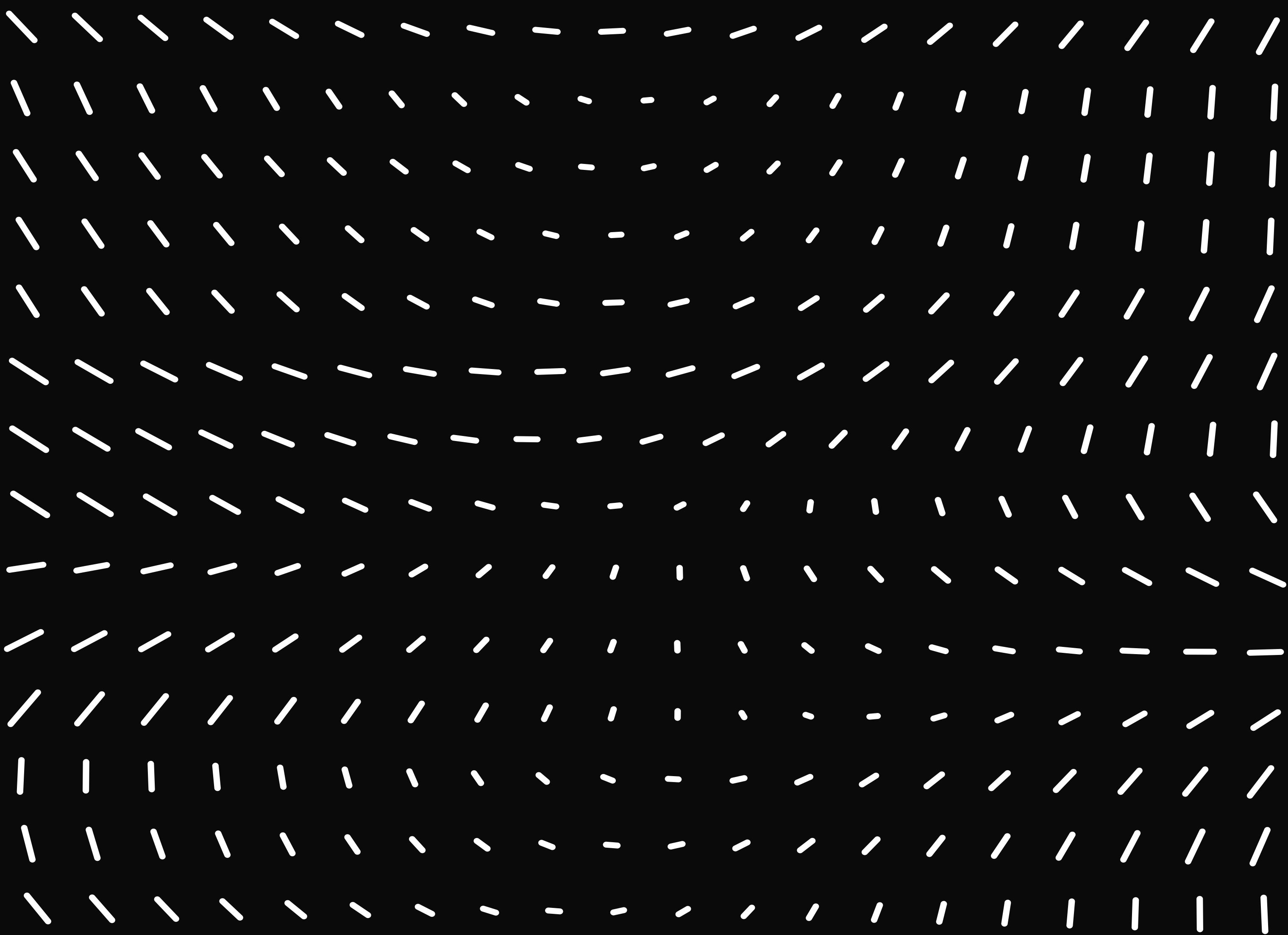


UNIT 3

chapter 1 - differentiation

product rule

if $y = f(x)g(x)$ then $\frac{dy}{dx} = f'(x)g(x) + g'(x)f(x)$

quotient rule

if $y = \frac{f}{g}$ then $\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

