

# Chapter 1: Correctness of Programs:

- ⇒ Programmers make mistakes but some systems must not have errors.  
(planes, airbags, rocket systems)
- ↳ Careful engineering, systematic testing, proof of correctness through verification.

## 1.0 Assertions:

- ⇒ In order to prove that the program is correct, we must know what to expect from the program execution. We write that down with logic using assertions.  
The static method assert() expects a boolean argument.  
The calls of assert are only evaluated if Java is launched with the options -ea.

If the argument is true → execution continues, else Assertion Error is thrown.

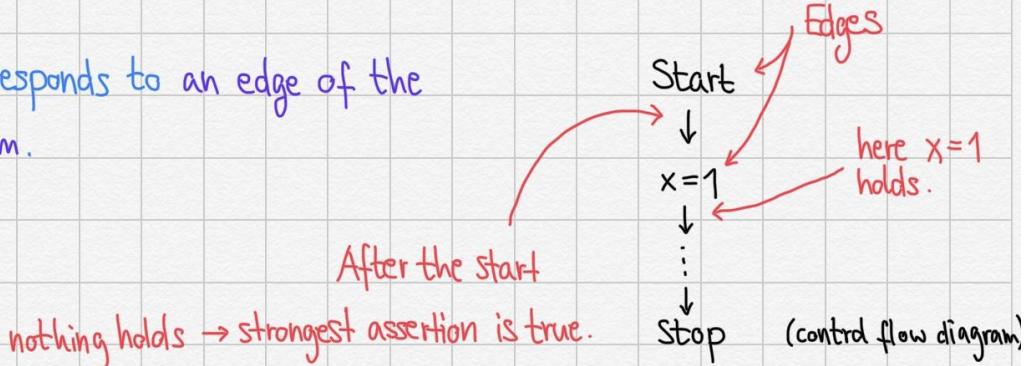
**Attention!** The check must not change the program state.

Use distinct inspector classes to inspect the properties of complicated data structures without interference.

## 1.1 Program Verification:

- ⇒ General method to guarantee a given assertion is valid.  
Each program point is annotated with a logic formula and proven to be valid.

A program point corresponds to an edge of the control-flow diagram.



We need to prove that the assertions are locally consistent.

## Assignments:

- Every assignment transforms a post-condition B into a minimal assumption that must be valid before the execution.

That minimal assumption is called the weakest precondition.

weakest precondition:  $\text{WP}[\text{assignment}] (\text{postcondition}) \equiv \text{postcondition}[\text{assignment-substitution}]$   
 (A given precondition A is valid, if  $A \Rightarrow \text{WP}[\cdot](B)$ )

Some special operations in MiniJava have defined weakest preconditions.

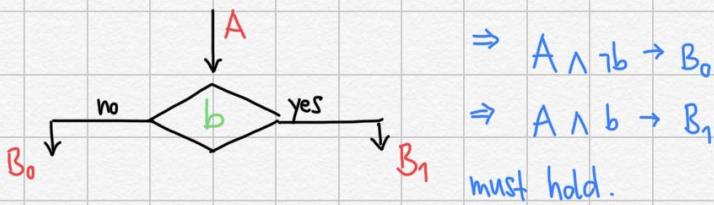
no operation:  $\text{WP}[\cdot](B) \equiv B$

assignment:  $\text{WP}[x=e](B) \equiv B[e/x] \leftarrow \text{substitute } e \text{ for } x$ .

read:  $\text{WP}[x=\text{read}();](B) \equiv \forall x. B \leftarrow B \text{ must be valid no matter what } x \text{ is.}$

write:  $\text{WP}[\text{write}(e);](B) \equiv B$

Conditionals:



weakest precondition:  $\text{WP}[b](B_0, B_1) \equiv ((\neg b) \Rightarrow B_0) \wedge (b \Rightarrow B_1) \equiv (\neg b \wedge B_0) \vee (b \wedge B_1)$

Summary of systematic local consistency:

1. Annotate each program point with an assertion.
2. The program start receives the annotation true.
3. If statement s between A and B then  $A \Rightarrow \text{WP}[s](B)$ .
4. If the conditional branch with condition b between A and  $B_0, B_1$ , then  $A \Rightarrow \text{WP}[b](B_0, B_1)$   
 $\Rightarrow$  locally consistent.

