

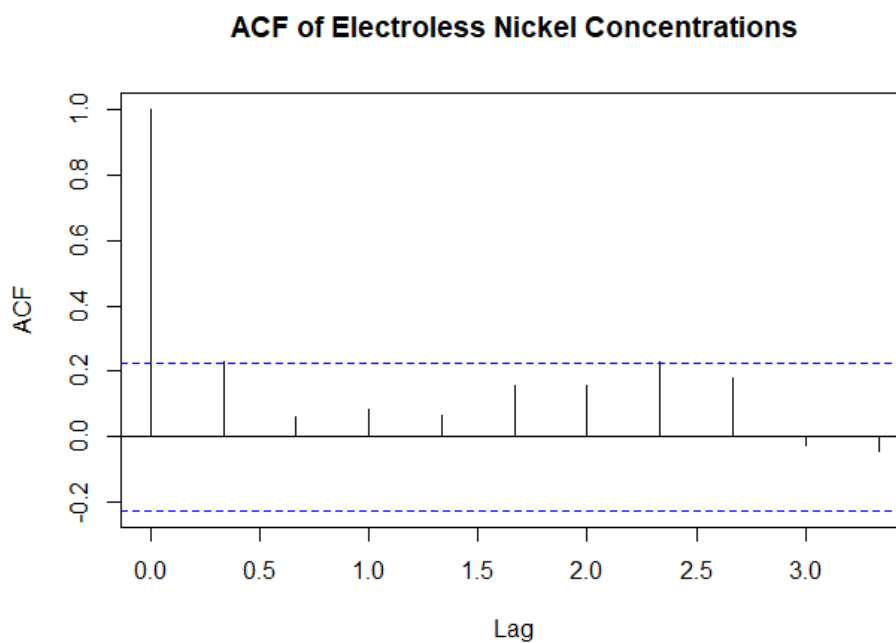
# DATA315 Assignment 3

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1. (a) 

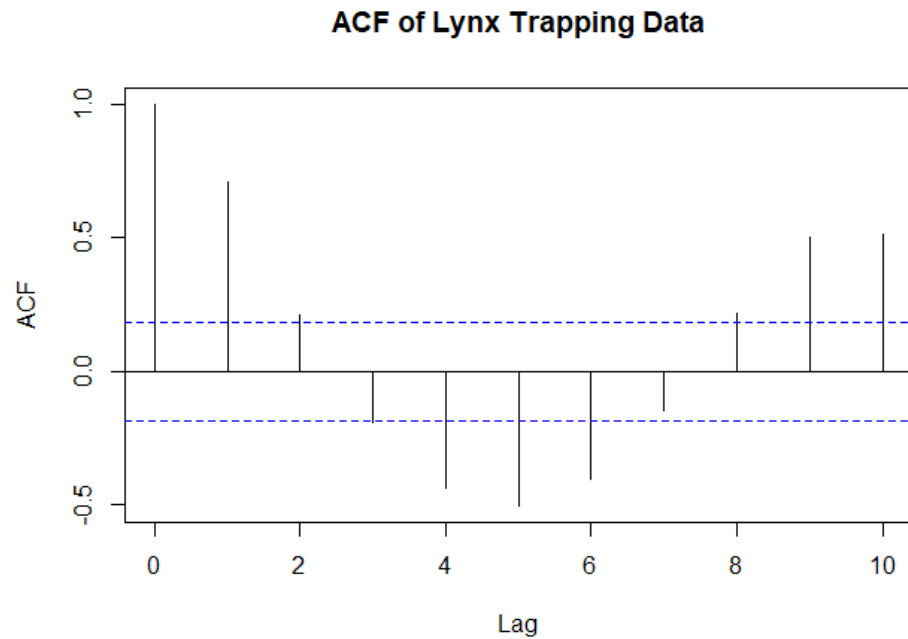
```
source("nickel.R")  
acf(nickel, lag.max = 10,  
main = "ACF of Electroless Nickel Concentrations")
```



The ACF plot seems to follow an MA(1) process, as significant correlation at lag 1 followed by immediate drop to near zero.

