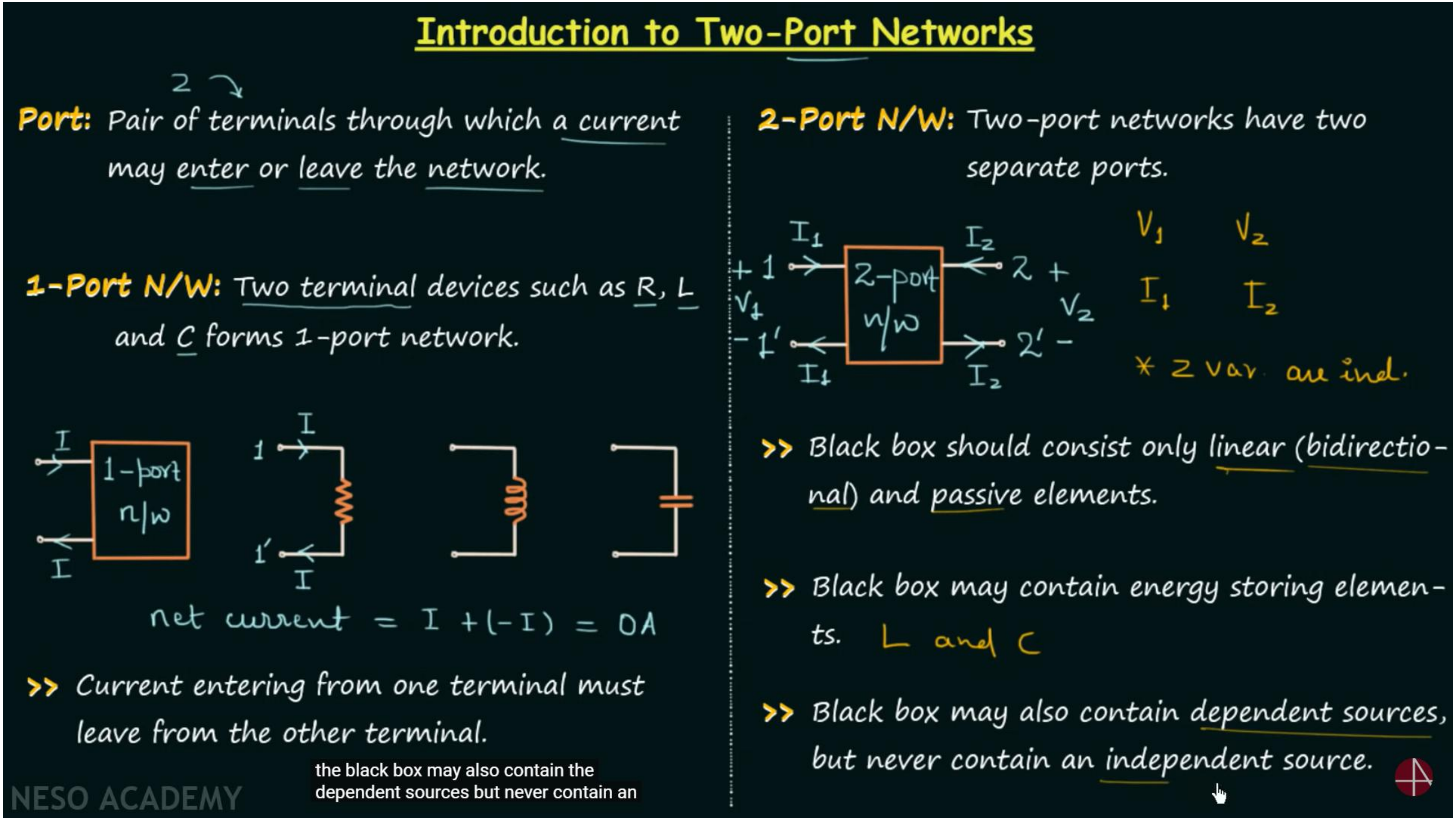


Introduction to Two-Port Networks

2

Port: Pair of terminals through which a current may enter or leave the network.

1-Port N/W: Two terminal devices such as R, L and C forms 1-port network.

net current = I + (−I) = 0A

>> Current entering from one terminal must leave from the other terminal.

the black box may also contain the
dependent sources but never contain an

2-Port N/W: Two-port networks have two separate ports.

V_1 V_2

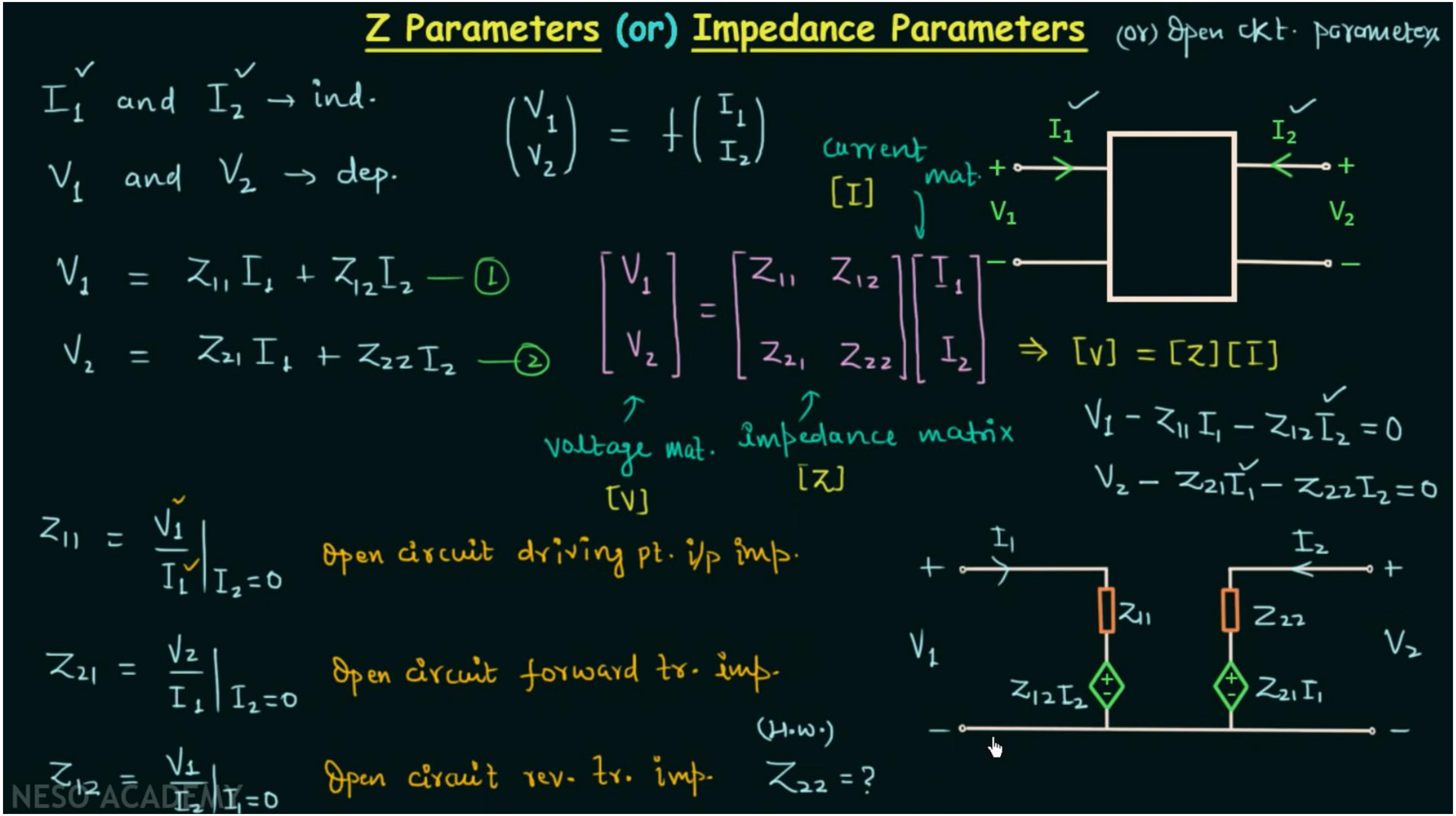
I_1 I_2

* 2 var. are ind.

>> Black box should consist only linear (bidirectional) and passive elements.

>> Black box may contain energy storing elements. L and C

>> Black box may also contain dependent sources, but never contain an independent source.



Z Parameters (or) Impedance Parameters (or) Open ckt. parameters

$I_1^{\checkmark}$ and $I_2^{\checkmark} \rightarrow$ ind.

V_1 and $V_2 \rightarrow$ dep.

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = f\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad — ①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad — ②$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

voltage mat. [V]

impedance matrix [Z]

current mat. [I]

$\Rightarrow [V] = [Z][I]$

$$V_1 - Z_{11} I_1 - Z_{12} I_2^{\checkmark} = 0$$

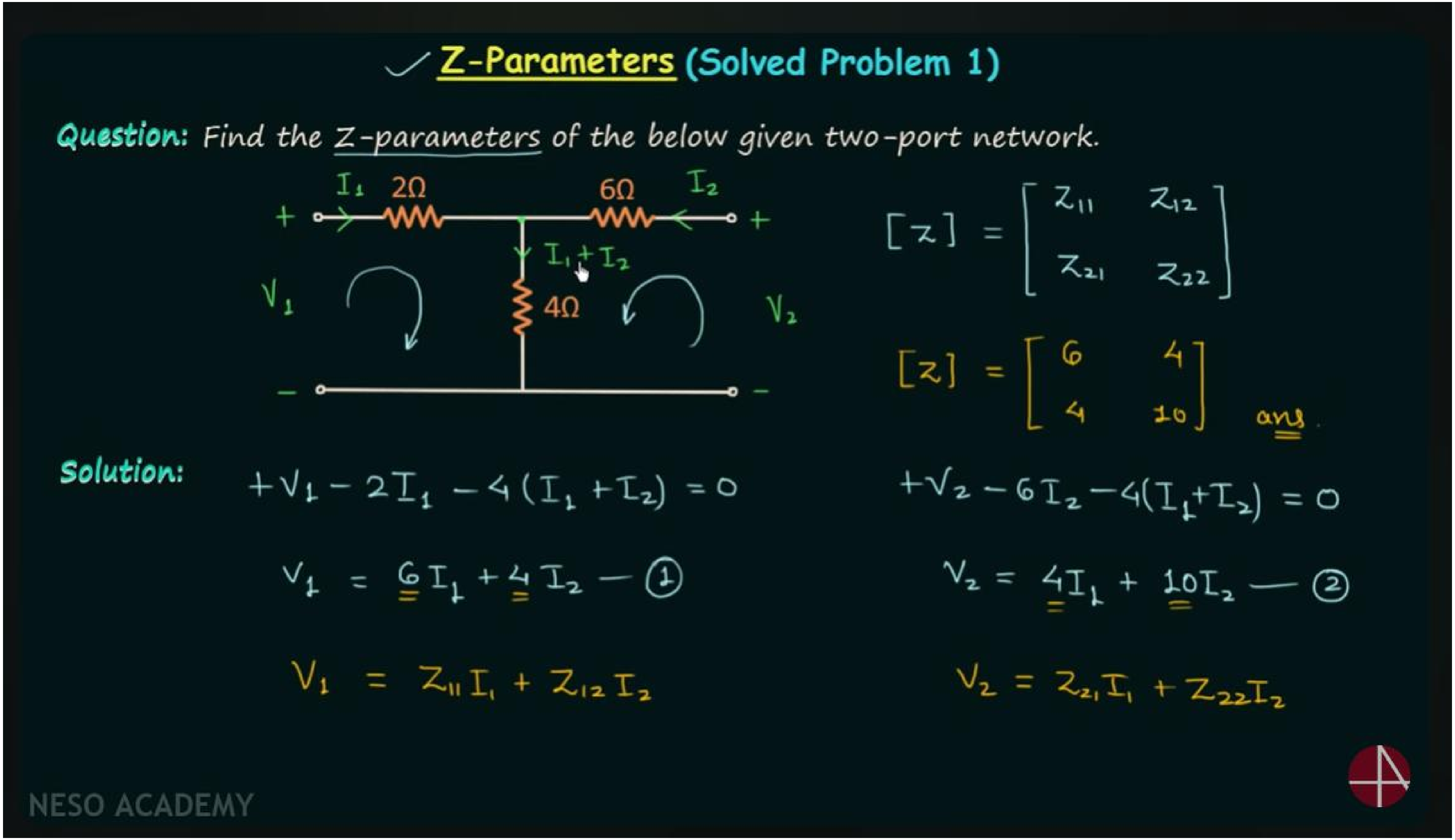
$$V_2 - Z_{21} I_1^{\checkmark} - Z_{22} I_2 = 0$$

$$Z_{11} = \left.\frac{V_1^{\checkmark}}{I_1^{\checkmark}}\right|_{I_2=0}$$ Open circuit driving pt. i/p imp.

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$ Open circuit forward tr. imp.

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$ Open circuit rev. tr. imp.

