

MARKING KEY 2CMAT/2DMAT

DRAFT

RESOURCE-FREE EXAMINATION

Question 1 [9 marks]

(a) Simplify: $4b - 2[3b - (b+7)]$

2 marks	Description
1	$4b - 2[3b - (b+7)]$
1	$= 4b - 2(2b - 7) \checkmark$
1	$= 14 \checkmark$

(b) Solve the following equation algebraically, showing working: $(2a - 4)(a + 1) = 0$.

3 marks	Description
1	$(2a - 4)(a + 1) = 0$
1,1	If a product is equal to zero, at least one of the terms must be zero. So either $(2a - 4) = 0$ or $(a + 1) = 0 \checkmark$
1,1	That is, $a = 2$ or $a = -1$ (numbers) \checkmark ('or' in the expression) \checkmark

(c) Show that $3k(3k - 1) = 4(4 - k) + k$ is equivalent to $9k^2 - 16 = 0$.

2 marks	Description
1	$3k(3k - 1) = 4(4 - k) + k$
1	Expanding each side gives: $9k^2 - 3k = 16 - 4k + k \checkmark$
1	That is, $9k^2 - 3k = 16 - 3k$
1	Adding $3k$ to each side gives the required result: $9k^2 - 16 = 0 \checkmark$

(d) Use factorisation to solve $9k^2 - 16 = 0$.

2 marks	Description
1	$9k^2 - 16 = 0$
1	$(3k - 4)(3k + 4) = 0 \checkmark$
1	If a product is equal to zero, at least one of the terms must be zero So either $(3k - 4) = 0$ or $(3k + 4) = 0$.
1	That is, $k = 4/3$ or $k = -4/3 \checkmark$ (do not penalise for lack of 'or' here)

Question 2 [6 marks]

Describe a graphical method of solving the equation $5 - x^2 = 2x + 1$. Provide a sketch as part of your answer.

6 marks	Description
1	Sketch showing $f(x) = -x^2 + 9$ and $g(x) = 2x + 1$ on the same axes
1	parabola shape, symmetrical about the y axis, inverted \checkmark
1	y intercept 9, x intercepts -3 and 3 \checkmark
1	straight line with positive gradient and positive y intercept \checkmark
1	y intercept 1, x intercept -0.5 \checkmark
1,1	Description: Graph $f(x) = -x^2 + 9$ and $g(x) = 2x + 1$ on the same axes Read off the x coordinates \checkmark of the points of intersection \checkmark

Question 3 [5 marks]

(a) One year later, environmental officers netted 56 bream from the river, of which 7 had tags. The fish were then released back into the river. Estimate the population of bream in the river at this time.

3 marks	Description
1	$7 \div 56 = 800 \div P \checkmark$
1	$P = 800 \cdot 56 \div 7 \checkmark$
1	Hence $P \approx 6400 \checkmark$

(b) What, if any, assumptions did you make in your answer to (a)?

2 marks	Description
1	Assumed:
1	All 800 of restocked fish still in river ✓
1	Restocked fish thoroughly mixed with existing bream (or similar) ✓

Question 4 [5 marks]

(a) Find the gradient of the line passing through A and B.

2 marks	Description
1,1	gradient = $-\frac{10}{6}$ ✓ (fraction) ✓ (negative sign)
	= $-\frac{5}{3}$

(b) Determine the equation of the line that is perpendicular to the side AB and that passes through the point C.

3 marks	Description
1	Perpendicular gradient to AB is $\frac{3}{5}$ ✓
1	Required line is $y - (-1) = \frac{3}{5} \times (x - (-3))$ ✓
1	Which simplifies to $y = \frac{3}{5}x + \frac{4}{5}$ ✓

Question 5 [8 marks]

(a) For each of the following conjectures, state whether it is true or false. If it is true, give three examples of when it is true. If it is false, give one example of when it is false. The set of counting numbers is {1, 2, 3, 4, ...}.

(i) If n is an odd counting number greater than 1, then $n^2 - 1$ is a multiple of 8.

