MARKING KEY 3AMAT/3BMAT

RESOURCE-FREE EXAMINATION

Question 1

Using calculus and algebra, determine the coordinates of any point(s) on the curve $y = x^3 - 3x^2 + 3x - 4$ where the gradient is zero.

4 marks	Description
1	$y = x^3 - 3x^2 + 3x - 4$ $dy/dx = 3x^2 - 6x + 3$ ✓
1	if $dy/dx = 0$ then $3x^2 - 6x + 3 = 0$ $\therefore 3(x^2 - 2x + 1) = 0$
1	$\therefore 3(x-1)(x-1) = 0$ ✓ so x = 1 ✓
1	substituting gives y = -3, $\therefore$ the gradient is 0 at (1, -3) ✓

Question 2

Determine the equation of the line tangential to the curve $y = 5x^4 - 3x^2 + 7$ at the point where x = 1.

4 marks	Description
1	$y = 5x^4 - 3x^2 + 7$ $dy/dx = 20x^3 - 6x$ ✓
1, 1	at x = 1, y = 9 ✓, $dy/dx = 14$ ✓
1	using $y = mx + c$, $9 = 14 \times 1 + c$, c = -5 the equation of the tangent $y = 14x - 5$ ✓
1	**or** using $y - y_1 = m(x - x_1)$, the equation of the tangent is: $y - 9 = 14(x - 1)$ $y = 14x - 5$ ✓

Question 3

Given $\frac{dy}{dx} = 4x^3 - 2x + 7$ and y = 5 when x = -1, determine y as a function of x.

3 marks	Description
1	$\frac{dy}{dx} = 4x^3 - 2x + 7$ $y = x^4 - x^2 + 7x + c$ ✓
1	substituting x = -1 and y = 5 $\quad 5 = 1 - 1 - 7 + c \quad \therefore c$ = 12 ✓
1	$y = x^4 - x^2 + 7x + 12$ ✓

Question 4

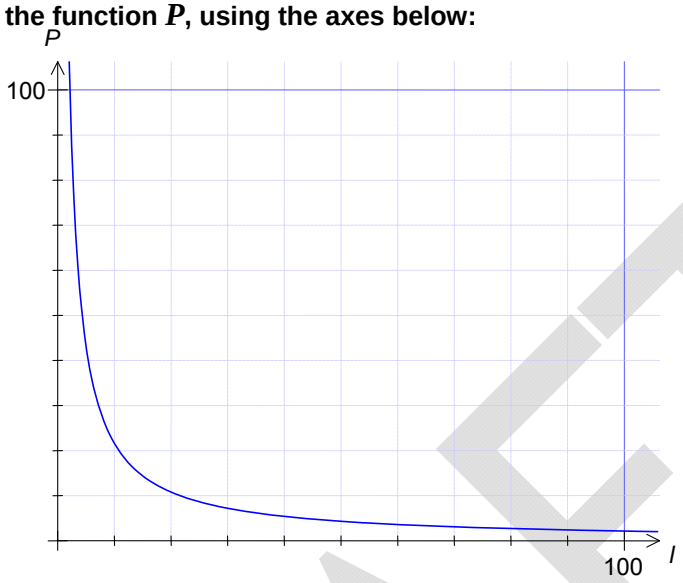
(a) Describe in words how P varies with l.

2 marks	Description
1	P is inversely proportional to l, ✓
1	As l increases, P decreases ✓ (or as l decreases, P increases).

(b) What would be a suitable domain for *l*? Justify your choice.

2 marks	Description
1	$l > 0$ ✓
1	as a pipe must have a length to exist ✓
or	**or**
1	$l > 0$ ✓
1	as $P = 216/l$ does not exist if l is zero and lengths would always be positive. ✓

(c) Graph the function P, using the axes below:



3 marks	Description
1	Sketch in first quadrant ✓
1	Hyperbola close to symmetrical ✓
1	Asymptotic behaviour as the curve approaches horizontal and vertical axes ✓

(d) What is the pitch for a pipe with length 3 metres?

2 marks	Description
1	P = 216/3 ✓
1	= 72 cycles per second ✓ (no units no mark)

(e) (i) Is P increasing or decreasing as l increases and l = 3?

1 mark	Description
1	Decreasing ✓

(ii) What units would the rate of change of P be measured in for the $P = \frac{216}{l}$ relationship?

1 mark	Description
1	Cycles per second per metre ✓

Question 5

(a) What restrictions are there on *S*?