(H.W.) $Z_{22} = ?$



✓ Z-Parameters (Solved Problem 1)

Question: Find the Z-parameters of the below given two-port network.

$$[z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$[z] = \begin{bmatrix} 6 & 4 \\ 4 & 10 \end{bmatrix} \quad \underline{\underline{\text{ans}}}$$

Solution:

$$+V_1 - 2I_1 - 4(I_1 + I_2) = 0$$

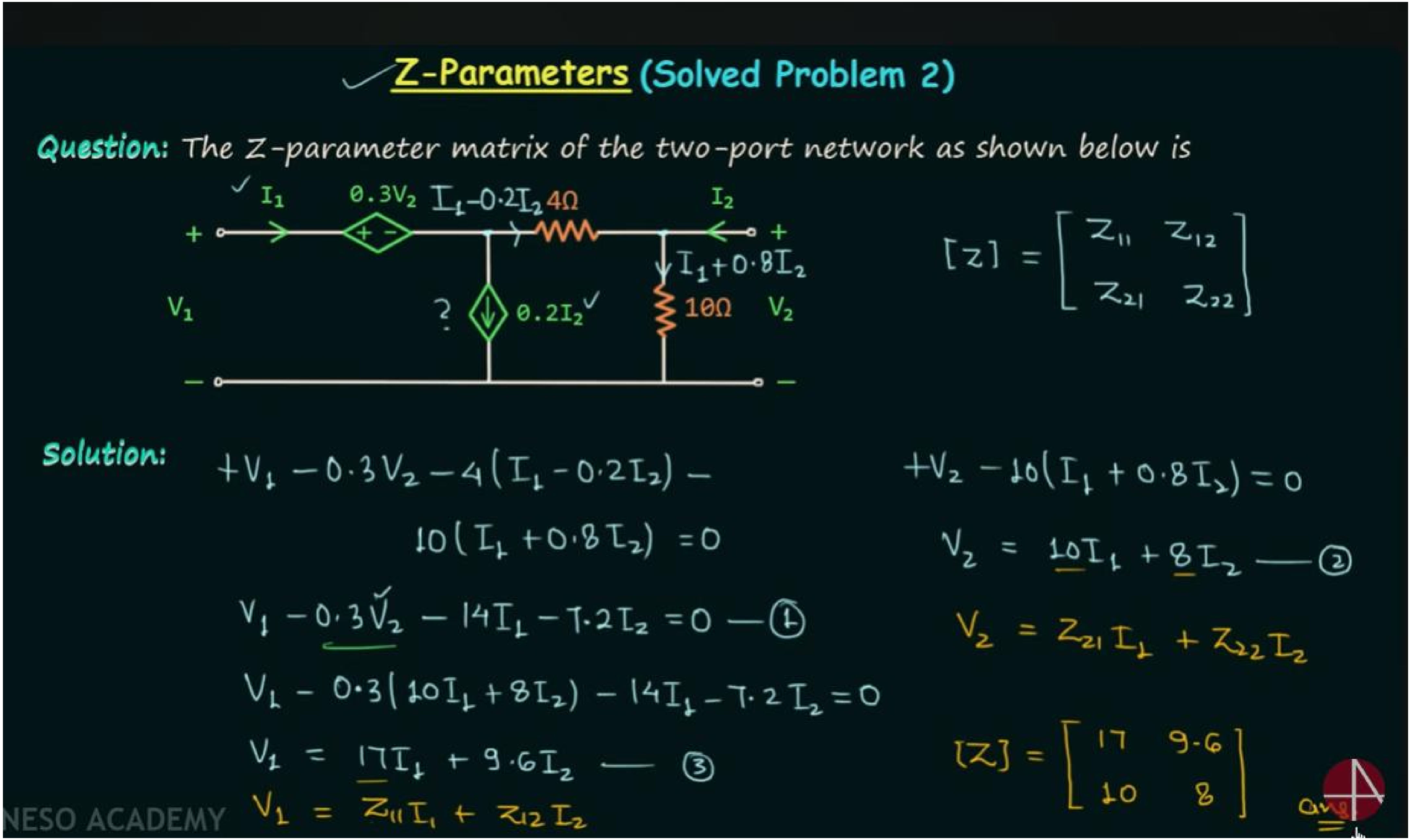
$$V_1 = \underline{\underline{6}}\,I_1 + \underline{\underline{4}}\,I_2 \quad \text{—} \ ①$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$+V_2 - 6I_2 - 4(I_1 + I_2) = 0$$

$$V_2 = \underline{\underline{4}}\,I_1 + \underline{\underline{10}}\,I_2 \quad \text{—} \ ②$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



Z-Parameters (Solved Problem 2)

Question: The Z-parameter matrix of the two-port network as shown below is

$$[z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Solution:

$$+V_1 - 0.3V_2 - 4(I_1 - 0.2I_2) - 10(I_1 + 0.8I_2) = 0$$

$$V_1 - \underline{0.3V_2} - 14I_1 - 7.2I_2 = 0 \text{ — ①}$$

$$V_1 - 0.3(10I_1 + 8I_2) - 14I_1 - 7.2I_2 = 0$$

$$V_1 = \underline{17}I_1 + 9.6I_2 \text{ — ③}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

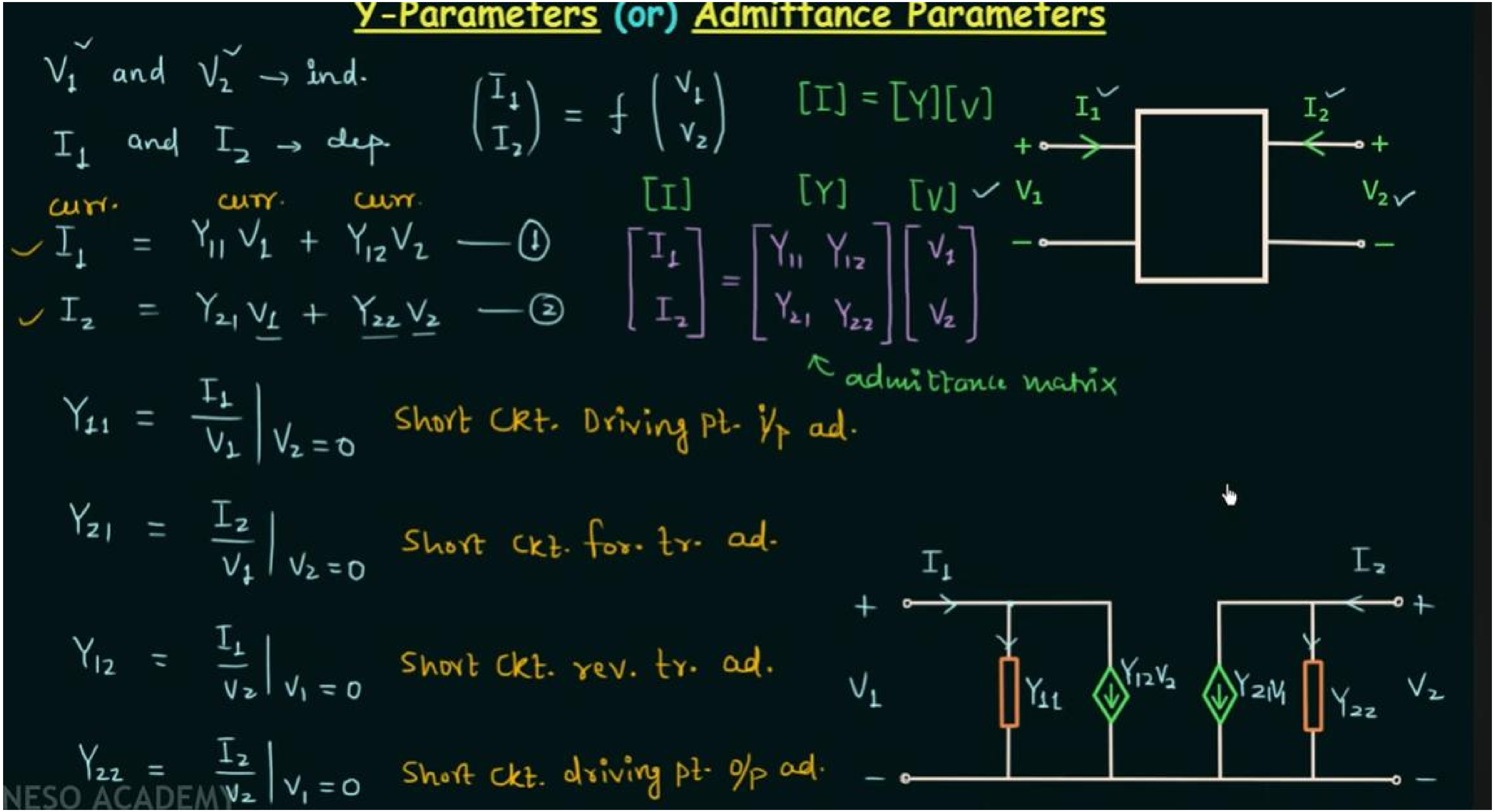
$$+V_2 - 10(I_1 + 0.8I_2) = 0$$

$$V_2 = \underline{10}I_1 + \underline{8}I_2 \text{ — ②}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$[Z] = \begin{bmatrix} 17 & 9.6 \\ 10 & 8 \end{bmatrix}$$

ans



Y-Parameters (or) Admittance Parameters

V_1 ✓ and V_2 ✓ → ind.

I_1 and I_2 → dep.

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$[I] = [Y][V]$$

curr. curr. curr.

✓ $I_1 = Y_{11}V_1 + Y_{12}V_2$ — ①

✓ $I_2 = Y_{21}\underline{V_1} + \underline{Y_{22}}\,\underline{V_2}$ — ②

[I] [Y] [V] ✓

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

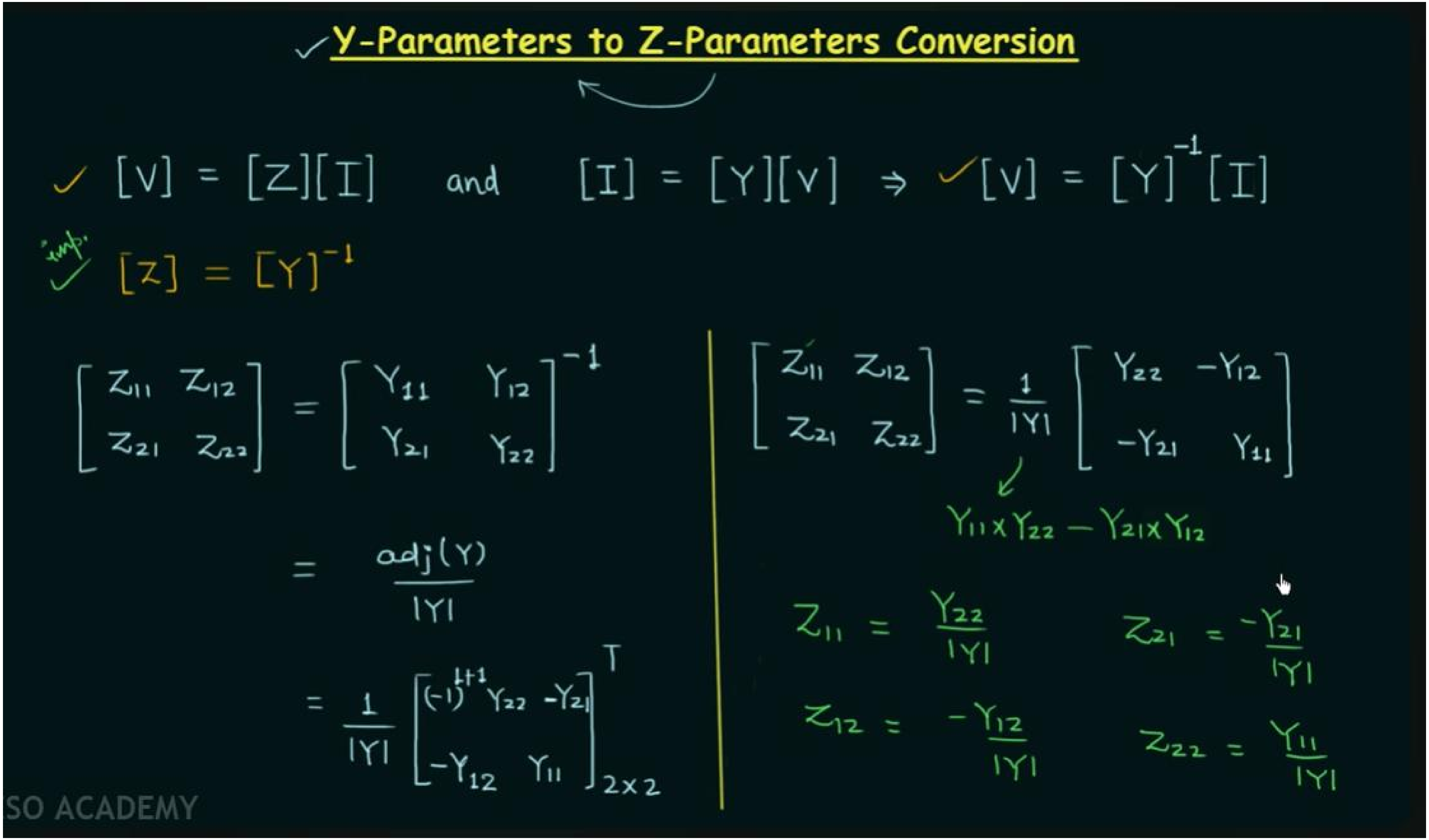
↖ admittance matrix

$Y_{11} = \dfrac{I_1}{V_1}\Big|_{V_2=0}$ Short Ckt. Driving pt. i/p ad.

$Y_{21} = \dfrac{I_2}{V_1}\Big|_{V_2=0}$ Short ckt. for. tr. ad.

$Y_{12} = \dfrac{I_1}{V_2}\Big|_{V_1=0}$ Short Ckt. rev. tr. ad.

$Y_{22} = \dfrac{I_2}{V_2}\Big|_{V_1=0}$ Short ckt. driving pt. o/p ad.



✓ Y-Parameters to Z-Parameters Conversion

✓ $[V] = [Z][I]$ and $[I] = [Y][V] \Rightarrow$ ✓ $[V] = [Y]^{-1}[I]$

imp. ✓ $[Z] = [Y]^{-1}$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$= \frac{\text{adj}(Y)}{|Y|}$$