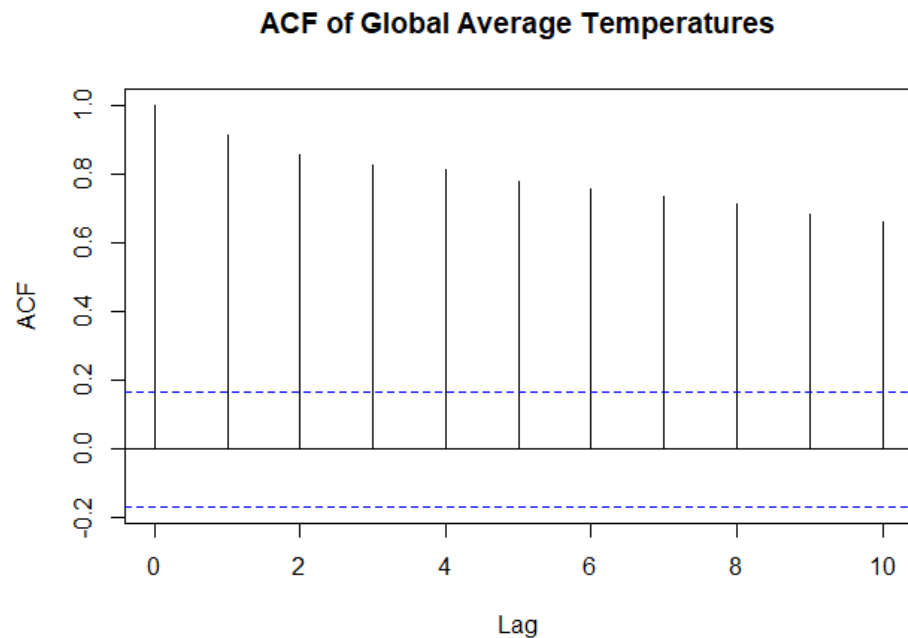
- (b) 

```
data(lynx)  
acf(lynx, lag.max = 10, main = "ACF of Lynx Trapping Data")
```



So far, there are no models that fit, because the plot shows a cyclic pattern between predator and prey populations.

```
(c) source("Globaltemps.R")
    temps <- ts(temps, start = 1880, end = 2016)
    acf(temps, lag.max = 10,
        main = "ACF of Global Average Temperatures")
```