## 1.2 Correctness:

- The program state  $\sigma$  in MiniJava consists only of variable assignments.

$$\sigma = \{x \mapsto \dots, y \mapsto \dots, \dots\}$$

A state  $\sigma$  satisfies an assertion A if every variable in A substituted by its value in  $\sigma$  is equivalent to true. ( $\sigma \models A$ )

## Trivial Properties:

$\sigma \models \text{true}$  For every  $\sigma$

$\sigma \models \text{false}$  For no  $\sigma$

$\sigma \models A_1 \text{ and } \sigma \models A_2 \equiv \sigma \models A_1 \wedge A_2$

$\sigma \models A_1 \text{ or } \sigma \models A_2 \equiv \sigma \models A_1 \vee A_2$

- An execution trace  $\pi$  traverses a path in the control-flow graph.

It starts in a program point  $u_0$  with an initial state  $\sigma_0$  and ends in a program point  $u_m$  with a final state  $\sigma_m$ .

Every step in the execution trace can change the program point and state.

$\pi = (u_0, \sigma_0) s_1 (u_1, \sigma_1) \dots (u_m, \sigma_m)$  ( $s_i$  are elements of the control flow diagram)

If  $p$  is a MiniJava program and  $\pi$  is the execution trace starting at point  $u$  and ending at point  $v$  then:

- ⇒ All program points in  $p$  are annotated with locally consistent assertions.
  - ⇒ The starting point  $u$  is annotated with  $A$
  - ⇒ The ending point  $v$  is annotated with  $B$
- } If the initial state of  $\pi$  satisfies  $A$  then the final state of  $\pi$  must satisfy  $B$ .

Must ensure, that:

- The assertion at the starting point is true
- Every further program point has an assertion
- Proof that all assertions are locally consistent.

## 1.3 Optimization:

⇒ In general, if a program has no loops, a weakest precondition can be calculated for each program point. ( $\uparrow$  Assignments, Conditionals)

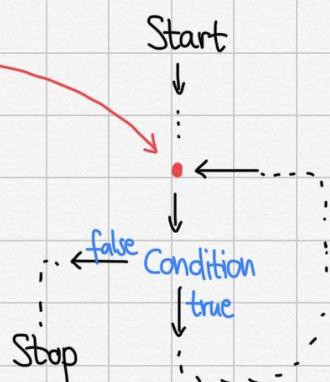
### Loops:

→ For loops, you have to find a loop invariant;

Which holds:

- before the condition
- at the entry of the loop
- and the exit of the loop

The assertions for all other program points can be obtained by means of  $WP[\dots]()$ ...



## 1.4 Termination:

⇒ The approaches above can only prove an assertion to be valid whenever that program point is reached. It is needed to guarantee termination of a program.

- Programs without loops terminate always!
  - Programs with "well behaving loops" also terminate.
    - That means there are only loops of the form
- ⇒ You can make sure a loop is executed finitely often and therefore well behaving by doing the following:

- For each loop
- introduce an auxiliary variable  $r$  that is always  $>0$  whenever the loop is entered
  - decrease  $r$  during every iteration of the loop

If you can verify those two properties, the program will always terminate.

```

 $r = e_0$ 
while (...) {
    assert ( $r > 0$ )
    :
    assert ( $r > e_1$ )
     $r = e_1$ 
}

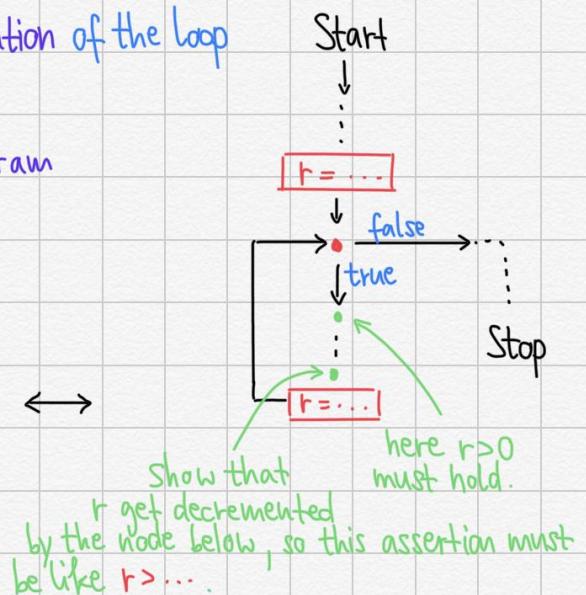
```

$i$  is always positive       $i$  get decremented in every iteration  
 $i$  is not modified in other matters

```

for( $i = x$ ;  $i > 0$ ;  $i--$ )
{
    ...
}

```



## Chapter 2 : The Basics in OCaml

⇒ The basic Syntax and style of programming in the functional programming language OCaml.

