[A]

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$ Open ckt. dr. pt. i/p ad.

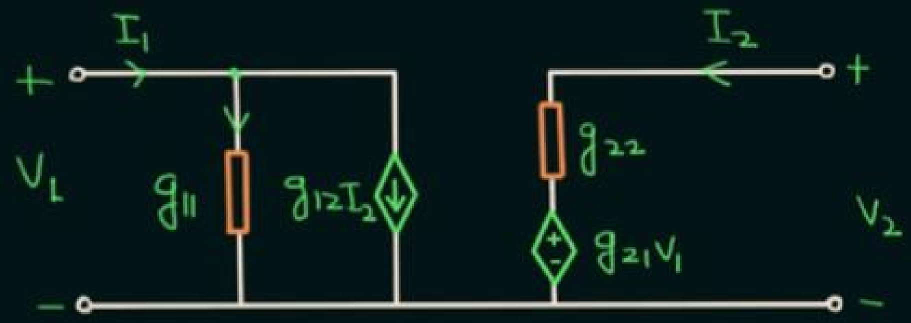
$$* \; g_{11} = \frac{1}{V_1/I_1 |_{I_2=0}} \Rightarrow g_{11} = \frac{1}{Z_{11}}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$ Open ckt. for. vol. gain

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$ Short Ckt. rev. curr. gain

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ Short ckt. dr. pt. o/p imp.

$$* \; g_{22} = \frac{1}{I_2/V_2 |_{V_1=0}} \Rightarrow g_{22} = 1/Y_{22}$$



$\rightarrow$ $[A] = [G][B]$

$[B] = [H][A] \Rightarrow [A] = [H]^{-1}[B] \Rightarrow [H] = [G]^{-1}$

$\Rightarrow [G] = [H]^{-1}$

$h \rightarrow g$:-

$g_{11} = h_{22}/|H| \quad g_{12} = -h_{12}/|H|$

$g_{21} = -h_{21}/|H| \quad g_{22} = h_{11}/|H|$

$g \rightarrow h$:-

$h_{11} = g_{22}/|G| \quad h_{12} = -g_{12}/|G|$

$h_{21} = -g_{21}/|G| \quad h_{22} = g_{11}/|G|$

Z-Parameters to g-Parameters Conversion

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{—— (I)}$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{—— (II)}$$

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \text{—— (III)}$$
$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \text{—— (IV)}$$

$$V_1 - Z_{12}I_2 = Z_{11}I_1$$

$$\Rightarrow I_1 = \frac{1}{Z_{11}}V_1 - \frac{Z_{12}}{Z_{11}}I_2 \quad \text{—— (V)}$$

$I_1 \rightarrow$ (II) :-

$$V_2 = Z_{21}\left[\frac{1}{Z_{11}}V_1 - \frac{Z_{12}}{Z_{11}}I_2\right] + Z_{22}I_2$$

$$V_2 = \frac{Z_{21}}{Z_{11}}V_1 - \frac{Z_{21}Z_{12}}{Z_{11}}I_2 + Z_{22}I_2$$

$$V_2 = \frac{Z_{21}}{Z_{11}}V_1 + \left[Z_{22} - \frac{Z_{21}Z_{12}}{Z_{11}}\right]I_2$$

$$\frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{11}} = \frac{|Z|}{Z_{11}}$$

$$V_2 = \frac{Z_{21}}{Z_{11}}V_1 + \frac{|Z|}{Z_{11}}I_2 \quad \text{—— (VI)}$$

$$g_{11} = \frac{1}{Z_{11}}$$

$$g_{12} = -\frac{Z_{12}}{Z_{11}}$$

$$g_{21} = \frac{Z_{21}}{Z_{11}}$$

$$g_{22} = \frac{|Z|}{Z_{11}}$$

$g \rightarrow z$:-

$$Z_{11} = 1/g_{11}$$

$$Z_{12} = -g_{12}/g_{11}$$

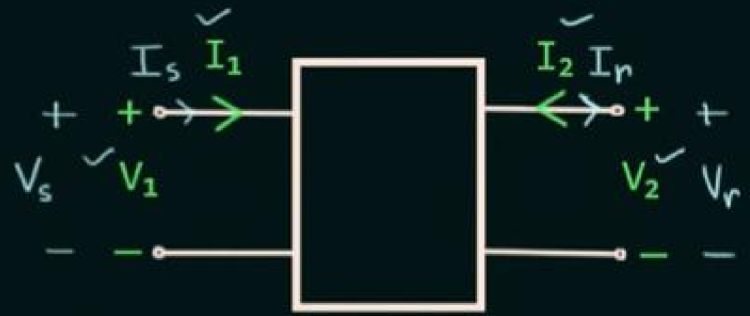
$$Z_{21} = g_{21}/g_{11}$$

$$Z_{22} = |G|/g_{11}$$

ABCD-Parameters (or) Transmission Parameters

>> Also known as T-Parameters, Cascade Parameters and Chain Parameters.

$V_s = V_1$, $I_s = I_1$, $V_r = V_2$, $I_r = -I_2$



V_r and $I_r \rightarrow$ ind.

V_s and $I_s \rightarrow$ dep.

$$\begin{pmatrix} V_s \\ I_s \end{pmatrix} = f\begin{pmatrix} V_r \\ I_r \end{pmatrix}$$

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{[T]} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 + B(-I_2) \text{ —— (i)}$$

$$I_1 = CV_2 + D(-I_2) \text{ —— (ii)}$$

$$V_1 = AV_2 - BI_2 \text{ —— (iii)}$$

$$I_1 = CV_2 - DI_2 \text{ —— (iv)}$$

t-parameters
(or)
I.T.P.
(or)
abcd par.

$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ Open ckt. rev. vol. gain

$C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$ Open ckt. rev. tr. ad.

$B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0}$ Short ckt rev. tr. imp.

