a1

2024-09-12

1 # Question 1).

```
unirand <- function(n, m=30269, a=171, seed=1) {
  x <- numeric(min(m-1,n))
  x[1] <- seed
  for (i in 1:min(m-1,n)){
    y <- x[i]
    x[i+1] <- (a*y)%%m
  }
  x[2:(n+1)]/m
}
unirand(5)
```

```
## [1] 0.005649344 0.966037861 0.192474148 0.913079388 0.136575374
```

```
# Code taken from SimulationI slide
```

This multiplicative congruential generator contains a maximal cycle length of $m - 1 = 30306$.

Question 2).

```
unirand2 <- function(n, m=30323, a=170, seed=1) {
  x <- numeric(min(m-1,n))
  x[1] <- seed
  for (i in 1:min(m-1,n)){
    y <- x[i]
    x[i+1] <- (a*y)%%m
  }
  x[2:(n+1)]/m
}
unirand2(5)
```

```
## [1] 0.005606305 0.953071926 0.022227352 0.778649870 0.370477855
```

```
# Code taken from SimulationI slide
```

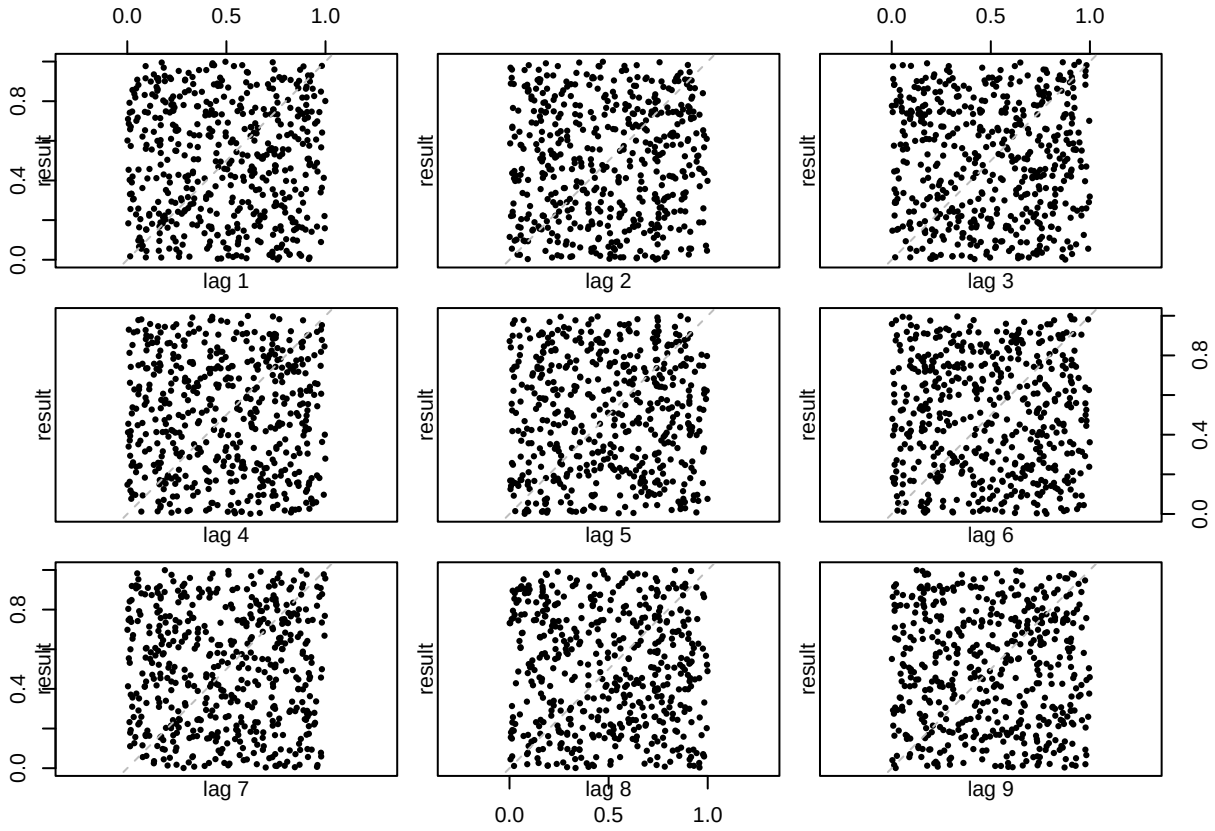
This multiplicative congruential generator contains a maximal cycle length of $m - 1 = 30322$.

Question 3).

```
unirand3 <- function(n, s1, s2) {
  u1 <- unirand(n, seed=s1)
  u2 <- unirand2(n, seed=s2)
  u3 <- u1 + u2 - floor(u1 + u2)
  return(u3)

}

result <- unirand3(500, 1, 1)
lag.plot(result, lag=9, pch=16, do.lines=FALSE)
```



By these lag graphs, we can tell that the plots are uniformly distributed, and they do not seem to be dependent of the past plots. The cycle length of this generator cannot exceed 300000, because the m of unirand and unirand2 are both below 300000, where the cycle length is determined by taking the minimum of $m-1$ and n where $n = 300000$.

Question 4).