### 2.1 Interpreter Environment:

⇒ You can call the ocaml interpreter with ocaml or using utop  
Variables, functions, etc... can be inserted and evaluated immediately

# use "file.ml";; lets you read from a file.

# means the interpreter is waiting for input

;; causes evaluation of the given input → The result gets returned with its type.

### 2.2 Expressions:

⇒ In OCaml everything has a type that is inferred automatically.

primitive datatypes:

int: +, -, \*, /, mod

float: +., -. , \*., /.

bool: not, ||, && ←

string: ^ (Stringconcatenation)

char: ...

OCaml uses "short-term evaluation"

comparison operators on the other hand are polymorphic: =, <> (not equal),  
<, <=, >=, >

### 2.3 Definition of values:

A variable can be assigned a value by means of let.

⇒ #let x = 3 + 4;; .

(val x : int = 7)

Later calls to that variable return the assigned value. ⇒ #x;;

(- : int = 7)

Variable names  
MUST start  
with small letters.

Variables in OCaml are immutable. Another "new definition" of an already existing variable is allowed, but will create a new variable with the same name that from then on hides the old variable.

## 2.4 Complex Datatypes:

⇒ In addition to the primitive datatypes there are also **Tupels** and **Records** predefined in OCaml.

### Tupels:

⇒  $(a, b, c, \dots, \dots)$  Elements in the tupel can each have different types. (int, float, etc...)

Tupel can have any length  $\geq 2$ . (There is also a tupel of length 0:  $() \rightarrow \text{unit}$ )

### Records:

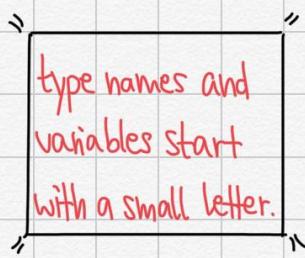
⇒ type newtype = { attriba : string ; attribb : int ; ... }

Attributes can be from any type.

The order in which the attributes are listed doesn't matter.

Records are tuples with named components whose ordering is irrelevant.

You can access record components via  
recordname . componentname



## 2.5 Pattern matching and case distinction:

⇒ To get information about the structure and its components of a complex datatype, you can do pattern matching.

match structure with pattern1 → ...  
 | pattern2 → ...  
 :  
 | \_ → ...  
 ↑  
 the underscore matches every other case (like default in switch)

The matched pattern could be a general tuple  
 (x,y,z), a record, etc... (for Lists see below)  
 All expressions must be of the same type

Make sure every match is not redundant nor has incomplete matches.

## 2.6 Lists:

⇒ All elements in a list must have the same type.  
 You can construct Lists by means of [] and ::

The type of the elements in a list can be arbitrary

[1; 2; 3] = 1 :: (2 :: (3 :: []))  
 ↑ right association of the :: operator

Pattern matching with a list is normally of the following structure:

match myList with [] → ...  
 | x :: xs → ...  
 ↑ first element      rest of the list

## 2.7 Functions in OCaml:

⇒ Functions in general are defined in the following pattern:

let rec func-name para1 para2 ... = func-body

↑  
 The function name is just a variable whose value is a function.  
 (val function-name : int → int)

You can alternatively also introduce a variable whose value is a function.

let func-name = fun para1 para2... → func-body

Mind that the values of the variables used in a function are evaluated at the point of the function definition! (static binding)

A function is recursive, if it calls itself (directly or indirectly) which is called mutually recursive

⇒ OCaml offers the keyword rec.

Case distinction with function definitions:

⇒ example...

let rec len = fun l → match l  
with [] → ...  
...  
...

syntactic sugar

let rec len = function [] → ...  
...

Local Definitions in functions:

⇒ Definitions by let may occur locally combined with the keyword in.