2 marks	Description
2	$0 \le S \le 300$ (no penalty for < instead of ≤) ✓ ✓

(b) (i) What is his fitness index?

2 marks	Description
1	$F = \frac{50 \times 4 \times 60}{(100 + 80 + 70)}$ ✓
1	= 48 ✓

(ii) What rating would he get?

1 mark	Description
1	Poor ✓

(c) (i) Pulse rates (*r*) are between 50 and 140. Express this as a restriction on *R*.

1 mark	Description
1	$150 < R < 420$ ✓ (no penalty for ≤ instead of <)

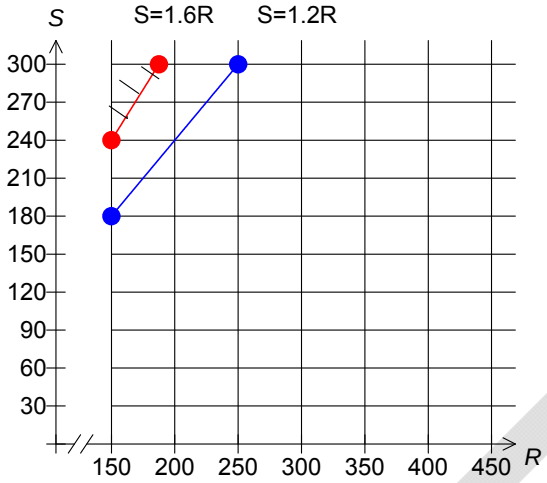
(ii) Give an example of 3 measured pulse rates after 4 minutes of exercise that would result in a 'Good' rating and verify this by calculation

2 marks	Description
1	55,52,53 ✓ (or any other valid set)
1	$F = \frac{50 \times 240}{(55 + 52 + 53)} = 75$ ✓

(d) Find the equation of the line between the categories 'Excellent' and 'Good'.

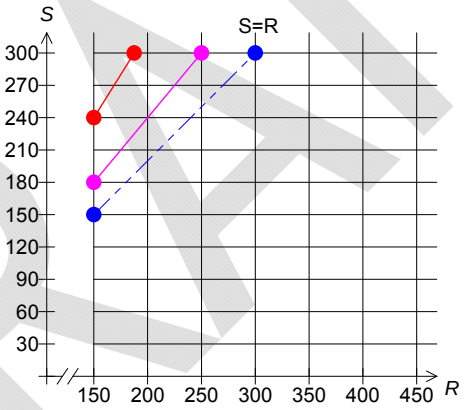
3 marks	Description
1	$F = 80$ ✓
1	$80 = \frac{50 \times S}{R}$ ✓
1	$S = 1.6R$ ✓

(e) Shade the region that represents an 'Excellent' rating, on the graph above.



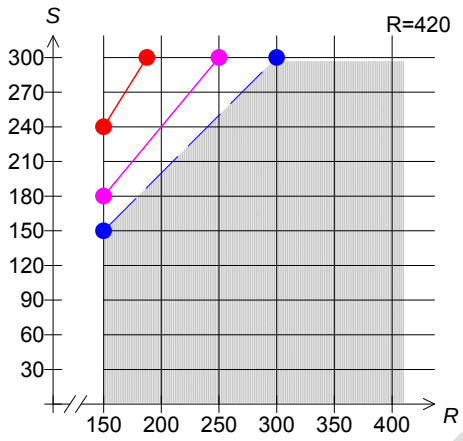
3 marks	Description
1	Line $S = 1.6R$ ✓
1	Restricted range ✓ $(S \leq 300)$
1	Shading above ✓

(f) An extra rating of 'Very Poor' is introduced for a fitness index of below 50. Explain why *S = R* is the line forming the boundary between 'Poor' and 'Very Poor' and add this line to the graph above.



3 marks	Description
1	$F = 50$ $50 = \frac{50S}{R}$ ✓
1	$\therefore S = R$ $S = R$ line on graph (dotted or solid) ✓
1	$S \leq 300$ ✓ (correct range)

(g) Shade the region on the graph that represents a 'Very Poor' rating.



1 mark	Description
1	Shade below and to the right of the line $S = R$, between $150 \leq R \leq 420$, for $S \leq 300$ ✓ (line should be dotted, no penalty for solid)

RESOURCE-RICH EXAMINATION

Question 1

(a) Display this data in the table below, then complete the table.

