MATHEMATICS
2A/2B

Calculator-assumed

WACE Examination 2012

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Section Two: Calculator-assumed **(100 Marks)**

Question 8 **(15 marks)**

(a) Charlotte earns $16 per hour waitressing during weekdays (Monday – Friday). From Monday to Friday, she works a total of 15 hours.

(i) How much does Charlotte earn for working Monday to Friday? (1 mark)

Solution
$16 \times 15 = \$240$
Specific behaviours
✓ correctly uses the hourly rate to calculate the wage

On Saturdays Charlotte's pay is 10% more than her weekday rate and on Sundays her pay is 20% more than her Saturday rate.

(ii) How much does Charlotte earn per hour on Sundays? (2 marks)

Solution
$16 \times 1.1 \times 1.2 = \$21.12/h$
Specific behaviours
✓ identifies the total percentage increase (i.e. 132% or 1.32) or correctly calculates Saturday's rate
✓ correctly calculates the Sunday rate

(iii) Determine Charlotte's weekend pay when she works 2 hours on Saturday and 5 hours on Sunday. (2 marks)

Solution
$16 \times 1.1 \times 2 + 21.12 \times 5 = 35.20 + 105.60$ $\qquad\qquad\qquad\qquad = \$140.80$
Specific behaviours
✓ calculates one day's wage
✓ calculates the other day's wage or the total wage

(b) The Earth has a land area of 148 940 000 km^2.

(i) Write this number using scientific notation. (1 mark)

Solution
1.4894×10^8
Specific behaviours
✓ correctly converts to scientific notation

(ii) Australia has a land area of 7 682 300 km². What percentage of the Earth's land area is this? Give your answer to **two (2)** decimal places. (2 marks)

Solution
$\dfrac{7\ 682\ 300}{148\ 940\ 000} \times 100\% = 5.16\%$
Specific behaviours
✓ represents the correct expression for calculating the percentage
✓ correctly rounds percentage to two decimal places

(c) Angelique sells chemical-free beauty products.

A body moisturiser is sold in bottles in two sizes:

$$375 \text{ mL for } \$19.70 \quad \text{or} \quad 600 \text{ mL for } \$32$$

Angelique needs to advise her clients which bottle is better value for money. Use calculations to show which bottle is the better buy. (3 marks)

Solution
$19.70 \div 0.375 = \$52.53\,/\,\text{litre}$ or $\dfrac{1970}{375} = 5.25\ c\,/\,mL$
$32 \div 0.600 = \$53.33\,/\,\text{litre}$ $\dfrac{3200}{600} = 5.33\ c\,/\,mL$
therefore the 375 mL bottle is the better buy
Specific behaviours
✓ calculates the unit pricing for the 375 mL bottle or cost per millilitre, in cents, for 375 mL bottle
✓ calculates the unit pricing for the 600 mL bottle or cost per millilitre, in cents, for 600 mL bottle
✓ compares the unit pricings and states the best buy

(d) Sonya receives a 22% commission on all she sells.

(i) Determine the amount of money Sonya earns if she sells $520 worth of products. (2 marks)

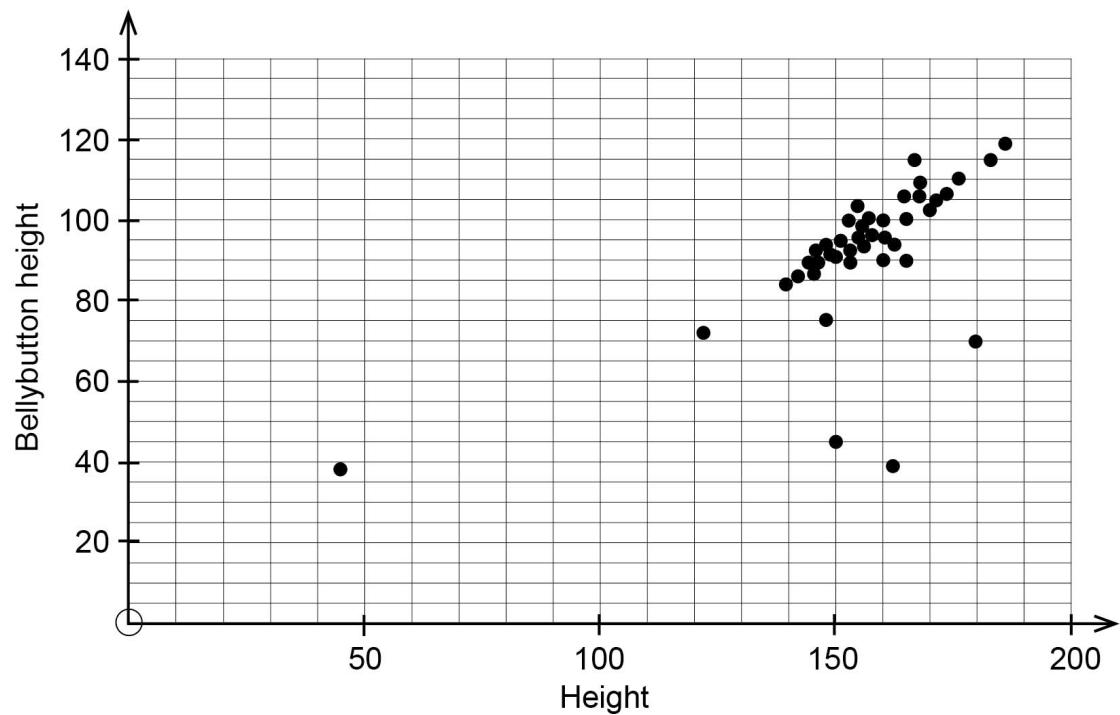| Solution |
|---|
| Commission = 22% of 520 |
| = $114.40 |
| **Specific behaviours** |
| ✓ identifies commission as the percentage of the sales |
| ✓ calculates the commission correctly |