The ACF plot seems to follow an AR(1) process, as significant correlation at lag 1 followed by gradual decay.

```
(d) data("EuStockMarkets")
```

```

dax <- EuStockMarkets[, 1]

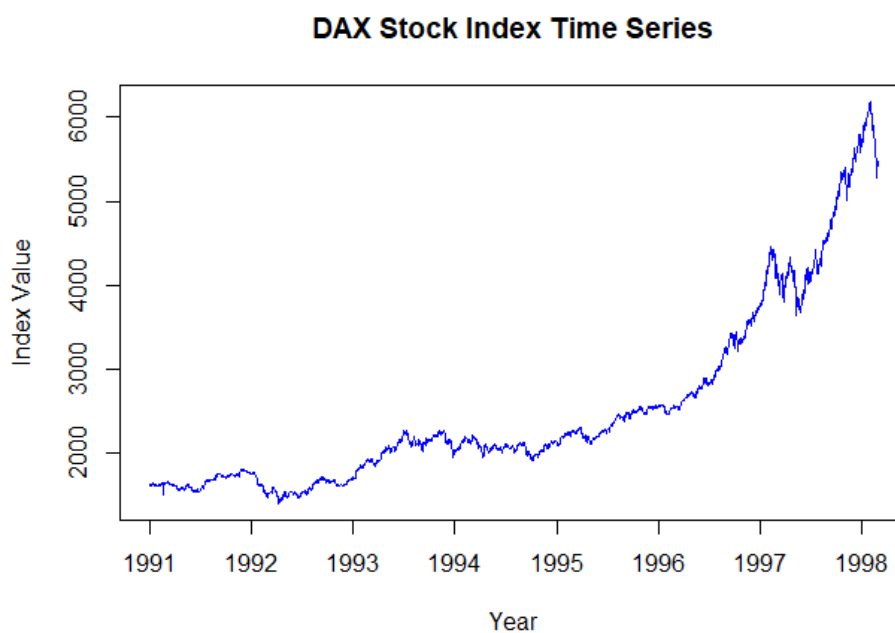
# 260 trading days per year
dax_ts <- ts(dax, start = c(1991, 1), frequency = 260)

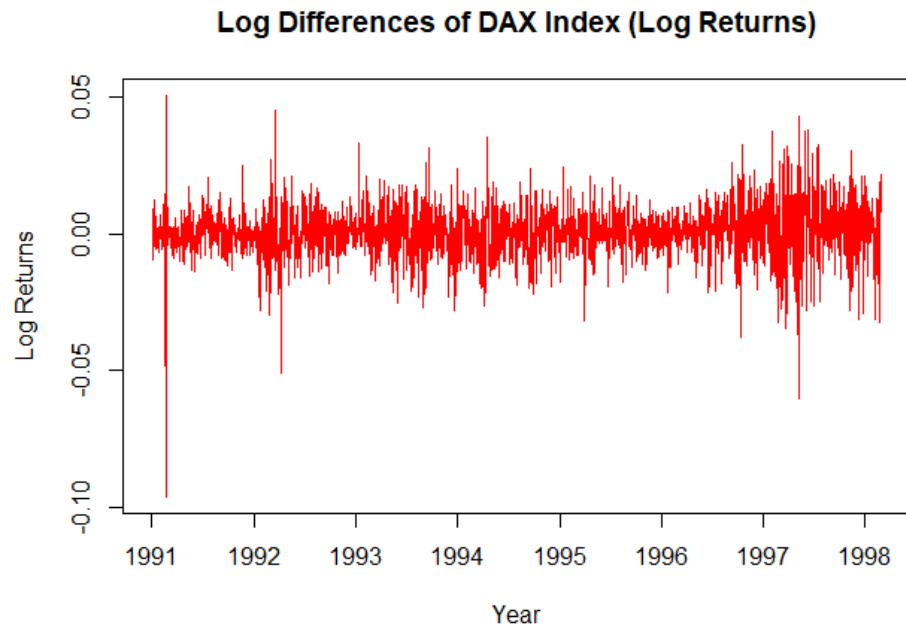
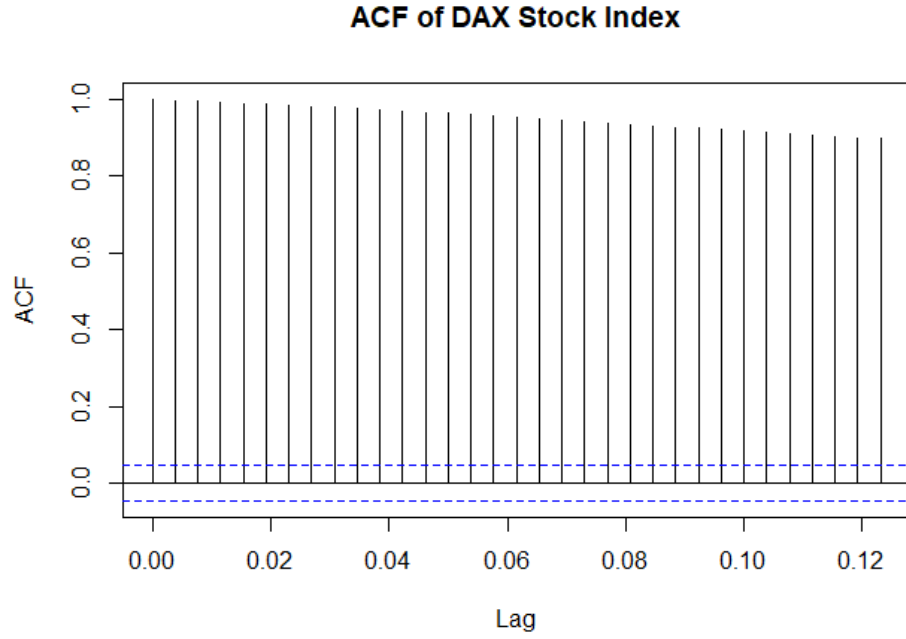
# Time series plot
plot(dax_ts,
     main = "DAX Stock Index Time Series",
     ylab = "Index Value", xlab = "Year",
     col = "blue", type = "l")

# ACF plot
acf(dax_ts, main = "ACF of DAX Stock Index")

# Take the natural log
log_dax <- log(dax_ts)
# Compute first differences (log returns)
diff_log_dax <- diff(log_dax)
acf(diff_log_dax, main = "ACF of Log Returns of DAX")