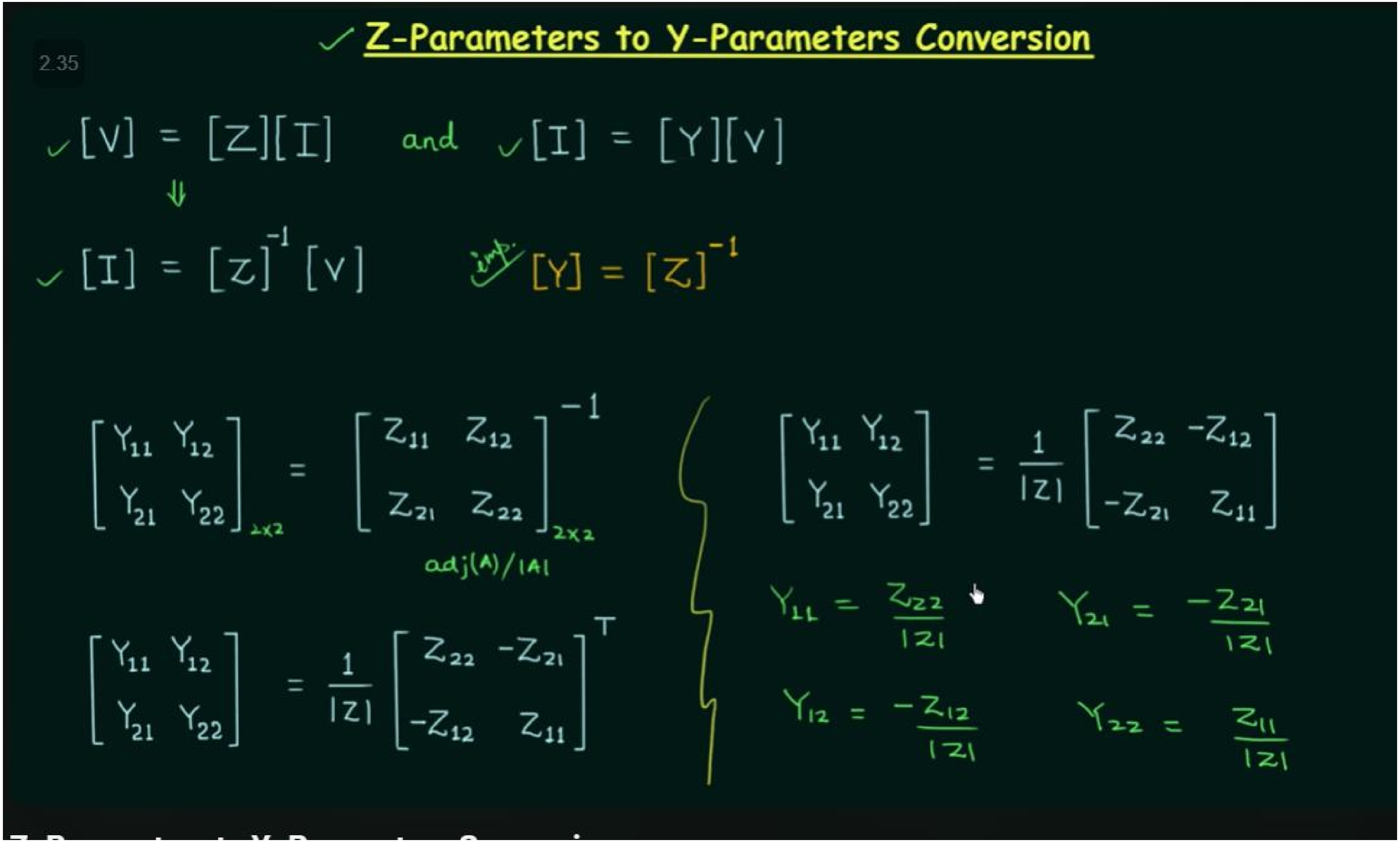
$$= \frac{1}{|Y|} \begin{bmatrix} (-1)^{1+1} Y_{22} & -Y_{21} \\ -Y_{12} & Y_{11} \end{bmatrix}^{T}_{2 \times 2}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{|Y|} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$|Y| \rightarrow Y_{11} \times Y_{22} - Y_{21} \times Y_{12}$$

$$Z_{11} = \frac{Y_{22}}{|Y|} \qquad Z_{21} = \frac{-Y_{21}}{|Y|}$$

$$Z_{12} = \frac{-Y_{12}}{|Y|} \qquad Z_{22} = \frac{Y_{11}}{|Y|}$$



✓ Z-Parameters to Y-Parameters Conversion

✓ $[V] = [Z][I]$ and ✓ $[I] = [Y][V]$

$\Downarrow$

✓ $[I] = [Z]^{-1}[V]$

imp. $[Y] = [Z]^{-1}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{2\times2} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_{2\times2}^{-1}$$

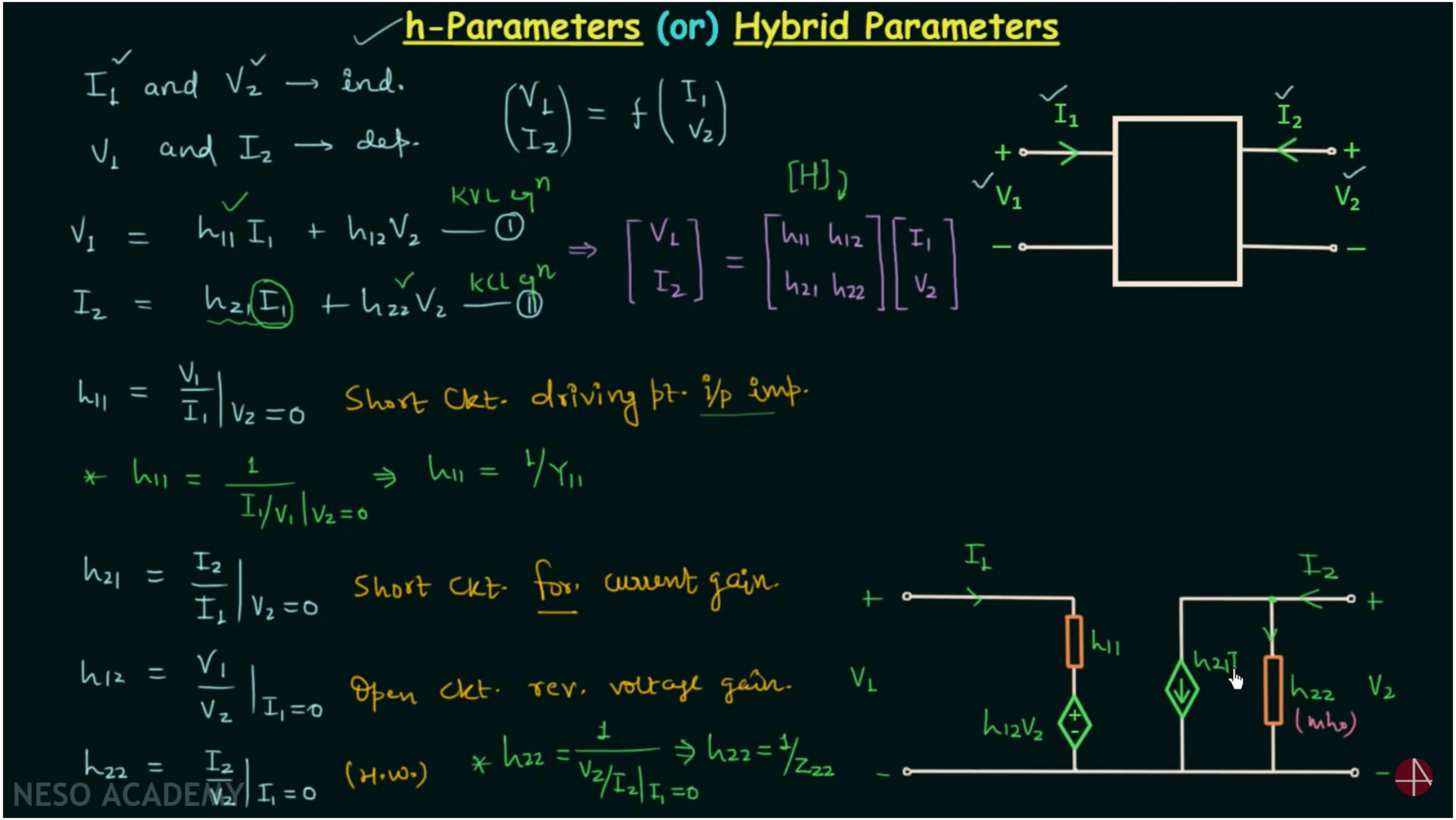
adj(A)/|A|

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{|Z|}\begin{bmatrix} Z_{22} & -Z_{21} \\ -Z_{12} & Z_{11} \end{bmatrix}^{T}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{|Z|}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$Y_{11} = \frac{Z_{22}}{|Z|}$ $\quad$ $Y_{21} = \frac{-Z_{21}}{|Z|}$

$Y_{12} = \frac{-Z_{12}}{|Z|}$ $\quad$ $Y_{22} = \frac{Z_{11}}{|Z|}$



h-Parameters (or) Hybrid Parameters

I_1 and V_2 → ind.

V_1 and I_2 → dep.

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = f\begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \xrightarrow{\text{KVL eq}^n} ①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \xrightarrow{\text{KCL eq}^n} ②$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \overset{[H]}{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0}$$ Short ckt. driving pt. i/p imp.

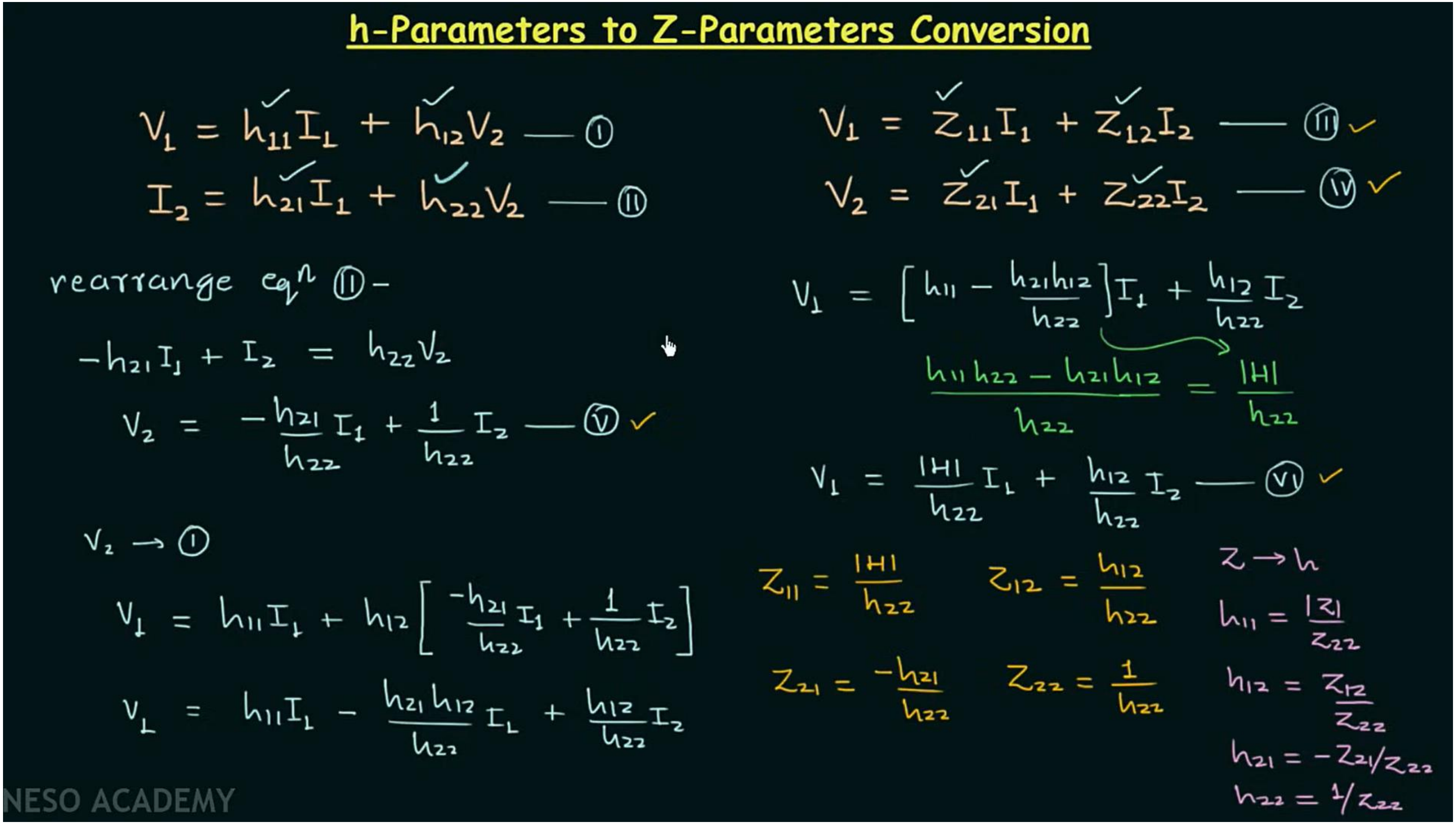
$$* \; h_{11} = \frac{1}{I_1/V_1 |_{V_2=0}} \Rightarrow h_{11} = 1/Y_{11}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}$$ Short ckt. fwd. current gain.

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}$$ Open ckt. rev. voltage gain.

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$ (H.W.)

$$* \; h_{22} = \frac{1}{V_2/I_2 |_{I_1=0}} \Rightarrow h_{22} = 1/Z_{22}$$



h-Parameters to Z-Parameters Conversion

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—} ①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—} ②$$

rearrange eq^n ② -

$$-h_{21} I_1 + I_2 = h_{22} V_2$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{—} ⑤ \checkmark$$

$V_2 \rightarrow ①$

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$V_1 = h_{11} I_1 - \frac{h_{21} h_{12}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—} ③ \checkmark$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—} ④ \checkmark$$

$$V_1 = \left[h_{11} - \frac{h_{21} h_{12}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\frac{h_{11} h_{22} - h_{21} h_{12}}{h_{22}} = \frac{|H|}{h_{22}}$$

$$V_1 = \frac{|H|}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \quad \text{—} ⑥ \checkmark$$

$$Z_{11} = \frac{|H|}{h_{22}} \qquad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} \qquad Z_{22} = \frac{1}{h_{22}}$$