$D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0}$ Short ckt rev. current gain

ABCD-Parameters to Z-Parameters Conversion

$$V_1 = AV_2 - BI_2 \quad \text{— (I)}$$
$$I_1 = CV_2 - DI_2 \quad \text{— (II)}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{— (III)}$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{— (IV)}$$

$$I_1 + DI_2 = CV_2$$
$$\Rightarrow V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2 \quad \text{— (V)}$$

$V_2 \rightarrow$ (I) :-

$$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2$$
$$V_1 = \frac{A}{C}I_1 + \frac{AD}{C}I_2 - BI_2$$
$$V_1 = \frac{A}{C}I_1 + \left[\frac{AD}{C} - B\right]I_2$$

$$\frac{AD}{C} - B = \frac{AD - BC}{C} = \frac{|T|}{C}$$

$$V_1 = \frac{A}{C}I_1 + \frac{|T|}{C}I_2 \quad \text{— (IV)}$$

$$Z_{11} = \frac{A}{C}$$
$$Z_{12} = \frac{|T|}{C}$$
$$Z_{21} = \frac{1}{C}$$
$$Z_{22} = \frac{D}{C}$$

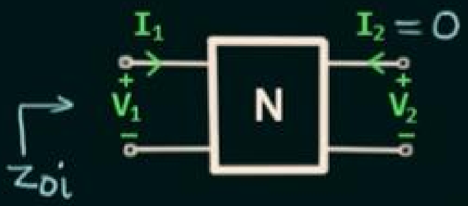
$Z \rightarrow ABCD$:-

$$A = \frac{Z_{11}}{Z_{21}}$$
$$B = \frac{|Z|}{Z_{21}}$$
$$C = \frac{1}{Z_{21}}$$
$$D = Z_{22}/Z_{21}$$

Open Circuit & Short Circuit Impedances (in terms of ABCD-Parameters)

$$V_1 = AV_2 - BI_2 \quad \text{—— (I)} \qquad\qquad I_1 = CV_2 - DI_2 \quad \text{—— (II)}$$

1) Open Circuit i/p Impedance:



$$Z_{oi} = \left.\frac{\overset{AV_2}{V_1}}{\underset{CV_2}{I_1}}\right|_{I_2=0}$$

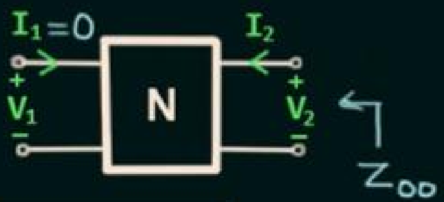
$$Z_{oi} = \frac{A\cancel{V_2}}{C\cancel{V_2}} \Rightarrow \boxed{Z_{oi} = \frac{A}{C}}$$

3) Short Circuit i/p Impedance:

$$Z_{si} = \left.\frac{\overset{-BI_2}{V_1}}{\underset{-DI_2}{I_1}}\right|_{V_2=0}$$

$$Z_{si} = \frac{-BI_2}{-DI_2} \Rightarrow \boxed{Z_{si} = \frac{B}{D}}$$

2) Open Circuit o/p Impedance:



$$Z_{oo} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$0 = CV_2 - DI_2$$

$$\Rightarrow \frac{V_2}{\ldots} = \frac{D}{\ldots}$$

$$\boxed{Z_{oo} = \frac{D}{\ldots}}$$

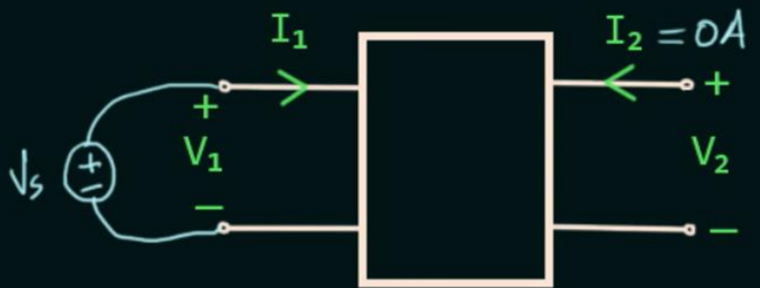
H.W. ✓

4) Short Circuit o/p Impedance:

$$Z_{so} = ?$$

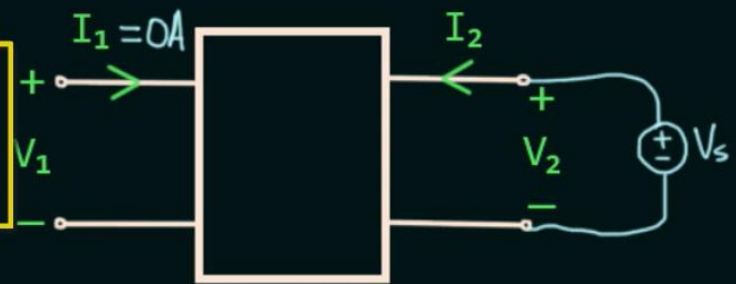
Concept of Symmetry in Two-Port Networks

>> A two-port network is said to be symmetrical if the ratio of excitation to response remains the same at both (① and ②) the ports independently w.r.t. the defined circuit conditions such as open circuit or short circuit.

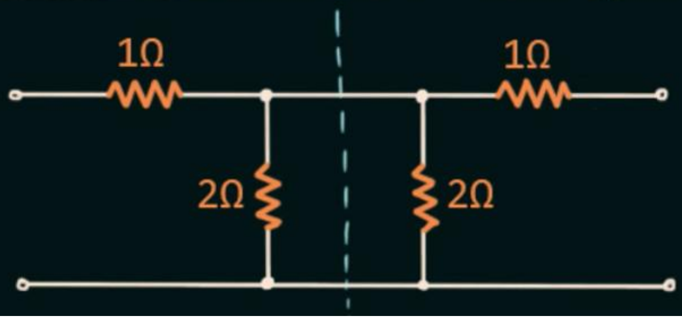


* imp

$$\boxed{\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}}$$



Note: For small independent networks we can identify the symmetry using the mirror image property.



Condition for Symmetry in Two-Port Networks

>> Condition for Symmetry:

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$

>> In terms of Z-Parameters:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{— (i)}$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{— (ii)}$$

$V_1 = V_s$ and $I_2 = 0$

$$V_s = Z_{11}I_1 \Rightarrow \left.\frac{V_s}{I_1}\right|_{I_2=0} = Z_{11}$$

$V_2 = V_s$ and $I_1 = 0$

$$\boxed{Z_{11} = Z_{22}}$$

$$V_s = Z_{22}I_2 \Rightarrow \left.\frac{V_s}{I_2}\right|_{I_1=0} = Z_{22}$$

>> In terms of Y-Parameters:

$$Z_{11} = Y_{22}/|Y|$$
$$Z_{22} = Y_{11}/|Y|$$
$$\Rightarrow \frac{Y_{22}}{|Y|} = \frac{Y_{11}}{|Y|} \Rightarrow Y_{11} = Y_{22}$$

>> In terms of h-Parameters:

$$Z_{11} = \frac{|H|}{h_{22}}$$
$$Z_{22} = 1/h_{22}$$
$$\Rightarrow \frac{|H|}{h_{22}} = \frac{1}{h_{22}} \Rightarrow |H| = 1$$

