The University of British Columbia
I.K. Barber Faculty of Science

COSC 405/DATA 405/DATA 505/COSC 505 Modelling and Simulation

Practice Term Test 1

1. (4 marks) Write out the R code required to

   (a) calculate the standard deviation of the following sample:

   ```
   ##  [1]  27  48  72 101  98  37  22  55  41  79  58  44  61
   ```

   (b) produce a boxplot of the sample above.

   (c) produce a normal QQ-plot of the sample above with reference line.

   (d) test whether the mean of the sample differs from 50. and constructs a 99% confidence interval for the mean of the population from which the sample was taken.

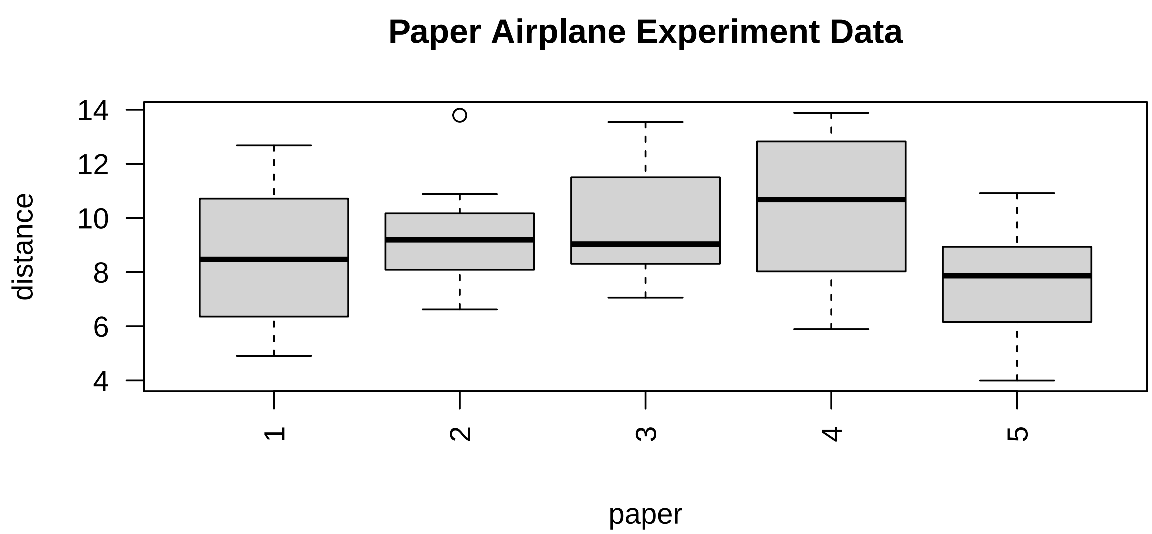
2. (10 marks) Consider the following simulated data and analysis. The simulation is to emulate a paper airplane throwing experiment involving a number of throws for each of a number of different types of paper.

```
papergroup <- factor(rep(1:5, 7))
distances <- rnorm(35, mean = 9, sd = 3)
```

(a) How many different types of paper are being simulated?

(b) Is there a true difference in the mean distance travelled by paper airplanes in the different groups?

(c) Provide the R code to obtain the following plot



(d) Fill in the blanks in the code below in order to get the following output.

```
_________(lm(____________ ~ _______________))
```

```
## Analysis of Variance Table
##
## Response: distances
##             Df Sum Sq Mean Sq F value Pr(>F)
## papergroup   4  33.50  8.3751   1.199 0.3316
## Residuals   30 209.55  6.9849
```

(e) Based on the output from part (d), what can you conclude about the simulated paper airplanes? Briefly support your conclusions.

3. (6 marks) Consider the gas mileage data in `table.b3` of the *MPV* package.

   (a) Fit a multiple regression model to estimate mean gas mileage y for cars with x_7 number of transmission speeds and having weight x_{10}.

   (b) Use the model to estimate mean gas mileage for cars having weight 5000 pounds and 4 transmission speeds.

4. (5 marks) Consider a data frame, such as the `women` object built into R, for which the heights could be taken as x values and the weights could be taken as y values.

   Write an R function called TukeySmooth which outputs a new data frame consisting of a column of equally spaced x values and a column of corresponding local medians, and which takes the following arguments

   - `x`: the vector of x values
   - `y`: the vector of y values
   - `x.min`: a constant which specifies the left boundary of the plotted curve
   - `x.max`: a constant which specifies the right boundary of the plotted curve
   - `window`: a constant which specifies the range of the x values used to calculate each of the moving medians.

   The output for this function will be a data frame with 2 columns: `x` and `y`, which will correspond to the y-medians and the corresponding x locations where the medians are taken.

5. (2 marks) Apply the function obtained in the previous question, using a window width of 5, to the data in `women`, plotting the data, and overlaying the smooth curve.

6. (5 marks) Consider the following iteration scheme:

$$x_{n+1} = f(x_n) := \frac{x_n}{2} - \frac{24.5}{x_n}$$

where x_0 is the initial value, say, $x_0 = 0.5$.

(a) Plot the graph of the function $f(x)$, for $x \in [0.1, 10]$ and overlay the straight line with intercept 0 and slope 1. Does $f(x)$ have a fixed point?

(b) Using a `for` loop in R, run 10 steps of the proposed scheme, printing out the value of x_n at each step.

(c) Based on your analysis, could the proposed scheme lead to a useful pseudorandom number generator?

7. (4 marks) Which of the following linear congruential pseudorandom number generators have maximal cycle length?

   (a) $a = 1025, c = 27, m = 2^{31}$
   (b) $a = 1025, c = 54, m = 2^{31}$
   (c) $a = 1025, c = 375, m = 2^{31}$
   (d) $a = 10241, c = 375, m = 2^{31}$

8. (8 marks) Write a function called `myrbinom()` to compute `N` independent binomial random variates which have parameter `n` and `p`. The function should use the `runif()` function in a single `for()` loop to simulate `N` vectors of `n` Bernoulli(`p`) random variates which are then added to obtain the binomial numbers:

$$X = \sum_{i=1}^{n} B_i$$

is a binomial (n, p) random variable if $B_1, \ldots, B_n$ are independent Bernoulli (p) random variables.