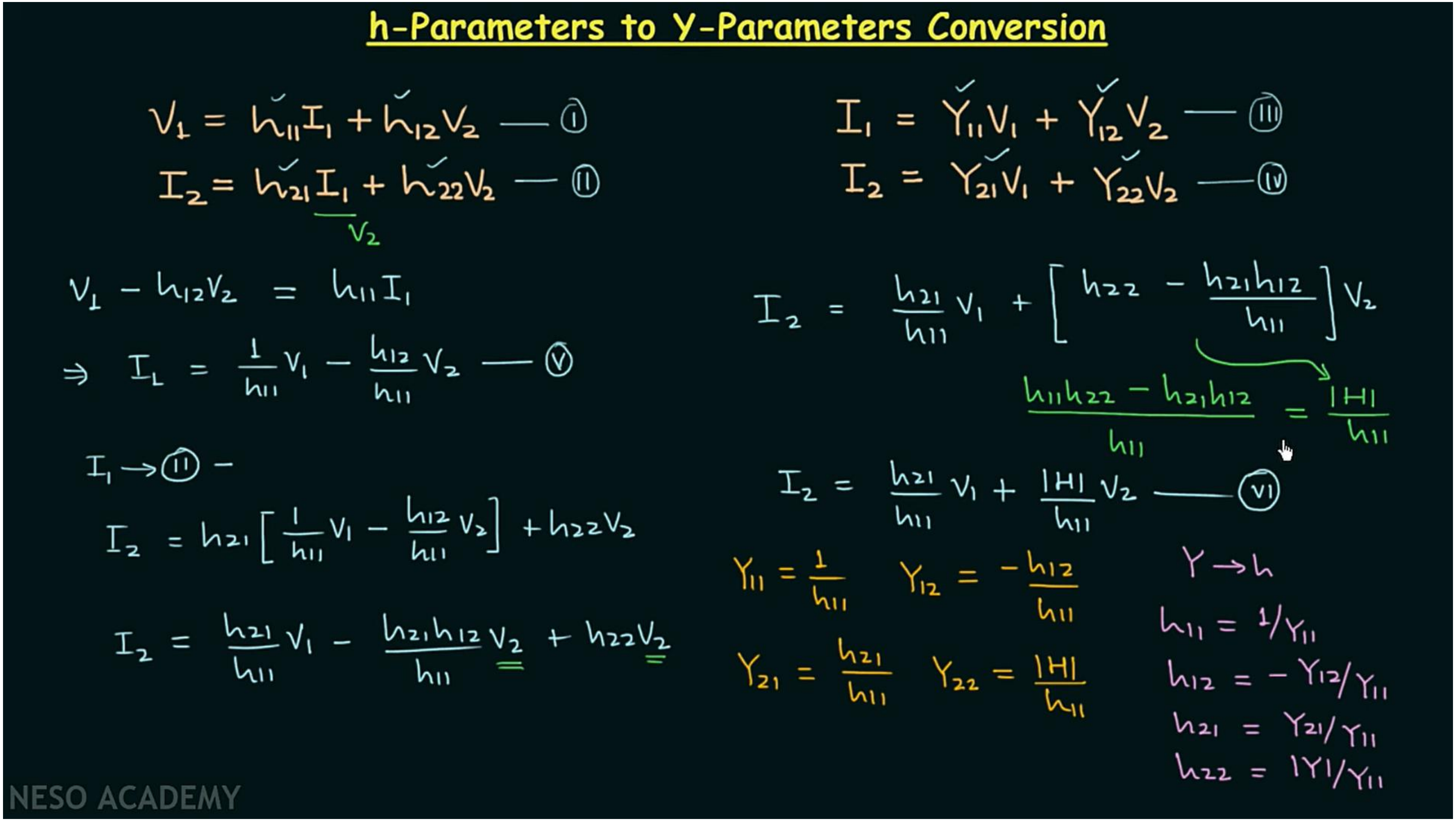
$Z \rightarrow h$

$$h_{11} = \frac{|Z|}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = -Z_{21}/Z_{22}$$

$$h_{22} = 1/Z_{22}$$



h-Parameters to Y-Parameters Conversion

$$V_1 = \overset{\checkmark}{h}_{11} I_1 + \overset{\checkmark}{h}_{12} V_2 \;—\; \text{(I)}$$

$$I_2 = \overset{\checkmark}{h}_{21} \underset{V_2}{\underline{I_1}} + \overset{\checkmark}{h}_{22} V_2 \;—\; \text{(II)}$$

$$V_1 - h_{12}V_2 = h_{11} I_1$$

$$\Rightarrow I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \;—\; \text{(V)}$$

$I_1 \rightarrow$ (II) -

$$I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 - \frac{h_{21}h_{12}}{h_{11}} \underline{\underline{V_2}} + h_{22}\underline{\underline{V_2}}$$

$$I_1 = \overset{\checkmark}{Y}_{11} V_1 + \overset{\checkmark}{Y}_{12} V_2 \;—\; \text{(III)}$$

$$I_2 = \overset{\checkmark}{Y}_{21} V_1 + \overset{\checkmark}{Y}_{22} V_2 \;—\; \text{(IV)}$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left[h_{22} - \frac{h_{21}h_{12}}{h_{11}}\right] V_2$$

$$\frac{h_{11}h_{22} - h_{21}h_{12}}{h_{11}} = \frac{|H|}{h_{11}}$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{|H|}{h_{11}} V_2 \;—\; \text{(VI)}$$

$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = -\frac{h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \frac{|H|}{h_{11}}$$

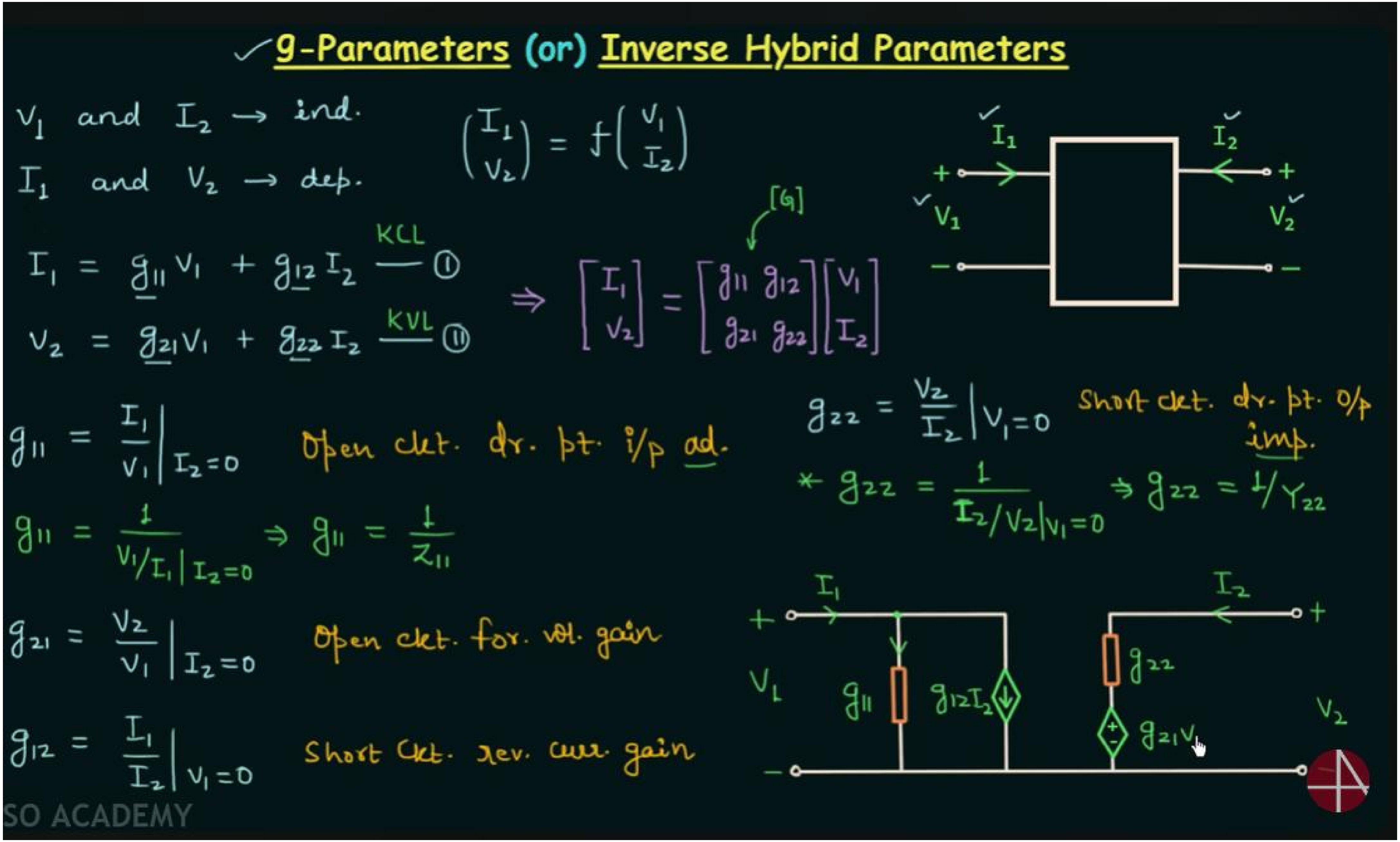
$Y \rightarrow h$

$h_{11} = 1/Y_{11}$

$h_{12} = -Y_{12}/Y_{11}$

$h_{21} = Y_{21}/Y_{11}$

$h_{22} = |Y|/Y_{11}$



✓g-Parameters (or) Inverse Hybrid Parameters

V_1 and I_2 → ind.

I_1 and V_2 → dep.

$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$

$$I_1 = \underline{g_{11}} V_1 + \underline{g_{12}} I_2 \xrightarrow{\text{KCL}} ①$$

$$V_2 = \underline{g_{21}} V_1 + \underline{g_{22}} I_2 \xrightarrow{\text{KVL}} ⑪$$

$$\Rightarrow \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \overset{[G]}{\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$$ Open ckt. dr. pt. i/p ad.

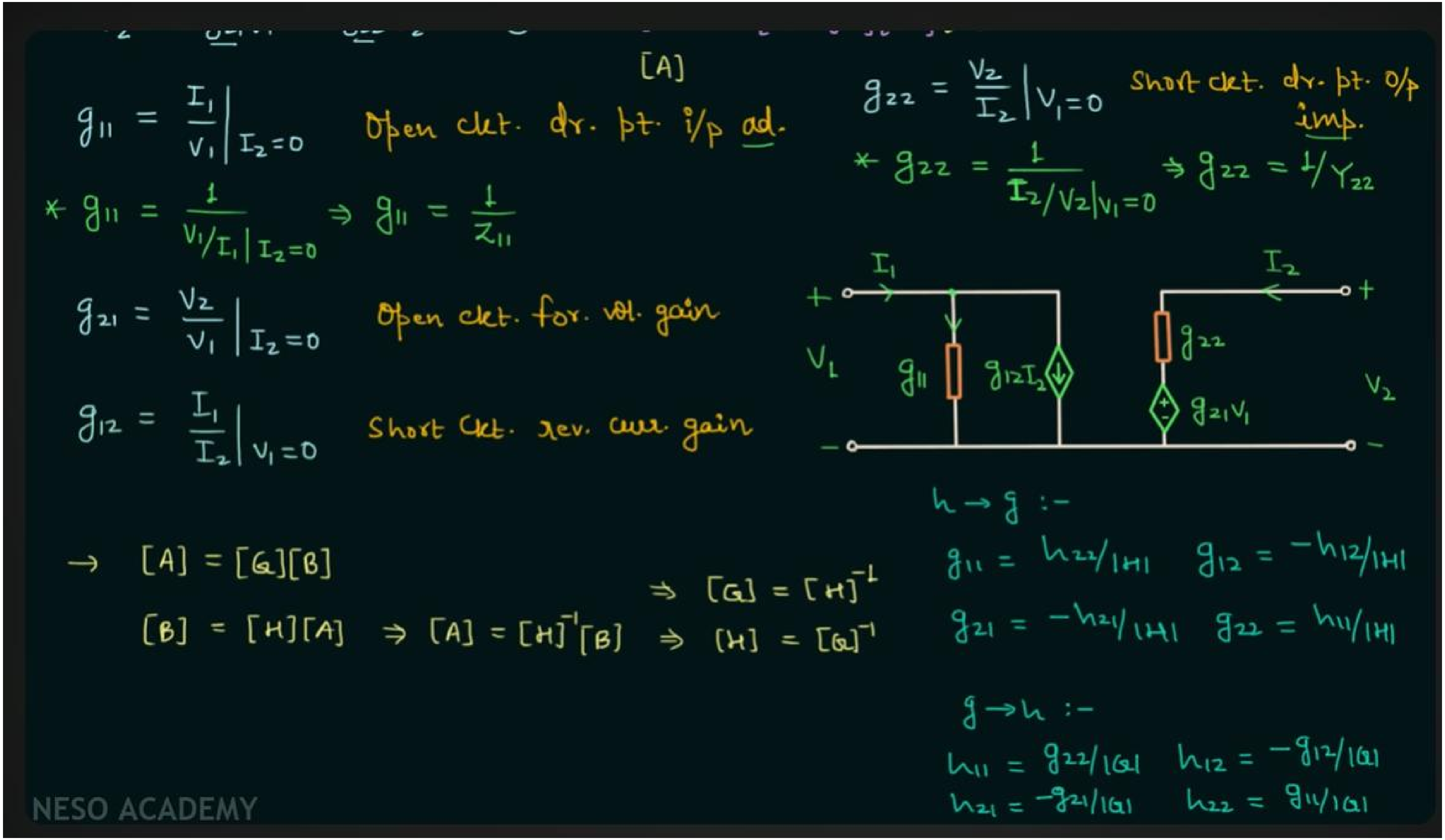
$$g_{11} = \frac{1}{V_1/I_1 \,|\, I_2=0} \Rightarrow g_{11} = \frac{1}{Z_{11}}$$

$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$$ Open ckt. for. vol. gain

$$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$$ Short ckt. rev. curr. gain

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$$ Short ckt. dr. pt. o/p imp.

$$* \; g_{22} = \frac{1}{I_2/V_2 |_{V_1=0}} \Rightarrow g_{22} = 1/Y_{22}$$



[A]

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$ Open ckt. dr. pt. i/p ad.

$$* \; g_{11} = \frac{1}{V_1/I_1|_{I_2=0}} \Rightarrow g_{11} = \frac{1}{Z_{11}}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$ Open ckt. for. vol. gain

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$ Short ckt. rev. curr. gain

$$\rightarrow \; [A] = [G][B]$$

$$[B] = [H][A] \Rightarrow [A] = [H]^{-1}[B] \Rightarrow [G] = [H]^{-1} \Rightarrow [H] = [G]^{-1}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ Short ckt. dr. pt. o/p imp.