	In favour	Against	Undecided	Total
Male	51	**40**	3	94
Female	**70**	31	**5**	**106**
Total	**121**	71	8	**200**

2 marks	Description
2	All entries correct ✓✓ (- 1 for 1 or 2 errors, -2 for 3 or more errors)

(b) If an adult from the survey is chosen at random, find the probability that this adult is:

(i) a male in favour of the proposal.

1 mark	Description
1	$\frac{51}{200}$ ✓

(ii) a male, given the adult is against the proposal.

2 marks	Description
2	$\frac{40}{71}$ 40 ✓ 71 ✓

(c) If a male is chosen at random from the male group surveyed and a female is chosen at random from the female group surveyed, who is more likely to be in favour of the proposal? Justify your answer.

3 marks	Description
1	P(favour I male)= 51/94 ✓ (≈ 0.543)
1	P(favour I female)= 70/106 ✓ (≈ 0.660)
1	70/106 > 51/94 hence the female is more likely to be in favour ✓

Question 2

(a) Determine the length of QS.

3 marks	Description
1	QS = $\sqrt{8^2 + 12^2 - 2 \times 8 \times 12 \cos 47^\circ}$ ✓ (states the rule with the correct values)
1	≈ 9m or 8.8m or 8.78 m ✓ (8.77817.. rounded)
1	✓ (rounding is correct and units stated throughout the question--in parts a **and** b **and** c)

(b) Given angle R is acute, determine the size of angle QRS.

2 marks	Description
1	sin ∠QRS = $\frac{QS \times \sin 35^\circ}{9}$ ✓ (states sine rule with the appropriate lengths/values)
1	∠QRS ≈ 34.02° ✓ (34.01702…rounded)

(c) Determine the area of quadrilateral PQRS.

2 marks	Description
1	Area PQRS=Area triangle PQS + area triangle QRS = ½ x 8 x 12 sin47° + ½ x QS x 9 x sin ∠QSR ✓
1	(states areas with appropriate lengths/angles/values) ≈ 72 m^2 ✓ (71.98726.. rounded)

Question 3

(a) Determine Kim's bicycle riding speed in metres/second.

2 marks	Description
1	Speed ≈ 1.25 km in 5 min ✓ (or use kilometres close to 1.25)
1	≈ $4.1\dot{6}$ m/s ✓

(b) Determine the speed of the car in kilometres per hour.

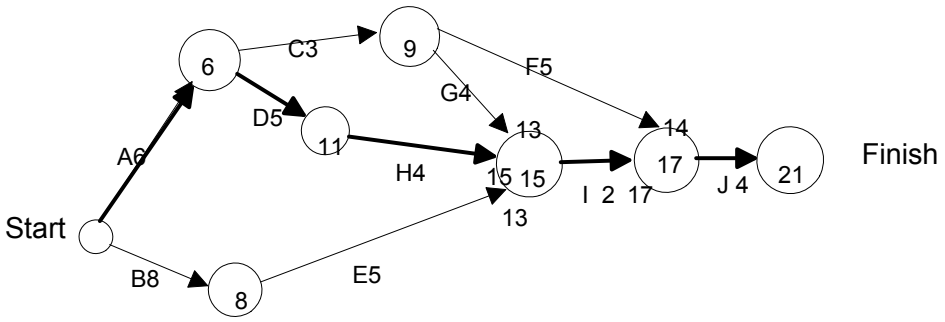
2 marks	Description
1	Speed ≈ (3 - 1.25)/2 km/min ✓
1	≈ 52.5 km/h ✓

(c) If Kim had ridden her bicycle all the way to school at the same speed as in the first leg of the trip, would she have arrived at school earlier, later or at the same time? Justify your answer.

3 marks	Description
1	extending the line for the bicycle ride ✓
1	shows she would have arrived at school after about 14 minutes, ✓
1	therefore she would have arrived earlier ✓
or	**or**
1	The graph indicates the school was 3.5 km from where Kim started, so travelling at (1.25/5 km per minute) ✓
1	she would have arrived after 14 minutes. ✓
1	Therefore she would have arrived earlier. ✓
	or
	other method (reasoning ✓✓ and conclusion ✓)

Question 4

(a) Draw a project network which satisfies the above conditions:



3 marks	Description
3	Draws full diagram correctly with letters, numbers, start/finish ✓✓✓ (-1 for each error or omission)

(b) Determine the minimum time to complete the project and state the critical path, if all tasks are completed on time.

2 marks	Description
1	21 days ✓
1	critical path as A-D-H-I-J ✓

(c) If task *C* were to be delayed by 3 days, what effect, if any, would this have on the completion time?

