Formula for finding Linear spline

Linear Spline is denoted by $S_i(x)$ for $1 \le i \le n$

And defined as

$$\boxed{S_i(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i)}$$

we put $i = 1, 2, 3,$ we get $S_1(x)$, $S_2(x)$, ... etc

Q.1 Let $f(x) = x^3$, Compute the Linear spline s which interpolates f at the Knots 0, 1, 2.

Sol: Here $f(x) = x^3$

$$\begin{array}{c|c|c|c} & \overleftrightarrow{\quad s_1 \quad} & & \overleftrightarrow{\quad s_2 \quad} \end{array}$$

x	0	1	2
y	0	1	8
	y_1	y_2	y_3

Now Let Linear spline is denoted by S_i and defined as

$$S_i(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i), \quad 1 \le i \le n$$

Note (Here 3 data Point is given so 2 spline is Possible)

$$S_1(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$S_1(x) = 0 + \frac{1-0}{1-0}(x-0) = x, \quad x \in [0,1] \text{ Ans}$$

And

$$S_2(x) = y_2 + \frac{y_3 - y_2}{x_3 - x_2}(x - x_2)$$

$$= 1 + \frac{8-1}{2-1}(x-1)$$

$$= 1 + 7(x-1)$$

$$S_2(x) = 7x - 6, \quad x \in [1,2] \text{ Ans}$$

Q2 What the value of the velocity at <u>16</u> is by using Linear spline interpolation, with the help of following table

Time(s) :	0	10	20	15	22.5
Velocity(m/s) :	0	227.04	517.35	362.78	602.97

<u>Solⁿ</u> First we write given data in ~~[illegible]~~ ~~ascending~~ ascending order.

Time(s) :	0	10	15	20	22.5
Velocity :	0	227.04	362.78	517.35	602.97

$$\because\ s_i(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i) \qquad x_i \le x \le x_{i+1}$$

$$15 \le x \le 20$$

$$= 362.78 + \frac{517.35 - 362.78}{20 - 15}(x - 15)$$

$$= 362.78 + 30.913\,(16 - 15) \qquad \because x = 16$$

$$= 393.7 \text{ m/s}$$

Q.3 find y at $x = 6$ if

x	22	42	52	82	100
y	4181	4179	4186	4199	4217

Quadratic spline

If $(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$ data are given then Quadratic spline is denoted by $S_i(x)$ and defined as

$$S_i(x) = \frac{z_{i+1} - z_i}{2[x_{i+1} - x_i]}(x - x_i)^2 + z_i(x - x_i) + y_i$$

Here z are unknown, and find as

$$z_{i+1} = -z_i + 2\left[\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right], \quad 0 \le i \le n-1$$

Note we take $z_0 = 0$

Q1 Find Quadratic spline function and also find approximate Value of $y(1.5)$, given data is

$(-2,0), (0,2), (2,7)$

Sol^n

We know that Quadratic spline is given as

$$s_i(x) = \frac{z_{i+1} - z_i}{2(x_{i+1} - x_i)}(x - x_i)^2 + z_i(x - x_i) + y_i \longrightarrow ①$$

Here z is given as

$$z_{i+1} = -z_i + 2\left[\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right], \quad 0 \le i \le n-1$$

Here $n = 3$ in question

Put $z_0 = 0$

$0 \le i \le 3-1$

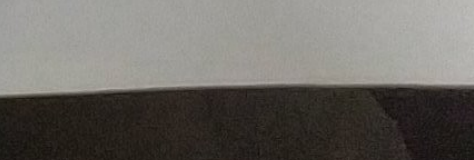
$0 \le i \le 2$

$$z_1 = -z_0 + 2\left[\frac{y_1 - y_0}{x_1 - x_0}\right]$$

$$z_1 = 0 + 2\left[\frac{2 - 0}{0 + 2}\right] = 2$$

$$z_2 = -z_1 + 2\left[\frac{y_2 - y_1}{x_2 - x_1}\right]$$

$$z_2 = -2 + 2\left[\frac{7 - 2}{2 - 0}\right] = 3$$



Here two ~~goal~~ quadratic spline are Possible.

