P = r^2 d_0 = r^3 d$.

if $f(3) = 3^r$

3 is not a root of ch. eqn.

$\therefore\ a_r^P = d\cdot 3^r$.

lecture 55

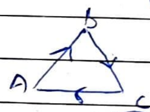
Graph Introduction

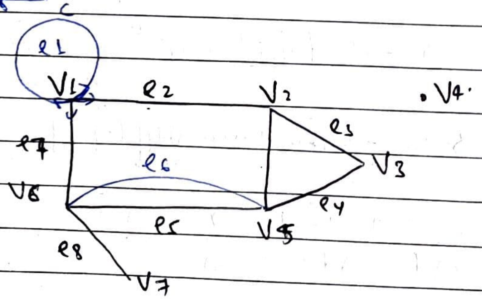
Graph consists of set of vertices.
Transport system Roads in one direction / two direction is example of Graph.

Graph

directed (one way)

Undirected (Two way Roads)





V → set of vertices $\{V_1, V_2 \cdots V_6\}$
E → set of Edges $\{e_2, e_3, e_4, e_5, e_6, e_7, e_8, e\}$

Parallel edges = edges created from same vertices
Self loop = $\{e_1\}$

Order = No. of vertices
Size of graph = No. of edges

Adjacent vertices:- an edge lies b/w two adjacent vertices.

Degree vertices:-

$\deg(V_1) = 4$

$\deg(V_2) = 3$.

for undirected graph they have two direction (say)

Isolated vertex:-
$\deg(V_\bullet) = 0$
ex V4.

pendent vertex:-
$\deg(v) = 1$.
V7.

lecture 56

let $G(V,E)$ = n vertices, e edges → then → $\sum_{i=1}^{n} \deg(v_i) = 2e$.

Q4 A graph has 12 edges, 2 vertices of degree 3, 2 vertices of degree 4 and other vertices of degrees 5 find no. of vertices.

$e = 12$, Total vertices $= n$.

$2 \times 3 + 2 \times 4 + (n-4) \times 5 = 2 \times 12$.

$14 + 5n - 20 = 24$.

$\therefore$ $n = 6$.

Types of Graph.

i) Simple Graph: without self loop and parallel edge

ii) Multi Graph: Having some parallel edges.

iii) Trivial Graph:- One vertex no edge.

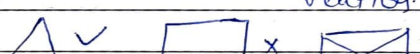
$\cdot V_1$

iv) Null Graph: No edge

$V_1 \cdot$ $\cdot V_2$

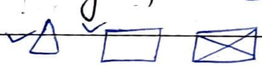
$V_3 \cdot$ $\cdot V_4$.

v) Complete Graph: An edge b/w every pair of vertices.



✓ × ✓

vi) Regular graph:- Degree of each vertex is same.



✓ ✓ ✓

vii) Bipartite graph:-

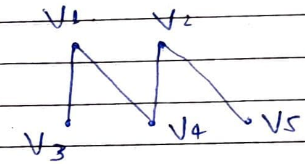
A graph $G(V,E)$

Partition of the set $(V(G))$ into two disjoint sets $V_1(G)$ and $V_2(G)$ such that each edge of the graph has its one end in $V_1(G)$ and other end in $V_2(G)$.

$V(G)$ $\{V_1, V_2, V_3, V_4, V_5, V_6\}$

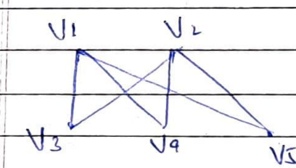
$\{V_1\ V_2\} \Rightarrow V_1(G)$ $\qquad$ $V_2(G)\ \{V_3, V_4, V_5\}$

$V_1 \cap V_2 = \phi \Rightarrow$ disjoint set.



Complete Bipartite graph.

Vertex of each set connect with all the vertex of other set.



Note:-

n vertices in a graph.

$\boxed{\text{edges} = \frac{n(n-1)}{2}}$ simple Complete graph

14/12/20

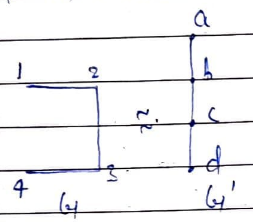
Lecture 57

ISOMORPHIC GRAPH:-

Two graph G(V,G) and G' are Isomorphic if there is a one to one correspondance (that is) one-one and onto) between their vertices and between their edges.