1 mark	Description
1	project would take 1 more day ✓ (or 22 days to complete)

(d) If task *D* were to be shortened by 3 days, what effect, if any, would this have on the completion time?

1 mark	Description
1	project would take 2 days less to complete ✓ (or 19 days to complete)

Question 5

(a) How many students sat for the examination?

1 mark	Description
1	119 ✓

(b) Describe student performance in the examination.

6 marks	Description
2	The average score was 67% (mean score) The student ranked in the middle of the class (60th) obtained a score between 61-70 inclusive (comments on an average ✓ gives an appropriate figure✓)
2	The spread of scores for the whole group of students was large (range about 89%) but most of the students scored above 50% and less than 91% resulting in a standard deviation of 18.6% (comments on the spread of the scores ✓ gives the range and standard deviation ✓)
2	The distribution of scores was skewed to the left (negatively skewed) because most students scored more than 50%, and a few students had very low scores (2 between 11–20 inclusive, 4 between 21–30 inclusive, 15 between 31–50 inclusive.) (identifies direction of the skew ✓, explains the skew which could include mentioning the modal class 71–80 ✓)

Question 6

(a) (i) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 4 on Test 1.

1 mark	Description
1	using line L2 and x = 4 $\hat{y} \approx 4$ ✓

(ii) Justify why you used that particular line of regression and comment on the reliability of your prediction.

2 marks	Description
1	Line L2 was used because it fits the central band of data better than line L1 and the prediction is within this band. ✓
1	Reliability is reasonable, judging by the correlation coefficient r = 0.928 and because the prediction involves interpolation ✓

(b) (i) Use the more appropriate of the two lines of regression to estimate the mark on Test 2 for a student who scored 11 on Test 1.

1 mark	Description
1	using line L1 and x = 11 , $\hat{y} \approx 11$ ✓

(ii) Explain why you used that particular line of regression and comment on the reliability of your prediction.

2 marks	Description
1	Line L1 was used because it is closer to data values near the prediction than is line L2 ✓
1	The prediction is not reliable because it involves extrapolation ✓

Question 7

(a) Show that the height, in terms of the base, is given by the expression

$h = \sqrt{(200 + b)(200 - b)}$

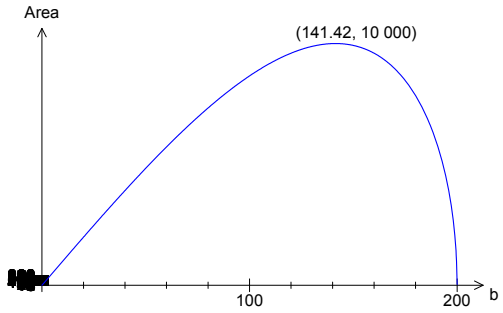
2 marks	Description
1	$h^2 = 200^2 - b^2$ ✓
1	$h = \sqrt{200^2 - b^2}$ $h = \sqrt{(200 + b)(200 - b)}$ ✓

(b) Complete the following table (to two decimal places):

Base, b (m)	Height, h (m)	Area, A (m^2)
90	178.61	8037.26
100	**173.21**	**8660.25**
110	**167.03**	**9186.81**

2 marks	Description
1	Both values correct for when b = 100 ✓
1	Both values correct for when b = 110 ✓ (no penalty for rounding final answers incorrectly, -1 if Area is incorrect because of carry through errors i.e. height to 2 decimal places was used)

(c) Determine the maximum area showing clearly the method used:

4 marks	Description
1	Area = $0.5 \times b \times \left(200^2 - b^2\right)^{\frac{1}{2}}$ ✓
2	 Graph ✓ maximum point labelled ✓
1	Max area = 10 000 m^2 ✓
or	**or**
1	Area = $0.5 \times b \times \left(200^2 - b^2\right)^{\frac{1}{2}}$ ✓
1	Max area = 10 000m^2 ✓ read from the table of function values on a calculator, at
2	b = 141.4 m ✓✓
	or by calculation (beyond the requirements of Units 3A/B) base height area 130 9879 140 9998 141 9999.8 141.4 999.9995 141.42 10000
	or by calculus (derivative is beyond Units 3A/B)

Question 8

(a) Complete the table below showing the starting population for each year from 1995 to 2007.

