Bay's Theorem

If $E_1, E_2, \ldots E_n$ are n mutually exclusive and exhaustive events in a sample space, and A is an arbitrary event for which $P(A) \neq 0$, then the conditional probability of occurence of E_i, given that A has already occured is given by
$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum P(E_i) \cdot P(A/E_i)} \quad i = 1, 2, \ldots n$$

Q Three urns I, II, III contain 6 red and 4 black balls, 2 red and 6 black balls; and 1 red and 8 black balls respectively. An urn is chosen randomly and a ball is drawn from the urn. If the ball drawn is red, find the probability that it was drawn from urn I.

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let A: Red ball is drawn

E_1 : Urn I is chosen, $P(E_1) = \frac{1}{3}$, $P(A/E_1) = \frac{6}{10} = \frac{3}{5}$

E_2 : Urn II is chosen, $P(E_2) = \frac{1}{3}$, $P(A/E_2) = \frac{2}{8} = \frac{1}{4}$

E_3 : Urn III is chosen, $P(E_3) = \frac{1}{3}$, $P(A/E_3) = \frac{1}{9}$

$\therefore$ Req. Probability $= P(E_1/A) = \dfrac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$

$$= \frac{\cancel{\frac{1}{3}} \times \frac{3}{5}}{\left(\cancel{\frac{1}{3}} \times \frac{3}{5}\right) + \left(\cancel{\frac{1}{3}} \times \frac{1}{4}\right) + \left(\cancel{\frac{1}{3}} \times \frac{1}{9}\right)} = \frac{108}{173}$$

Q An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter driver is 0.01, a car driver is 0.03 and a truck driver is 0.15. If one of an insured person meets with an accident, what is the probability that he is a car driver?

Sol. Let A: An insured person meets with an accident.

E_1: Person is an insured scooter driver.

$$P(E_1) = \frac{2000}{2000+4000+6000} = \frac{2}{12}, \quad P(A/E_1) = 0.01$$

E_2: Person is an insured car driver.

$$P(E_2) = \frac{4000}{2000+4000+6000} = \frac{4}{12}, \quad P(A/E_2) = 0.03$$

E_3: Person is an insured truck driver.

$$P(E_3) = \frac{6000}{2000+4000+6000} = \frac{6}{12}, \quad P(A/E_3) = 0.15$$

$$\therefore P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1).P(A/E_1) + P(E_2)P(A/E_2) + P(E_3).P(A/E_3)}$$

$$= \frac{\frac{4}{12} \cdot (0.03)}{\frac{2}{12}(0.01) + \frac{4}{12}(0.03) + \frac{6}{12}(0.15)} = \frac{3}{26}$$

Q Two urns I and II contain 3 Red and 4 black balls, 2 red and 5 black balls respectively. A ball is transfered from urn I to urn II and then a ball is drawn from urn II. If the ball drawn is found to be red, find the probability that the ball transferred from urn I is red.

Sol Let A: A Red ball is drawn from urn II.

E_1: Ball transferred from urn I is red

$$P(E_1) = \frac{3}{7}, \quad P(A/E_1) = \frac{3}{8}$$

E_2: Ball transferred from urn I is black

$$P(E_2) = \frac{4}{7}, \quad P(A/E_2) = \frac{2}{8}$$

$$\therefore P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{3}{7} \cdot \frac{3}{8}}{\left(\frac{3}{7} \cdot \frac{3}{8}\right) + \left(\frac{4}{7} \cdot \frac{2}{8}\right)} = \frac{9}{17}$$

Q A card from a deck of 52 cards is missing. 2 cards are drawn from the remaining deck of 51 cards and are found to be of spade. Find the probability that the missing card is of spade.

Sol

Let A: 2 cards of spade are drawn from a deck of 51 cards.

E_1: missing card is of spade

$$P(E_1) = \frac{13}{52}, \quad P(A/E_1) = \frac{^{12}C_2}{^{51}C_2}$$

E_2: missing card is a non-spade card.

$$P(E_2) = \frac{39}{52}, \quad P(A/E_2) = \frac{^{13}C_2}{^{51}C_2}$$

$$\therefore P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$$

$$= \frac{\frac{13}{\cancel{52}} \times \frac{12 \times 11}{\cancel{51 \times 50}}}{\left(\frac{13}{\cancel{52}} \times \frac{12 \times 11}{\cancel{51 \times 50}}\right) + \left(\frac{39}{\cancel{52}} \times \frac{13 \times 12}{\cancel{51 \times 50}}\right)}$$

$$= \frac{11}{11+39} = \frac{11}{50}$$