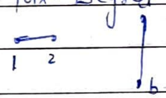
(u,v) ∈ E(G) iff (f(u), f(v)) ∈ E(G')

① (Isomorphic)

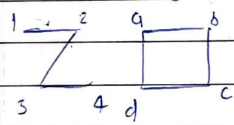


① No. of vertices and edges are same.
② Equal no. of vertices with given degree.
③ Max. Degree.

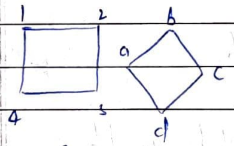
Ex② (isomorphic) length does not Matter.



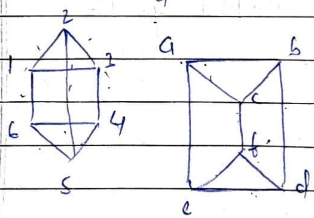
③ Not isomorphic.

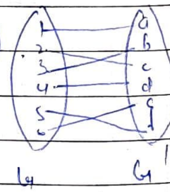


④ Isomorphic.

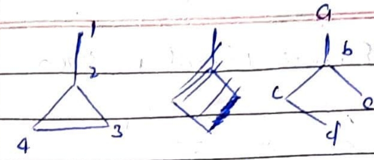


⑤ (Isomorphic)

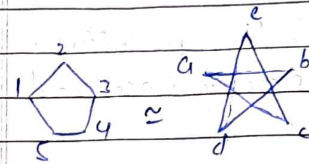




⑥ (Non-Isomorphic)
Vertices are not same.



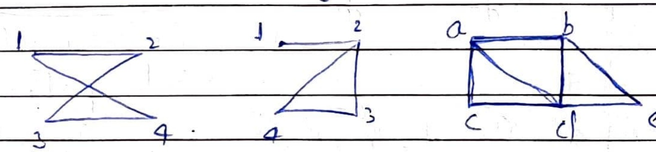
⑦ (ISOMORPHIC)



Lecture 58

EULERS PATH:

Eulers path in a graph G is a path that traverse every edge exactly one & only once.



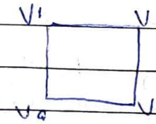
$V_1 \to V_2 \to V_3 \to V_4 \to V_2$.

Vertex repetition is allowed where as edge repetition not allowed.

a → c → d → b → a → d → e (Euler's Path)

Degree = No. of edge with which verticle are connected.

In Eulers path if 1st & last vertex degree are odd.
if starting and last vertex are same then that is called Eulers Circuit



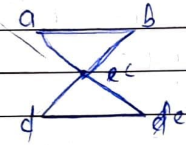
$V_1 \to V_2 \to V_3 \to V_4 \to V_1$

$V_1 \to 2$.
$V_2 \to 2$
$V_3 \to 2$
$V_4 \to 2$

each
degree of vertices in Eulers circuit is even

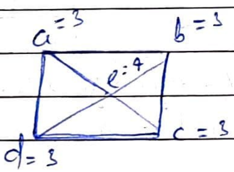
Euler Circuit:-
Euler Circuit in a graph G is a circuit that traverse each edge exactly once and only once.

A Connect graph is Euler graph iff it has atmost 2 odd degree vertices.



a→b→c→d→e→c→a

Euler circuit also Euler graph.



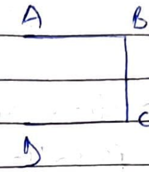
No. of odd vertices = 4
but only atmax 2 odd degree vertices are allow ∴ it is not a Euler graph and also not a Euler path.

lecture 59

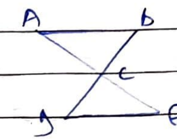
Hamiltonion Path and cycle:-

A path that contains each vertices exactly once.

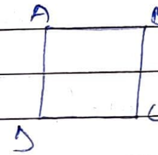
A Hamiltonion Circuit is a circuit that contains each vertex ~~on~~ exactly once except for the first vertex which is also the last vertex.