2 marks	Description
1	Three trials with correctly simplified answers ✓
	When $n = 1$, $n^2 - 1 = 1 - 1 = 0$
	$n = 3$, $n^2 - 1 = 9 - 1 = 8$
	$n = 5$, $n^2 - 1 = 25 - 1 = 24$ which are multiples of 8
1	It appears this conjecture will hold true ✓

(ii) The product of two consecutive counting numbers is a multiple of 4.

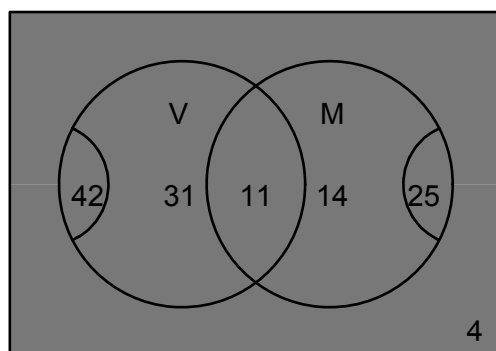
2 marks	Description
1	One trial ✓
	$1 \times 2 = 2$ (False)
	or $2 \times 3 = 6$ (False)
	or $5 \times 6 = 30$ (False)
1	This conjecture is false ✓

(b) Justify the claim that the sum of three consecutive numbers is always a multiple of 3.

4 marks	Description
1	Let the first number be x ✓
1	The next consecutive numbers would be $x + 1$ and $x + 2$ ✓
1	Sum = $x + (x + 1) + (x + 2)$ ✓
	= $3x + 3$
1	= $3(x + 1)$ which is a multiple of 3 ✓

Question 6 [7 marks]

(a) Use the above information to complete the Venn diagram below.



3 marks	Description
1	correct placement of 4, 42 and 25 on the Venn diagram ✓
1	obtains 11 for the intersection ✓
1	obtains the 31 and 14 ✓

(b) (i) What is the probability that one of the tourists selected at random from those who intend visiting The Valley of the Giants also intends to visit Monkey Mia?

2 marks	Description
1,1	$\frac{11}{60}$ 11✓ 60✓

(ii) What is the probability that one of the whole group of tourists selected at random intends to visit Monkey Mia, only?

1 mark	Description
1	$\frac{14}{60}$ ✓

(iii) Write the question in (ii) in using probability notation.

1 mark	Description
1	$P(\overline{V} \cap M)$ ✓

RESOURCE-RICH EXAMINATION

Question 1 [6 marks]

- (a) For the suburb of Riverton, six houses were sold in January. The prices were:
\$180 000, \$200 000, \$165 000, \$210 000, \$175 000, \$220 000
For these prices, find the
(i) mean
(ii) median

2 marks	Description
1	Mean: \$191 667 ✓
1	Median: \$190 000 ✓

- (b) Give an example of the five prices that fit this description.

2 marks	Description
2	an example \$310 000, \$320 000, \$330 000, \$340 000, \$2 000 000

- (c) Explain why the median is generally a better representative value than the mean for this type of data.

2 marks	Description
2	One extreme value can make the mean increase so that it is no longer representative of the majority of the values.

Question 2 [6 marks]

- (a) What is the total numbers of words she will have learnt after:
(i) 7 days?

1 mark	Description
1	$31 + 7 \times 8 = 87$ ✓

- (ii) n days?

1 mark	Description
1	$31 + 8n$ ✓

- (b) Use algebraic recursive notation to describe:
(i) T_n , the total number of words she will have learnt after n days;

2 marks	Description
1,1	$T_n = T_{n-1} + 8$ ✓ $T_0 = 31$ ✓

- (ii) T_w , the total number of words she will have learnt after w weeks.

2 marks	Description
1,1	$T_w = T_{w-1} + 56$ ✓ $T_0 = 31$ ✓

Question 3 [13 marks]

- (a) Give TWO different reasons for having confidence in the data in the table.