⇒ example...

let x=5 in  
let func-name para1 para2... = ...

(x is only visible in the  
function func-name)

## 2.8 User-defined Datatypes:

Enumeration Types: type type-name = Enum1 | Enum2 | Enum3 | ...

⇒ This representation is intuitive and internally efficient

A constructor  
usually start with a capital letter

(example → playing cards)

Sum Types: type type-name = Enum of 'a-type | ...

⇒ Sum types generalize of enums in that constructors now may have arguments.

Those datatypes can be defined recursively!

example: type nat = Zero | Succ(nat)

Option Datatype: OCaml provides the built in datatype option with constructors None and Some.

⇒ This datatype of choice is used when a function is only partially defined.

## Chapter 3 : A closer Look at Functions in OCaml

⇒ Since OCaml is a functional programming language , it is important to have a closer look at functions.

### 3.1 Last Calls :

« A last call in the body of a function is a call whose value provides the whole value for the body »

⇒ You don't have to return to the calling function from the last call.  
Therefor the stack space can immediatly be recycled.

A recursive function is called tail recursive if all calls to it are last calls.

(→ Tail-recursive functions can be executed efficiently as loops )

### 3.2 Higher Order Functions:

⇒ OCaml follows the principle of Currying .

Consider the following two functions:

f has one single argument  
⇒  $f: \text{int}^* \text{int} \rightarrow \text{int}$

g has the argument a . The result of this is again a function that is then applied to the argument b.  
⇒  $g: \text{int} \rightarrow (\text{int} \rightarrow \text{int})$

Tupel  
 $\text{let } f(a, b) = a + b + 1$   
 $\text{let } g a \underbrace{b} = a + b + 1$

Sequence

So technically g is a higher order function because its result is again a function.

The application of a higher order function to one argument is called partial .

### 3.3 List Functions:

⇒ The List module implemented in OCaml provides some useful methods for handling Lists. (→ called list functionals)

List.hd mylist , List.tl mylist : Returns the first or last element of the list.

List.length mylist : Returns the amount of elements in the list.

List.rev mylist : Returns the reversed list.

List.map func mylist : Applies the function func to every element of the list.

List.fold\_left func acc mylist : Traverses the list from left to right and applies func acc element to every element of the list.

The accumulator is defined at the start, then updated to the result of func in every iteration and at the end returned.

List.fold\_right func acc mylist : Does the same thing as fold\_left but traverses the list from right to left.

List.find\_opt func mylist : Returns the first element of the list that satisfies the predicate func. None if there isn't any in the list.

### 3.4 Polymorphic Functions:

< Functions which operate on equally structured data of various types are called polymorphic >

OCaml shows the types 'a, 'b, ... Those are type variables which can be instantiated by any type. But each occurrence then must have the same type!

Example: Every list functional above is polymorphic since it does matter what kind of list you are dealing with...

### 3.5 Polymorphic Datatypes:

< User-defined datatypes may be polymorphic, too. Those are called type constructors because they allow to create a new type from another type 'a >

Example: 'a tree = Leaf of 'a | Node of ('a tree \* 'a tree)

→ The functions on that tree are again polymorphic.

### 3.6 Queues:



Representation by a list:

- type 'a queue = 'a list
- is\_empty : If list is empty
- queue\_of\_list and list\_of\_queue  
are trivial...
- dequeue : access top element
- enqueue : concatenate the old list  
with new element at the end.

⇒ Insertion requires list length many calls...

—

Representation by two lists:

type 'a queue = Queue of 'a list \* 'a list

- is\_empty : If both lists are empty

Idea:

⇒ Insertion is in the second list

- enqueue : Add to the front of the  
second list

⇒ Extraction is normally from the first list  
unless it's empty

- dequeue : Returns tuple of  
(dequeued element, new queue)

↪ option datatype (could not exist)

⇒ Amortized cost analysis shows only constant  
costs for insertion and extraction.

### 3.7 Anonymous Functions:

⇒ Since functions are only data, they can be used without naming them.

λ-calculus for functions: fun para1 para2... → func-body

Anonymous functions are convenient if just used once in a  
program.

Recursive functions  
cannot be defined  
that way.

# Chapter 5 : Practical Features in OCaml

⇒ Even though OCaml is a functional programming language , Side Effects ,etc.. can be useful in some scenarios...