$$s_i(x) = \frac{z_{i+1}-z_i}{2(x_{i+1}-x_i)}(x-x_i)^2 + z_i(x-x_i) + y_i$$

$$s_1(x) = \frac{z_2-z_1}{2(x_2-x_1)}(x-x_1)^2 + z_1(x-x_1) + y_1$$

$$= \frac{3-2}{2(2-0)}(x-0)^2 + 2(x-0) + 2$$

$$s_1(x) = \frac{1}{4}x^2 + 2x + 2$$

And $s(x) =$

$$S_0(x) = \frac{z_1-z_0}{2(x_1-x_0)}(x-x_0)^2 + z_0(x-x_0) + y_0$$

$$= \frac{2-0}{2(0+2)}(x+2)^2 + 0 + 0$$

$$s_0(x) = \frac{1}{2}(x+2)^2$$

And

$$s_1(x) = \frac{z_2-z_1}{2(x_2-x_1)}(x-x_1)^2 + z_1(x-x_1) + y_1$$

$$= \frac{3-2}{2(2-0)}(x-0)^2 + 2(x-0) + 2$$

$$s_1(x) = \frac{1}{4}(x)^2 + 2x + 2$$

Hence

$$s_0(x) = \frac{1}{2}(x+2)^2, \quad x \in [-2, 0]$$

$$s_1(x) = \frac{1}{4}x^2 + 2x + 2, \quad x \in [0, 2]$$

So $y(1.5) = \frac{1}{4}(1.5)^2 + 2(1.5) + 2 = 5.5627$ Ans

Given data

x :	0	1	2	3
$f(x)$:	1	2	33	244

Fit Quadratic spline with $M(0) = f''(0) = 0$.
Hence find an estimate of $f(2.5)$.

Sol^n As we know that Quadratic spline is given as

$$s_i(x) = \frac{z_{i+1}-z_i}{2(x_{i+1}-x_i)}(x-x_i)^2 + z_i(x-x_i) + y_i \longrightarrow (1)$$

Here z is given as

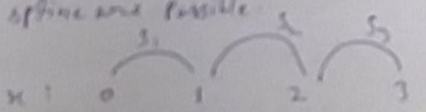
$$z_{i+1} = -z_i + 2\left[\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right], \quad 0 \le i \le n-1$$

Here $n = 4$ in question

$0 \le i \le 4-1$

$0 \le i \le 3$

Here three quadratic spline are possible



$$\therefore z_1 = -z_0 + 2\left[\frac{y_1-y_0}{x_1-x_0}\right] \quad \text{take } z_0 = 0$$

$$z_1 = 0 + 2\left[\frac{2-1}{1-0}\right] = 2$$

$$z_2 = -z_1 + 2\left[\frac{y_2-y_1}{x_2-x_1}\right]$$

$$= -2 + 2\left[\frac{33-2}{2-1}\right] = -2 + 62 = 60$$

$$\begin{array}{r} 422 \\ 60 \\ \hline 362 \end{array}$$

$$z_3 = -z_2 + 2\left[\frac{y_3 - y_2}{x_3 - x_2}\right]$$

$$= -60 + 2\left[\frac{244 - 33}{3 - 2}\right]$$

$$= -60 + 2[211] = -60 + 422$$

$$z_3 = 362$$

Now
$$s_i(x) = \frac{z_{i+1} - z_i}{2(x_{i+1} - x_i)}(x - x_i)^2 + z_i(x - x_i) + y_i$$

$$s_0(x) = \frac{z_1 - z_0}{2(x_1 - x_0)}(x - x_0)^2 + z_0(x - x_0) + y_0$$

$$= \frac{2 - 0}{2(1 - 0)}(x - 0)^2 + 0 + 1$$

$$s_0(x) = x^2 + 1$$

And
$$s_1(x) = \frac{z_2 - z_1}{2(x_2 - x_1)}(x - x_1)^2 + z_1(x - x_1) + y_1$$

$$= \frac{60 - 2}{2(2 - 1)}(x - 1)^2 + 2(x - 1) + 2$$

$$= 29(x - 1)^2 + 2(x - 1) + 2$$

$$s_1(x) = 29(x - 1)^2 + 2x$$

And
$$s_2(x) = \frac{z_3 - z_2}{2(x_3 - x_2)}(x - x_2)^2 + z_2(x - x_2) + y_2$$

$$= \frac{362 - 60}{2(3 - 2)}(x - 2)^2 + 60(x - 2) + 33$$

$$s_2(x) = \frac{302}{2}(x - 2)^2 + 60(x - 2) + 33 =$$

$$s(2.5) = \frac{302}{2}(2.5 - 2)^2 + 60(2.5 - 2) + 33 = \underline{\underline{101}}$$

Cubic Spline Interpolation

A cubic spline satisfies the following Properties

(i) $S(x_i) = f_i$, $i = 0, 1, 2, \ldots n$

(ii) on each sub interval $[x_{i-1}, x_i]$, $1 \le i \le n$, $F(x)$ is a third degree polynomial.