P_t	Year	Beginning population
P_0	1995	400
P_1	1996	**440**
P_2	1997	**484**
P_3	1998	**39**
P_4	1999	**89**
P_5	2000	**183**
P_6	2001	**290**
P_7	2002	**304**
P_8	2003	**298**
P_9	2004	**301**
P_{10}	2005	**299**
P_{11}	2006	**300**
P_{12}	2007	**300**

4 marks	Description
2	440 ✓ 484 ✓
2	89, 183, 290 ….. ✓✓ (truncated answers—rounded answers also acceptable)

(b) (i) What type of function is $P = k(a)^t$?

1 mark	Description
1	Exponential ✓

(ii) Give the values of k and a.

2 marks	Description
1	k = 400 ✓
1	a =1.1 ✓

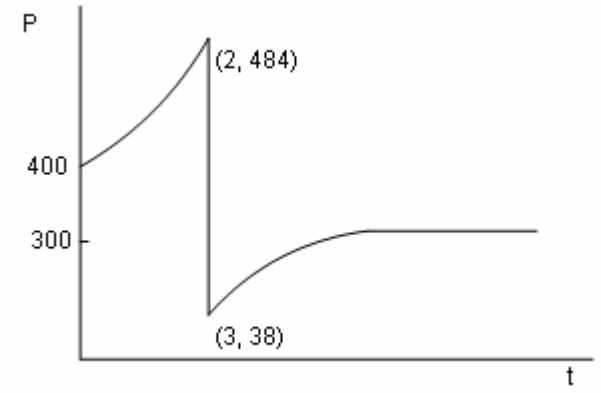
(c) (i) What population size will the European wasps level out at if the sterilisation program continues?

1 mark	Description
1	300 ✓

(ii) Explain mathematically why the population levels out.

3 marks	Description
2	When $P_{t-1} = 300$ $P_t = 2.5 \times 300 - 0.005 \times 300^2 = 300$ ✓✓
1	and every following P_t term will calculate to be 300. ✓

(e) Sketch the population graph.



4 marks	Description
1	y intercept 400 ✓
1	First section an exponential curve ✓
1	Highest point level with 484✓and lowest point level with 38 (or 38.72) and
1	Levels off level with 300 ✓

Question 9

(a) Calculate the values in the cells denoted by A and B.

2 marks	Description
2	A = 584 ✓ B = -38 ✓

(b) The equation of the trend line for the moving averages is y = 1.476t + 563.497 (with correlation r ≈ 0.944). Predict the sales (to the nearest 50 cents) for Monday of Week 5.

3 marks	Description
1	t = 21 From the trend line $\hat{y}$ = 594.493 ✓
1	Seasonal component for Mondays = (70+68+72)/3 = 70 ✓
1	Predicted sales with seasonal adjustment = \$594.493+ \$70 = \$664.50 to the nearest 50c ✓

(c) Comment on the reliability of your prediction in (b), using mathematical reasoning.

2 marks	Description
1,1	• the high correlation coefficient and • the fact that the prediction is close to provided data indicate the prediction should be reliable. However, the requirement for extrapolation brings reliability into doubt. ✓✓ (Two valid observations)

(d) Comment on the reliability of your prediction in (b), in terms of the context.

1 mark	Description
1	The sales in a school canteen do not always follow a pattern because of special events such as sports carnivals, and sales could be zero on Monday because of a long weekend. Thus, the prediction might not be reliable because the school calendar has not been taken into account. ✓ (One plausible observation and a conclusion about reliability)

Question 10

(a) What proportion of students scored less than 100 marks?

2 marks	Description
2	X : mark scored , X~N(124, 30^2) P(X<100) ≈ 0.21186 ✓✓ (1 for probability notation, 1 for correct decimal)

(b) The top 2% of students were awarded a certificate of distinction. What minimum mark was needed for a certificate of distinction?

2 marks	Description
1,1	Let x be the minimum mark for a distinction P(X> x)=0.02 x ≈ 185.6 ✓✓ (Answer only sufficient, 1 mark if correct setup and wrong answer)

(c) The middle 40% of students received a grade of C. What were the minimum and maximum marks in order for a student to be awarded a C?

3 marks	Description
1	Let maximum mark be k and minimum mark be c P(X < k) = 0.7, k ≈ 139.7 ✓
1	P(X < c) = 0.3, c ≈ 108.3 ✓
1	If whole marks only then maximum mark is 139 (or 140) and minimum mark is 108 (or 109) ✓ Or if half marks allowed, maximum mark is 139.5 (or 140) and minimum mark is 108.5 (or 108)