(ii)　In one week Sonya earns \$92.40 commission. What was the value of her sales
in this week?　　　　　　　　　　　　　　　　　　　　　　　(2 marks)

Solution
Commission　= 22% of sales
92.40　= 22% × sales
Sales　= 92.40/0.22
= \$420
Alternative solution
22% = 92.40
$1\% = \dfrac{92.4}{22} = \$4.20$
$100\% = \dfrac{92.4}{22} \times 100$
= \$420
Specific behaviours
✓ substitutes correct information into equation to be solved
✓ correctly calculates sales
or
✓ correctly calculates 1%
✓ correctly calculates 100%

Question 9 **(8 marks)**

Fifty students were randomly chosen from those who completed the 2011 CensusAtSchool survey. Their heights and bellybutton heights are plotted below.

(a) Circle the **four (4)** outliers, then ignore them for the rest of the question. (2 marks)

Solution

Specific behaviours

✓ identifies at least two outliers

✓ identifies all four outliers

(b) What is the approximate difference between someone's height and the height of their bellybutton? (1 mark)

Solution
60 cm (55 cm – 65 cm)
Specific behaviours
✓ calculates the approximate difference

(c) Describe the trend in this data. (1 mark)

Solution
increasing
Specific behaviours
✓ identifies the trend

(d) Draw a trend line for this data. (1 mark)

Solution
Specific behaviours
✓ draws an appropriate trend line
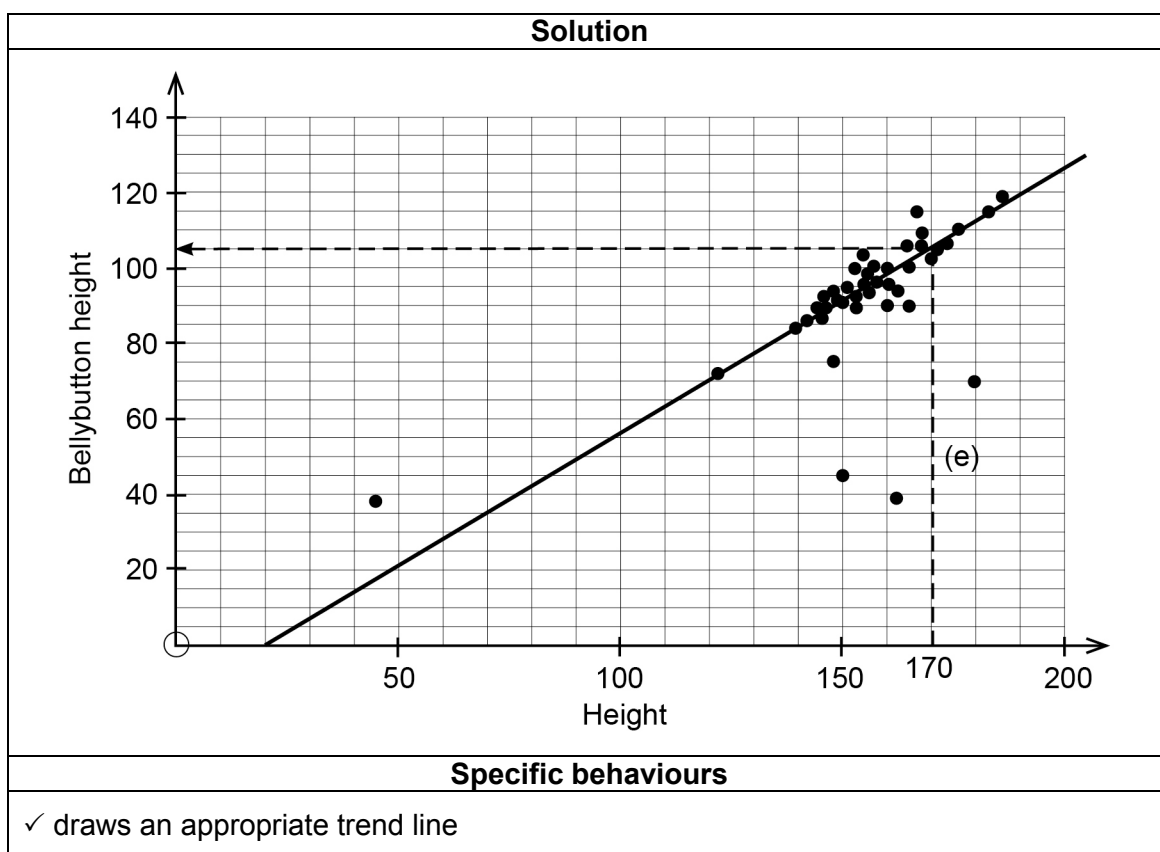
(e) Show use of your trend line to predict the bellybutton height for someone who is 170 cm tall and state your prediction. (2 marks)