$$* \; g_{22} = \frac{1}{I_2/V_2|_{V_1=0}} \Rightarrow g_{22} = 1/Y_{22}$$

$h \rightarrow g$:-

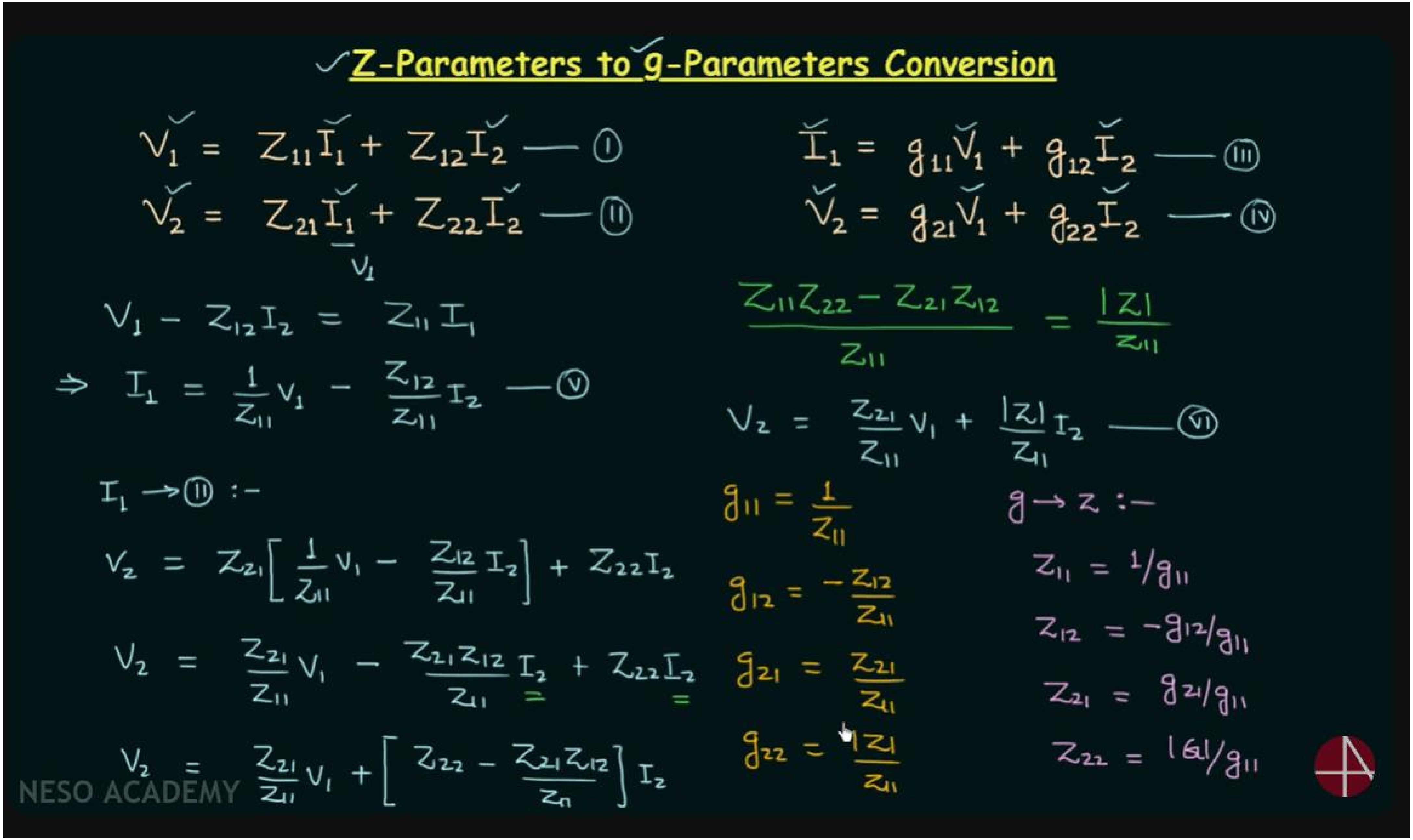
$g_{11} = h_{22}/|H|$ $\quad g_{12} = -h_{12}/|H|$

$g_{21} = -h_{21}/|H|$ $\quad g_{22} = h_{11}/|H|$

$g \rightarrow h$:-

$h_{11} = g_{22}/|G|$ $\quad h_{12} = -g_{12}/|G|$

$h_{21} = -g_{21}/|G|$ $\quad h_{22} = g_{11}/|G|$



✓Z-Parameters to ✓g-Parameters Conversion

$$\overset{\checkmark}{V_1} = Z_{11}\overset{\checkmark}{I_1} + Z_{12}\overset{\checkmark}{I_2} \text{ — ①}$$

$$\overset{\checkmark}{V_2} = Z_{21}\underset{V_1}{\underline{\overset{\checkmark}{I_1}}} + Z_{22}\overset{\checkmark}{I_2} \text{ — (II)}$$

$$\overset{\checkmark}{I_1} = g_{11}\overset{\checkmark}{V_1} + g_{12}\overset{\checkmark}{I_2} \text{ — (III)}$$

$$\overset{\checkmark}{V_2} = g_{21}\overset{\checkmark}{V_1} + g_{22}\overset{\checkmark}{I_2} \text{ — (IV)}$$

$$V_1 - Z_{12}I_2 = Z_{11}I_1$$

$$\Rightarrow I_1 = \frac{1}{Z_{11}}V_1 - \frac{Z_{12}}{Z_{11}}I_2 \text{ — (V)}$$

$I_1 \rightarrow$ (II) :-

$$V_2 = Z_{21}\left[\frac{1}{Z_{11}}V_1 - \frac{Z_{12}}{Z_{11}}I_2\right] + Z_{22}I_2$$

$$V_2 = \frac{Z_{21}}{Z_{11}}V_1 - \frac{Z_{21}Z_{12}}{Z_{11}}\underset{=}{I_2} + Z_{22}\underset{=}{I_2}$$

$$V_2 = \frac{Z_{21}}{Z_{11}}V_1 + \left[Z_{22} - \frac{Z_{21}Z_{12}}{Z_{11}}\right]I_2$$

$$\frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{11}} = \frac{|Z|}{Z_{11}}$$

$$V_2 = \frac{Z_{21}}{Z_{11}}V_1 + \frac{|Z|}{Z_{11}}I_2 \text{ — (VI)}$$

$$g_{11} = \frac{1}{Z_{11}}$$

$$g_{12} = -\frac{Z_{12}}{Z_{11}}$$

$$g_{21} = \frac{Z_{21}}{Z_{11}}$$

$$g_{22} = \frac{|Z|}{Z_{11}}$$

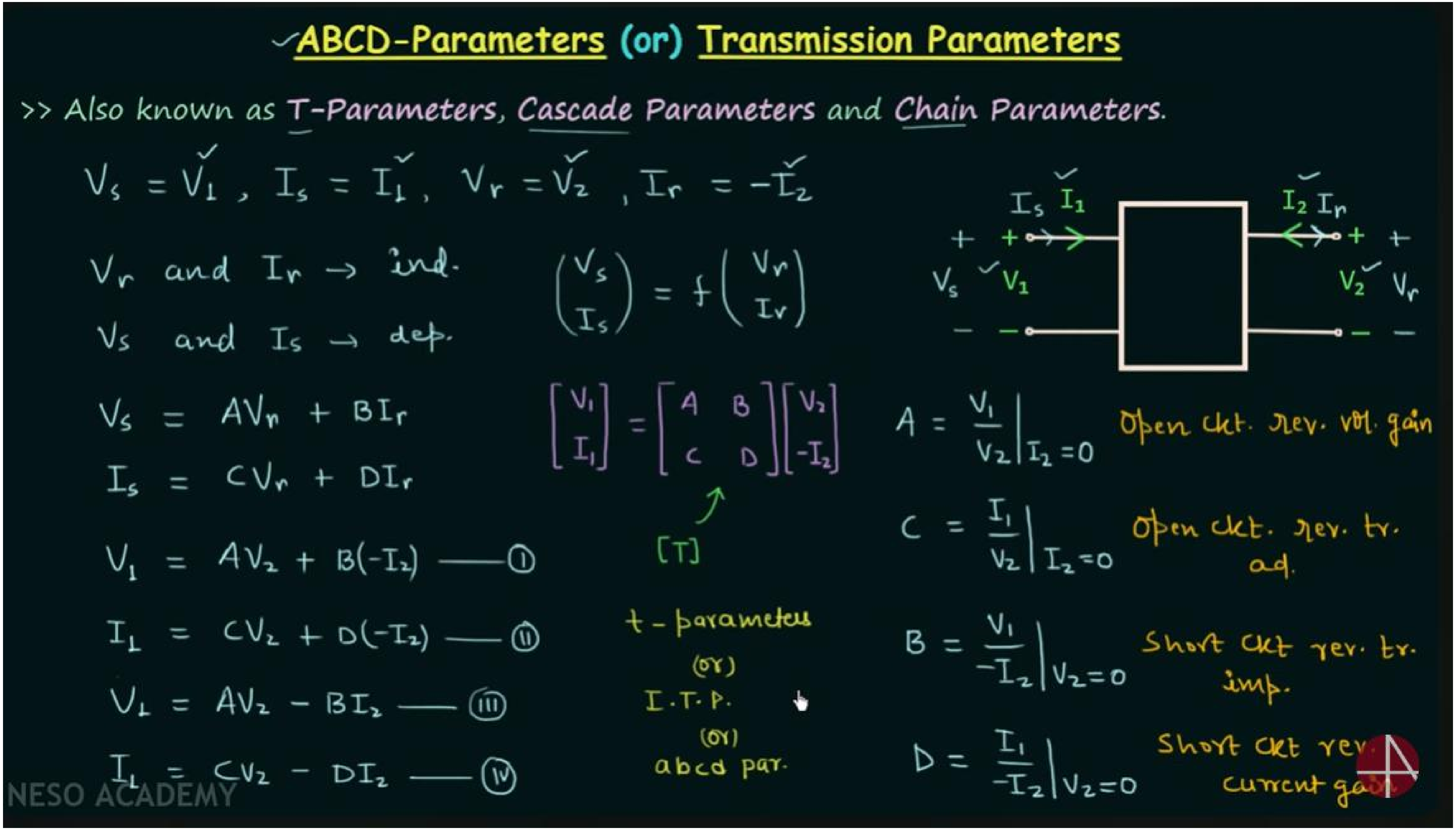
$g \rightarrow z$:-

$$Z_{11} = 1/g_{11}$$

$$Z_{12} = -g_{12}/g_{11}$$

$$Z_{21} = g_{21}/g_{11}$$

$$Z_{22} = |G|/g_{11}$$



ABCD-Parameters (or) Transmission Parameters

>> Also known as T-Parameters, Cascade Parameters and Chain Parameters.

$V_s = V_1$, $I_s = I_1$, $V_r = V_2$, $I_r = -I_2$

V_r and I_r → ind.

V_s and I_s → dep.

$$\begin{pmatrix}V_s\\I_s\end{pmatrix} = f\begin{pmatrix}V_r\\I_r\end{pmatrix}$$

$V_s = AV_r + BI_r$

$I_s = CV_r + DI_r$

$V_1 = AV_2 + B(-I_2)$ —— (i)

$I_1 = CV_2 + D(-I_2)$ —— (ii)

$V_1 = AV_2 - BI_2$ —— (iii)

$I_1 = CV_2 - DI_2$ —— (iv)

$$\begin{bmatrix}V_1\\I_1\end{bmatrix} = \underbrace{\begin{bmatrix}A & B\\C & D\end{bmatrix}}_{[T]}\begin{bmatrix}V_2\\-I_2\end{bmatrix}$$

t-parameters

(or)

I.T.P.

(or)

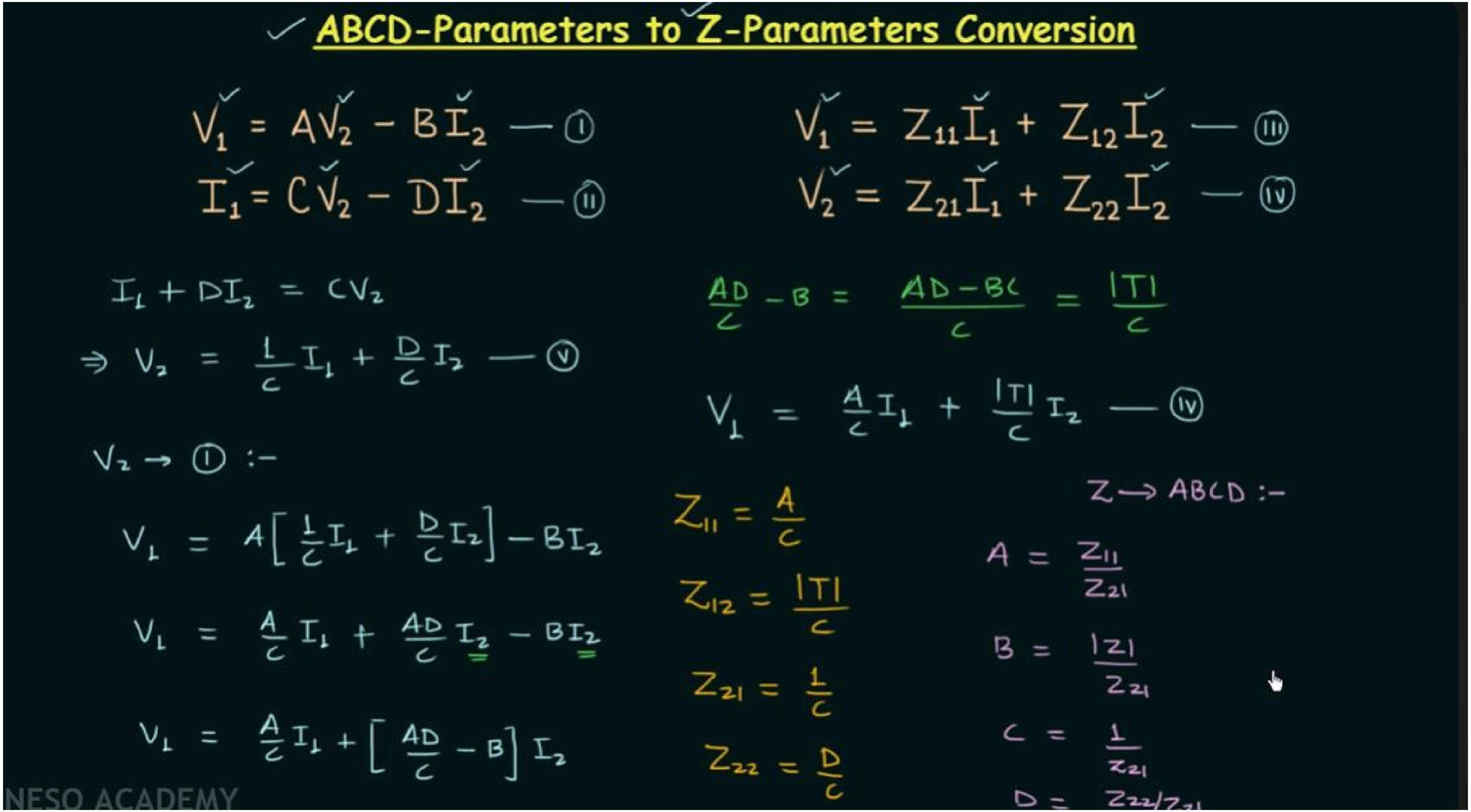
abcd par.

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ Open ckt. rev. vol. gain

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ Open ckt. rev. tr. ad.

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$ Short ckt rev. tr. imp.