(iii) $S(x)$, $S'(x)$, $S''(x)$ are continuous on (a, b)

Working steps

(i) $M_0 = M_n = 0$ (Natural spline)

(ii) $M_0 = M_n$, $M_1 = M_{n+1}$, $f_0 = f_n$, $f_1 = f_{n+1}$, $h_1 = h_{n+1}$
(A spline satisfying these conditions is called a periodic spline)

(iii) For Equispaced knots $h_i = h$ for all i.

Cubic spline

$$S(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right] + (x_i - x)\left[y_{i-1} - \frac{1}{6} M_{i-1}\right] + (x - x_{i-1})\left[y_i - \frac{1}{6} M_{i-1}\right]$$

cubic spline formula is

$$S(x) = \frac{(x_i - x)^3}{6h} M_{i-1} + \frac{(x - x_{i-1})^3}{6h} M_i + \frac{(x_i - x)}{h}\left(y_{i-1} - \frac{h^2}{6} M_{i-1}\right) + \frac{(x - x_{i-1})}{h}\left(y_i - \frac{h^2}{6} M_i\right)$$

Q.1 Calculate cubic splines

x	1	2	3	4
y	1	5	11	8

$y(1.5)$, $y'(2)$

Sol Here $h=1$, $n=3$, $M_0=0$, $M_3=0$

We know that cubic spline formula is

$$F(x) = \frac{(x_i-x)^3}{6h} M_{i-1} + \frac{(x-x_{i-1})^3}{6h} M_i + \frac{(x_i-x)}{h}\left(y_{i-1} - \frac{h^2}{6} M_{i-1}\right)$$

$$+ \frac{(x-x_{i-1})}{h}\left(y_i - \frac{h^2}{6} M_i\right) \longrightarrow ①$$

We also know that

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right] \longrightarrow ②$$

Putting $i=1$ in ②

$$M_0 + 4M_1 + M_2 = \frac{6}{1}\left[y_0 - 2y_1 + y_2\right]$$

$$0 + 4M_1 + M_2 = 6\left[1 - 2\times5 + 11\right]$$

$$4M_1 + M_2 = 12 \longrightarrow (i)$$

Putting $i=2$ in ②

$$M_1 + 4M_2 + M_3 = \frac{6}{1}\left[y_1 - 2y_2 + y_3\right]$$

$$M_1 + 4M_2 + 0 = 6\left[5 - 22 + 8\right]$$

$$M_1 + 4M_2 = -54 \longrightarrow (ii)$$

On solving

$$4M_1 + M_2 = 12$$
$$M_1 + 4M_2 = -54$$
$$16M_1 + 4M_2 = 48$$
$$\underset{-}{M_1} + \underset{-}{4M_2} = \underset{+}{-54}$$

$$15M_1 = 102$$
$$\boxed{M_1 = 6.8}$$
$$M_2 = 12 - 4(6.8)$$
$$\boxed{M_2 = -15.2}$$

Putting $i=1$ in equation (1) we get cubic spline in 1st interval $[1,2]$

$$S_1(x) = \frac{(x_1-x)^3}{6h_1}M_0 + \frac{(x-x_0)^3}{6h_1}M_1 + \frac{(x_1-x)}{h_1}\left(y_0 - \frac{h_1^2}{6}M_0\right)$$
$$+ \frac{(x-x_0)}{h_1}\left(y_1 - \frac{h_1^2}{6}M_1\right)$$

$$= \frac{(2-x)^3}{6}\cdot 0 + \frac{(x-1)^3}{6}(6.8) + \frac{(2-x)}{1}\left(1-\frac{1}{6}\times 0\right)$$
$$+ \frac{(x-1)}{1}\left(5-\frac{1}{6}(6.8)\right)$$

$$= \frac{6.8}{6}(x-1)^3 + (2-x) + (x-1)\left(5-\frac{6.8}{6}\right)$$

$$S(x) = 1.1333x^3 - 3.4x^2 + 6.2667x - 3 \quad \text{for } 1 \le x \le 2$$

Ans $\longrightarrow$ (A)

