Worksheet 1: SMP

Rin Meng 51940633

January 9, 2025

1 Build intuition through examples.

From this point onwards, we will read **Prefers**(x, y) as x prefers y over some other option.

1. Small and trivial instances of the problem, consider these instances:

   (a) Only one employer e and one sutdent s:
   - Students: s_1
   - Employers: e_1
   - Preferences:
     A. **Prefers**(e_1, s_1)
     B. **Prefers**(s_1, e_1)
   - Triviality: Only one student and one employer, so there is only one match.

   (b) Equal preferences for all students and employers:
   i. Students: s_1, s_2
   ii. Employers: e_1, e_2
   iii. Preferences:
     A. **Prefers**$(e_1, s_1 \vee s_2)$
     B. **Prefers**$(e_2, s_1 \vee s_2)$
     C. **Prefers**$(s_1, e_1 \vee e_2)$
     D. **Prefers**$(s_2, e_1 \vee e_2)$
   iv. Triviality: All students and employers have indifferent preferences, so there are multiple stable matchings.

(c) Perfectly matched preferences between students and employers:

i. Students: s_1, s_2

ii. Employers: e_1, e_2

iii. Preferences:

A. **Prefers**(e_1, s_1)

B. **Prefers**(e_2, s_2)

C. **Prefers**(s_1, e_1)

D. **Prefers**(s_2, e_2)

iv. Triviality: All are perfectly matched, therefore there is only one stable matching.

2. Potential solutions to these instances:

(a) Only one employer e and one sutdent s:

- The only stable matching is (e_1, s_1).
- This solution is trivial but is optimal as well.

(b) Equal preferences for all students and employers:

- There are multiple stable matchings: $(e_1, s_1), (e_2, s_2)$ and $(e_1, s_2), (e_2, s_1)$.
- This solution is optimal, but not unique.

(c) Perfectly matched preferences between students and employers:

- The only stable matching is $(e_1, s_1), (e_2, s_2)$.
- This solution is optimal and unique.

There are many ways to conclude if a solution is better. When we consider fairness and satisfication, we may value **Instance b** more than others. However, when we consider uniqueness and optimality, we may value **Instance c** more than others.

2 Developing a Formal Problem Specification

1. Notation for describing the problem instance.

(a) Let $S = \{s_1, s_2, \ldots, s_n\}$ be the set of students.

(b) Let $E = \{e_1, e_2, \ldots, e_n\}$ be the set of employers

(c) Student's preference list $P(s_i)$, which is a ranked list of employers from most preferred to least preferred.

(d) Employer's preference list $P(e_j)$, which is a ranked list of students from most preferred to least preferred.

2. Notation for describing a potential solution

   (a) A set of potential matches $M = (e_1, s_1), (s_2, e_2)$ of employer-student pairs.

   (b) Valid if and only if every student and employer is assigned exactly one (uniqueness).

3. Good solutions

   (a) Optimality

      i. A solution is student-optimal if it provides the best possible match for every student.

      ii. A solution is employer-optimal if it provides the best possible match for every employer.

   (b) Uniqueness

      i. If a unique stable matching exists, it is the only correct solution.

      ii. If multiple stable matchings exist, we will pick the one that maximizes a criteria (e.g., student happiness, fairness).

   (c) Stability

      i. A matching M is stable if there exists no employer-student pair (e_i, s_j) such that

         A. e_i prefers s_j over $M(e_i)$, the student currently matched with e_i

         B. s_j prefers e_i over $M(s_j)$, the employer currently matched with s_j