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$ Short ckt rev. current gain



ABCD-Parameters to Z-Parameters Conversion

$$V_1 = AV_2 - BI_2 \quad — \textcircled{i}$$

$$I_1 = CV_2 - DI_2 \quad — \textcircled{ii}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad — \textcircled{iii}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad — \textcircled{iv}$$

$$I_1 + DI_2 = CV_2$$

$$\Rightarrow V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2 \quad — \textcircled{v}$$

$V_2 \rightarrow \textcircled{i}$:-

$$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2$$

$$V_1 = \frac{A}{C}I_1 + \frac{AD}{C}\underline{\underline{I_2}} - B\underline{\underline{I_2}}$$

$$V_1 = \frac{A}{C}I_1 + \left[\frac{AD}{C} - B\right]I_2$$

$$\frac{AD}{C} - B = \frac{AD-BC}{C} = \frac{|T|}{C}$$

$$V_1 = \frac{A}{C}I_1 + \frac{|T|}{C}I_2 \quad — \textcircled{iv}$$

$$Z_{11} = \frac{A}{C}$$

$$Z_{12} = \frac{|T|}{C}$$

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

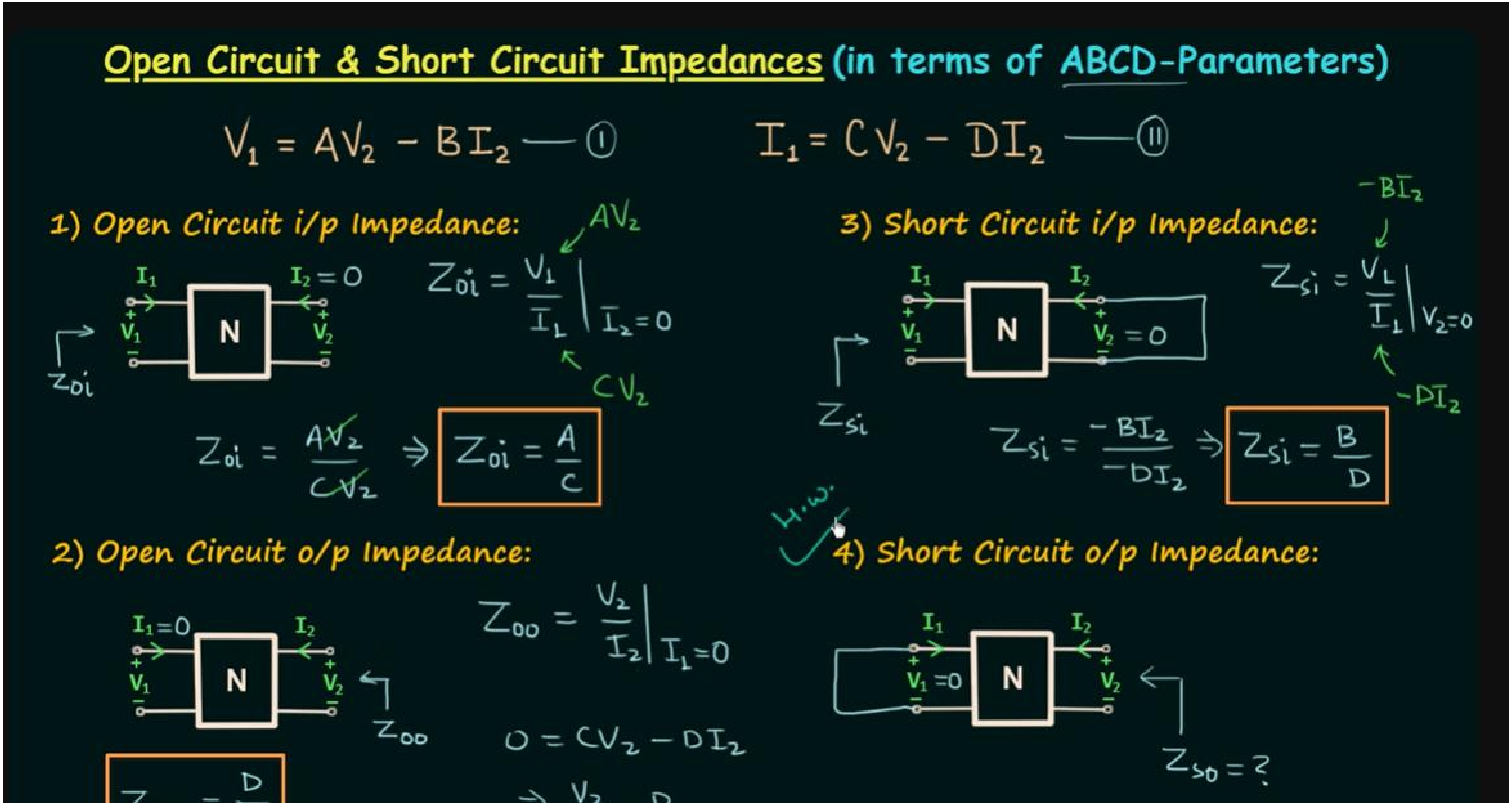
$Z \rightarrow ABCD$:-

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{|Z|}{Z_{21}}$$

$$C = \frac{1}{Z_{21}}$$

$$D = Z_{22}/Z_{21}$$



Open Circuit & Short Circuit Impedances (in terms of ABCD-Parameters)

$V_1 = AV_2 - BI_2$ —— ①

$I_1 = CV_2 - DI_2$ —— (II)

1) Open Circuit i/p Impedance:

$$Z_{oi} = \frac{\overset{AV_2}{V_1}}{\underset{CV_2}{I_1}}\Bigg|_{I_2=0}$$

$$Z_{oi} = \frac{A\cancel{V_2}}{C\cancel{V_2}} \Rightarrow \boxed{Z_{oi} = \frac{A}{C}}$$

2) Open Circuit o/p Impedance:

$$Z_{oo} = \frac{V_2}{I_2}\Bigg|_{I_1=0}$$

$$0 = CV_2 - DI_2$$

$\Rightarrow V_2 \quad D$

$Z \quad = \quad D$

3) Short Circuit i/p Impedance:

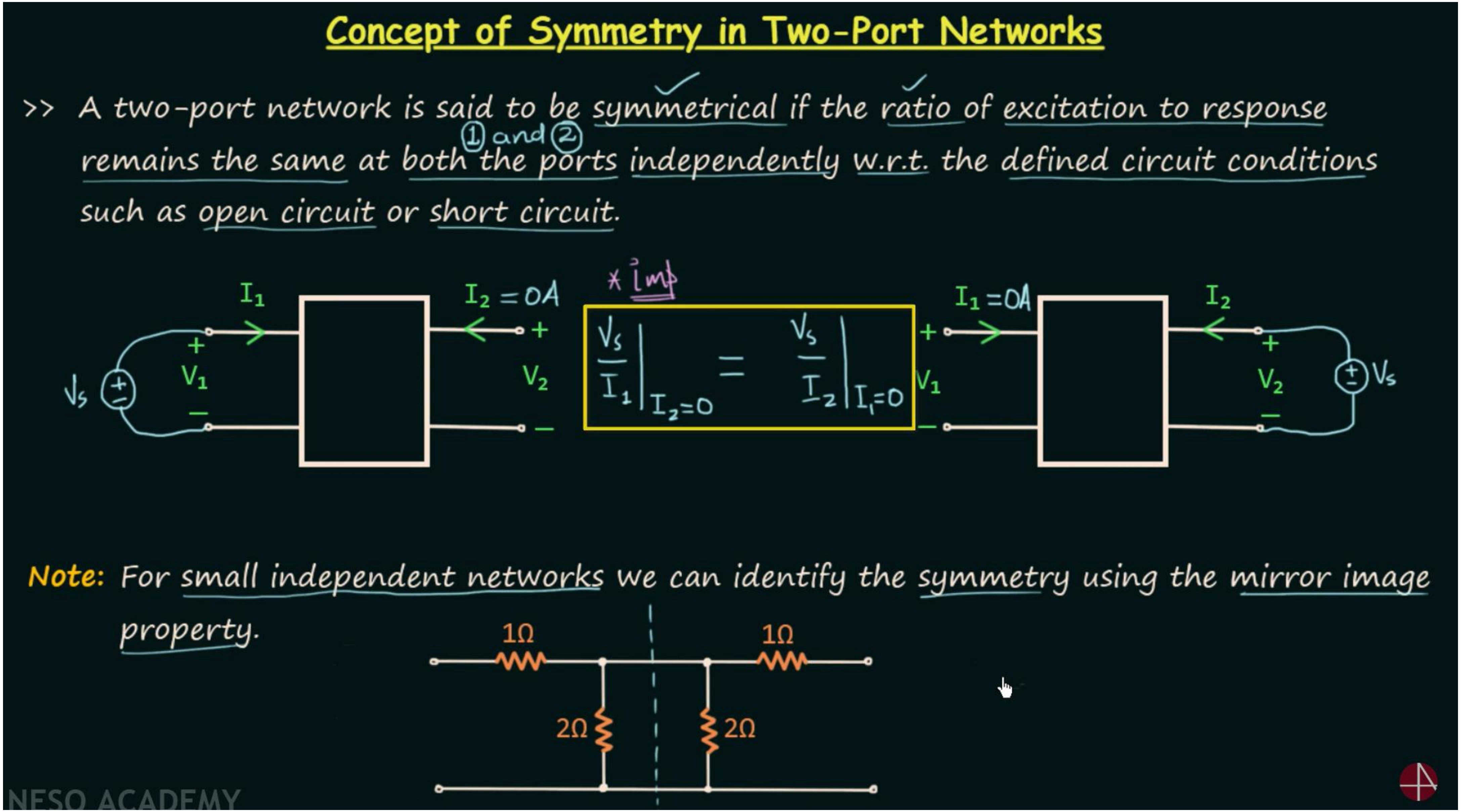
$$Z_{si} = \frac{\overset{-BI_2}{V_1}}{\underset{-DI_2}{I_1}}\Bigg|_{V_2=0}$$

$$Z_{si} = \frac{-BI_2}{-DI_2} \Rightarrow \boxed{Z_{si} = \frac{B}{D}}$$

H.W. ✓

4) Short Circuit o/p Impedance:

$Z_{so} = ?$



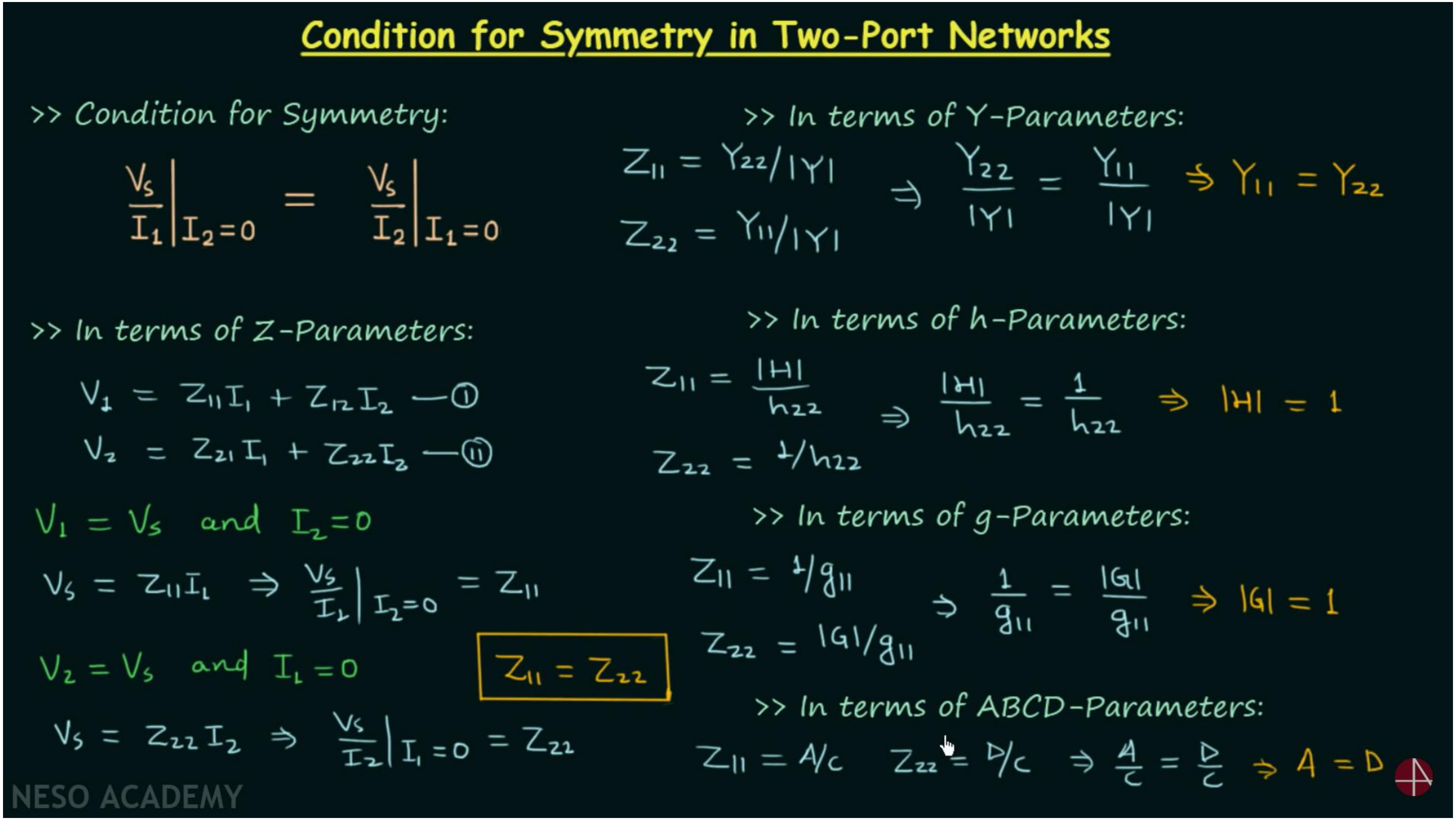
Concept of Symmetry in Two-Port Networks

>> A two-port network is said to be symmetrical if the ratio of excitation to response remains the same at both ① and ② the ports independently w.r.t. the defined circuit conditions such as open circuit or short circuit.

* imp

$$\boxed{\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}}$$

Note: For small independent networks we can identify the symmetry using the mirror image property.