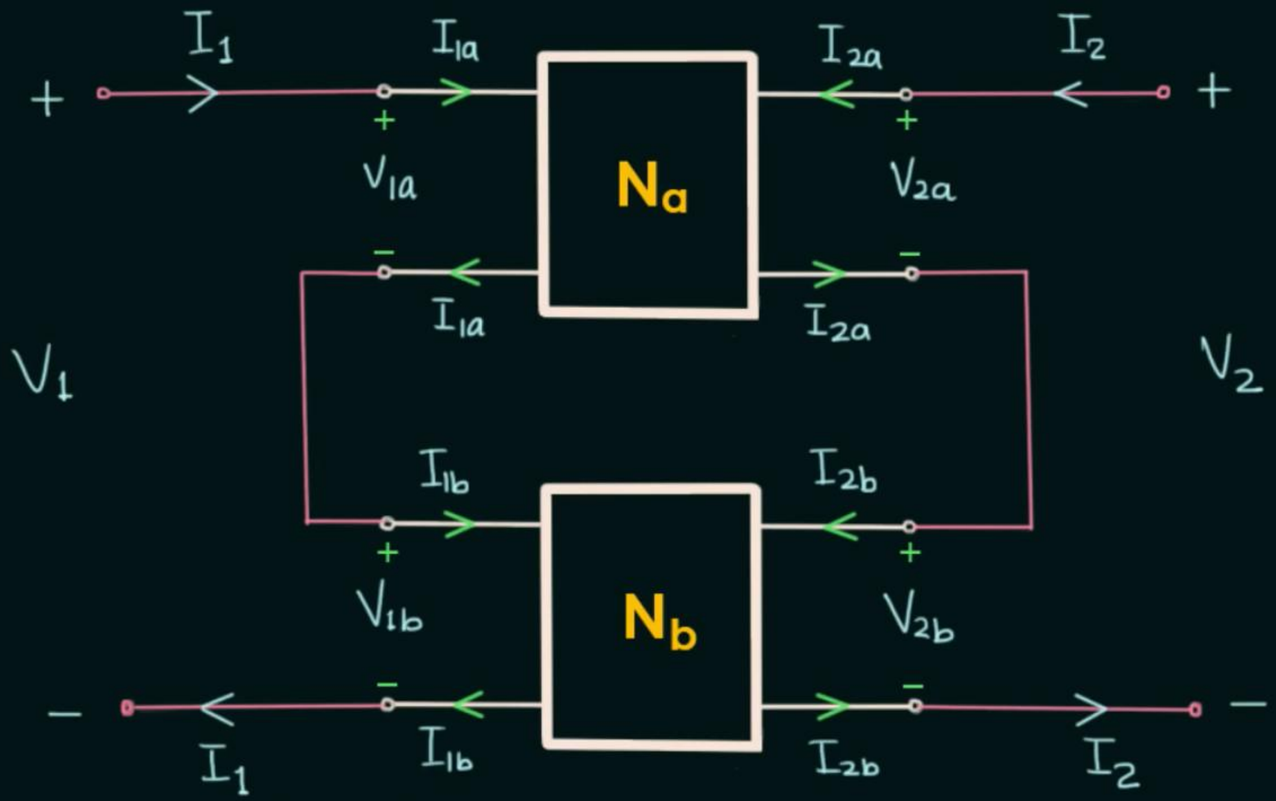
>> In terms of g-Parameters:

$$Z_{11} = 1/g_{11}$$
$$Z_{22} = |G|/g_{11}$$
$$\Rightarrow \frac{1}{g_{11}} = \frac{|G|}{g_{11}} \Rightarrow |G| = 1$$

>> In terms of ABCD-Parameters:

$$Z_{11} = A/C \quad Z_{22} = D/C \Rightarrow \frac{A}{C} = \frac{D}{C} \Rightarrow A = D$$

Series-Series Interconnection



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z_a]\begin{bmatrix} \cancelto{I_1}{I_{1a}} \\ \cancelto{I_2}{I_{2a}} \end{bmatrix} + [Z_b]\begin{bmatrix} \cancelto{I_1}{I_{1b}} \\ \cancelto{I_2}{I_{2b}} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\{[Z_a]+[Z_b]\}}_{[Z]}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

imp.