2 marks	Description
2	Two good reasons, e.g. It would have been a large sample Credibility of ABS, national government agency Published official results (likely to be checked more carefully)

- (b) Describe as precisely as you can the meaning of the 37 in the second column of figures in the table.

3 marks	Description
1,1,1	Of the children questioned, 37% of 9-11 year olds accessed the internet at school. ✓✓✓ (three elements %, age, school needed for three marks)

- (c) What percentage of the children (aged 5-14) did *not* access the internet over the twelve months represented in the table?

1 mark	Description
1	53% (i.e., $1 - 0.47$, from bottom right of table, converted to %) ✓

- (d) Describe, in detail, the internet access of the 9-11 year olds.

3 marks	Description
3	(i) School use of 37% > home use of 30% > other home use of 10% ✓ (ii) 55% access internet at any site ✓ (iii) Public library access negligible (3%) ✓

- (e) Explain how to use the table to conclude that some students must have accessed the internet both at home and at school.

2 marks	Description
1	If students accessed in home or school, but not both, then 'Any site' percentage would be greater than 'home' + 'school' percentages ✓
1	In each row of the table, 'home' + 'school' < 'any site', so some students must be counted twice and thus have accessed internet in both home and school ✓

- (f) What conclusions can you draw from the table about a relationship between internet access and age of children? Justify your answer.

2 marks	Description
1	It seems that access increases with age for children aged 5-14 ✓
1	This effect is evident in the first three rows of every column of the table. ✓

Question 4 [5 marks]

- (a) Greg simulated 20 rolls of a fair eight-sided die and averaged the results of the rolls. He repeated this process two times. Give examples of the averages he is likely to produce and justify your choices.

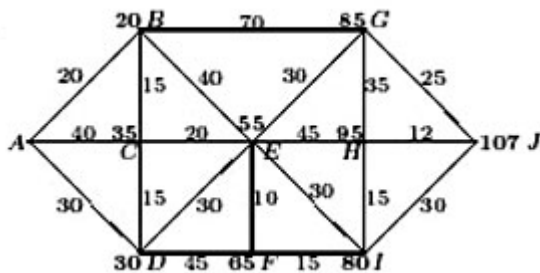
3 marks	Description
1	4 and 5 ✓
1,1	Expect a different result each time, averages between 1 and 8 are possible, expect them to be fairly close to the average of 1, 2 ... 8 (4.5) ✓✓ (two properties)

- (b) Next Greg simulated 200 rolls of the die and averaged the results of the rolls. Describe how averages produced in this way are likely to be different from those in part (b). Justify your description.

2 marks	Description
1	With 200 rolls, the averages would most likely be closer to 4.5 than with 20 rolls (or narrow range stated) ✓
1	because of larger sample size ✓

Question 5 [8 marks]

(a) Find the shortest path from *A* to *J* and state its length.



3 marks	Description
1	Bold paths on diagram
1	Shortest path: ABCEFIHJ ✓ Distance: 107 km ✓

(b) Find the shortest path from *A* to *J* that passes through *D*. What is the length of this path?

2 marks	Description
1	Shortest path: ADEFIHJ ✓
1	Distance: 112 km ✓

(c) A bypass under construction will reduce the distance along the section connecting centres *I* and *J* by x km. For what values of x will the shortest path from *A* to *J* use the bypass?

3 marks	Description
1	$30 - (15 + 12) = 3$ (some indication of working) ✓
1,1	∴ for any $x \geq 3$ bypass will be used ✓✓

Question 6 [8 marks]

(a) The table below shows the annual interest and the total amount owing after t years. Complete the last two rows.

Year (t)	Interest (\$)	Amount owing (\$)
1	3 600.00	48 600.00
2	3 888.00	52 488.00
3	\$4 199.04	\$56 687.04
4	\$4 534.96	\$61 222.00

2 marks	Description
1	Interest and amount correct for Year 3 ✓
1	Interest and amount correct for Year 4 ✓

(b) What amount would be owed after 20 years?