A→B→C→D. Hamiltonion path
n = 4.
2+2 ≥ 4



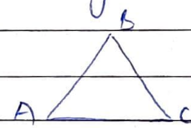
each vertex visited only once.
Hamiltonion path



Hamiltonion Circuit ∵ 1st vertex visited 2 times.

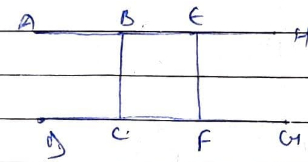
Theorem:- let G be a graph of n vertices then G has a Hamiltonion path if for any only two vertices u & v and v of G.

deg(u) + deg(v) ≥ n. (for any two vertices of a graph



n = 3
2+2 ≥ 3.

Hamiltonion path can be find



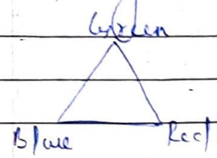
Not Hamiltonion path

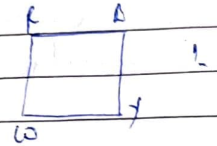
3+3 ≱ 8.

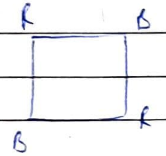
Lecture 60.

Colouring of Graph:-

Colouring of vertices so that no two adjacent vertices have the same colour is called vertex colouring.







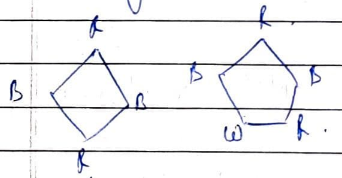
for min. of color used.

Chromatic No.:-

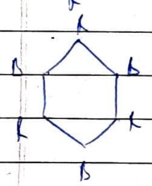
The min. no. of colours required to colour to colour a graph properly is called the chromatic no. of the graph.

denoted by $\chi(G)$ or $X(G)$

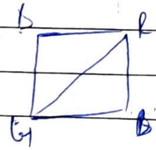
Every bipartite graph is 2 coloring.



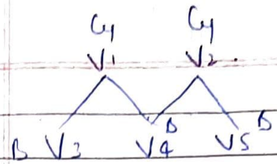
$X(G) = 3.$



$X(G) = 3$



$X(G) = 3.$



$V(G) = \{V_1, V_2, V_3, V_4, V_5\}$

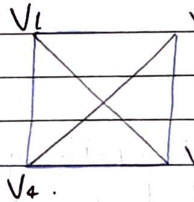
$\swarrow V_1, V_2$ $\qquad \searrow V_3, V_4, V_5$

2 colourable.

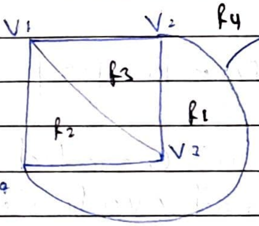
Lecture 61. 15/12/20.

Planar Graph:-

A graph is said to be planar if there exists some geometric representation of G that can be drawn on a plane such that no two of its edges intersect.

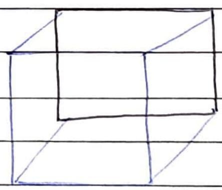


$\cong$

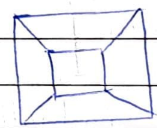


edge

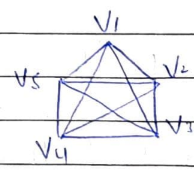
Planar.
Region decided by boundary.
like kitne boundary se ghira hua hai.



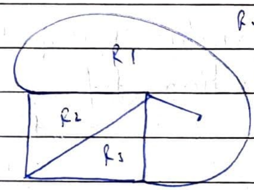
$\equiv$



= Planar.



X

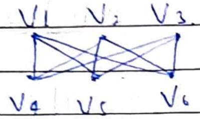


Euler's formula:-

Let G be a connected planar simple graph with e edges and v vertices. Let r be the no. of the regions in a planar representation of G. Then.

$\boxed{r = e - v + 2}$

Bipartite graph :-
K(3,3)



(Not a planar graph)
edge should not be intersect after expanding them.

lecture 62.