$$[Z] = [Z_a] + [Z_b]$$

$$\rightarrow Z_{11} = Z_{11a} + Z_{11b}$$

$$[Z] = [Z_a] + [Z_b] + \cdots + [Z_n]$$

$$I_1 = I_{1a} = I_{1b}$$

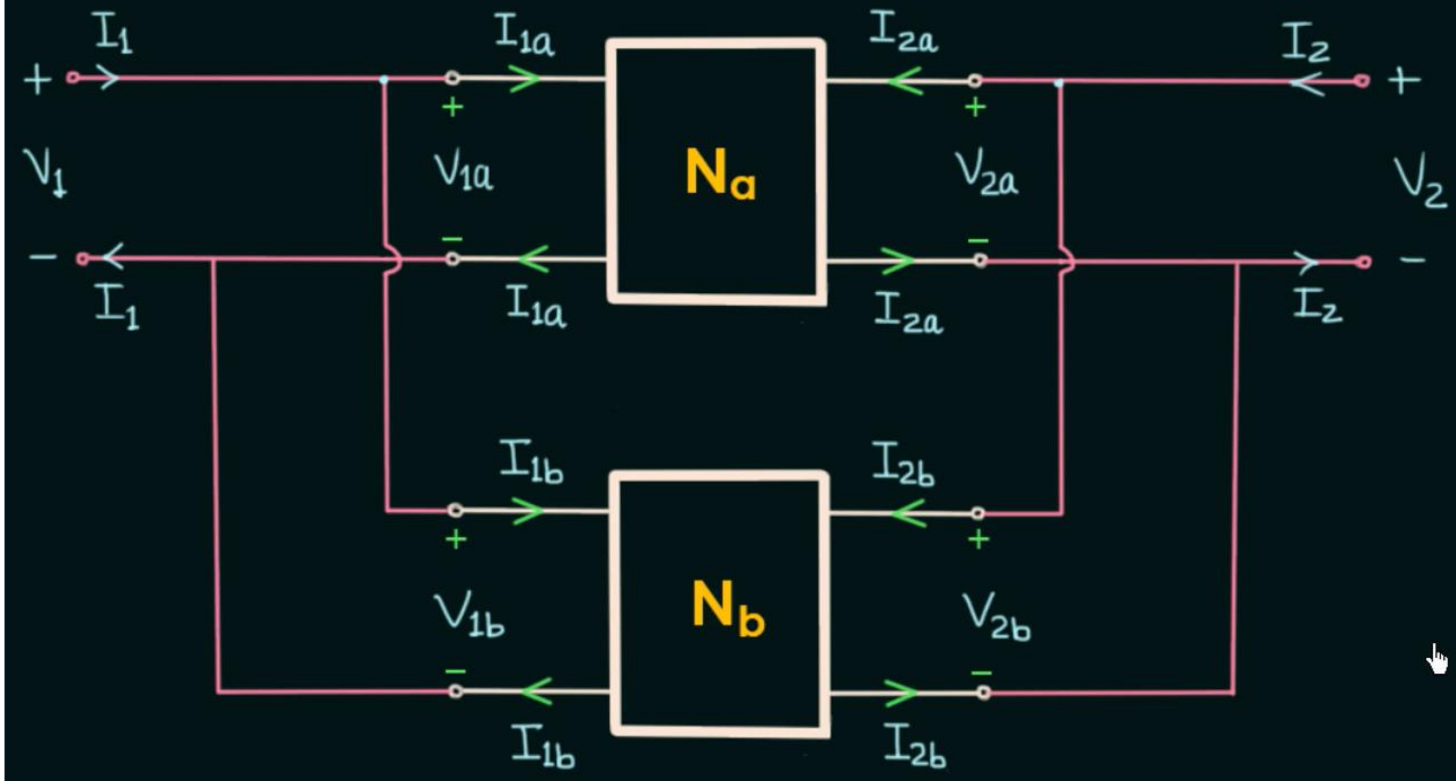
$$I_2 = I_{2a} = I_{2b}$$

$$+V_1 - V_{1a} - V_{1b} = 0$$

$$V_1 = V_{1a} + V_{1b} \quad \text{— (I)}$$

$$V_2 = V_{2a} + V_{2b} \quad \text{— (II)}$$

Parallel-Parallel Interconnection



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y_a]\begin{bmatrix} \overset{V_1}{\cancel{V_{1a}}} \\ \underset{V_2}{\cancel{V_{2a}}} \end{bmatrix} + [Y_b]\begin{bmatrix} \overset{V_1}{\cancel{V_{1b}}} \\ \underset{V_2}{\cancel{V_{2b}}} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\{[Y_a] + [Y_b]\}}_{[Y]}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[Y] = [Y_a] + [Y_b]$$

$$[Y] = [Y_a] + [Y_b] + \cdots + [Y_n]$$

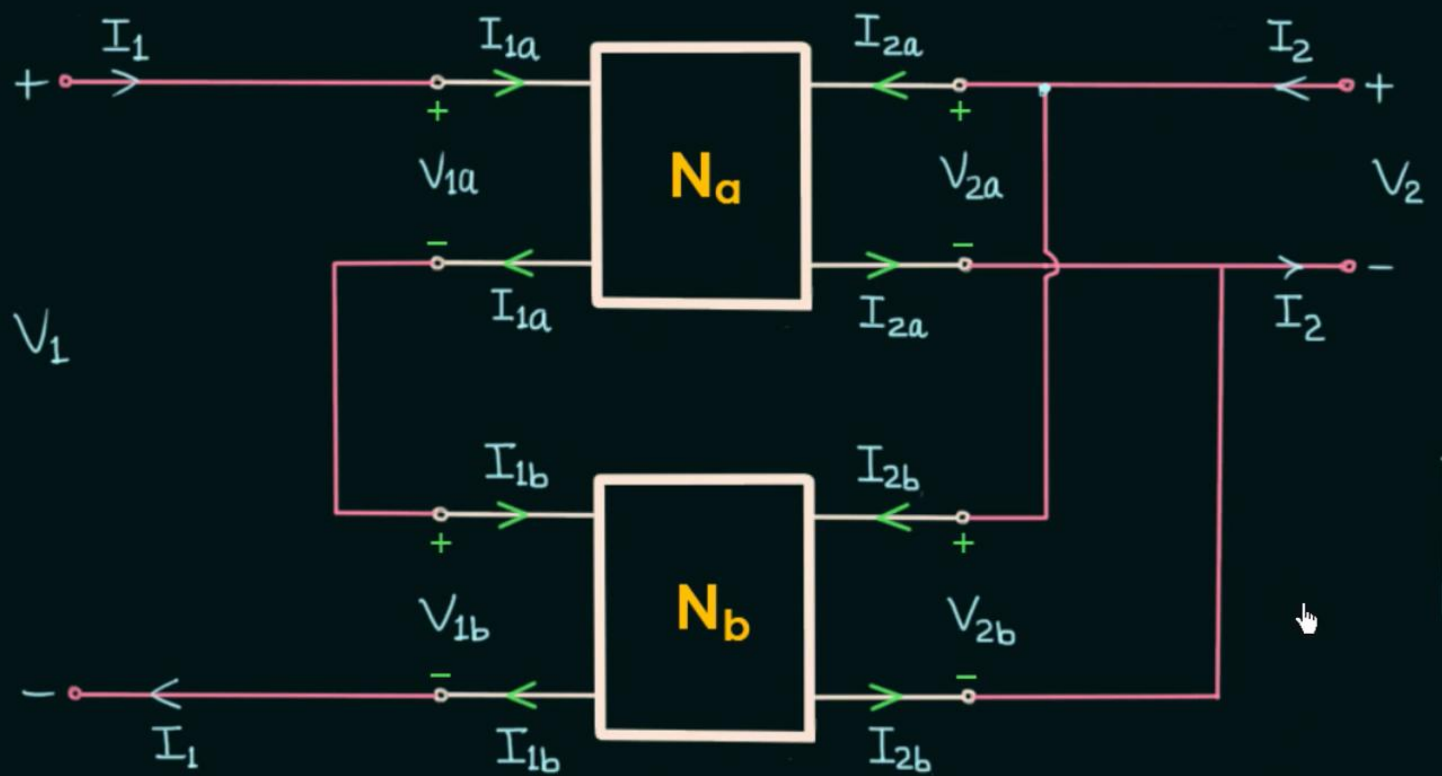
$$V_1 = V_{1a} = V_{1b}$$

$$V_2 = V_{2a} = V_{2b}$$

$$I_1 = I_{1a} + I_{1b} \quad \text{—— (i)}$$

$$I_2 = I_{2a} + I_{2b} \quad \text{—— (ii)}$$

Series-Parallel Interconnection



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [H_a] \begin{bmatrix} \overset{I_1}{\cancel{I_{1a}}} \\ \underset{V_2}{\cancel{V_{2a}}} \end{bmatrix} + [H_b] \begin{bmatrix} \overset{I_1}{\cancel{I_{1b}}} \\ \underset{V_2}{\cancel{V_{2b}}} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underbrace{\{[H_a] + [H_b]\}}_{[H]} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$[H] = [H_a] + [H_b]$$