Putting $i=2$ in equation (1), we get cubic spline in 2nd interval $[2,3]$

$$S_2(x) = \frac{(x_2-x)^3}{6h_1}M_1 + \frac{(x-x_1)^3}{6h_1}M_2 + \frac{(x_2-x)}{h_1}\left(y_1 - \frac{h_1^2}{6}M_1\right)$$
$$+ \frac{(x-x_1)}{h_1}\left(y_2 - \frac{h_1^2}{6}M_2\right)$$

$$= \frac{(3-x)^3}{6}(6.8) + \frac{(x-2)^3}{6}(-15.2) + \frac{(3-x)}{1}\left(5-\frac{1}{6}6.8\right)$$
$$+ \frac{(x-2)}{1}\left(11-\frac{1}{6}(-15.2)\right)$$

$$= -3.666x^3 + 25.4x^2 - 51.333 + 35.4 \quad \text{for } 2 \le x \le 3$$

Putting $i = 3$, in equation (1) we get cubic spline in interval $[3, 4]$

$$S_3(x) = \frac{(x_3 - x)^3}{6h} M_2 + \frac{(x - x_2)^3}{6h} M_3 + \frac{(x_3 - x)}{h}\left(y_2 - \frac{h^2}{6} M_2\right)$$

$$+ \frac{(x - x_2)}{h}\left(y_3 - \frac{h^2}{6} M_3\right)$$

$$= \frac{(4 - x)^3}{6}(-15.2) + \frac{(x - 3)^3}{6} \cdot 0 + \frac{(4 - x)}{1}\left(11 - \frac{1}{6}(-15.2)\right)$$

$$+ \frac{(x - 3)}{1}\left(8 - \frac{1}{6} \cdot 0\right)$$

$$= 2.5333x^3 - 30.4x^2 + 116.0667x - 132, \quad 3 \le x \le 4$$

$\therefore y(1.5) =$

i.e $1.5 \in [1, 2]$ so putting $x = 1.5$ in (A)

$\boxed{y(1.5) = 2.575}$ Ans

Q.2 Using cubic spline, find $y(.5)$ and $y'(1)$, given $M_0 = M_2 = 0$ and the table

x	0	1	2
y	-5	-4	3

Here $h = 1$, $n = 2$, $M_0 = 0$, $M_2 = 0$

We know that

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

Putting $i = 1$

$$M_0 + 4M_1 + M_2 = 6\left[y_0 - 2y_1 + y_2\right]$$

$$0 + 4M_1 + 0 = 6[-5 + 8 + 3] = 36$$

$$M_1 = 9$$

And we know that cubic spline formula is

$$F(x) = \frac{(x_i-x)^3}{6h} M_{i-1} + \frac{(x-x_{i-1})^3}{6h} M_i + \frac{(x_i-x)}{h}\left(y_{i-1} - \frac{h^2}{6}M_{i-1}\right)$$
$$+ \frac{(x-x_{i-1})}{h}\left(y_i - \frac{h^2}{6}M_i\right) \longrightarrow (1)$$

Putting $i=1$ in (1) we get cubic spline in Ist interval $[0,1]$

$$F(x) = \frac{(x_1-x)^3}{6}M_0 + \frac{(x-x_0)^3}{6}M_1 + (x_1-x)\left(y_0 - \frac{h^2}{6}M_0\right)$$
$$+ (x-x_0)\left(y_1 - \frac{1}{6}M_1\right)$$

$$= \frac{(1-x)^3}{6}(0) + \frac{(x-0)^3}{6}(9) + (1-x)\left(-5 - \frac{1}{6}\times 0\right)$$
$$+ (x-0)\left(-4 - \frac{1}{6}\times 9\right)$$

$$= \frac{3}{2}x^3 - 5(1-x) + x\left(-4 - \frac{3}{2}\right)$$

$$= \frac{3}{2}x^3 - 5(1-x) - \frac{11}{2}x$$

$$= \frac{3}{2}x^3 - 5 + 5x - \frac{11}{2}x$$

$$F(x) = \frac{3}{2}x^3 - \frac{1}{2}x - 5, \quad [0,1]$$

$$y(.5) = \frac{3}{2}(.5)^3 - \frac{1}{2}(.5) - 5 = -\frac{81}{16}$$

$$F'(x) = \frac{9}{2}x^2 - \frac{1}{2}$$

$$y'(1) = \frac{9}{2}(1) - \frac{1}{2} = \frac{9}{2} - \frac{1}{2} = 4 \text{ Ans}$$