Euler's Formula proof:-

Let G be a connected planar simple graph e edges and v vertices. Let r be the no. of regions in a planar representation of G then.

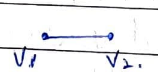
$$\boxed{r = e - v + 2}$$

Method of induction:-

Base value True if P(1) → True.
Induction step → true for k (supposition)
+ verify for k+1
if true then our supposition is true.

Euler's proof by Method of induction.

Base step, n = 1
$e = 1,\ v = 2,\ r = 1$



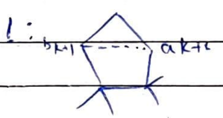
R.H.S $r = e - v + 2$.
$1 - 2 + 2$
L.H.S = 1. {True for base value}.

Induction step assume true for k.

$r_k = e_k - v_k + 2$ —(1) for graph G_k.

Let (a_{k+1}, b_{k+1}) be vertices and edge formed by these two vertices added to G_k then we get G_{k+1}.

Case 1:



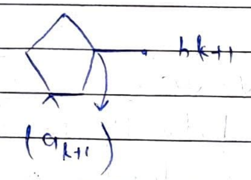
$r_{k+1} = r_k + 1$.
edges : $e_{k+1} = e_k + 1$.
$v_{k+1} = v_k$

thus

$r_{k+1} = e_{k+1} - v_{k+1} + 2$.
$r_k + \cancel{1} = e_k + \cancel{1} - v_k + 2$.
$r_k = e_k - v_k + 2$. —(II)

eqn (I) + (II) are same
∴ Result is true.

Case 2:



$r_{k+1} = r_k$,
$e_{k+1} = e_k + 1$.
$v_{k+1} = v_k + 1$.

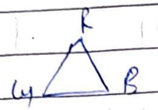
Now $r_{k+1} = e_{k+1} - v_{k+1} + 2$.
$r_k = e_k + \cancel{1} - v_k - \cancel{1} + 2$.
$r_k = e_k - v_k + 2$. —(III)

eqn (I) + (III) are equal

Atif Raza.
CSE/19/112.

∴ p(n) Induction value for n value is true.

* lecture 63



5-COLOR THEOREM:-

Every planar graph with n vertices can be colored using at most 5 colours.

proof by Induction Method:-

Base $P(1) \rightarrow$ true.
I.S k assume true.
$k+1 \rightarrow$ verify.

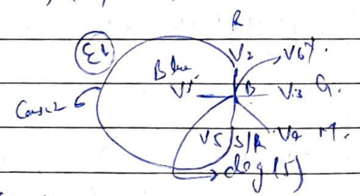
Base

$P(n) \leq 5 \rightarrow$ graph can be coloured using 5 colours.

Induction step:

$k=4$. degree of $(v) \leq 4 \rightarrow$ True

$k+1$ deg(5)



Case:-1.

let ~~V1 and V2, V3, V4, V5~~

V2 & V5 are not directly connected to an edge. Max 5 colours required.
True.

Case:-2. So V2 & V5 can't coloured with same colour

∵ V1 and V3 can't be connected directly but if we try to connect them then it will intersect with edge which is violated for planar graph.

∴ it is also true for 2nd case.

lemma:-
Every planar graph contains a vertex with deg(v) ≤ 5

Method to colour a graph:-

Step 1. Arrange the

UNIT- IV .

16/12/20

lecture-33

GROUP THEORY : BINARY OPERATION.

(i) Closure law:

we take a set and operator.

$A \cdot (A, \ast)$ → operator

$(Z, +)$ = set of ~~pos~~ integers closed under + operator. ✓

ex $2+3=5$

$(Z, \times)$ = ✓ → operator = multiplication.

$\left[\begin{array}{l} x \in X, \; y \in X \\ x \ast y \in X \end{array} \right]$ rule of closure law

→ operator.

$(N, +)$ $2+3=5$ ✓

$(N, -)$ × ∵ $2-5=-3$ (Not a Natural no.)