$$[H] = [H_a] + [H_b] + \cdots + [H_n]$$

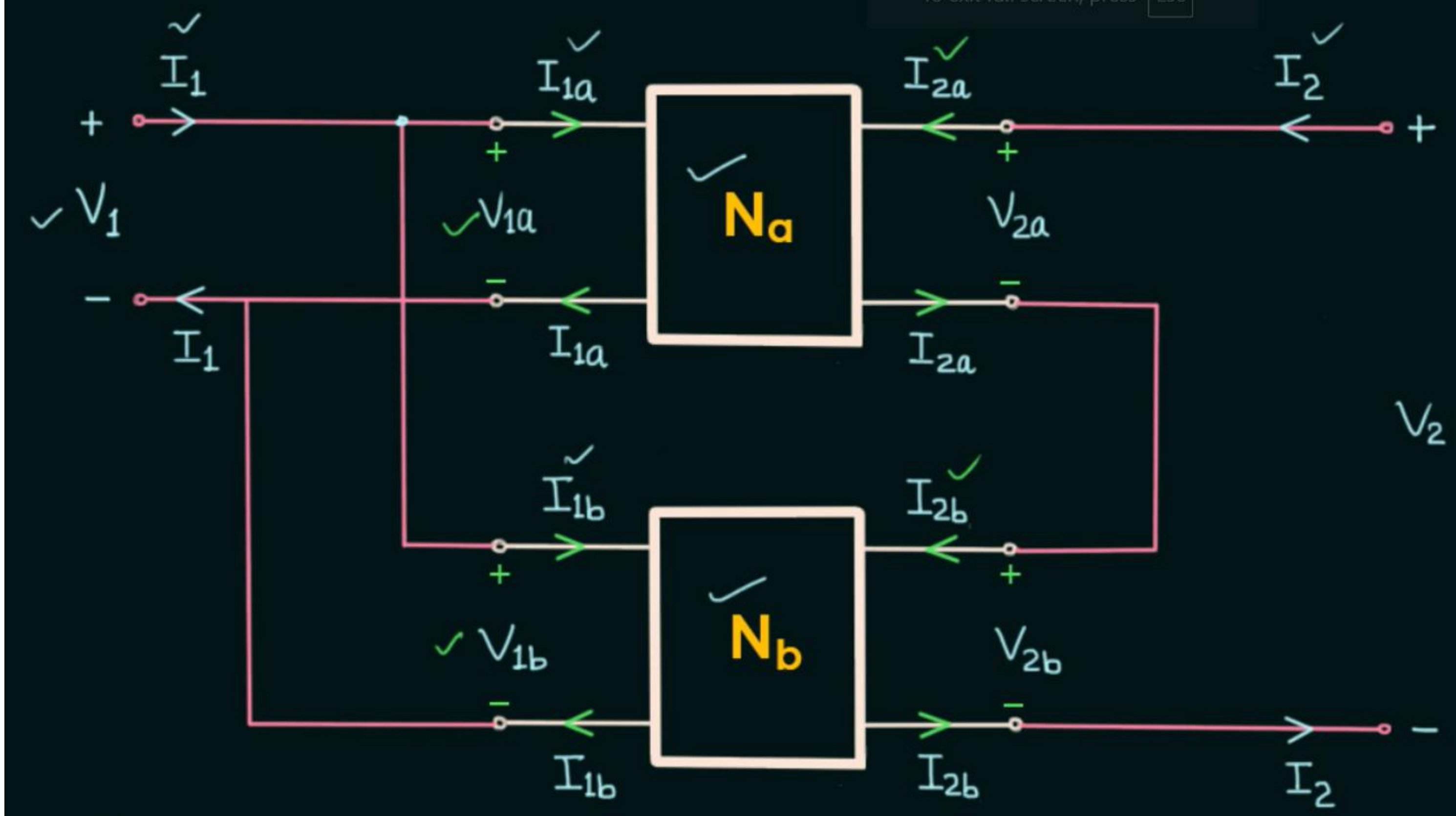
$$I_1 = I_{1a} = I_{1b}$$

$$V_2 = V_{2a} = V_{2b}$$

$$V_1 = V_{1a} + V_{1b} \quad \text{—— (i)}$$

$$I_2 = I_{2a} + I_{2b} \quad \text{—— (ii)}$$

Parallel-Series Interconnection



$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$[G_a]\begin{bmatrix} \cancel{V_{1a}} \\ \cancel{I_{2a}} \end{bmatrix} \qquad [G_b]\begin{bmatrix} \cancel{V_{1b}} \\ \cancel{I_{2b}} \end{bmatrix}$$

V_1 I_2 V_1 I_2

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [G_a]\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} + [G_b]\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \underbrace{\{[G_a] + [G_b]\}}_{[G]}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$[G] = [G_a] + [G_b]$$

$$[G] = [G_a] + [G_b] + \cdots + [$$

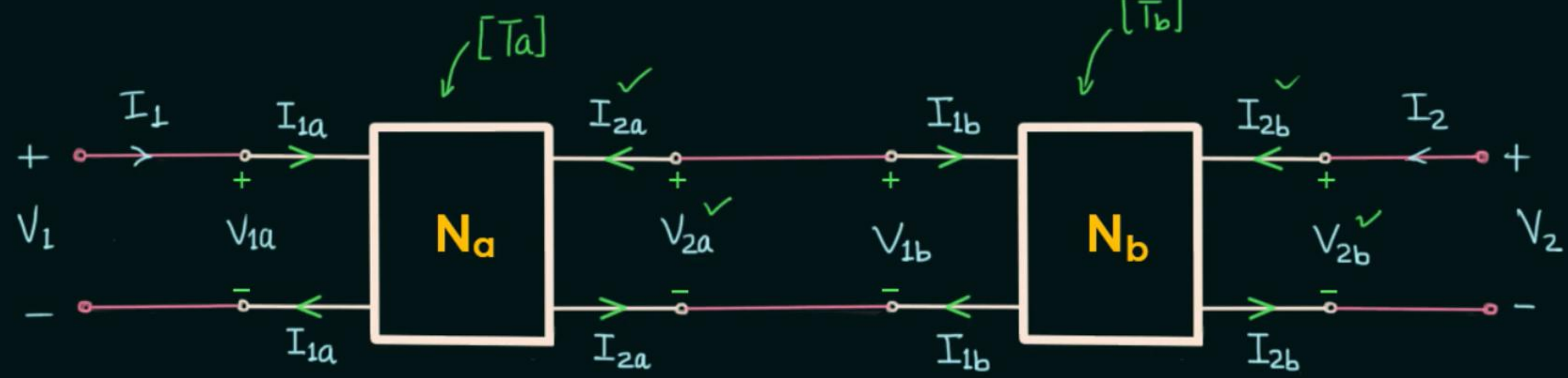
$$V_1 = V_{1a} = V_{1b}$$

$$I_2 = I_{2a} = I_{2b}$$

$$I_1 = I_{1a} + I_{1b} \quad \text{—— (I)}$$

$$V_2 = V_{2a} + V_{2b} \quad \text{—— (II)}$$

Cascade Interconnection



$$V_1 = V_{1a}, \quad I_1 = I_{1a}, \quad V_{2a} = V_{1b}, \quad I_{2a} = -I_{1b}, \quad V_{2b} = V_2 \text{ and } I_{2b} = I_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a] \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

V_{1b} $\quad$ I_{1b}

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a] \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a][T_b] \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

V_2 $\quad$ $-I_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{[T_a] \times [T_b]}_{[T]} \times \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

imp.

$$[T] = [T_a] \times [T_b]$$

Bartlett's Bisection Theorem

Z-par.