chain rule

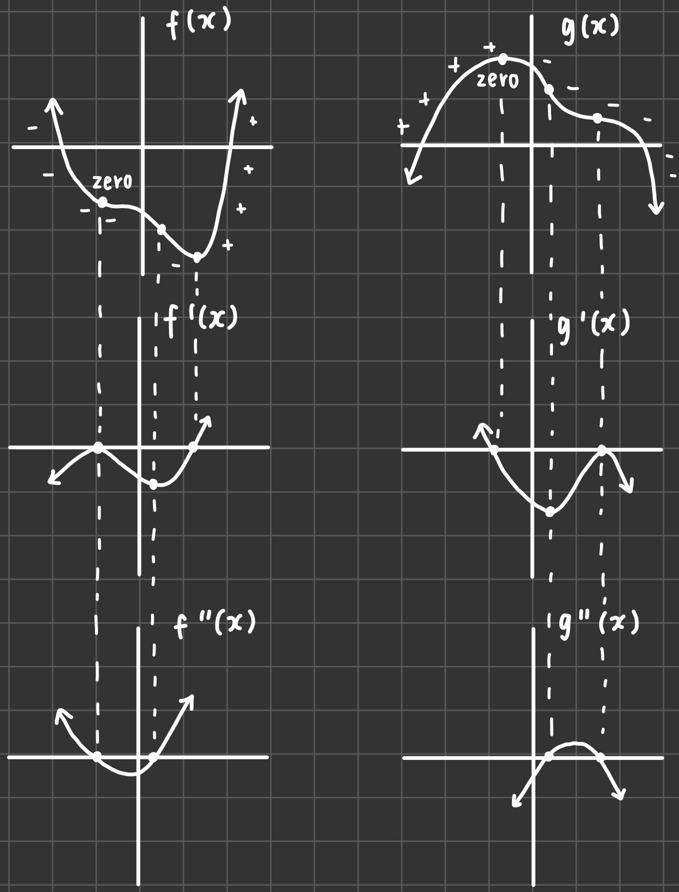
if $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

product rule for chain rule

if $y = [f(x)]^n$ then $\frac{dy}{dx} = n[f(x)]^{n-1} \cdot f'(x)$

chapter 2 - applications of differentiation

examining the 2nd derivative



$f''(x) < 0$ $f(x)$ = concave down

$f''(x) > 0$ $f(x)$ = concave up

$f''(x) = 0$ POI

acceleration

$v = \frac{dx}{dt}$ — dx — displacement, dt — time

$a = \frac{dv}{dt}$ — dv — velocity

a — acceleration

displacement
velocity
acceleration

] differentiate

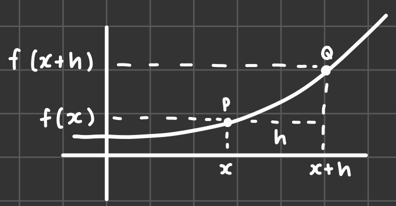
rates of change

the rate of change of y with respect to $x = \frac{dy}{dx}$

optimisation

1. draw a diagram
2. what is being max/minimised? + find an equation (e.g. C = ???)
3. if C involves 2 variables, find another equation to sub in
4. C in terms of 1 variable, find t.p's by making $dC/dx = 0$ + solving
5. use $f''(x)$ to determine max or min
6. check that it falls within domain/range

small changes



δx in place of h
(small change in x coordinate)
δy in place of $f(x+h)-f(x)$
(small change in y coordinate)

gradient function $= \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$

which is written as $\frac{dy}{dx}$

gradient @ P : $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\frac{dy}{dx} \approx \frac{\delta y}{\delta x}$ ← if δx is small

$\therefore \delta y \approx \frac{dy}{dx} \delta x$

example small % changes

$V = 2x^3$, find % change in V when x changes by 2%.

$\frac{\delta x}{x} = \frac{2}{100}$, find $\frac{\delta V}{V}$

if $V = 2x^3$ then $\frac{dV}{dx} = 6x^2$

$\therefore \frac{\delta V}{\delta x} \approx 6x^2$

$\frac{\delta V}{V} \approx \frac{6x^2 \delta x}{V} = \frac{6x^2 \delta x}{2x^3}$

$= 3 \frac{\delta x}{x} = \frac{6}{100}$

$\therefore$ x changes by 2%, V changes by $\approx$ 6%.

marginal rates of change

$\frac{\delta C}{1} \approx \frac{dC}{dx}$

dC/dx : marginal cost
(cost of producing 1 more)

dR/dx : marginal revenue
(revenue from 1 more after the x^{th} item has been sold)

dP/dx : marginal profit
(extra profit produced from 1 more produced)

example marginal rates of change

$C(x) = 6x + 10\sqrt{x} + 500$
determine approx. cost of producing 1 more item

$dC/dx = 6 + \frac{5}{\sqrt{x}}$

$\therefore \delta C/\delta x = 6 + \frac{5}{\sqrt{x}}$

with $\delta x = 1$, $x = 100$ $\quad \delta C = 6 + \frac{5}{\sqrt{100}}$

$= 6.5$

$\therefore$ it will cost approx \$6.50 to produce one more item when $x = 100$

chapter 3 - antidifferentiation

antidifferentiation

$\int 2x \, dx$
$= x^2 + c$

if $\frac{dy}{dx} = ax^n$

then $y = \frac{ax^{n+1}}{n+1} + c$

integrals of this form involve an unknown constant - indefinite integrals

if $\frac{dy}{dx} = f'(x)[f(x)]^n$

then $y = \frac{[f(x)]^{n+1}}{n+1} + c$

example $\int (2x+1)^4$
(derivative of $2x+1 = 2$)