3 From the table

x	1	2	3
y	-8	-1	18

Compute $y(1.5)$, and $y'(1)$ Using cubic spline.

Sol Here $h=1$, $n=2$ [$\because$ Intervals are $(1,2)$ and $(2,3)$]

$M_0=0$, $M_2=0$

We know that

$$M_{i-1}+4M_i+M_{i+1}=\frac{6}{h^2}[y_{i-1}-2y_i+y_{i+1}]$$

Putting $i=1$

$$M_0+4M_1+M_2=6[y_0-2y_1+y_2]$$

$$0+4M_1+0=6[-8+2+18]=72$$

$$\boxed{M_1=18}$$

And we know that cubic spline in 1st interval [1,2]

$$F(x)=\frac{(x_1-x)^3}{6}M_0+\frac{(x-x_0)^3}{6}M_1+(x_1-x)\left(y_0-\frac{h^2}{6}M_0\right)$$

$$+(x-x_0)\left(y_1-\frac{1}{6}M_1\right)$$

$$=\frac{(2-x)^3}{6}\times 0+\frac{(x-1)^3}{6}(18)+(2-x)\left(-8-\frac{1}{6}\times 0\right)$$

$$+(x-1)\left(-1-\frac{1}{6}\times 18\right)$$

$$=3(x-1)^3-8(2-x)+(x-1)(-4)$$

$$=3(x-1)^3-8(2-x)-4(x-1)$$

$$=3(x^3-1-3x^2+3x)-16+8x-4x+4$$

$$=3x^3-9x^2+9x-3+4x-12$$

$$F(x)=3x^3-9x^2+13x-15$$

$$y(1.5)=3(1.5)^3-9(1.5)^2+13(1.5)-15=-5.625 \text{ Ans}$$

Ans 6

$y'(x) = 9x^2 - 18x + 13$

$y'(1) = 9 - 18 + 13 = 4$ Ans

Q.4 Obtain the cubic spline approximation for the function $y = f(x)$ from the following data, given that $y''(0) = y''(3) = 0$

x:	-1	0	1	2
y:	-1	1	3	35

Solⁿ Here $h = 1$ (Since the values of x are equally spaced with $h=1$)

$n = 3$

$M_0 = 0,\ M_3 = 0$

We know that cubic spline formula is

$$F(x) = \frac{(x_i - x)^3}{6h} M_{i-1} + \frac{(x - x_{i-1})^3}{6h} M_i + \frac{(x_i - x)}{h}\left[y_{i-1} - \frac{h^2}{6} M_{i-1}\right] + \frac{(x - x_{i-1})}{h}\left(y_i - \frac{h^2}{6} M_i\right) \longrightarrow ①$$

And we find out M as

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right] \longrightarrow ②$$

Putting $i = 1$ in ②

$M_0 + 4M_1 + M_2 = \frac{6}{1}[y_0 - 2y_1 + y_2]$ $\quad \because M_0 = M_3 = 0$

$0 + 4M_1 + M_2 = 6[-1 - 2\times 1 + 3]$

$4M_1 + M_2 = 6[-3+3] = 0$

$4M_1 + M_2 = 0 \longrightarrow (i)$

Putting $i = 2$ in (2)