## 5.1 Exceptions:

⇒ In case of a runtime error, the OCaml system generates an exception.

## Predefined Constructors for Exceptions:

- |                                     |           |  |
|-------------------------------------|-----------|--|
| ⇒ Division_by_zero                  | raised by | division by 0                                |
| Invalid_argument of string          | raised by | wrong usage                                  |
| Failure of string                   | raised by | general error                                |
| Match_failure of string * int * int | raised by | incomplete match (match case not exhaustive) |
| Not_found                           | raised by | not found                                    |
| ⇒ Out_of_memory                     | raised by | memory exhausted                             |
| ⇒ End_of_file                       | raised by | end of file                                  |
| ⇒ Exit                              | raised by | for the user...                              |

These constructors for the first class citizen datatype `exn` are extensible...  
exception `exc_name` can add the new exception `exc_name`...

Exception-Handling : Exceptions can be handled and raised through the keywords:

try following the keyword with , the exception value can be inspected  
... by means of pattern matching on the exn datatype  
with Exception → ... (→ several exceptions can be caught at the same time)

The programmer can trigger exceptions on his/her own through the keyword `raise`

let div x y = if  $y=0$  then raise Division-by-zero ← the exception can  
else  $x/y$  replace any expression!

## 5.2 Textual Input and Output:

⇒ Since reading from the input and writing to the output violates the paradigm of functional programming ⇒ Those operations are realized by means of side-effects.  
We use functions whose return value is as irrelevant as unit...

Console output → `print_string "..."` (Returns unit)  
input → `read_line ()` (Takes unit and returns string)

Reading from a file → `let file = open-in "filename" in`  
`try` ← file has type in-channel.  
`let line = input-line file in`  
with End-of-file → ...  
← if there is no further line available,  
End-of-file is raised.  
`close-in file` ← You MUST always make sure, that every  
channel no longer needed gets closed!

⇒ stdin is the console as text input  
input-char returns the next char of the input and  
input-line returns the next line of the input as a string.  
in-channel\_length returns the length of the channel.

Writing to a file → `let file = open-out "filename" in`  
file has type out-channel.

Each function  
writes a string into  
the file...

{ `let variant1 = output-string file "message..." in`  
`let variant2 = Printffprintf file "message..." in`

`close_out file` ← You MUST always make sure, that every  
channel no longer needed gets closed!  
(The written words may only appear inside  
the file once it has been closed)

### 5.3 Sequences:

⇒ Since side-effects can now occur, ordering matters!

Actions can be sequenced by the sequence operator ;

example: print\_string "Test01";  
print\_string "Test02";  
print\_string "Test03"

⇒ List.iter func mylist

can be used to do something with every  
element of a list ...

# Chapter 6 : The Module System of OCaml

⇒ To structure bigger projects , OCaml offers the concept of modules.

## 6.1 Modules:

### Creating and Accessing Modules:

```
module Mod-name =  
  struct  
    type ...  
    let func-name ...  
    let ab ...  
  end
```

Definitions inside a module can only be accessed via qualification: Mod-name . func-name  
→ Several function with the same name can be defined in different modules.

⇒ The compiler returns the signature of the module.

### Opening and Including Modules:

open Mod-name → To avoid explicit qualification , you can open a module to make all definitions directly accessible  
( just like import in Java )

include Mod-name → Includes the definitions of another module into the present one.

⇒ Only include can change the signature of a module.

### Nested Modules :

module Outer = struct → Module can again contain other modules.  
 module Inner = struct  
 :  
 Access through: Outer.Inner.func-name

## 6.2 Types and Signatures:

← The signature of a module is inferred automatically.

An explicit indication of the signature allows to restrict what a module exports. ⇒

module Mod1 = struct

:

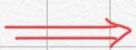
let func = ...

let help-func = ...

end

⇒ The automatically inferred signature allows access to for example help-func ...

to hide functions



we introduce a new signature

module type Mod1\_sig = sig

val func = ...

:

end

module Mod2 : Mod1\_sig = Mod1

⇒ The function help-func is no longer exported / visible from the outside.

