Activity 23   Auxiliary angle

1.

a)
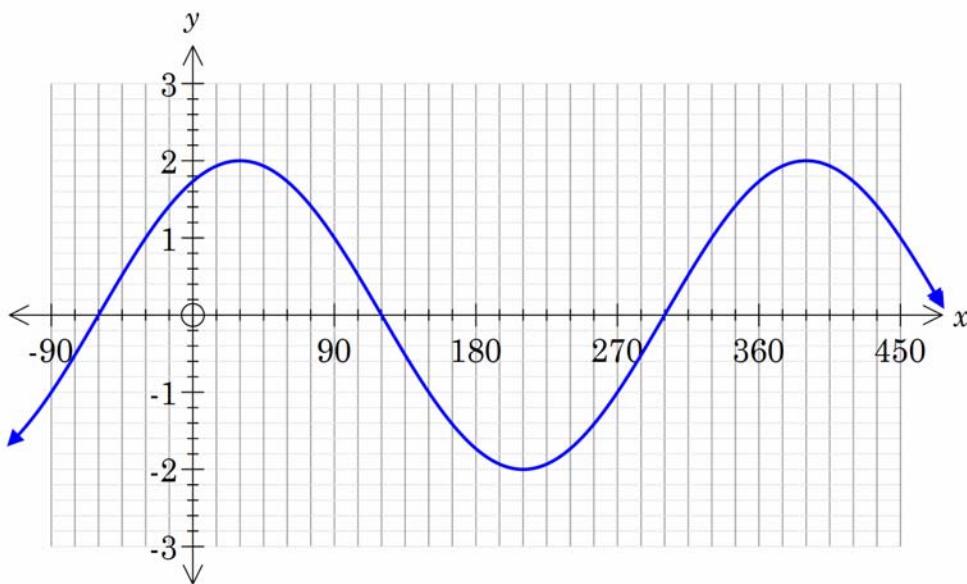
b)   Period $= 360°$

c)   Maximum 2 units, occurs when $x = 30°$

d) $y = 2\cos(x - 30°)$

2.   a)   (i) $R^2 = \left(\sqrt{3}\right)^2 + 1^2$

$R = 2$

(ii) $\cos(\alpha) = \dfrac{\sqrt{3}}{2}$

(iii) $\sin(\alpha) = \dfrac{1}{2}$

(iv) $\alpha = 30°$

b) $R(\cos x \cos \alpha + \sin x \sin \alpha)$

c)

$\sqrt{3}\cos(x) + \sin(x)$

$= 2\left(\dfrac{\sqrt{3}}{2}\cos(x) + \dfrac{1}{2}\sin(x)\right)$

$= 2\left(\cos 30° \cos x + \sin 30° \sin x\right)$

$= 2\cos(x - 30°)$

Note that this can also be written $2\cos(30° - x)$ since $\cos(-x) = \cos x$

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3.

a)

$$\sin(x) - \cos(x) = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin(x) - \frac{1}{\sqrt{2}}\cos(x)\right)$$

$$= \sqrt{2}\left(\sin(x)\cos\left(\frac{\pi}{4}\right) - \cos(x)\sin\left(\frac{\pi}{4}\right)\right)$$

$$= \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$

b)
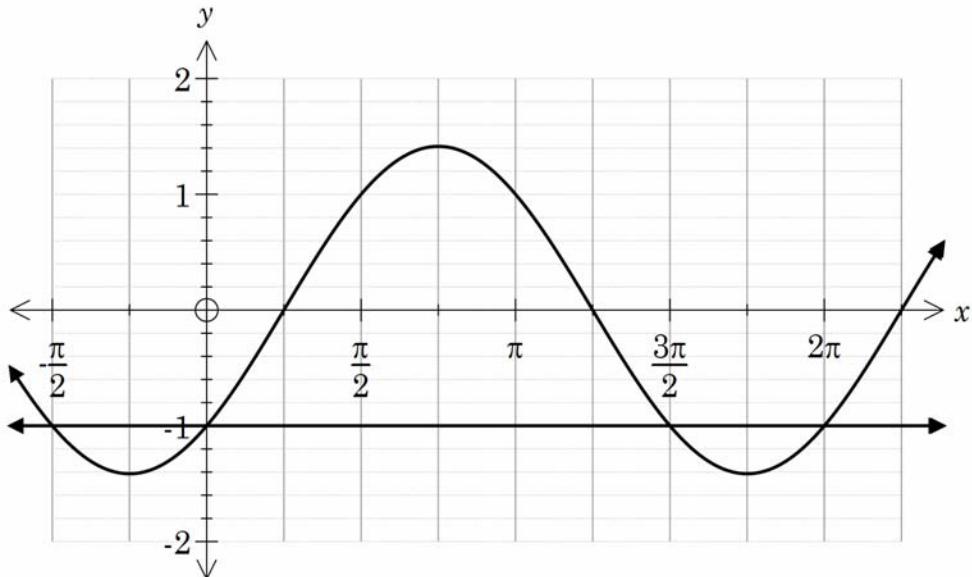
c)

(i)   From graph, $x = 0,\ \dfrac{3\pi}{2},\ 2\pi$ (iii)

(ii)   Algebraically:

$$\sin(x) - \cos(x) = -1$$

$$\sqrt{2}\sin(\ ) = -1$$

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

$$x - \frac{\pi}{4} = -\frac{\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$$

$$x = 0,\ \frac{3\pi}{2},\ 2\pi$$