TRINITY COLLEGE

Mathematics Methods Unit 3,4
Test 2 2019

Section 1 Calculator Free

Fundamental Theorem, The Exponential Function & Trigonometry

STUDENT'S NAME _____

DATE: Thursday 4 April **TIME:** 30 minutes **MARKS**: 32

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (3 marks)

Given $g'(x) = e^{2x}\cos(2x)$, determine a simplified value for the rate of change of g′(x) when $x = \pi$

2. (9 marks)

(a) $\frac{d}{dx}\left(\frac{e^{-x}}{\sin x}\right)$ [2]

(b) $\int_0^2 x\, e^{4-x^2}\, dx$ [3]

(c) $\frac{d}{dx}\int_x^0 \sin(t) + e^t \ \mathrm{dt}$ [2]

(d) $\int 4\cos(3x)\ dx$ [2]

3. (4 marks)

Given $\int_1^6 f(x)\ dx = 10$, determine

(a) $\int_6^1 e^2\ f(x)\ dx$ [2]

(b) $\int_1^3 (f(x) - 2)\ dx - \int_6^3 f(x)\ dx$ [2]

4. (5 marks)

Determine the exact area enclosed by $y = \cos x - \frac{1}{2}$ and the x-axis from 0 to 2π

5. (12 marks)

Consider the function: $f(x) = xe^x$.

(a) Determine the co-ordinates of any axis intercepts. [1]

(b) Show that the function has only one stationary point. [3]

(c) Determine the co-ordinates and nature of the stationary point. [3]

(d) Determine the co-ordinates of the point of inflection. [2]

(e) Determine $\int x e^x dx$ [3]

TRINITY COLLEGE

Mathematics Methods Unit 3,4
Test 2 2019

Section 2 Calculator Assumed

Fundamental Theorem, The Exponential Function & Trigonometry

STUDENT'S NAME _____

DATE: Thursday 4 April **TIME:** 20 minutes **MARKS**: 22

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

6. (5 marks)

A radioactive material decays such that it has a half-life of 25 years, meaning it takes twenty-five years for the amount of material to be halved.

(a) State a formula for the mass m(t) remaining after t years where the initial amount is 8000 kg. (Give your answer in the form: $M = M_0 e^{kt}$ where M_0 and k need to be stated to 4 significant figures). [2]

(b) Calculate the mass remaining after 30 years. [3]

7. (7 marks)

A particle moves along a straight line such that its displacement, x metres at time t seconds is given by the equation $x = 3\sin(2t) + 4$. Determine:

(a) An equation for the velocity of the particle at time t. [1]

(b) The distance from the origin the particle comes to a stop for the first time. [2]

(c) The distance travelled in the first three seconds. [2]

(d) The acceleration when $t = \frac{3\pi}{4}$ seconds. [2]

8. (4 marks)

Given that $r = \sqrt{t}$ $t = 4x$ and $x = \cos\theta$,

Prove that $\frac{dr}{d\theta} = -\frac{\sin\theta}{\sqrt{\cos\theta}}$

9. (6 marks)

Element J has a quantity of radioactive material present that has the following relationship $\frac{dA}{dt} = -0.03A$ where A is the amount (in grams) of the element remaining after t days. There is 200 g present initially.

(a) Give an expression for the amount of radioactive material remaining at t days. [2]

(b) How much of the original element is remaining after 2 weeks? [2]

(c) Determine when 150 g of material has decayed. [2]