⇒ ⚡ The types in the signature must fit to the module (→ raises Signature mismatch)

## 6.3 Information Hiding:

⇒ You can use signatures to hide the structure / implementation of a datatype from the user. (obfuscate the true nature)

example: Define the module Queue with type 'a queue = 'a list.

To hide, that we used a list as a queue we define the signature

module type Queue = sig

type 'a queue

:



now the methods can only be applied to queues...

## 6.4 Functors:

« Modules of higher order are called **Functors**.

They receive a sequence of modules as parameters which can be used in the module's body »

Example:

module type Deconstruct = sig

type 'a t

val decons : 'a t → ('a \* 'a t) option

end

This module offers the function to split a sequence of something into one element and the rest.

module type Folds = functor (X: Deconstruct) → sig

val fold\_left : ...

val fold\_right : ...

This module is a functor, which can you functions of the module Deconstruct. (ex. X.decons)

We can then apply this functor to models to obtain new ones...

module FoldXY = Folds (XY)

← By that we define FoldXY.fold\_left ...

↓ The function XY.decons must exist in the module.

## 6.5 Separate Compilation:

⇒ OCaml programs can be compiled by calling `ocamlc test.ml`

Here `test.ml` is treated as a sequence of definitions for a new module `test`.

The compiler will create bytecode for the module, bytecode for the signature and an executable.

You can compile a module `test.ml` with a specific signature `test.mli` by calling `ocamlc test.mli test.ml`

You can also combine modules by calling `ocamlc B.mli B.ml A.mli A.ml`

## Chapter 7 : Formal Verification for OCaml

⇒ To make sure an OCaml Program behaves as it should , we require formal semantics to prove assertions about programs.

### 7.1 Introducing MiniOCaml:

⇒ We only consider a fraction of OCaml . That means only base type int, bool, tuples and lists, recursive functions at top level but no input, output or modifiable datatypes.

Expressions := const | name | op<sub>1</sub>E | E<sub>1</sub>opE<sub>2</sub> | (E<sub>1</sub>,...,E<sub>k</sub>) | let name = E<sub>1</sub> in E<sub>2</sub> |  
match E with P<sub>1</sub> → E<sub>1</sub> | ... | P<sub>k</sub> → E<sub>k</sub> | fun name → E

Patterns := const | name | (P<sub>1</sub>,...,P<sub>k</sub>) | P<sub>1</sub>::P<sub>2</sub>

Values := const | fun name<sub>1</sub> ... name<sub>k</sub> → E | (V<sub>1</sub>,...,V<sub>k</sub>) | V<sub>1</sub>::V<sub>2</sub> | []

### 7.2 Big-step operational semantics:

⇒ A value is returned for every expression . It is an expression that cannot be further evaluated.

We define a relation  $e \Rightarrow v$  between every expression and its value by means of axioms and rules . (Big-Step)

Tupel:  $T \frac{e_1 \Rightarrow v_1 \dots e_k \Rightarrow v_k}{(e_1, \dots, e_k) \Rightarrow (v_1, \dots, v_k)}$

Lists:  $L \frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2}{[v_1 :: v_2] \Rightarrow v}$

every x is substituted  
by v<sub>1</sub>

Global Definitions : GD  $\frac{f = e \quad e \Rightarrow v}{f \Rightarrow v}$   
functionname  
/ don't forget " = "  
often predefined as tcf

Local Definitions : LD  $\frac{e_1 \Rightarrow v_1 \quad e[v_1/x] \Rightarrow v_0}{\text{let } x = e_1 \text{ in } e_0 \Rightarrow v_0}$

**Function Calls:** APP  $\frac{e_0 \Rightarrow \text{fun } x_1 \dots x_k \rightarrow e \quad e_1 \Rightarrow v_1 \dots e_k \Rightarrow v_k \quad e[v_1/x_1 \dots v_k/x_k] \Rightarrow v}{e_0 e_1 \dots e_k \Rightarrow v}$

use  $\pi_f$  here  
/ function arguments  
function name

substitute every variable by its value

**Pattern Matching:** PM  $\frac{e_0 \Rightarrow v_0 \quad e_i[v_1/x_1, \dots, v_k/x_k] \Rightarrow v}{\text{match } e_0 \text{ with } p_1 \rightarrow e_1 \mid \dots \mid p_m \rightarrow e_m \Rightarrow v}$

only the pattern that matches  $v_0$   
(- if no patterns match)

**Operators:** OP  $\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1 \text{ op } v_2 \Rightarrow v}{e_1 \text{ op } e_2 \Rightarrow v}$

Here you use the "real" Operator to add/mul/div/  
...etc. the values of the expressions.

