TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 3 2016

Section 1 Calculator Free

Vectors in Two & Three Dimensions and Systems of Equations

STUDENT'S NAME: _____

DATE: Thursday 28th April **TIME:** 20 minutes **MARKS**: 23

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

Solve the following system of equations, explaining what the equations and their solution represent in space:

$$3x + 2y - z = 19$$
$$4x - y + 2z = 4$$
$$2x + 4y - 5z = 32$$

2. (7 marks)

By considering the value(s) of *lambda*, λ, determine the number of points of intersection of the line, $\boldsymbol{r} = (1-3\lambda)\boldsymbol{i} + (4+9\lambda)\boldsymbol{j}$, with the circle, $|\boldsymbol{r} - (8\boldsymbol{i} + 3\boldsymbol{j})| = \sqrt{50}$. Hence state the coordinates of any point(s) of intersection.

3. (9 marks)

Given the three points: $P(0,-2,1)$, $Q(4,1,3)$ and $R(-1,0,2)$

(a) (i) State the vector equation of the line through points P and Q in terms of λ. [2]

(ii) What will be the impact of restricting *lambda* such that $0 \le \lambda \le 1$? [1]

(iii) Hence determine the Cartesian form of the equation of the line stated in part (i).

[2]

(b) (i) Calculate the normal $\boldsymbol{n} = \mathbf{PR} \times \mathbf{PQ}$ [2]

Hint: $\boldsymbol{a} \times \boldsymbol{b} = (a_2 b_3 - a_3 b_2)\boldsymbol{i} + (a_3 b_1 - a_1 b_3)\boldsymbol{j} + (a_1 b_2 - a_2 b_1)\boldsymbol{k}$

(ii) Hence determine the Cartesian equation of the plane that contains the three points P, Q and R.

[2]

End of Questions

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Mathematics Specialist Units 3 & 4
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Section 2 Calculator Assumed

Vectors in Two & Three Dimensions and Systems of Equations

STUDENT'S NAME: _____

DATE: Thursday 28th April **TIME:** 25 minutes **MARKS**: 27

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet retained from Section 1.

Special Items: Drawing instruments, templates, three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment).

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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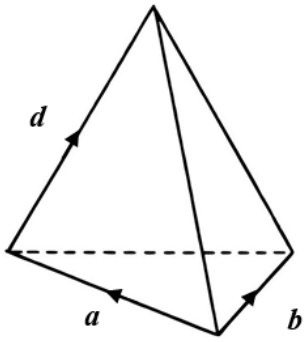
4. (4 marks)

Determine the vector equation of the sphere which has Cartesian equation:

$$x^2 + y^2 + z^2 + 2x - 4y + 6z - 11 = 0$$

5. (7 marks)

The diagram shows a tetrahedron with three edges described by vectors $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{d}$.



(a) Prove the area of the bottom face is given by:

$$A = \frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b}|$$

[2]

(b) Prove that the volume of the tetrahedron is given by:

$$V = \frac{1}{6}|\boldsymbol{d} \bullet (\boldsymbol{a} \times \boldsymbol{b})|$$

[5]

6. (10 marks)

Consider the system of equations:

$$x+y+z=3, \quad x-2y+z=6 \quad \text{and} \quad x-y+\mathrm{k}z=\mathrm{m}$$

(a) Determine the value(s) of k and m so that the system has:

(i) a unique solution [3]

(ii) more than one solution [2]

(iii) no solution [2]

(b) For case (ii) above:

(i) describe the solution in words [1]

(ii) illustrate the solution with a small sketch [1]

(iii) state the solution in parametric form, with parameter, $t \in \mathbb{R}$. [1]

7. (6 marks)

Consider two aircraft A and B, flying with constant velocities in m/s and initial positions as stated below:

A: $\boldsymbol{r}_0 = (5, -2, 1)$ km $\qquad \boldsymbol{v}_A = (-30, 50, 5)$

B: $\boldsymbol{r}_0 = (-8, -4, 2.5)$ km $\qquad \boldsymbol{v}_B = (40, 70, 15)$

State the closest distance these two aircraft come to each other and the time at which this happens.

End of Questions