(ii) Associative law:-

$x, y, z \in X \Rightarrow x \ast (y \ast z) = (x \ast y) \ast z.$

(iii) Existence of identity Element:-

$x \in X, \quad e \in X$

$x \ast e = e \ast x = x$

ex:- $1 \cdot a = a \cdot 1 = a$

(iv) Existence of inverse element:-

$x \in X, \; y \in X$

$x \ast y = y \ast x = e.$

Satisfied for multipⁿ.

Ex: $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$

(v) Commutative law:-

$\forall x, y \in X$

$x \ast y = y \ast x$ ✓

$x + y = y + x$ ✓

$x - y \neq y - x$ ✗

$x \ast y = y \ast x$ ✓

Closure → Algebraic structure

Associative → semi group

Identity → Monoid.

Inverse → group.

Commutative → abilion group.

lecture 34

Algebraic structure : If a set X w.r.t operation $\ast$ satisfy closure property then it is A.S.

Closure property: A set X w.r.t operation $\ast$ is said to satisfy closure property if $\forall a, b \in X$ then $a \ast b \in A$.

Ex	A.S.	Semi group (Next lecture)	(Monoid) (Next lecture)	Ex	A.S.	Mon
$(N, +)$	✓	✓	×	$(R, +)$	✓	✓
$(N, -)$	∵ (−)1 is not a Natural no. (×)		×	$(R, -)$	✓	×
$(N, \times)$	✓	✓	✓	$(R, \times)$	✓	✓
$(N, \div)$	×	×	×	$(R, \div)$	×	×
$(Z, +)$	✓	✓	✓	∞ = a/b ∵ $b=0$		
$(Z, -)$	✓	×	×	$(M, +)$	✓	✓
$(Z, \ast)$	✓	✓	✓	$(M, -)$	✓	✓
$(Z, \div)$	×	×	×			

17/12/20

lecture 35

SEMI GROUP:- (closure property + follow Associative law)

Algebraic structure should hold the closure property.

A set S with binary operation * is called a semi group if following cond^n hold

i) S is closed w.r.t to binary operation *.
ii) S is associative w.r.t to binary operation *.

ex:

(S, *) S = {a, b, c}

$a*(b*c) = (a*b)*c$ (for associative)

if S = Z.

$(Z^+, +)$ Semi group

↓

$(2+3)+4 = 2+(3+4)$

lecture 36

MONOID:-

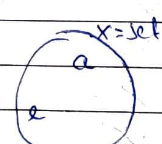
A set M a binary operation * is called a monoid if following cond^n hold:-

i) M is closed w.r.t binary operation *
ii) M is associative w.r.t *
iii) There exists identity element in M w.r.t *.

identity property

$a*e = e*a = a$

i.e. $a+0 = 0+a = a$.



$(N, \times)$ $2\times1 = 1\times2 = 2$. (with one) Monoid

contains 0

$(Z, +)$ $a+0 = 0+a = a$ Monoid

lecture 37

Example Monoid, semigroup, A.S.

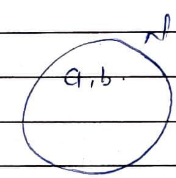
Q) Check whether set of +ve integers N is a monoid w.r.t binary operation * defined as

a*b = l.c.m (least common ~~factor~~ multiple)
∀ lcm(a,b) ∀ a,b ∈ N.

Sol^n

Closure P:- ✓

l.c.m(3,4) = 12 → Natural no.
↑
operator.



Associative:- ✓

$a*(b*c) = (a*b)*c$,

~~lcm(a, lcm(b,c))~~ = l.c.m [a, lcm(b,c)]

lcm(lcm(a,b), c)

let say 3, 4, 5

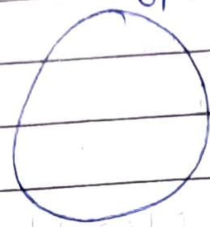
lcm(60) = lcm(60) ✓

Identity: ✓

l.c.m of (a,1) = ~~l.c.m a~~ l.c.m(1,a)

Q) Let $(G, *)$ be a semigroup where $G = \{x, y\}$ & $x * x = y$. Then P.T. $x * y = y * x$.

G



given $\Rightarrow x * x = y$

$$x * y = x * \underbrace{x * x}$$
$$= (x * x) * x$$
$$= y * x.$$