DATA315 Assignment 4

Rin Meng 51940633

March 29, 2025

1. (a) $z_t = 0.4z_{t-3} + \varepsilon_t$ is an order 3 autoregressive model, AR(3). Since the $\phi = 0.4$ is less than 1, the process is stationary.

   (b) $z_t = 0.7z_{t-1} - 1.2z_{t-2} + \varepsilon_t$ is an order 2 autoregressive model, AR(2). The characteristic polynomial is $1 - 0.7B - 1.2B^2 = 0$. The roots of the characteristic polynomial are $B_1 = -\frac{5}{4}$ and $B_2 = \frac{2}{3}$, thus the process is stationary.

   (c) $z_t = -0.2z_{t-1} - 1.2z_{t-3} + \varepsilon_t$ is an order 3 autoregressive model, AR(3). The characteristic polynomial is $1 + 0.2B + 1.2B^3 = 0$. The roots of the characteristic polynomial are $B \simeq -0.88$, thus the process is stationary.

   (d) $z_t = 0.7z_{t-1} - 0.2z_{t-2} + 0.1z_{t-3} + 0.4z_{t-4} + \varepsilon_t$ is an order 4 autoregressive model, AR(4). The characteristic polynomial is $1 - 0.7B + 0.2B^2 - 0.1B^3 - 0.4B^4 = 0$. The roots of the characteristic polynomial are $B_1 \simeq 1$, $B_2 \simeq -1.6852$, thus the process is non-stationary.

   (e) $z_t = z_{t-1} - 2z_{t-2} + 3z_{t-3} + 4z_{t-4} + \varepsilon_t$ is an order 4 autoregressive model, AR(4). The characteristic polynomial is $1 - B + 2B^2 - 3B^3 - 4B^4 = 0$. The roots of the characteristic polynomial are $B_1 \simeq 0.59105$, $B_2 \simeq -1.35538$, thus the process is non-stationary.

2. (a) Given that
   $$z_t = -0.2z_{t-2} + \varepsilon_t, \sigma_\varepsilon^2 = 9$$
   - $E[z_t] = 0$ since $E[\varepsilon_t] = 0$.
   - $Var(z_t) = 0.2^2 Var(z_{t-2}) + Var(\varepsilon_t)$, since this is a stationary process, we have $Var(z_t) = Var(z_{t-2})$. $\gamma_0 = 0.2^2\gamma_0 + 9$, thus $\gamma_0 = \frac{9}{1-0.2^2} = \frac{9}{0.96} = 9.375$.
   - $\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{0}{9.375} = 0$.
   - $\rho_2 = \frac{\gamma_2}{\gamma_0}$, where $\gamma_2 = -0.2 \times 9.375 = -1.875$, thus $\rho_2 = \frac{-1.875}{9.375} = -0.2$.

   (b) Given that
   $$z_t = -0.8z_{t-1} + 0.1z_{t-2} + \varepsilon_t, \sigma_\varepsilon^2 = 4$$
   - $E[z_t] = 0$ since $E[\varepsilon_t] = 0$.
   - $Var(z_t) = (-0.8)^2 Var(z_{t-1}) + (0.1)^2 Var(z_{t-2}) + Var(\varepsilon_t)$, since this is a stationary process, we have $Var(z_t) = Var(z_{t-1}) = Var(z_{t-2})$. $\gamma_0 = 0.64\gamma_0 + 0.01\gamma_0 + 4$, thus $\gamma_0 = \frac{4}{1-0.65} = \frac{4}{0.35} = 11.42857$.
   - $\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{-0.8 \times 11.42857}{11.42857} = -0.8$.
   - $\rho_2 = \frac{\gamma_2}{\gamma_0}$, where $\gamma_2 = (0.8)^2 \times 11.42857 + (0.1)^2 \times 11.42857 = 7.142857$, thus $\rho_2 = \frac{7.142857}{11.42857} = 0.62376$.