```





Some observations for the DAX stock index time series plot; visually, we can see that there is a general upward trend with some fluctuations. The ACF plot shows that there is a significant correlation at lag 1, followed by a gradual, slow decay, which means that it may be following other process we have not covered yet. The log returns plot shows that the data revolves around 0, which is a good sign for stationarity.

2. (a) Given that the MA(1) process is defined as

$$X_t = \mu_t + \epsilon_t + \theta\epsilon_{t-1}$$

How we have to test whether the MA parameter is equal to 0.

```

# Print the model summary
summary(ma1_model)

Call:
arima(x = nickel, order = c(0, 0, 1))

Coefficients:
            ma1  intercept
            0.2260      4.6223
s.e.    0.1099      0.0277

sigma^2 estimated as 0.03857:
log likelihood = 15.63,  aic = -25.26

# Extract MA(1) coefficient and its standard error
theta_hat <- ma1_model$coef["ma1"]
se_theta <- sqrt(ma1_model$var.coef["ma1", "ma1"])

# Compute t-statistic
t_value <- theta_hat / se_theta

# Compute p-value (two-tailed test)
p_value <- 2 * (1 - pnorm(abs(t_value)))

# Print results
t_value
      ma1
2.05681
p_value
      ma1
0.03970446

```

The fitted model is  $X_t = 4.6223 + \epsilon_t + 0.2260\epsilon_{t-1}$ . Since the p-value is less than 0.05, we reject the null hypothesis that the MA parameter is equal to 0.

- (b) The portmanteau test checks whether the residuals from our fitted MA(1) model behave like white noise, meaning they are uncorrelated. We can use the Box-Ljung test to check this.

```

# Perform the Box-Ljung test
Box.test(ma1_model$residuals, lag = 10, type = "Ljung-Box")

library(forecast)
checkresiduals(ma1_model)

```

Ljung-Box test

```

data:  Residuals from ARIMA(0,0,1) with non-zero mean
Q* = 2.5221, df = 5, p-value =
0.7732

```

Model df: 1. Total lags used: 6

Since the p-value is greater than 0.05, we fail to reject the null hypothesis that the residuals are uncorrelated.

- (c) If the 75th value is missing, we can forecast it using the fitted MA(1) model.

$$X_{75} = 4.6223 + \epsilon_{75} + 0.2260\epsilon_{74}$$

Now we can extract the last residuals

```
# Extract last residual
epsilon_74 <- residuals(ma1_model)[74]

# Compute forecast
X_75_hat <- 4.6223 + (0.2260 * epsilon_74)
X_75_hat
[1] 4.545646
```

The forecasted value for  $X_{75}$  is 4.545646. The standard deviation of the forecast is given by

$$\sigma_{\text{forecast}} = \sqrt{\hat{\sigma}^2} \sqrt{1 + \theta^2}$$

From the model, we have

$$\hat{\sigma}^2 = 0.03857$$

$$\theta = 0.2260$$

So we can calculate the standard deviation of the forecast.

$$\sigma_{\text{forecast}} = \sqrt{0.03857} \sqrt{1 + 0.2260^2} = 0.2013455$$

The error in terms of standard deviations is given by

$$Z = \frac{X_{75} - X_{75}^{\text{forecast}}}{\sigma_{\text{forecast}}} = \frac{4.3 - 4.545646}{0.2013455} = -1.220022$$

Since  $|Z| < 2$ , the forecast is within the 95% confidence interval.

