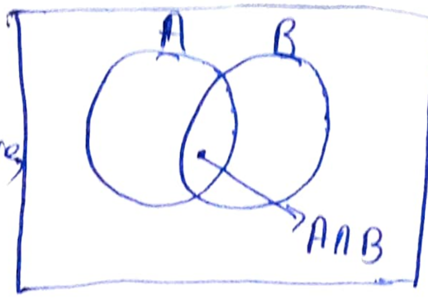
Addition Law of Probability

If A and B are any 2 events,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



If A and B are mutually exclusive

$A \cap B = \emptyset$, Then $P(A \cap B) = 0$

$\therefore$ In case of mutually exclusive events,

$P(A \cup B) = P(A) + P(B)$

For any 3 events,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Q Find the probability of getting a heart or an ace, when a card is drawn from a well shuffled pack of 52 cards.

Sol Let A: Getting a heart card, $P(A) = \frac{13}{52}$

B: " an Ace, $P(B) = \frac{4}{52}$

$A \cap B$: Getting an Ace of heart, $P(A \cap B) = \frac{1}{52}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Q. Find the probability of getting neither spade nor Ace, when a card is drawn from a well shuffled pack of 52 cards.

Sol. Let A: Getting ~~an ace~~ a spade, $P(A) = \frac{13}{52}$

B: " an Ace, $P(B) = \frac{4}{52}$

$A \cap B$: " an Ace of spade, $P(A \cap B) = \frac{1}{52}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{4}{13}$

Again, Probability of neither spade nor Ace is given by $P(A^c \cap B^c)$.

$P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$

$= 1 - \frac{4}{13} = \frac{9}{13}$

Q. 3 Newspapers A, B, C are published in a city and a survey on readers reveals following information: 25% read A, 30% read B, 20% read C. 10% read both A and B, 5% read both A and C, 8% read both B and C, 3% read all 3 newspapers. For a person chosen at random, find the probability that he reads none of the newspapers.

Sol $P(A) = \frac{25}{100}$, $P(B) = \frac{30}{100}$, $P(C) = \frac{20}{100}$,

$P(A \cap B) = \frac{10}{100}$, $P(A \cap C) = \frac{5}{100}$, $P(B \cap C) = \frac{8}{100}$,

$P(A \cap B \cap C) = \frac{3}{100}$.

Now $P(A \cup B \cup C) = \frac{25}{100} + \frac{30}{100} + \frac{20}{100} - \frac{10}{100} - \frac{8}{100} - \frac{5}{100} + \frac{3}{100}$

$$= \frac{55}{100} = \frac{11}{20}$$

$\therefore P(A \cup B \cup C)^c = 1 - P(A \cup B \cup C)$

$$= 1 - \frac{11}{20} = \frac{9}{20}$$

Conditional Probability

The probability of occuence of an event A, when event B has already occured is called conditional probability of A and is denoted by $P(A/B)$.

* If $P(A/B) = P(A)$, then event A is said to be independent of event B.

Multiplicative Law of probability

$$P(A \cap B) = P(A) \cdot P(B/A)$$
$$= P(B) \cdot P(A/B)$$

$P(A \cap B)$ is also written as $P(AB)$

If A and B are independent events, then $P(B/A) = P(B)$ and $P(A/B) = P(A)$

$\therefore$ If A and B are independent,

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

\# ~~From~~ Formulae for conditional probability:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Q An unbiased coin is ~~tossed~~ tossed thrice. In which of the following cases events A and B are independent

(i) A: The first throw results in a tail
B: The last " " " " head

(ii) A: The number of tails is two
B: The last throw results in a tail.

Sol $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

(i) $A = \{TTT, TTH, THT, TTT\}$, $P(A) = \frac{4}{8} = \frac{1}{2}$

$B: \{HHH, HTH, THH, TTH\}$, $P(B) = \frac{4}{8} = \frac{1}{2}$

$A \cap B = \{THH, TTH\}$, $P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

Now $P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\therefore P(A \cap B) = P(A) \cdot P(B)$

Hence events A and B are independent.

(ii) $A = \{HTT, THT, TTH\}$, $P(A) = \frac{3}{8}$

$B = \{HHT, HTT, TTT, THT\}$, $P(B) = \frac{4}{8} = \frac{1}{2}$

$A \cap B = \{HTT, THT\}$, $P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

Now, $P(A) \cdot P(B) = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16} \neq P(A \cap B)$

Hence A and B are not independent events.

Q If $P(A) = 0.3$, $P(B) = 0.5$ and $P(A/B) = 0.4$

Find (i) $P(AB)$, (ii) $P(B/A)$ (iii) $P(A^c \cup B^c)$

Sol (i) $P(AB) = P(A \cap B) = P(B) \cdot P(A/B) = 0.5 \times 0.4 = 0.2$

(ii) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.3} = \frac{2}{3}$

(iii) $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B) = 1 - 0.2 = 0.8$

Q Cards are dealt one by one from a well shuffled pack of 52 cards until an ace appears. Show that the probability that exactly n cards are dealt before the first ace appears is

$$\frac{4(51-n)(50-n)(49-n)}{52 \cdot 51 \cdot 50 \cdot 49}$$

Sol Let A: Drawing n non-ace cards

B: Drawing an ace in $(n+1)^{th}$ draw

$$P(A) = \frac{{}^{48}C_n}{{}^{52}C_n} = \frac{48!}{n!\,(48-n)!} \cdot \frac{n!\,(52-n)!}{52!}$$

$$= \frac{\cancel{48!} \cdot (52-n)(51-n)(50-n)(49-n) \cdot \cancel{(48-n)!}}{\cancel{(48-n)!} \cdot 52.51.50.49.\cancel{48!}}$$

$$\Rightarrow P(A) = \frac{(52-n)(51-n)(50-n).(49-n)}{52.51.50.49}$$

$$\therefore P(B/A) = \frac{{}^4C_1}{{}^{52-n}C_1} = \frac{4}{52-n}$$

Hence $P(A \cap B) = P(A).P(B/A) = \dfrac{4(51-n)(50-n)(49-n)}{52.51.50.49}$

Q Two dice are thrown and sum of number appearing is observed to be 6. Find the conditional probability that number 2 has appeared at least once.

Sol Let A: Number 2 has appeared at least once

B: Sum of numbers on two dice is 6.

$A \equiv \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2),$
$(2,1), (2,3), (2,4), (2,5), (2,6)\}$

$B \equiv \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$A \cap B \equiv \{(2,4), (4,2)\}$

Now $P(A \cap B) = \frac{2}{36}$, $P(B) = \frac{5}{36}$

$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{\cancel{36}} \times \frac{\cancel{36}}{5} = \frac{2}{5}$

Q A bag contains 7 red, 5 black balls; another bag contains 5 red and 8 black balls. A ball is drawn from the first bag and without noticing its colour, is put in the 2nd bag and then a ball is drawn from 2nd bag. Find the probability that the ball drawn is red in colour.

Sol Case 1 A: Red Ball is drawn from 1st bag
B: Red Ball is drawn from 2nd bag

$P(A \cap B) = P(A) \cdot P(B/A) = \frac{7}{12} \cdot \frac{6}{14} = \frac{42}{168}$

Case 2 C: Black Ball is drawn from 1st bag
B: Red " " " " 2nd bag

$P(C \cap B) = P(C) \cdot P(B/C) = \frac{5}{12} \times \frac{5}{14} = \frac{25}{168}$

Req. Prob $= P(A \cap B) + P(C \cap B) = \frac{42}{168} + \frac{25}{168} = \frac{67}{168}$