Condition for Symmetry in Two-Port Networks

>> Condition for Symmetry:

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$

>> In terms of Z-Parameters:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{—①}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{—(ii)}$$

$V_1 = V_s$ and $I_2 = 0$

$$V_s = Z_{11}I_1 \Rightarrow \left.\frac{V_s}{I_1}\right|_{I_2=0} = Z_{11}$$

$V_2 = V_s$ and $I_1 = 0$

$$\boxed{Z_{11} = Z_{22}}$$

$$V_s = Z_{22}I_2 \Rightarrow \left.\frac{V_s}{I_2}\right|_{I_1=0} = Z_{22}$$

>> In terms of Y-Parameters:

$$Z_{11} = Y_{22}/|Y|$$

$$Z_{22} = Y_{11}/|Y|$$

$$\Rightarrow \frac{Y_{22}}{|Y|} = \frac{Y_{11}}{|Y|} \Rightarrow Y_{11} = Y_{22}$$

>> In terms of h-Parameters:

$$Z_{11} = \frac{|H|}{h_{22}}$$

$$Z_{22} = 1/h_{22}$$

$$\Rightarrow \frac{|H|}{h_{22}} = \frac{1}{h_{22}} \Rightarrow |H| = 1$$

>> In terms of g-Parameters:

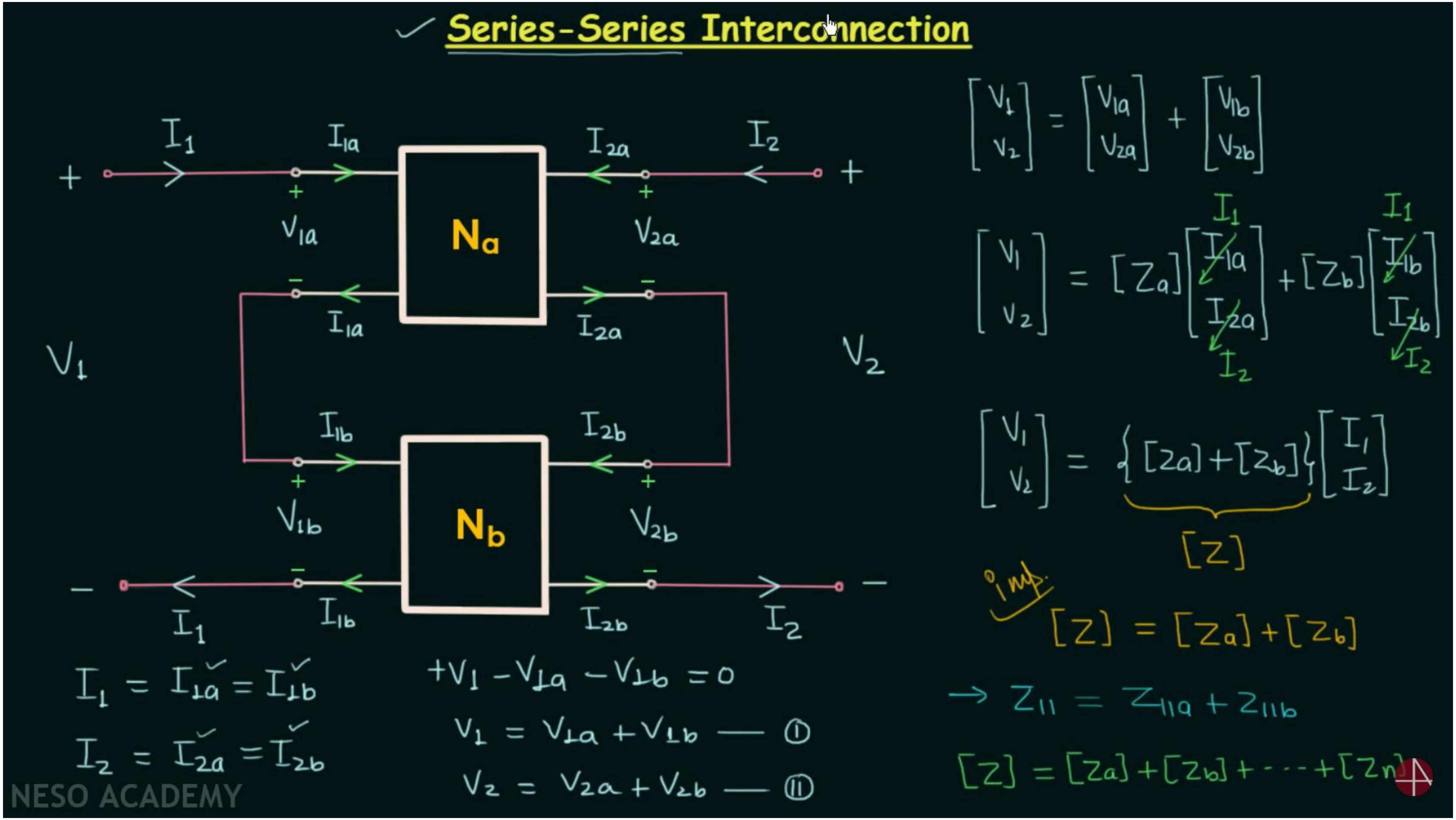
$$Z_{11} = 1/g_{11}$$

$$Z_{22} = |G|/g_{11}$$

$$\Rightarrow \frac{1}{g_{11}} = \frac{|G|}{g_{11}} \Rightarrow |G| = 1$$

>> In terms of ABCD-Parameters:

$$Z_{11} = A/C \quad Z_{22} = D/C \Rightarrow \frac{A}{C} = \frac{D}{C} \Rightarrow A = D$$



✓ Series-Series Interconnection

$$\begin{bmatrix}V_1\\V_2\end{bmatrix} = \begin{bmatrix}V_{1a}\\V_{2a}\end{bmatrix} + \begin{bmatrix}V_{1b}\\V_{2b}\end{bmatrix}$$

$$\begin{bmatrix}V_1\\V_2\end{bmatrix} = [Z_a]\begin{bmatrix}\overset{I_1}{\cancel{I_{1a}}}\\\underset{I_2}{\cancel{I_{2a}}}\end{bmatrix} + [Z_b]\begin{bmatrix}\overset{I_1}{\cancel{I_{1b}}}\\\underset{I_2}{\cancel{I_{2b}}}\end{bmatrix}$$

$$\begin{bmatrix}V_1\\V_2\end{bmatrix} = \underbrace{\{[Z_a]+[Z_b]\}}_{[Z]}\begin{bmatrix}I_1\\I_2\end{bmatrix}$$

imp.

$$[Z] = [Z_a]+[Z_b]$$

$$\rightarrow Z_{11} = Z_{11a} + Z_{11b}$$

$$[Z] = [Z_a]+[Z_b]+\cdots+[Z_n]$$

$I_1 = \overset{\checkmark}{I_{1a}} = \overset{\checkmark}{I_{1b}}$

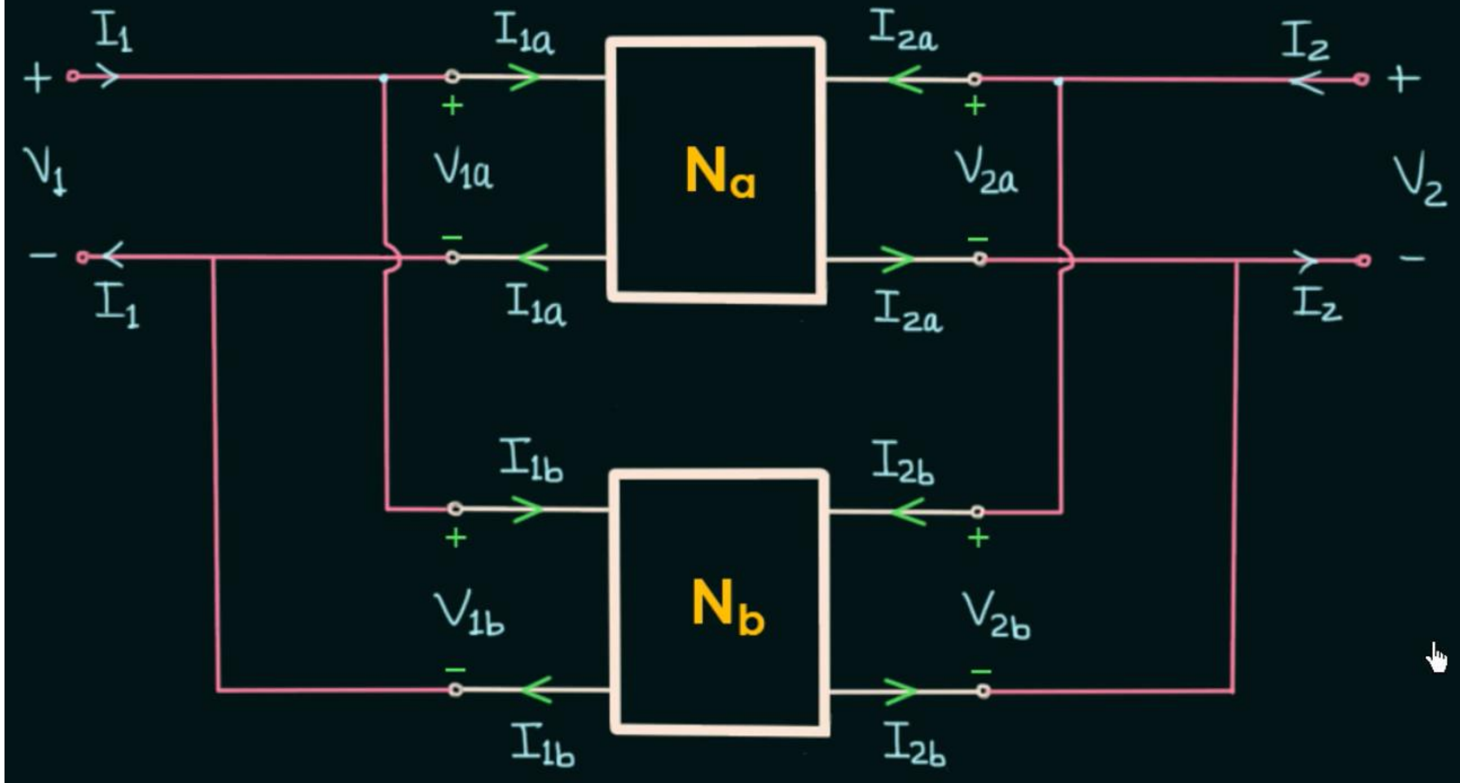
$I_2 = \overset{\checkmark}{I_{2a}} = \overset{\checkmark}{I_{2b}}$

$+V_1 - V_{1a} - V_{1b} = 0$

$V_1 = V_{1a} + V_{1b}$ —— ①

$V_2 = V_{2a} + V_{2b}$ —— ②

Parallel-Parallel Interconnection



$V_1 = V_{1a} = V_{1b}$

$V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} + I_{1b}$ —— ⓘ

$I_2 = I_{2a} + I_{2b}$ —— ⓘⓘ

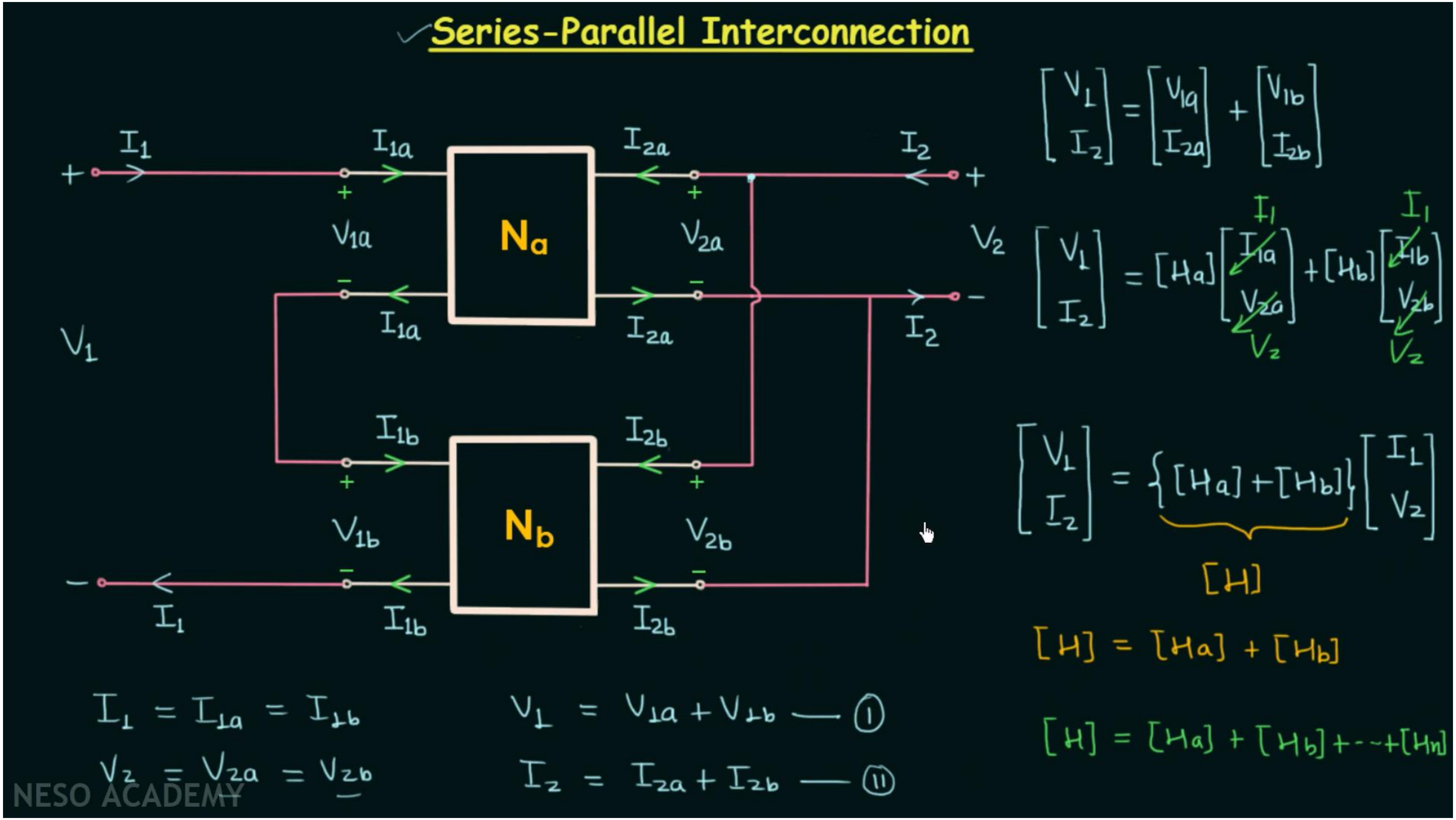
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y_a]\begin{bmatrix} \cancelto{V_1}{V_{1a}} \\ \cancelto{V_2}{V_{2a}} \end{bmatrix} + [Y_b]\begin{bmatrix} \cancelto{V_1}{V_{1b}} \\ \cancelto{V_2}{V_{2b}} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\{[Y_a] + [Y_b]\}}_{[Y]} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[Y] = [Y_a] + [Y_b]$$

$$[Y] = [Y_a] + [Y_b] + \cdots + [Y_n]$$



Series-Parallel Interconnection

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [H_a]\begin{bmatrix} \overset{I_1}{\cancel{I_{1a}}} \\ \underset{V_2}{\cancel{V_{2a}}} \end{bmatrix} + [H_b]\begin{bmatrix} \overset{I_1}{\cancel{I_{1b}}} \\ \underset{V_2}{\cancel{V_{2b}}} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underbrace{\{[H_a]+[H_b]\}}_{[H]} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$[H] = [H_a] + [H_b]$$