- (d) First we fit an AR(1) model to the data.

```
ar1_model <- arima(nickel, order = c(1, 0, 0))
```

```
# Print model summary
summary(ar1_model)
```

Call:

```
arima(x = nickel, order = c(1, 0, 0))
```

Coefficients:

```
          ar1  intercept
          0.2363      4.6221
s.e.    0.1139      0.0295
```

sigma^2 estimated as 0.03845:

log likelihood = 15.74, aic = -25.47

Then we forecast for the 2nd and 3rd values after the end of the series.

```
# Forecast 2 steps ahead
ar1_forecast <- predict(ar1_model, n.ahead = 3)

# Print forecasted values
ar1_forecast$pred
Time Series:
Start = c(26, 1)
End = c(26, 3)
Frequency = 3
[1] 4.545956 4.604083 4.617820
```

Now we check if the residuals are white noise.

```
# Perform Ljung-Box test on AR(1) residuals
Box.test(ar1_model$residuals, lag = 10, type = "Ljung-Box")
```

Box-Ljung test

```
data: ar1_model$residuals
X-squared = 7.1631, df = 10, p-value = 0.71
```

Since the p-value is greater than 0.05, we fail to reject the null hypothesis that the residuals are white noise.

3. The given time series model is

$$y_t = \mu + \phi(y_{t-1} - \mu) + \varepsilon_t$$

The expected value of  $y_t$  is

$$\begin{aligned} E(y_t) &= \mu + \phi(E(y_{t-1}) - \mu) \\ &= \mu + \phi(\mu - \mu) = \mu \end{aligned}$$

$\mu$  can be estimated by the sample mean:

$$\hat{\mu} = \frac{1}{n} \sum_{t=1}^n y_t$$

for the given data  $\{3.2, 3.2, 2.2, 2.3, 1.8, 1.3, 2.2, 2.7\}$

$$\hat{\mu} = \frac{1}{8}(3.2 + 3.2 + 2.2 + 2.3 + 1.8 + 1.3 + 2.2 + 2.7) = 2.3625$$

$$\hat{\mu} = 2.3625$$

Now, we will estimate  $\phi$ . The autocovariance at lag 1 is given by

$$\gamma_1 = E[(y_t - \mu)(y_{t-1} - \mu)]$$

which can be estimated by

$$\hat{\gamma}_1 = \frac{1}{n} \sum_{t=2}^n (y_t - \hat{\mu})(y_{t-1} - \hat{\mu})$$

similarly, the variance is

$$\gamma_0 = E[(y_t - \mu)^2]$$

which can be estimated by

$$\hat{\gamma}_0 = \frac{1}{n-1} \sum_{t=1}^n (y_t - \hat{\mu})^2$$

Since for this process, the autocorrelation at lag 1 is given by we can estimate  $\phi$  by

$$\hat{\phi} = \frac{\hat{\gamma}_1}{\hat{\gamma}_0}$$

Computing the estimates for  $\gamma_1$  and  $\gamma_0$ :

$$\hat{\gamma}_0 = \frac{1}{8} \sum_{t=1}^8 (y_t - 2.3625)^2 = 0.3773438$$

$$\hat{\gamma}_1 = \frac{1}{8} \sum_{t=2}^8 (y_t - 2.3625)(y_{t-1} - 2.3625) = 0.189442$$

So the estimate for  $\phi$  is

$$\hat{\phi} = \frac{0.189442}{0.3773438} = 0.502$$

Now we can estimate  $\sigma$  by

$$\hat{\sigma}^2 = \hat{\gamma}_0(1 - \hat{\phi}^2) = 0.282$$

$$\hat{\sigma} = \sqrt{0.282} = 0.531$$