3 marks	Description
1,1	$\$45\,000 \times 1.08^{20}$ ✓✓
1	$= \$209\,743.07$ ✓ (correct answer to 2 dp for the mark)

(c) The house was initially valued at \$300 000, increasing at a rate of 3% per annum. Will the increase in value of the house exceed the amount owing after 20 years? If so, by how much?

3 marks	Description
1	$\$300\,000 \times 1.03^{20} = \$541\,833.37$ ✓
1	Increase is $\$541\,833.37 - \$300\,000 = \$241\,833.37$ ✓
1	Since loan is only \$209 743.07 then YES, it can be fully repaid ✓

Question 7 [8 marks]

(a) Find the distance BD.

2 marks	Description
1	$BD^2 = 27.4^2 + 31.5^2 - 2(27.4)(31.5)\cos(85^\circ)$ ✓ (mark for stating the cosine rule with correct substitutions)
1	$BD = 39.9\text{m}$ ✓

(b) Find the area of triangle ABD.

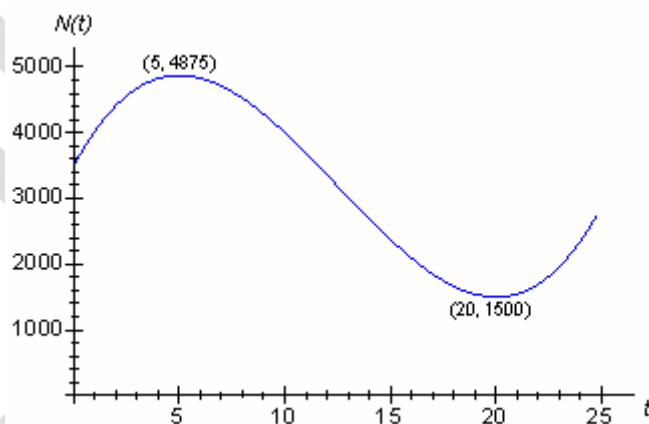
2 marks	Description
1	$\text{Area} = 0.5 \times 31.5 \times 27.4 \times \sin(85^\circ)$ ✓ (mark for stating the area expression with correct substitutions)
1	$= 429.9\text{m}^2$ ✓

(c) Find the area of the whole block of land ABCD.

4 marks	Description
1	Treats polygon as two triangles Area of ABD = 429.9m^2
1	In triangle CBD $19.2^2 + 41.8^2 - 2(19.2)(41.8)\cos(C^\circ) = 39.9^2$ ✓ (cosine rule stated with correct substitutions)
1	Hence $C = 71^\circ$ ✓
1	Area of CBD = $0.5 \times 19.2 \times 41.8 \times \sin(71^\circ)$ $= 379.4\text{m}^2$ ✓
1	Total area = $429.9 + 379.4 = 809.3\text{m}^2$ ✓

Question 8 [10 marks]

(a) Sketch a graph to show how the number of weeds varies from day to day, indicating clearly any maximum or minimum values.



3 marks	Description
1	shape, domain and y intercept correct ✓
1	Indicates max (5, 4875) clearly ✓
1	Indicates min (20, 1500) clearly ✓

(b) For how many of the 25 days was the recording more than 4 200 weeds? Show reasoning.

4 marks	Description
1	The number of weeds = 4200 when $4200 = 2t^3 - 75t^2 + 600t + 3500$ ✓
1	or $N(t) = 2t^3 - 75t^2 + 600t + 3500$ and $N(t) = 4200$
1	solving this gives $t \approx 1.4, 9.3$ and not again in $[0, 25]$ ✓ (1.4 and 9.3 sufficient)
1	so days when the recording exceeded 4200 were day 2 to day 9 inclusive ✓
1	Therefore, there were 8 days in which the number of weeds recorded exceeded 4200 ✓ (valid reasoning shown on the graph --4 marks)

(c) Write a short descriptive report on the success (or otherwise) of the weed-killing process during the above time period. This report should refer to the number of weeds recorded and the number of days since the poisoning occurred.