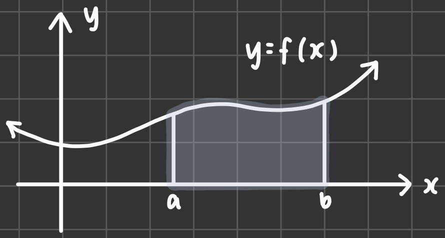
$= \int \frac{1}{2} \times 2 \times (2x+1)^4 \, dx$ ← rearranged to the form $af'(x)[f(x)]^n$

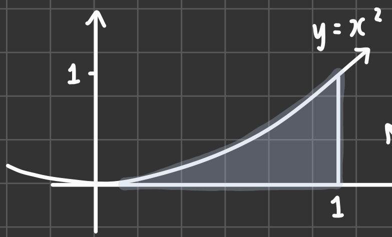
$= \frac{1}{2} \times \int 2 \times (2x+1)^4 \, dx$

$= \frac{1}{2} \times \frac{(2x+1)^5}{5} + c$

$= \frac{(2x+1)^5}{10} + c$

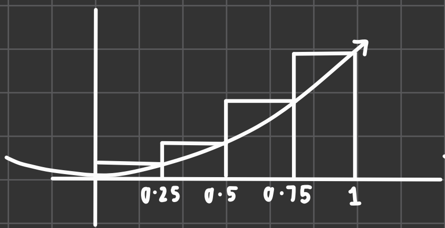
chapter 4 - area under a curve

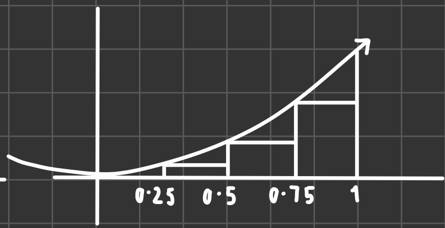




$\frac{17}{50} = 0.34 \text{ units}^2$

50 squares total
17 squares in area





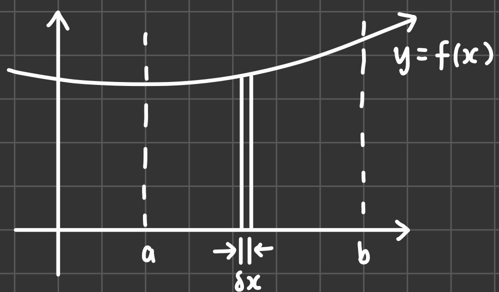
overestimate

1st sect $= 0.25(0.25)^2 = 0.02$
2nd sect $= 0.06$
3rd sect $= 0.14$
4th sect $= 0.25$
0.47

underestimate

1st sect $= 0.25(0)^2 = 0$
2nd sect $= 0.02$
3rd sect $= 0.06$
4th sect $= 0.14$
0.22

mean of the two : 0.34 units^2
more sectors = better approximation of area



$$\lim_{\delta x \to 0} \sum_{x=a}^{x=b} f(x)\,\delta x$$

$$\therefore \int_a^b f(x)\,dx$$

to evaluate $\int_a^b f(x)\,dx$
1. antidifferentiate $f(x)$ with respect to x (omit the $+c$)
2. sub b into the answer from 1
3. sub a into the answer from 1
4. calculate: part (2) answer - part (3) answer

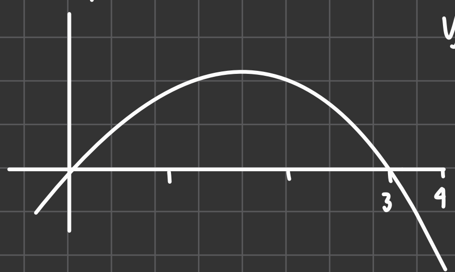
$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

integrals of this form will not involve the constant of integration - definite integrals

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \qquad \int_a^a f(x)\,dx = 0 \qquad \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

regions wholly or partially below the x-axis

example:



$y = 3x - x^2$

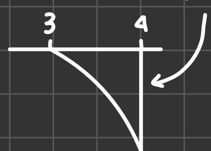
from $x=0 \to x=3$

$$\int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = 4.5$$

from $x=3 \to x=4$

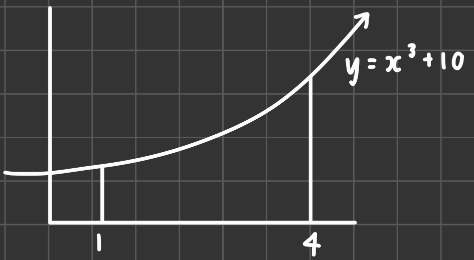
$$\int_3^4 (3x - x^2)\,dx = -\frac{11}{6}$$

below the x-axis



from $x=0 \to x=4$

another example:



$$\int_1^4 (x^3+10)\,dx$$

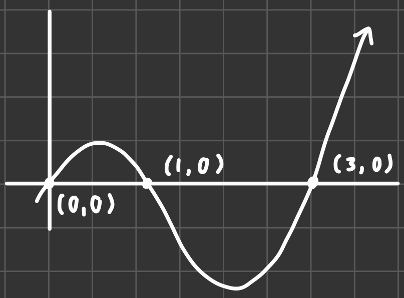
$$= \left[\frac{x^4}{4} + 10x\right]_1^4$$

$$= (64+40) - \left(\tfrac{1}{4}+10\right)$$

$$= 93\tfrac{3}{4}$$

area

one more example:



$$\int_0^1 (x^3-4x^2+3x)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right]_0^1$$

$$= \frac{5}{12}$$

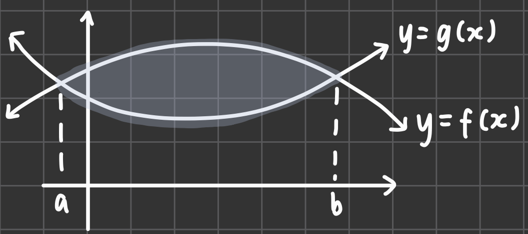
$$\int_1^3 (x^3-4x^2+3x)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right]_1^3$$

$$= -\frac{8}{3}$$

total area $= \frac{5}{12} + \frac{8}{3} = \frac{37}{12}$ units²

area between curves



shaded area =

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b [f(x) - g(x)]\,dx$$

example area between curves

between $y = 2x^2$ and $y = 12 - x^2$

$$\int_{-2}^{2} (12-x^2)\,dx - \int_{-2}^{2} 2x^2\,dx$$

$$= \int_{-2}^{2} (12-x^2-2x^2)\,dx$$

$$= \int_{-2}^{2} (12-3x^2)\,dx$$

$= 32$ units²

chapter 5 - the fundamental theorem of calculus

if asked to determine $\int_1^2 (3x^2+4)\,dx$

$$\int_1^2 (3x^2+4)$$

$$= \left[x^3+4x\right]_1^2$$

$$= [(2)^3+4(2)] - [(1)^3+4(1)]$$

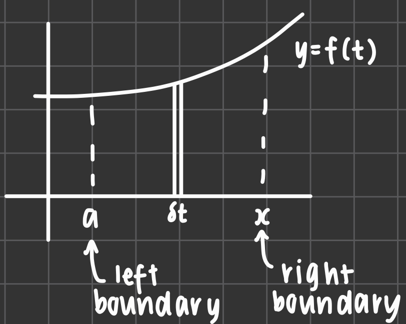
$$= 16-5$$

$$= 11$$

the fundamental theorem

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$F(x)$ = antiderivative of $f(x)$



$$A(x) = \lim_{\delta t \to 0} \sum_{t=a}^{t=x} y\,\delta t$$

$$= \int_a^x f(t)\,dt$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

can be written as:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

$$A(x) = \int_a^x f(t)\,dt$$
$$A'(x) = f(x)$$

using these

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

$$\therefore \int_a^b f'(x)\,dx = f(b) - f(a)$$

integrating the derivative 'gives us the function back'

and $$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

differentiating the integral of the function 'gives us the function back'

chapter 6 - the exponential function

$$e = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]$$

(approximately 2·71828)

derivative of e^x

if $y = e^x$

then $\frac{dy}{dx} = e^x$

example deriving e^x

$x^3 + e^x$ $dy/dx = 3x^2 + e^x$

$5e^x$ $dy/dx = 5e^x$

e^{x^2-5x+1} $u = x^2-5x+1$, $y = e^u$

chain rule

$\frac{dy}{du} = e^u$ $\frac{du}{dx} = 2x-5$

$\therefore \frac{dy}{dx} = e^u(2x-5)$
$= (2x-5)e^{x^2-5x+1}$

if $y = e^{f(x)}$

then $\frac{dy}{dx} = f'(x)e^{f(x)}$

growth and decay

$A = A_0 e^{kt}$

A — amount @ time (t); A_0 — initial amount; k — constant

if $\frac{dP}{dt} = kP$

then $P = P_0 e^{kt}$

integrating exponential functions

$$\int e^x\,dx = e^x + c$$

also by the chain rule:

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

example integrating e^x

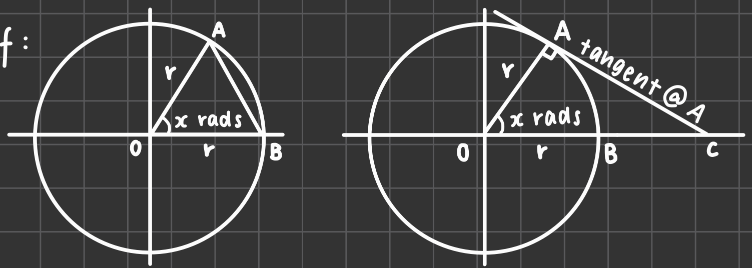
$$\int e^{6x}\,dx = \int \frac{1}{6}\times 6e^{6x}\,dx$$
$$= \frac{1}{6}\times \int 6e^{6x}\,dx$$
$$= \frac{1}{6}\times e^{6x} + c$$
$$= \frac{e^{6x}}{6} + c$$

$$\int 10xe^{x^2}\,dx = \int 5\times 2xe^{x^2}\,dx$$
$$= 5\times \int 2xe^{x^2}\,dx$$
$$= 5e^{x^2} + c$$

chapter 7 - calculus of trigonometric functions

$\lim_{h \to 0} \frac{\sin h}{h}$ and $\lim_{h \to 0} \frac{1 - \cos h}{h}$ IN RADIANS

proof:



$\lim_{x \to 0} \frac{\sin x}{x} = 1$

differentiation of sine and cosine

if $y = \sin x$ $\quad dy/dx = \cos x$

if $y = \cos x$ $\quad dy/dx = -\sin x$

— examples dy/dx of sin + cos —

$y = \cos(2x+3)$

$\cos \to -\sin$
$2x+3 \to 2$] $(2)(-\sin(2x+3))$
$= -2\sin(2x+3)$

$y = \sin(5-2x)$

$\sin \to \cos$
$5-2x \to -2$] $(-2)(\cos(5-2x))$
$= -2\cos(5-2x)$

$\frac{d}{dx}(\sin x \cos x)$ product rule.

$\cos x \cos x + \sin x(-\sin x)$

$= \cos^2 x - \sin^2 x$

antidifferentiation of sine and cosine

$\int f'(x) \cos f(x)\, dx = \sin f(x) + c$

$\int f'(x) \sin f(x)\, dx = -\cos f(x) + c$

— examples ∫sin and ∫cos —

$\cos 5x + \sin 2x$

$dy/dx = 5\cos 5x + 2\sin 2x$

$\therefore \int = \frac{1}{5}\sin 5x - \frac{1}{2}\sin 2x + c$

$8\cos(2x+1)$

$dy/dx = 2\cos(2x+1)$

$\therefore \int = 4\sin(2x+1) + c$

$15\cos 5x$

$3 \times \int 5x \cos 5x\, dx$

$3 \times \sin 5x + c$

$3\sin 5x + c$

$\cos^4 x \sin x$

$dy/dx = 5\cos^4 x(-\sin x)$

$= -5\cos^4 x \sin x$

$\therefore$ if $y = -\frac{1}{5}\cos^5 x$

$dy/dx = \cos^4 x \sin x$

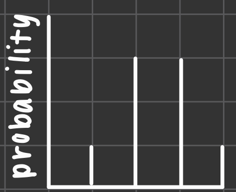
$\int = -\frac{1}{5}\cos^5 x + c$

chapter 8 - discrete random variables

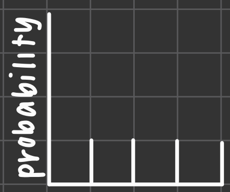
- sum of probabilities $= 1$

$P(X \leq x)$ is cumulative probability

$^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$



DRV



uniform

mean (expected value)

$E(X) = \bar{x} = (\Sigma x \cdot P(x))$

variance

$var(X) = (x - \mu)^2 (P(x))$

standard deviation

$s.d. = \delta = \sqrt{var}$

— example DRV's —

x	1	2	3	4
$P(X=x)$	0·1	0·2	0·4	0·3

$E(X) = (1 \times 0{\cdot}1) + (2 \times 0{\cdot}2) + (3 \times 0{\cdot}4) + (4 \times 0{\cdot}3)$

$= 2{\cdot}9$

$var(X) = [0{\cdot}1 \times (1 - 2{\cdot}9)^2] + [0{\cdot}2 \times (2 \times 2{\cdot}9)^2]$
$+ [0{\cdot}4 \times (3 - 2{\cdot}9)^2] + [0{\cdot}3 \times (4 - 2{\cdot}9)^2]$