$M_1 + 4M_2 + M_3 = \frac{6}{1}[y_1 - 2y_2 + y_3]$

$M_1 + 4M_2 + 0 = 6[1 - 2\times 3 + 35]$

$M_1 + 4M_2 = 6 \times 30 = 180$

$$M_1 + 4M_2 = 180 \longrightarrow (ii)$$

On solving (i) and (ii)

$$4M_1 + M_2 = 0 \longrightarrow (i)$$
$$M_1 + 4M_2 = 180 \longrightarrow \times (4)$$

$$\cancel{16 M_1 + 4M_2 = 0}$$
$$\cancel{-15 M_1 = +180}, \quad \cancel{M_1 = +/60}$$

$$4\cancel{M_1} + M_2 = 0$$
$$4\cancel{M_1} + 16M_2 = 4 \times 180$$
$$-15M_2 = -4 \times 180$$
$$M_2 = \frac{4 \times \cancel{180}^{12}}{\cancel{15}} = 48, \quad \boxed{M_2 = 48}$$

$$4M_1 + 48 = 0$$
$$4M_1 = -48 \Rightarrow \boxed{M_1 = -12}$$

$$M_1 = -12, \quad M_2 = 48$$

Now cubic spline in 1st ~~spline~~ interval $[-1, 0]$

$$F(x) = \frac{(x_1 - x)^3}{6} M_0 + \frac{(x - x_0)^3}{6} M_1 + (x_1 - x)\left(y_0 - \frac{h^2}{6} M_0\right) + (x - x_0)\left(y_1 - \frac{1}{6} M_1\right)$$
$$= 0 + \frac{(x+1)^3}{6}(-12) + \frac{(0-x)}{1}\left[-1 - \frac{1}{6} \times 0\right] + (x+1)\left(1 - \frac{1}{6} \times -12\right)$$
$$= -2(x+1)^3 + x + 3(x+1)$$
$$= -2\left[x^3 + 3x^2 + 3x + 1\right] + x + 3x + 3$$
$$= -2x^3 - 6x^2 - 6x - 2 + 4x + 3$$

$$\boxed{F(x) = -2x^3 - 6x^2 - 2x + 1}$$ This is spline in $(-1, 0)$ interval

Again spline in interval $(0,1)$

$$\because f(x) = \frac{(x_i - x)^3}{6h} M_{i-1} + \frac{(x - x_{i-1})^3}{6h} M_i + \frac{(x_i - x)}{h}\left[y_{i-1} - \frac{h^2}{6} M_{i-1}\right]$$
$$+ \frac{(x - x_{i-1})}{h}\left(y_i - \frac{h^2}{6} M_i\right) \longrightarrow (A)$$

Putting $i = 2$

$$f(x) = \frac{(x_2 - x)^3}{6} M_1 + \frac{(x - x_1)^3}{6} M_2 + \frac{(x_2 - x)}{1}\left[y_1 - \frac{1}{6} M_1\right] + (x - x_1)\left[y_2 - \frac{1}{6} M_2\right]$$

$$= \frac{(1-x)^3}{6}(-12) + \frac{(x-0)^3}{6}(48) + (1-x)\left[1 - \frac{1}{6}(-12)\right] + (x-0)\left[3 - \frac{1}{6}48\right]$$

$$= -2(1-x)^3 + 8x^3 + (1-x)(3) + x(-5)$$

$$= -2(1 - x^3 + 3x^2 - 3x) + 8x^3 + 3 - 3x - 5x$$

$$= -2 + 2x^3 - 6x^2 + 6x + 8x^3 + 3 - 8x$$

$$\boxed{f(x) = 10x^3 - 6x^2 - 2x + 1} \text{ in Interval } (0,1)$$

Again spline in Interval $(1,2)$

Putting $i = 3$ in (A)

$$f(x) = \frac{(x_3 - x)^3}{6h} M_2 + \frac{(x - x_2)^3}{6h} M_3 + \frac{(x_3 - x)}{h}\left[y_2 - \frac{h^2}{6} M_2\right]$$
$$+ \frac{(x - x_2)}{h}\left(y_3 - \frac{h^2}{6} M_3\right)$$

$$= \frac{(2-x)^3}{6}(48) + \frac{(x-1)^3}{6} \times 0 + \frac{(2-x)}{1}\left[3 - \frac{1}{6}(48)\right] + (x-1)\left(35 - \frac{1}{6} \cdot 0\right)$$

$$= 8(2-x)^3 - 5(2-x) + 35(x-1)$$

$$= 8(8 - x^3 + 6x^2 - 12x) - 10 + 5x + 35x - 35$$

$$= 64 - 8x^3 + 48x^2 - 96x - 10 + 40x - 35$$

$$f(x) = -8x^3 + 48x^2 - 56x + 19 \text{ in interval } [1,2] \text{ Ans}$$