>> Albert Charles Bartlett

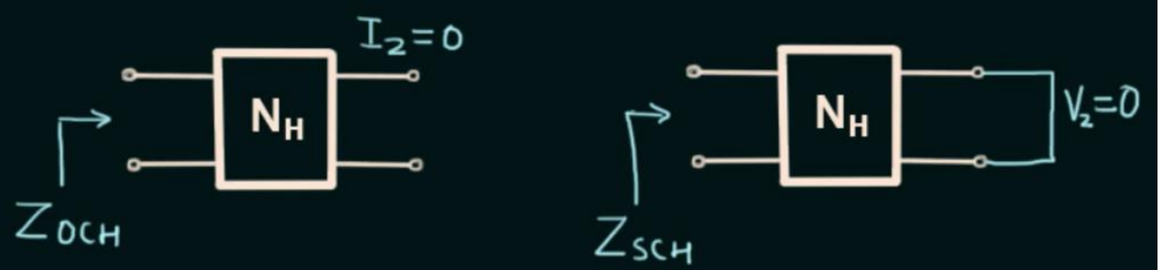
lattice n/w ? Shunt el. ?

>> Applicable to symmetrical two-port networks

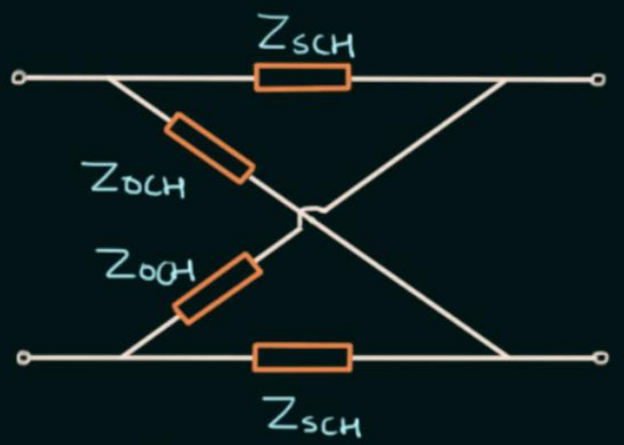
>> Basic symmetrical lattice

Step 1: Separate the n/w into two equal parts.

Step 2: Bisect and find out Z_{OCH} & Z_{SCH}.



Step 3: Construct basic symmetrical lattice using Z_{OCH} & Z_{SCH}.

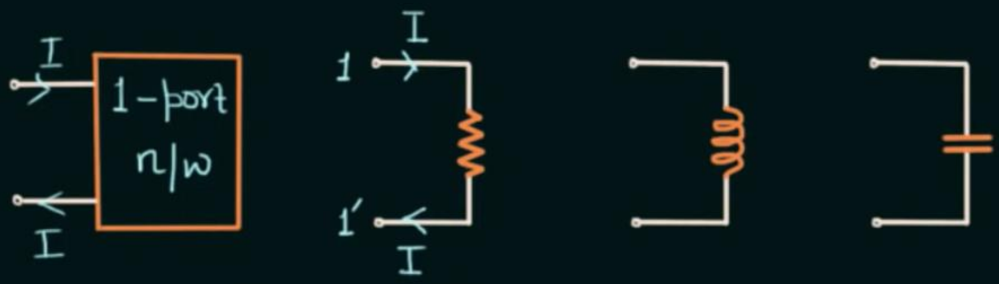


$$Z_{11} = Z_{22} = \frac{Z_{OCH} + Z_{SCH}}{2} \qquad Z_{12} = Z_{21} = \frac{Z_{OCH} - Z_{SCH}}{2}$$

Introduction to Two-Port Networks

Port: Pair of terminals through which a current may enter or leave the network.

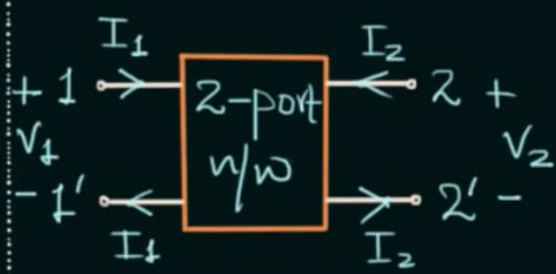
1-Port N/W: Two terminal devices such as R, L and C forms 1-port network.



net current $= I + (-I) = 0A$

>> Current entering from one terminal must leave from the other terminal.

2-Port N/W: Two-port networks have two separate ports.



V_1 V_2

I_1 I_2

* 2 var. are ind.

>> Black box should consist only linear (bidirectional) and passive elements.

>> Black box may contain energy storing elements. L and C

>> Black box may also contain dependent sources, but never contain an independent source.

the black box may also contain the dependent sources but never contain an

Z Parameters (or) Impedance Parameters (or) Open ckt. parameters

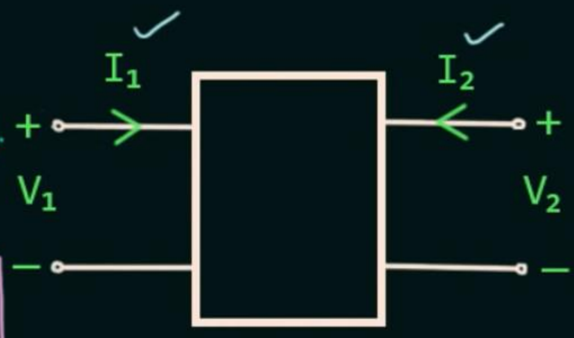
I_1 and $I_2 \rightarrow$ ind.

V_1 and $V_2 \rightarrow$ dep.

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = f\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

current mat.