$= 0{\cdot}89$

$\delta = \sqrt{var} = \sqrt{0{\cdot}89}$

$= 0{\cdot}9434$

change of scale: $Y = 3X$

3	6	9	12
0·1	0·2	0·4	0·3

$E(Y) = 3 \times E(X)$
$= 3 \times 2{\cdot}9$
$= 8{\cdot}7$

$var(Y) = 3^2 \times var(X)$
$= 9 \times 0{\cdot}89$
$= 8{\cdot}01$

$Z = 3X + 4$

$E(Z) = 3 \times E(X) + 4$
$= 3 \times 2{\cdot}9 + 4$
$= 12{\cdot}7$

$var(Z) = 3^2 \times var(X)$
$= 9 \times 0{\cdot}89$
$= 8{\cdot}01$

chapter 9 - bernoulli and binomial distributions

bernoulli distribution

x	1	2	3	4	5	6
P(X=x)	1/6	1/6	1/6	1/6	1/6	1/6

	failure	success
x	0	1
P(X=x)	5/6	1/6

bernoulli trial:
an associated random value with 2 possible variables (failure and success)

	failure	success
X	0	1
P(X=x)	1-p	p

$E(X) = 0 \times (1-p) + 1 \times p$
$= p$
$var(X) = (1-p) \times (0-p)^2 + p \times (1-p)^2$
$= p(1-p)(p+1-p)$
$= p(1-p)$

binomial distribution

- trials are independent

${}^nC_x\, p^x (1-p)^{n-x}$

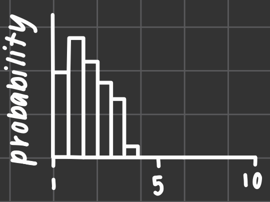
$\binom{n}{x} p^x (1-p)^{n-x}$

mean $(E(X)) = np$
$var(X) = np(1-p)$
$s.d(X) = \sqrt{np(1-p)}$

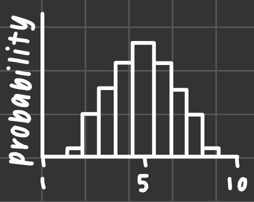
BinPDF (x, n, p)
BinCDF (x_1, x_2, n, p)
↑cumulative

x - number you want
n - number of trials
p - probability of success
x_1 x_2 = numbers between (e.g. $x \leq 2$ = 0, 2)

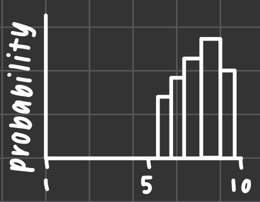
graphs of binomial distributions



p = 0·2



p = 0·5



p = 0·9

UNIT 4

chapter 1 - logarithmic functions

exponential form — **log form**

if $a^x = b$ then $\log_a b = x$

$a^x = b \quad \log_a b = x$

$\log_{10} 1000 = 3$
since $10^3 = 1000$

example:

$\log_2 16 = x \quad \therefore 2^x = 16$

$2^x = 2^4 \quad x = 4$

$\log_2 16 = 4$

laws of logarithms

$\log_a b = x$
$b = a^x$
$a^x a^y = a^{x+y}$
$bc = a^{x+y}$

$\log_a c = y$
$c = a^y$
$a^x \div a^y = a^{x-y}$
$b \div c = a^{x-y}$

$$\boxed{\begin{aligned}\log_a(bc) &= x+y \\ \log_a(bc) &= \log_a b + \log_a c\end{aligned}}$$

$$\boxed{\begin{aligned}\log_a\left(\tfrac{b}{c}\right) &= x-y \\ \log_a\left(\tfrac{b}{c}\right) &= \log_a b - \log_a c\end{aligned}}$$

$$\boxed{\log_a(b^c) = c\log_a b}$$

$$\boxed{\log_a\left(\tfrac{1}{b}\right) = -\log_a b}$$

$$\boxed{\begin{aligned}\log_a a &= 1 \\ \log_a 1 &= 0\end{aligned}}$$

using logs to solve equations

example:

$2^{5x-1} = 3^x$

$\log(2^{5x-1}) = \log(3^x)$

$(5x-1)\log 2 = x\log 3$

$5x\log 2 - \log 2 = x\log 3$

$x(5\log 2 - \log 3) = \log 2$

$x = \dfrac{\log 2}{5\log 2 - \log 3}$

natural logs

$b = e^x$ then $x = \log_e b$

$\ln = \log_e x$

$$\boxed{\log_a b = \frac{\log_c b}{\log_c a}}$$

$a^x = b$

$\log_c(a^x) = \log_c b$

$x\log_c a = \log_c b$

$x = \dfrac{\log_c b}{\log_c a}$

graphs of logarithmic functions

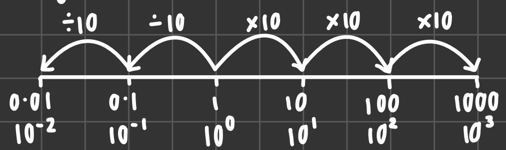
$y = af(x)$ — a = dilation
$-a$ = reflected over x-axis