3 marks	Description
1,1	<ul style="list-style-type: none"> initially process appeared unsuccessful with number of weeds increasing from 3500 to 4875 in the first 5 days after 5 days the number of weeds started to decrease until the 20th day when there was a minimum number of 1500 weeds the number of weeds increased after the 20th day so eventually may have exceeded the original number ✓✓ (2 valid statements describing the weed killing)
1	1500 was fewer than the original number of weeds, so the poisoning could be judged successful ✓ (any valid summary observation and a conclusion that is consistent with it)

Question 9 [6 marks]

(a) John's score was the second highest in the Geography test. What score did he get in History?

1 mark	Description
1	Score for History is 80 ✓

(b) If G represents the Geography mark and H the corresponding History mark of a student, which of the following could be the equation of the line that best fits the scattergraph? Explain your answer.

3 marks	Description
1	Correct equation is C ✓
1	Justification e.g. The line of best fit would have positive gradient ✓ and
1	The y-intercept would be around 40 on the graph provided but there is a break in scale. Therefore, if the line was extended further the y - intercept would be well below 40. ✓ Line C is the only line that fits both criteria.

(c) Ashley was sick on the day that the Geography and History tests were held. She sat the Geography test during the lunch-hour the next day, and her History mark was estimated from her Geography mark. Ashley's mark for Geography is 62. Use your answer for (c) to determine the mark Ashley was given for History.

2 marks	Description
1	$H = 0.9(62) + 8 = 63.8$ ✓
1	Therefore, Ashley should receive a mark of 64 (or 63.8) ✓

Question 10 [10 marks]

(a) (i) Four ways in which Frank can pay an account of \$40 are listed below:

\$10, \$10, \$10, \$10

\$10, \$10, \$20

\$10, \$20, \$10

\$20, \$10, \$10

Find another way.

1 mark	Description
1	\$20 \$20 ✓

(ii) Are there more than 5 ways in which Frank can feed the notes to pay \$40? Explain.

1 mark	Description
1	No, all combinations of only \$10 notes, both \$10 and \$20, only \$20 have been listed in a systematic way ✓

(b) (i) Consider other amounts that Frank could pay and complete the table below.

Amount to pay	\$10	\$20	\$30	\$40	\$50	\$60	\$70
Number of ways to feed into machine	1	2	3	5	8	13	21

A systematic way to approach this is to start with number of ways for \$10 and build up by \$10 amounts by adding a \$10 note to each of the previous examples and adding \$20 to the examples two steps back.

\$10 10	\$20 10 10 (add 10 to previous 10) 20 (1 with 20)	\$30 10 10 10 (add 10 to \$20 examples) 20 10 10 20 (add 20 to \$10 example)
\$40 10 10 10 10 (add 10 to \$30 examples) 20 10 10 10 20 10 10 10 20 (add 20 to \$20 examples) 20 20 20 10 20	\$50 10 10 10 10 10 (add 10 to \$40 examples) 20 10 10 10 10 20 10 10 10 10 20 10 20 20 10 10 10 10 20 (add 20 to \$30 examples) 10 20 20	

2 marks	Description
2	Number of ways 1 2 3 ✓ 5 8 13 21 ✓

(ii) Describe the pattern in the number of ways to feed the notes into the machine.

1 mark	Description
1	To get the next term you add the previous two terms ✓

(iii) How do you know the pattern will continue?

3 mark	Description
1	The amounts are going up by \$10 ✓
1	and
1	The ways are obtained by adding \$10 to each of the ways in the previous step ✓
1	and adding \$20 to the examples two steps back ✓
	(in the absence of the above, one mark for stating the pattern is a Fibonacci sequence)

- (c) Find the number of ways to feed \$10 and \$20 notes into the machine to pay \$130. Explain why the number of ways follows the pattern.

Amount	\$60	\$70	\$80	\$90	\$100	\$110	\$120	\$130
Number of ways to pay	13	21	34	55	89	144	233	377

2 marks	Description
1	The number of ways is 377 ✓
1	\$130 is in the pattern (+10) for the amounts ✓ so the sequence 1, 2, 3, 5, 8 can be used to calculate the number of ways