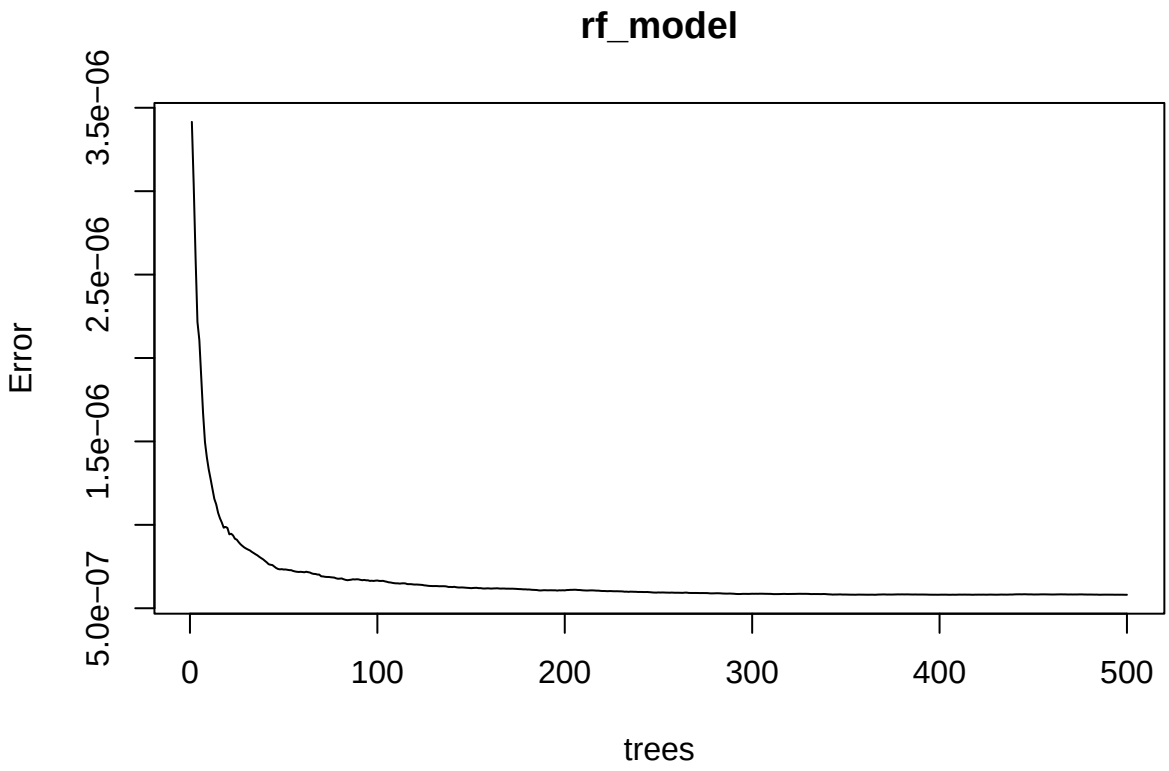
```
result1000 <- unirand3(1000, 1, 1)
df <- data.frame(y = result1000, x1 = result1000, x2 = result1000, x3 = result1000,
```

```
                        x4 = result1000, x5 = result1000, x6 = result1000, x7 = result1000,
                        x8 = result1000, x9 = result1000, x10 = result1000)
library(randomForest)
```

```
## randomForest 4.7-1.1

## Type rfNews() to see new features/changes/bug fixes.
```

```
rf_model <- randomForest(y~.,data = df)
plot(rf_model)
```




```
print(rf_model)
```

```
##
## Call:
##  randomForest(formula = y ~ ., data = df)
##                Type of random forest: regression
##                      Number of trees: 500
## No. of variables tried at each split: 3
##
##           Mean of squared residuals: 5.807857e-07
##                     % Var explained: 100
```

With the plot and the printed information, we can safely say that the randomForest follows a very high percentage of variance at 100%, which makes the model overfitting, that means the model is allowing less error as more trees are generated.

Question 5

```r
# find greatest common denominator
gcd <- function(a, b) {
  while (b != 0) {
    temp <- b
    b <- a %% b
    a <- temp
  }
  return(a)
}

# helper function
gcd_set <- function(numbers) {
  result <- numbers[1]
  for (i in 2:length(numbers)) {
    result <- gcd(result, numbers[i])
  }
  return(result)
}

# example
sequence <- c(2205, 21065, 5241, 12752, 25817, 6724, 18604, 7158, 21788, 20601)
gcd_result <- gcd_set(sequence)
print(gcd_result)
```

```
## [1] 1
```

```r
diffs <- diff(sequence)
print(diffs)
```

```
## [1]  18860 -15824   7511  13065 -19093  11880 -11446  14630  -1187
```

So, with the difference printed out, $diffs = [-19093, 18860]$ and since linear congruential generators often use powers of 2 for modulus values, we can start testing values that are closer to 19093, such as $m = 2^{15} = 32768$. Now we can use brute force to find a and c.

```r
m <- 32768

# define the known values from the sequence
X0 <- 2205
X1 <- 21065
X2 <- 5241

# Set up the equations (modular arithmetic)
# Equation 1: X1 = (a * X0 + c) %% m
# Equation 2: X2 = (a * X1 + c) %% m

# Solve for a and c using brute force or linear algebra in modular arithmetic
for (a in 1:m) {
  c <- (X1 - a * X0) %% m
```

```
  if ((a * X1 + c) %% m == X2) {
    print(paste("a:", a, "and c:", c))
    break
  }
}
```

```
## [1] "a: 3236 and c: 29109"
```

So now that we have our a and c value, we can start to predict the next 10 numbers after 20601.

```
a <- 3236
c <- 29109
m <- 32768

lastX <- 20601

generateNum <- function(a,c,m,xn){
  return((a*xn + c) %% m)
}

next10 <- c(20601)

for (i in 2:11){
  next10[i] <- generateNum(a, c, m, next10[i-1])
}

# Display the generated numbers
print(next10)
```

```
##  [1] 20601 11065 20025 14905 27193 10809 10809 10809 10809 10809 10809
```