$y = f(x-c)$ — $+c$ = left
$-c$ = right

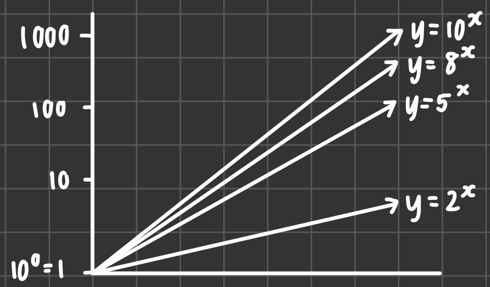
$y = f(x) - d$ — $+d$ = up
$-d$ = down

$y = f(bx)$ — b = dilation of the graph parallel to the x-axis w/ a factor of $\frac{1}{b}$

logarithmic scale



graphs of log scales



chapter 2 - calculus involving logarithmic functions

differentiation

gradient at $P(x, f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

thus $y = \ln x$ then $\frac{dy}{dx} = \lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$ $\qquad \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$

$y = \log_e x$ then $x = e^y$

$\frac{dy}{dx} = e^y$

thus $\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$

$\boxed{\therefore\ y = \log_e x \quad \frac{dy}{dx} = \frac{1}{x}}$

$\boxed{\text{if } y = \log_e f(x) \text{ then by the chain rule: } \frac{dy}{dx} = \frac{f'(x)}{f(x)}}$

integration

$y = \ln x$ then $\frac{dy}{dx} = \frac{1}{x}$

thus $\int \frac{1}{x}\, dx = \ln x + c$

if $y = \ln f(x)$ then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

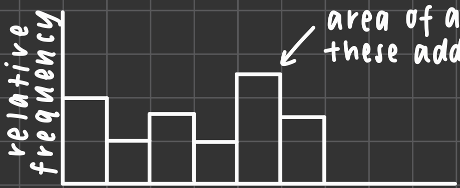
$\boxed{\text{thus } \int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c}$

$\int \frac{1}{x}\, dx = \ln x + c$ for $x > 0$ $\qquad \int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c$ for $f(x) > 0$

$\int \frac{1}{x}\, dx = \ln|x| + c \quad x \neq 0$

chapter 3 - continuous random variables

distribution as a histogram



area of all of these adds to 1

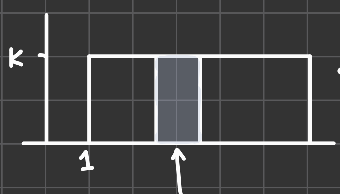
continuous random variables

- value can be anything within the limits
- from measuring something (e.g. heights, time etc.)

pdf = probability density function

probability density function (pdf)

if $f(x)$ is the probability density function for a continuous random variable, X, then the area under $f(x)$ from $x = a \to x = b$ gives $P(a < X < b)$



uniform (rectangular) distribution

example: $P(2.5 \leq X \leq 3.5)$

$k = 0.2$ because area $= 1$

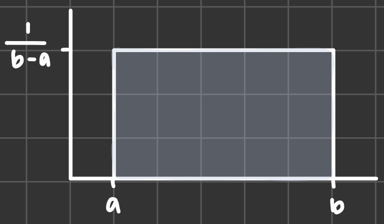
this can be written as: $\begin{cases} 0.2 & \text{for } 1 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$

- since $P(X = a)$ is negligible, $P(X \geq a) = P(X > a)$
- the probability density function must not dip below the x-axis because that would suggest a negative probability, which is meaningless

$f(x) \geq 0$ for all $a < x < b$

area under $f(x)$ for $a < x < b$ must $= 1$ $\quad$ i.e. $\int_a^b f(x)\, dx = 1$

uniform (or rectangular) distributions



$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

the mean, expected value or long-term average = halfway between a and b, i.e. = $\frac{a+b}{2}$

non-uniform distributions

$f(x) \geq 0$ for all x in $a < x < b$

area under the graph = 1 i.e. $\int_a^b f(x)\,dx = 1$

$$f(x) = \begin{cases} ke^{-kx} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

for $0 < x < \infty$ and $k > 0$ then $f(x) > 0$ $\int_0^\infty f(x)\,dx = 1$

expected value, variance and standard deviation

DRV X has possible values x_i, with $P(X = x_i) = p_i$ then $E(X)$

the expected or long term mean value is given by: $E(X) = \Sigma(x_i, p_i)$

the summation being carried out over all possible values of x_i

using μ to represent $E(X)$ then the variance, $var(X)$ is given by:

$var(X) = \Sigma[p_i(x_i - \mu)^2]$

standard deviation is the square root of the variance

for a continuous random variable X, w/ probability density function $f(x)$

$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$ $var(x) = \int_{-\infty}^{\infty} [f(x)(x-\mu)^2]\,dx$

change of scale and origin

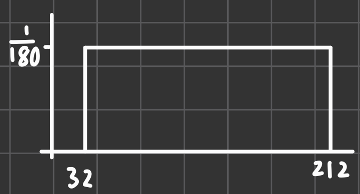
example: suppose that the temp. of something, recorded in °C is a CRV X (between 0 → 100)