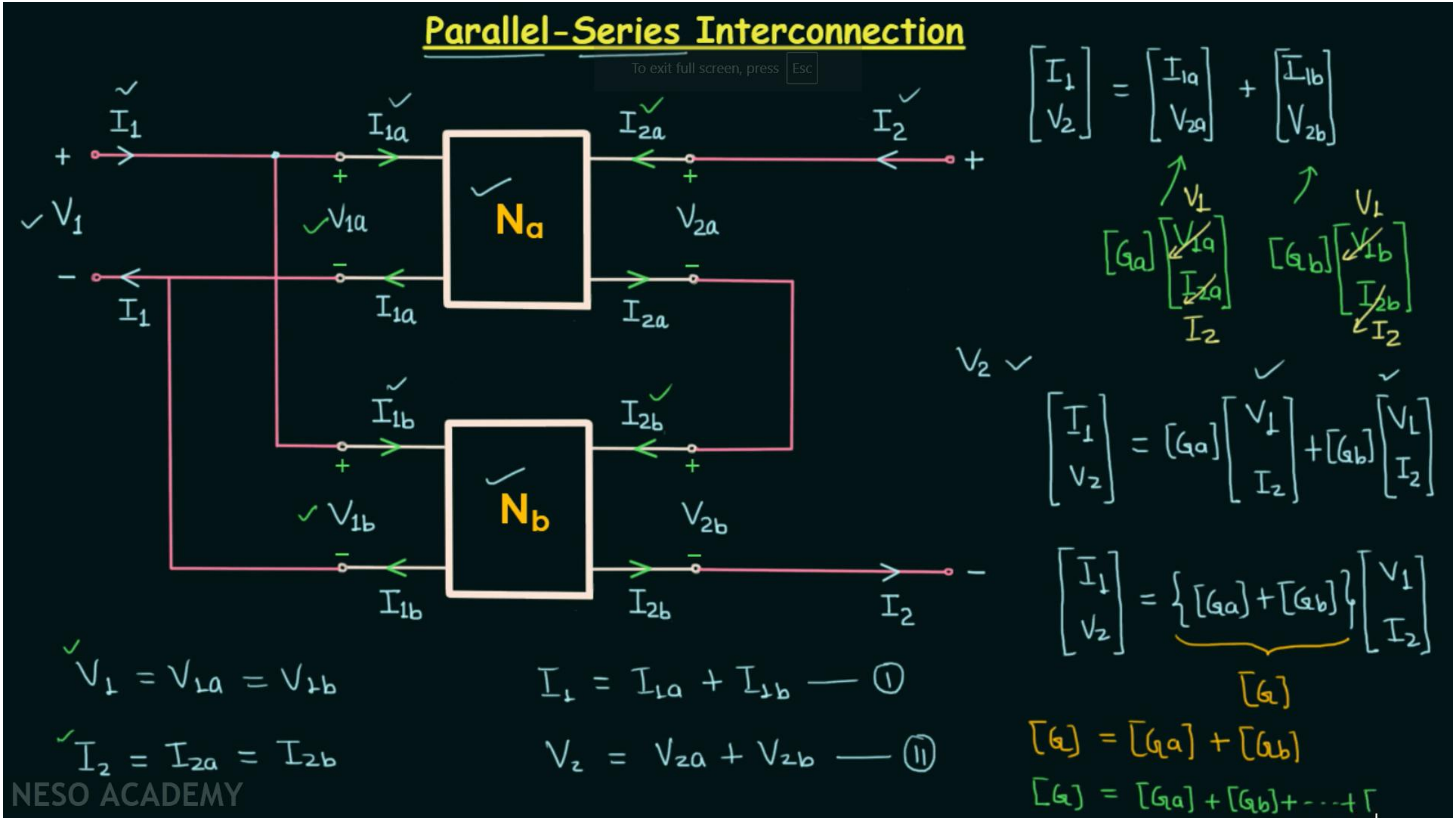
$$[H] = [H_a] + [H_b] + \cdots + [H_n]$$

$I_1 = I_{1a} = I_{1b}$

$V_2 = V_{2a} = V_{2b}$

$V_1 = V_{1a} + V_{1b}$ —— ①

$I_2 = I_{2a} + I_{2b}$ —— ⑪



Parallel-Series Interconnection

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$[G_a]\begin{bmatrix} \overset{V_1}{\cancel{V_{1a}}} \\ \underset{I_2}{\cancel{I_{2a}}} \end{bmatrix} \qquad [G_b]\begin{bmatrix} \overset{V_1}{\cancel{V_{1b}}} \\ \underset{I_2}{\cancel{I_{2b}}} \end{bmatrix}$$

V_2 ✓

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [G_a]\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} + [G_b]\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \underbrace{\{[G_a] + [G_b]\}}_{[G]}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$[G] = [G_a] + [G_b]$$

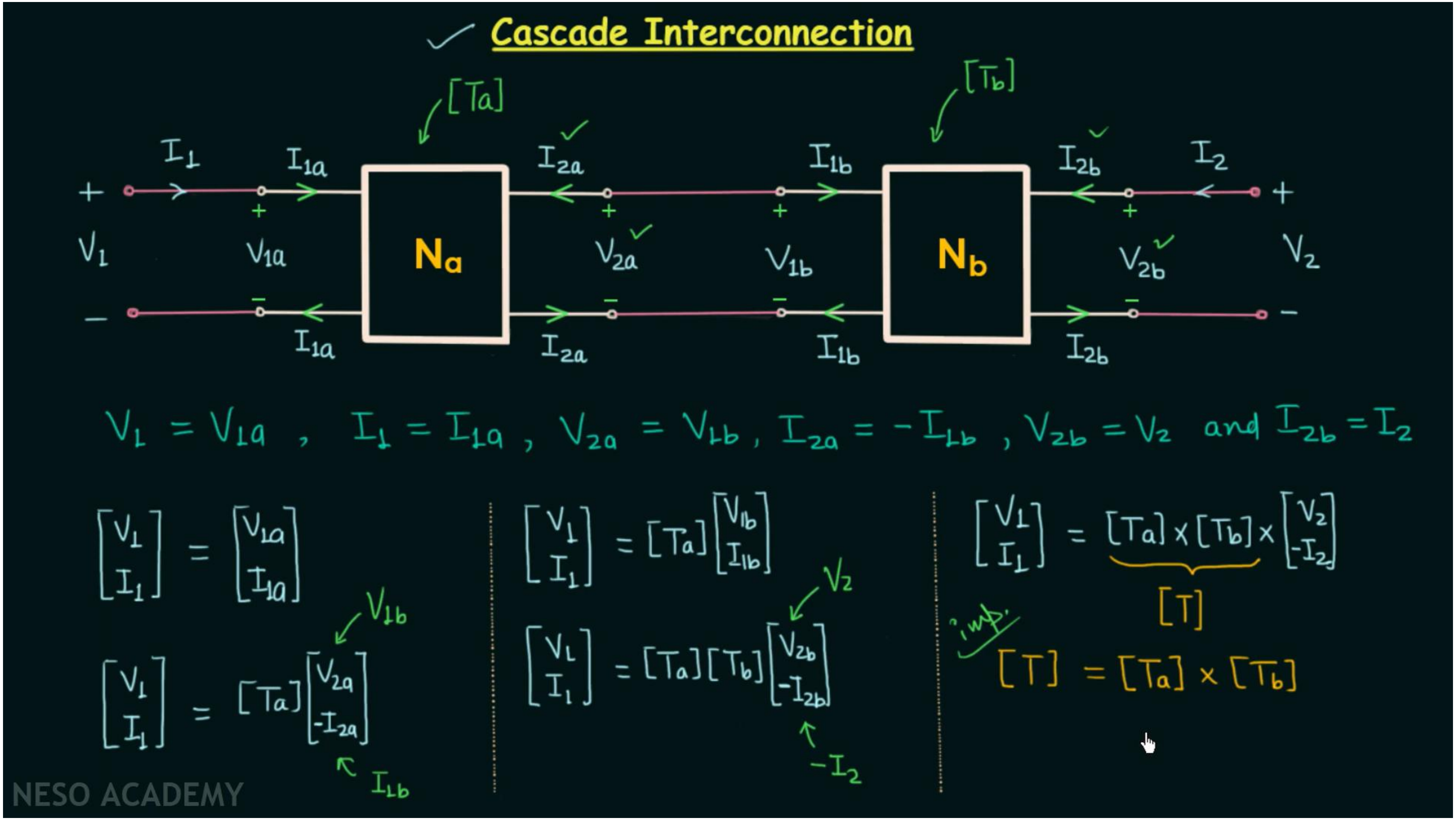
$$[G] = [G_a] + [G_b] + \cdots + [$$

$$V_1 = V_{1a} = V_{1b}$$

$$I_2 = I_{2a} = I_{2b}$$

$$I_1 = I_{1a} + I_{1b} \text{ — ①}$$

$$V_2 = V_{2a} + V_{2b} \text{ — (ii)}$$



Cascade Interconnection

$V_1 = V_{1a}$, $I_1 = I_{1a}$, $V_{2a} = V_{1b}$, $I_{2a} = -I_{1b}$, $V_{2b} = V_2$ and $I_{2b} = I_2$

$$\begin{bmatrix}V_1\\I_1\end{bmatrix} = \begin{bmatrix}V_{1a}\\I_{1a}\end{bmatrix}$$

$$\begin{bmatrix}V_1\\I_1\end{bmatrix} = [T_a]\begin{bmatrix}\overset{V_{1b}}{V_{2a}}\\ \underset{I_{1b}}{-I_{2a}}\end{bmatrix}$$

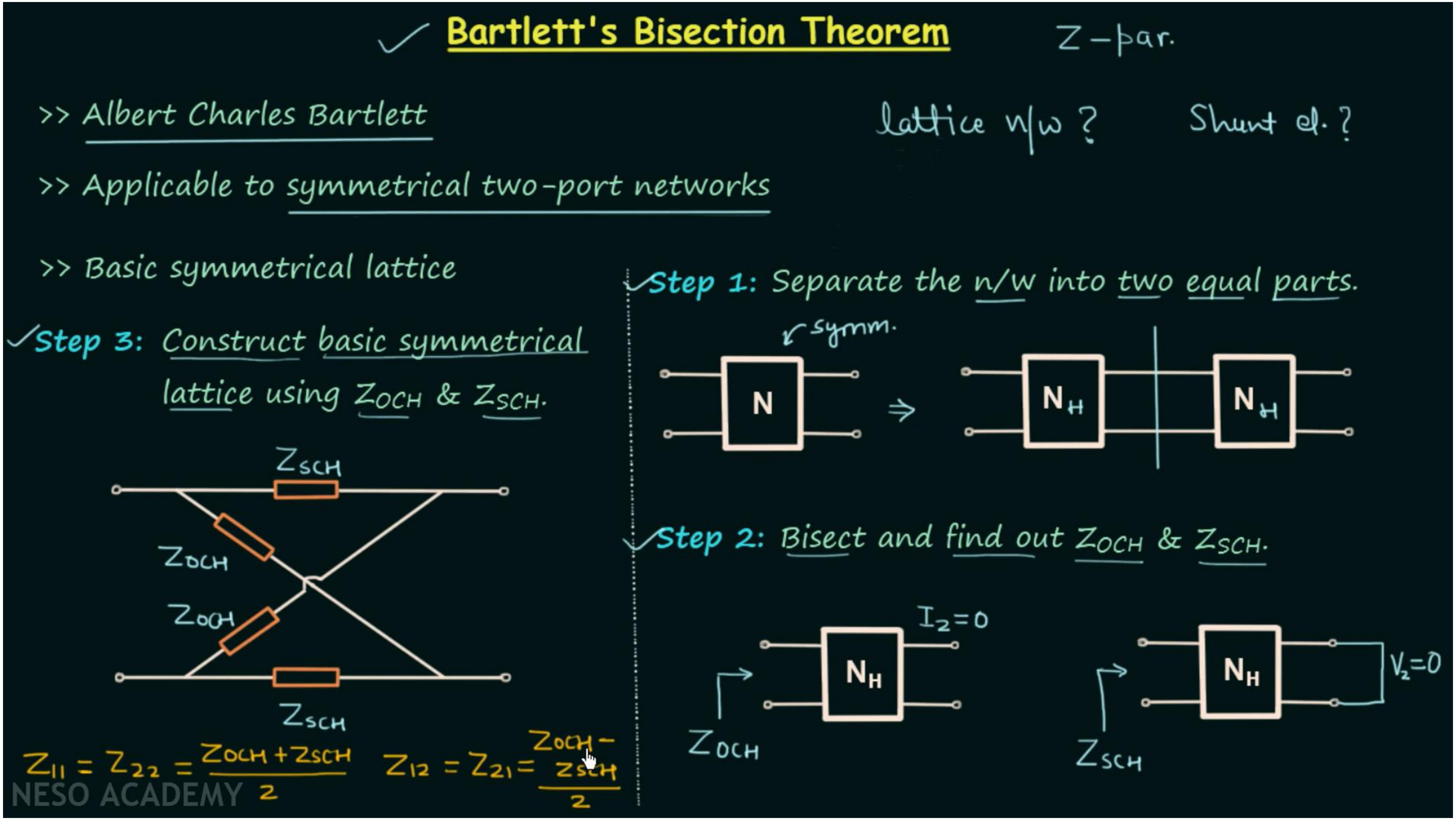
$$\begin{bmatrix}V_1\\I_1\end{bmatrix} = [T_a]\begin{bmatrix}V_{1b}\\I_{1b}\end{bmatrix}$$

$$\begin{bmatrix}V_1\\I_1\end{bmatrix} = [T_a][T_b]\begin{bmatrix}\overset{V_2}{V_{2b}}\\ \underset{-I_2}{-I_{2b}}\end{bmatrix}$$

$$\begin{bmatrix}V_1\\I_1\end{bmatrix} = \underbrace{[T_a]\times[T_b]}_{[T]}\times\begin{bmatrix}V_2\\-I_2\end{bmatrix}$$

imp.

$$[T] = [T_a]\times[T_b]$$



✓ Bartlett's Bisection Theorem

Z-par.

>> Albert Charles Bartlett

lattice n/w ? Shunt el. ?

>> Applicable to symmetrical two-port networks

>> Basic symmetrical lattice

✓Step 1: Separate the n/w into two equal parts.

✓Step 2: Bisect and find out Z_{OCH} & Z_{SCH}.

✓Step 3: Construct basic symmetrical lattice using Z_{OCH} & Z_{SCH}.

$$Z_{11} = Z_{22} = \frac{Z_{OCH} + Z_{SCH}}{2} \qquad Z_{12} = Z_{21} = \frac{Z_{OCH} - Z_{SCH}}{2}$$