[I]



$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—} (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—} (2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow [V] = [Z][I]$$

voltage mat. [V]

impedance matrix [Z]

$$V_1 - Z_{11} I_1 - Z_{12} I_2 = 0$$

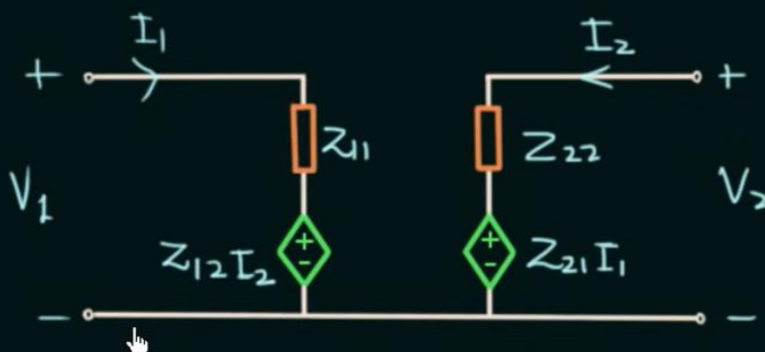
$$V_2 - Z_{21} I_1 - Z_{22} I_2 = 0$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ Open circuit driving pt. i/p imp.

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$ Open circuit forward tr. imp.

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$ Open circuit rev. tr. imp.

(H.W.) $Z_{22} = ?$



Z-Parameters (Solved Problem 1)

Question: Find the Z-parameters of the below given two-port network.

$$[z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

$$[z] = \begin{bmatrix} 6 & 4 \\ 4 & 10 \end{bmatrix} \quad \text{ans.}$$

Solution:

$$+V_1 - 2I_1 - 4(I_1 + I_2) = 0$$

$$V_1 = 6I_1 + 4I_2 \quad \text{— (1)}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$+V_2 - 6I_2 - 4(I_1 + I_2) = 0$$

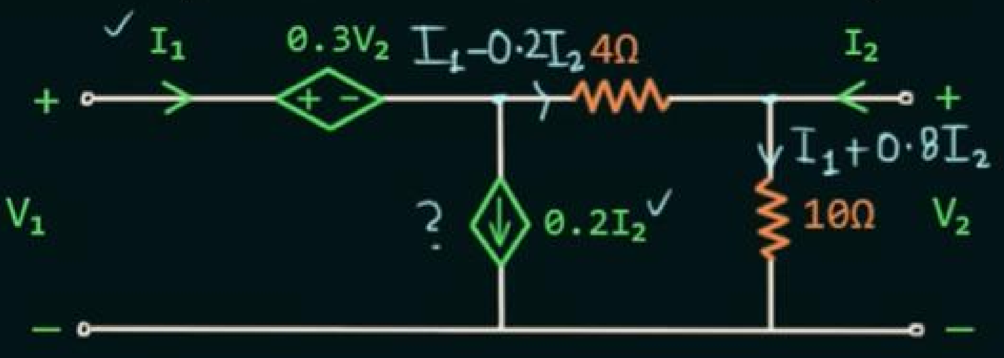
$$V_2 = 4I_1 + 10I_2 \quad \text{— (2)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

NESO ACADEMY

Z-Parameters (Solved Problem 2)

Question: The Z-parameter matrix of the two-port network as shown below is



$$[z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Solution:

$$+V_1 - 0.3V_2 - 4(I_1 - 0.2I_2) - 10(I_1 + 0.8I_2) = 0$$

$$V_1 - 0.3V_2 - 14I_1 - 7.2I_2 = 0 \quad \text{— (1)}$$

$$V_1 - 0.3(10I_1 + 8I_2) - 14I_1 - 7.2I_2 = 0$$

$$V_1 = 17I_1 + 9.6I_2 \quad \text{— (3)}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$+V_2 - 10(I_1 + 0.8I_2) = 0$$

$$V_2 = 10I_1 + 8I_2 \quad \text{— (2)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$[Z] = \begin{bmatrix} 17 & 9.6 \\ 10 & 8 \end{bmatrix}$$

ans

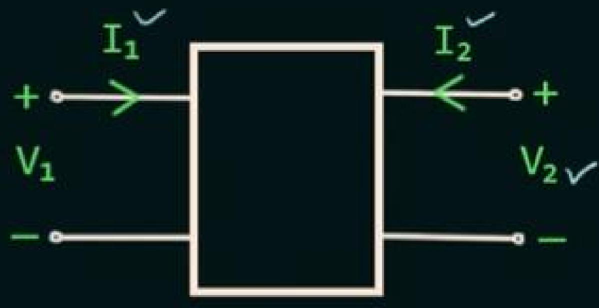
Y-Parameters (or) Admittance Parameters

V_1 and V_2 → ind.

I_1 and I_2 → dep.

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = f \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$[I] = [Y][V]$$



curr. curr. curr.

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{— (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{— (2)}$$

$$\begin{array}{ccc} [I] & [Y] & [V] \\ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} & = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} & \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \end{array}$$

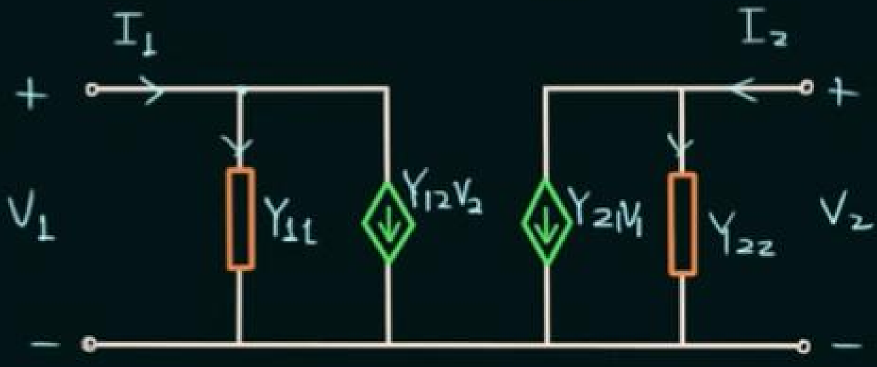
↖ admittance matrix

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$ Short ckt. Driving pt. i/p ad.

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$ Short ckt. for. tr. ad.

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$ Short ckt. rev. tr. ad.

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$ Short ckt. driving pt. o/p ad.