$E(X) = \int_0^{100} \left(\frac{1}{100} x\right) dx = 50$

$var(x) = \int_0^{100} \left(\frac{1}{100}(x-50)^2\right) dx = \frac{2500}{3}$

$SD(X) = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3}$

example: $°F = (°C \times 1.8) + 32$ $\therefore 0°C = 32°F$, $100°C = 212°F$



$E(X) = \int_{32}^{212} \left(\frac{1}{180} x\right) dx = 122$

$var(x) = \int_{32}^{212} \left(\frac{1}{180}(x-122)^2\right) dx = 2700$

$SD(X) = 30\sqrt{3}$

if the random variable X has mean μ and standard deviation δ (and variance δ^2) then the random variable $aX + b$ will have mean $a\mu + b$ and standard deviation $|a|\delta$ (and variance $a^2\delta^2$)

cumulative distribution function

uniform pdf

$$f(x) = \begin{cases} 0\cdot 2 & \text{for } 1 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

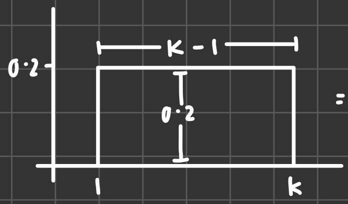
for $1 \le k \le 6$, $P(X \le k) = {}_1\int^k 0\cdot 2\,dx$

$= [0\cdot 2x]_1^k$

$= 0\cdot 2(k-1)$

CDF formula

alternative method for formula



$= 0\cdot 2(k-1)$

hence cumulative distribution function:

$$P(X \le x) = \begin{cases} 0 & \text{for } x < 1 \\ 0\cdot 2(x-1) & \text{for } 1 \le x \le 6 \\ 1 & \text{for } x > 6 \end{cases}$$

$\boxed{\text{example:}}$ determine $P(3 \le X \le 5)$

$P(3 \le X \le 5) = P(X \le 5) - P(X < 3)$

$= 0\cdot 2(5-1) - 0\cdot 2(3-1)$

$= 0\cdot 8 - 0\cdot 4$

$= 0\cdot 4$

same as ${}_3\int^5 0\cdot 2\,dx$

$P(a \le X \le b) = {}_a\int^b p(x)\,dx$

expected value + variance (from formula sheet)

$E(X) = \mu = {}_{-\infty}\int^{\infty} x\,p(x)\,dx$

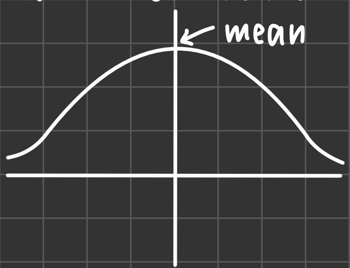
variance $= \sigma^2 = {}_{-\infty}\int^{\infty} (x-\mu)^2\,p(x)\,dx$

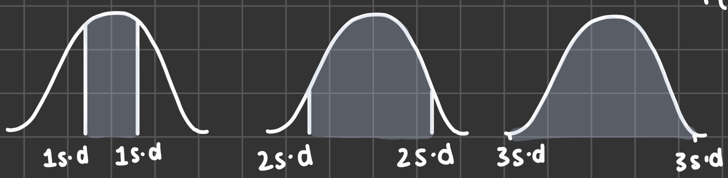
chapter 4 - the normal distribution

standard scores

standardised score $= \dfrac{\text{raw score} - \text{mean}}{\text{standard deviation}}$

normal distribution





68·3%   95·4%   99·7%

$\boxed{\text{example:}}$ 500g of breakfast cereal, weight is normally distributed, mean = 512g s·d = 8 grams

a) determine probability that a randomly chosen box contains between 504g & 520g

504 = 1s·d· below

520 = 1s·d· above

$\therefore P = 0\cdot 68$

b) determine the probability that a randomly chosen box < 500g

$P(x < 500) = 0\cdot 0668$

c) in a sample of 100 boxes, how many boxes should be expected to contain < 500g?

$P(x < 500) \approx 0\cdot 07$

$\therefore$ in 100 sample = 7 boxes

$$z \text{ score} = \frac{x \text{ score} - \text{mean of } x \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

standardised score

$\boxed{\text{example:}}$ if $X \sim N(63, 25)$ determine $P(x<55)$

mean, variance

$\therefore \sqrt{\text{var}} = \text{s.d}$

$P(x<55) = 0{\cdot}0548$

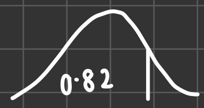
quantiles

values which a particular proportion of the distribution falls below

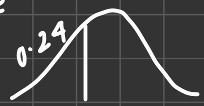
e.g. if 0·7 of the distribution is below 55 then 55 is the 0·7 quantile or 55 is in the 70th percentile

$X \sim N(20, 3^2)$

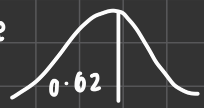
a) 0·82 quantile
= 22·7



b) 0·24 quantile
= 17·9



c) 0·62 quantile
= 20·9



the normal distribution pdf

$X \sim N(\mu, \sigma^2)$

using the normal distribution to model data