**Equality:**  $v_1 = v_2 \rightarrow \text{false}$  and  $v = v \rightarrow \text{true}$  Only if  $v_1, v_2, v$  do not contain functions.

Big-step semantics is well suited for proving that the evaluation of a function terminates for a particular argument.

You can use induction for that purpose.

- Base case:  $n=0$  —  $n$  is often an int that can also represent the length of a list.
- Induction step: We assume that ... terminates with ... for all  $n \geq 0$   
Now, we show that it also terminates with ... for input  $n+1$ .

always do induction  
of the argument that  
is changed in function.

by I.H.

⋮

...

□

Equality can only be proven for values that do not contain function (=comparables)  
The Big-step semantics allow to verify that optimizing transformations are correct.

### 7.3 Extended notation of equality:

⇒ If you want to prove that  $f \circ b = g \circ b$  holds for the functions  $f, g$  you have to extend the notion of equality:

We assume:

$$\begin{array}{c} \text{Termination:} \\ \frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1 = v_2}{e_1 = e_2} \\ \text{both terminating} \end{array}$$

Non-termination:

$$\frac{e_1, e_2 \text{ both not terminating}}{e_1 = e_2}$$

Tupel:

$$\frac{\begin{array}{c} e_1 = e'_1 \dots e_k = e'_k \\ (e_1, \dots, e_k) = (e'_1, \dots, e'_k) \end{array}}{e_1[v/x_1] = e_2[v/x_2] \text{ for all } v}$$

Functions:

$$\text{fun } x_1 \rightarrow e_1 = \text{fun } x_2 \rightarrow e_2$$

Substitution Lemma:

$$e[e_1/x] = e[e_2/x]$$

$e_1$  terminates

Local definitions:

$$\text{let } x = e_1 \text{ in } e = e[e_1/x]$$

evaluate functions for all arguments  
(theoretically, maybe by induction)

$$e_1 = e_2$$

if attributes  $e_1$  and  $e_2$  are equal,

they can be interchanged everywhere

1.  $e[v_1/x]$  terminates  $\Rightarrow e[e_1/x] = e[v_1/x] = v$

2.  $e[v_1/x]$  does not terminate  $\Rightarrow$  both sides do not terminate,  
equality is true.

$$e_0 = \text{fun } x \rightarrow e \quad e_1 \text{ terminates}$$

$$e_0 = \text{fun } x_1 \dots x_k \rightarrow e \quad e_1, \dots, e_k \text{ terminate}$$

Function calls:

$$e_0 e_1 = e[e_1/x]$$

$e_0 e_1 \dots e_k = e[e_1/x_1, \dots, e_k/x_k]$   
(multiple arguments)

$$e_0 = \dots$$

Pattern matching:

$$\text{match } e_0 \text{ with } [] \rightarrow e_1 | \dots | p_m \rightarrow e_m = e_1$$

## 7.4 Proofs for MiniOCaml Programs:

- ⇒ For a given definition of  $f \text{ arg}_1 \dots g \text{ arg}_1 \dots$  you can verify that  $f \text{ arg}_1 \dots = g \text{ arg}_1 \dots$  for all possible arguments.
- ⇒ Induction over the changing argument if List, int, etc... require a different base case.
- ⇒ Almost always generalize the claim by adding accumulators
- ⇒ Sometimes case distinction is needed to prove a claim.
- ⇒ It is always assumed the functions terminate.

Example: We prove the more general statement:

$$\text{acc} + f \dots = g \text{ acc} \dots$$

maybe length of a list

We do so by induction over on  $n$  ↗

- Base case:  $n=0$

$$\text{acc} + f \dots [] \dots$$

$$= \dots$$

:

$$= g \text{ acc} [] \dots$$

- Inductive step: We assume the statement holds for  $n \geq 0$

Now, we prove it for  $n+1$

$$\text{acc} + f x :: xs \dots$$

$$= \dots$$

: by induction hypothesis

$$= g \text{ acc } x :: xs$$

□

## Chapter 8 : Parallel Programming

⇒ OCaml provides support for creating multiple threads and communicating between them.