Y-Parameters to Z-Parameters Conversion

$$[V] = [Z][I] \quad \text{and} \quad [I] = [Y][V] \Rightarrow [V] = [Y]^{-1}[I]$$

imp.

$$[Z] = [Y]^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$= \frac{adj(Y)}{|Y|}$$

$$= \frac{1}{|Y|} \begin{bmatrix} (-1)^{1+1} Y_{22} & -Y_{21} \\ -Y_{12} & Y_{11} \end{bmatrix}^{T}_{2\times 2}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{|Y|} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$|Y| = Y_{11} \times Y_{22} - Y_{21} \times Y_{12}$$

$$Z_{11} = \frac{Y_{22}}{|Y|} \qquad Z_{21} = \frac{-Y_{21}}{|Y|}$$

$$Z_{12} = \frac{-Y_{12}}{|Y|} \qquad Z_{22} = \frac{Y_{11}}{|Y|}$$

2:35

Z-Parameters to Y-Parameters Conversion

$[V] = [Z][I]$ and $[I] = [Y][V]$

$\Downarrow$

$[I] = [Z]^{-1}[V]$

imp. $[Y] = [Z]^{-1}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{2\times 2} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_{2\times 2}^{-1}$$

adj(A)/|A|

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{|Z|}\begin{bmatrix} Z_{22} & -Z_{21} \\ -Z_{12} & Z_{11} \end{bmatrix}^{T}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{|Z|}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$Y_{11} = \dfrac{Z_{22}}{|Z|}$ $\qquad$ $Y_{21} = \dfrac{-Z_{21}}{|Z|}$

$Y_{12} = \dfrac{-Z_{12}}{|Z|}$ $\qquad$ $Y_{22} = \dfrac{Z_{11}}{|Z|}$

h-Parameters (or) Hybrid Parameters

I_1 and $V_2 \longrightarrow$ ind.

V_1 and $I_2 \longrightarrow$ dep.

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = f\begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \text{ —— (I)} \quad \text{KVL eq}^n$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \text{ —— (II)} \quad \text{KCL eq}^n$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{[H]} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$ Short ckt. driving pt. i/p imp.

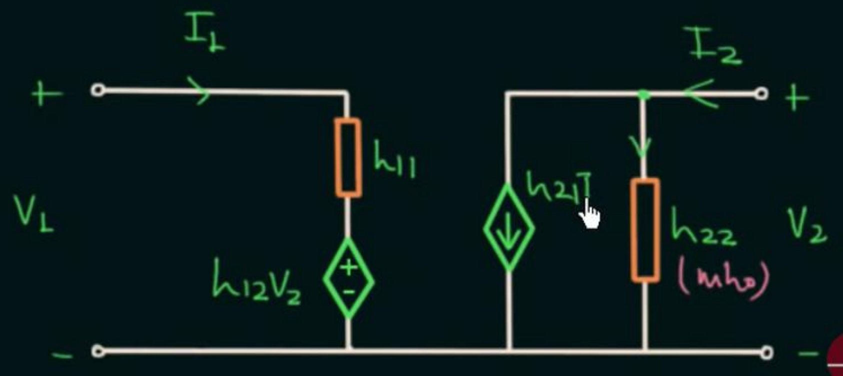
$$* \; h_{11} = \frac{1}{\left.I_1/V_1\right|_{V_2=0}} \Rightarrow h_{11} = 1/Y_{11}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$ Short ckt. fwd. current gain.

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$ Open ckt. rev. voltage gain.

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$ (H.W.)

$$* \; h_{22} = \frac{1}{\left.V_2/I_2\right|_{I_1=0}} \Rightarrow h_{22} = 1/Z_{22}$$



h-Parameters to Z-Parameters Conversion

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{— (I)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{— (II)}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{— (III)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{— (IV)}$$

rearrange eq^n (II) -

$$-h_{21} I_1 + I_2 = h_{22} V_2$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{— (V)}$$

$V_2 \rightarrow$ (I)

$$V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$V_1 = h_{11} I_1 - \frac{h_{21} h_{12}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left[h_{11} - \frac{h_{21} h_{12}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\frac{h_{11} h_{22} - h_{21} h_{12}}{h_{22}} = \frac{|H|}{h_{22}}$$

$$V_1 = \frac{|H|}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \quad \text{— (VI)}$$

$$Z_{11} = \frac{|H|}{h_{22}} \qquad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} \qquad Z_{22} = \frac{1}{h_{22}}$$

$Z \rightarrow h$

$$h_{11} = \frac{|Z|}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = -Z_{21}/Z_{22}$$

$$h_{22} = 1/Z_{22}$$

h-Parameters to Y-Parameters Conversion

$$V_1 = h_{11}I_1 + h_{12}V_2 \text{ --- (I)}$$
$$I_2 = h_{21}\underbrace{I_1}_{V_2} + h_{22}V_2 \text{ --- (II)}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \text{ --- (III)}$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \text{ --- (IV)}$$

$$V_1 - h_{12}V_2 = h_{11}I_1$$
$$\Rightarrow I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2 \text{ --- (V)}$$

$I_1 \rightarrow$ (II) -

$$I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 - \frac{h_{21}h_{12}}{h_{11}}V_2 + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 + \left[h_{22} - \frac{h_{21}h_{12}}{h_{11}}\right]V_2$$

$$\frac{h_{11}h_{22} - h_{21}h_{12}}{h_{11}} = \frac{|H|}{h_{11}}$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 + \frac{|H|}{h_{11}}V_2 \text{ --- (VI)}$$

$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = -\frac{h_{12}}{h_{11}}$$
$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \frac{|H|}{h_{11}}$$

$Y \rightarrow h$

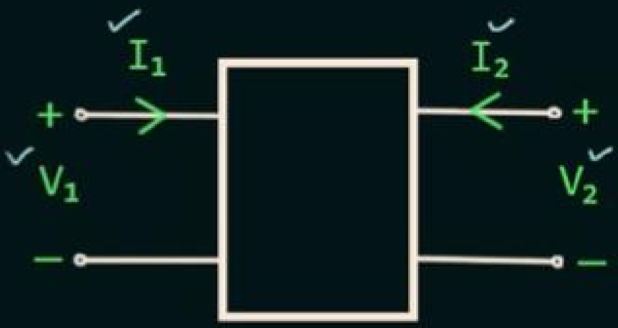
$$h_{11} = 1/Y_{11}$$
$$h_{12} = -Y_{12}/Y_{11}$$
$$h_{21} = Y_{21}/Y_{11}$$
$$h_{22} = |Y|/Y_{11}$$

g-Parameters (or) Inverse Hybrid Parameters

V_1 and $I_2 \rightarrow$ ind.

I_1 and $V_2 \rightarrow$ dep.

$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$



$$I_1 = g_{11} V_1 + g_{12} I_2 \quad \xrightarrow{\text{KCL}} \text{(I)}$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \quad \xrightarrow{\text{KVL}} \text{(II)}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}}_{[G]} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$ Open ckt. dr. pt. i/p ad.

$$g_{11} = \frac{1}{V_1/I_1 \,|\, I_2=0} \Rightarrow g_{11} = \frac{1}{Z_{11}}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$ Open ckt. for. vol. gain

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$ Short ckt. rev. curr. gain

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ Short ckt. dr. pt. o/p imp.

$$* \; g_{22} = \frac{1}{I_2/V_2 \,|\, V_1=0} \Rightarrow g_{22} = 1/Y_{22}$$