### 8.0 Threads:

The module `Thread` collects all basic functionality for creation of concurrency.

- `create task function arg` → creates a new thread
- `self ()` → returns thread executing the function
- `id t` → returns id of the thread as an int (Use: `id(self())`)
- `exit ()` → terminates current thread
- `join t` → waits for thread t to terminate
- `delay seconds` → delay current thread for some seconds

To compile a OCaml program using threads , you use :

⇒ `ocamlc -thread unix.cma threads.cma file.ml`

The program can be tested by `./a.out`

### 8.1 Channels:

⇒ OCaml offers a module `Event` for creating polymorphic channels on which the threads communicate on.

- `new_channel ()` → creates a new polymorphic channel
- `send channel data` → sends data on a channel
- `receive channel` → returns the data received on a channel
- `sync event` → returns the value of events and blocks current thread until event has "happened"  
↳ rendez-vous for synchronous communication.

⚡ Ordering of send and receive must be designed carefully, since threads can get blocked (Deadlock possible)

### General Structure:

```
module X = struct  
  type 'a req = A of .. | B of ...  
  type 'a t = 'a req channel
```

always create a new channel

this is the "server task"

let create arg1 arg2... =  
let c = new\_channel () in  
let task () =  
 let result = ... in  
 sync (send c result) in  
 let \_ = create\_task () in c

dummy variable to create the thread

return the channel

### 8.2 Examples of using threads:

#### Global Memory Cell:

```
module Cell = struct
```

type 'a req = Get of 'a channel | Put of 'a  
type 'a cell = 'a req channel

let get cell = let reply = new\_channel ()  
in sync (send cell (Get reply));  
sync (receive reply) request for server to send data

let put cell x = sync (send cell (Put x)) on that channel

let new\_cell x =

let cell = new\_channel () in

let rec serve x = match sync (receive cell) with  
| Get(reply) → sync (send reply x);

| Put(y) → serve y; in tail recursive and always calls  
create serve x; cell itself to keep the server running..

## Locks:

↳ Two threads aquiring two seperate locks in different order may lead to a deadlock!

```
module Lock = struct
  type ack = unit channel
  type lock = ack channel
  let acquire lock =
    let ack = new_channel () in
    sync (send lock ack); ack
  let release ack = sync (send ack ())
  let new_lock () = wait for another thread to acquire
  let lock = new_channel () in
  let rec acq_server () = tel-server (sync (receive lock))
  and let rel_server ack = wait for release of the lock
    sync (receive ack); acq_server () in
  create acq_server (); lock
```

unit channel since you just send acquire/release  
/ the lock

## Semaphores:

↳ When you have more than one copy of a resource...

```
module Sema = struct
```

```
  type sema = unit channel option channel
```

```
  let up sema = ... ← release a resource
```

```
  let down sema = ... ← acquire a resource
```

```
  let new_sema n = ...
```

↑ accumulating a parameter,  
maintaining the number of free resources

## 8.3 Selective Communication:

⇒ Since a thread does not know which communication rendez-vous will occur first, you need a non-deterministic choice between several actions.

- Select [ wrap (receive c1) (fun ... → ...);  
 ]  
 wrap (receive c2) (fun ... → ...) ]
- list of event waiting to  
once the event is received, the function is computed...
- First event that appears will be used...

- let select = sync (choose)

⚡ An exception can only be handled within the thread where it has been raised.

#### 8.4 Buffered Communication:

⇒ A channel for sending without blocking can be realized by the module Mailbox.

```
module mailbox = struct
  type 'a mbox = 'a channel * 'a channel
  let send (in, -) x = sync (send in x)
  let receive (-, out) = receive out
  let new_mailbox () =
    let in_ = new_channel () and out = new_channel () in
    let rec server q =
      if (is_empty q) then server (enqueue (sync (receive in_)) q)
      else select [
        wrap (receive in_) (fun y → server (enqueue y q));
        wrap (send out) (first q) (fun () → let (-, q) = dequeue q in server q)
      ]
    in create server (new_queue ());
    (in_, out)
end
```

One channel for sending  
and one for receiving

the server will answer  
immediately

Queue

⇒ Channels with synchronous communication allows to simulate asynchronous communication.