Solution
105 cm
Specific behaviours
✓ shows use of trend line for height of 170 cm
✓ correctly states predicted value from their trend line

(f) Why would your prediction in Part (e) be considered reliable? (1 mark)

Solution
The prediction is reliable as it requires interpolation or as most dots are near the trend line.
Specific behaviours
✓ identifies prediction as interpolation or a similar valid explanation
or
✓ makes reference to the strength of the relationship

Question 10   **(7 marks)**

The table below shows travel times (in minutes) along lines of a rail network connecting stations at P, Q, R, S, T, U and V. The diagram (not drawn to scale) shows the positions of the stations.

	P	Q	R	S	T	U	V
P	-						
Q	32	-					
R	28	20	-				
S	23	-	-	-			
T	-	34	18	21	-		
U	-	-	-	28	16	-	
V	-	64	-	-	29	23	-
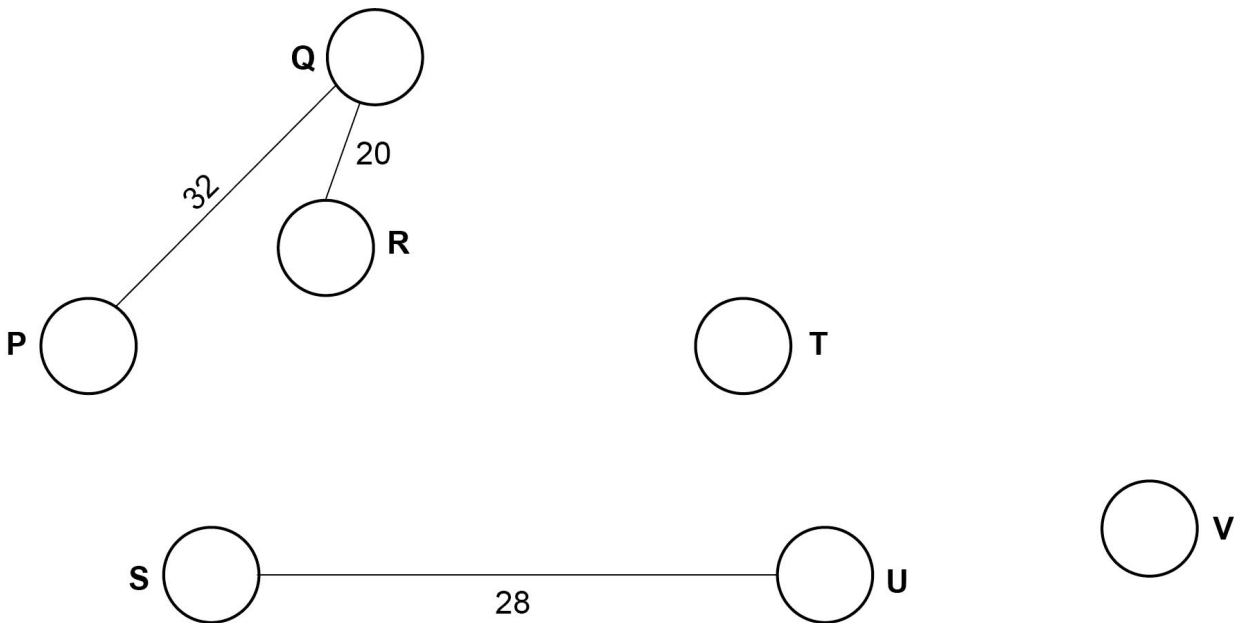
(a)   Complete the diagram by drawing all rail lines and labelling with the travel times.

(2 marks)

Solution

Specific behaviours

✓ correctly draws and labels at least four lines
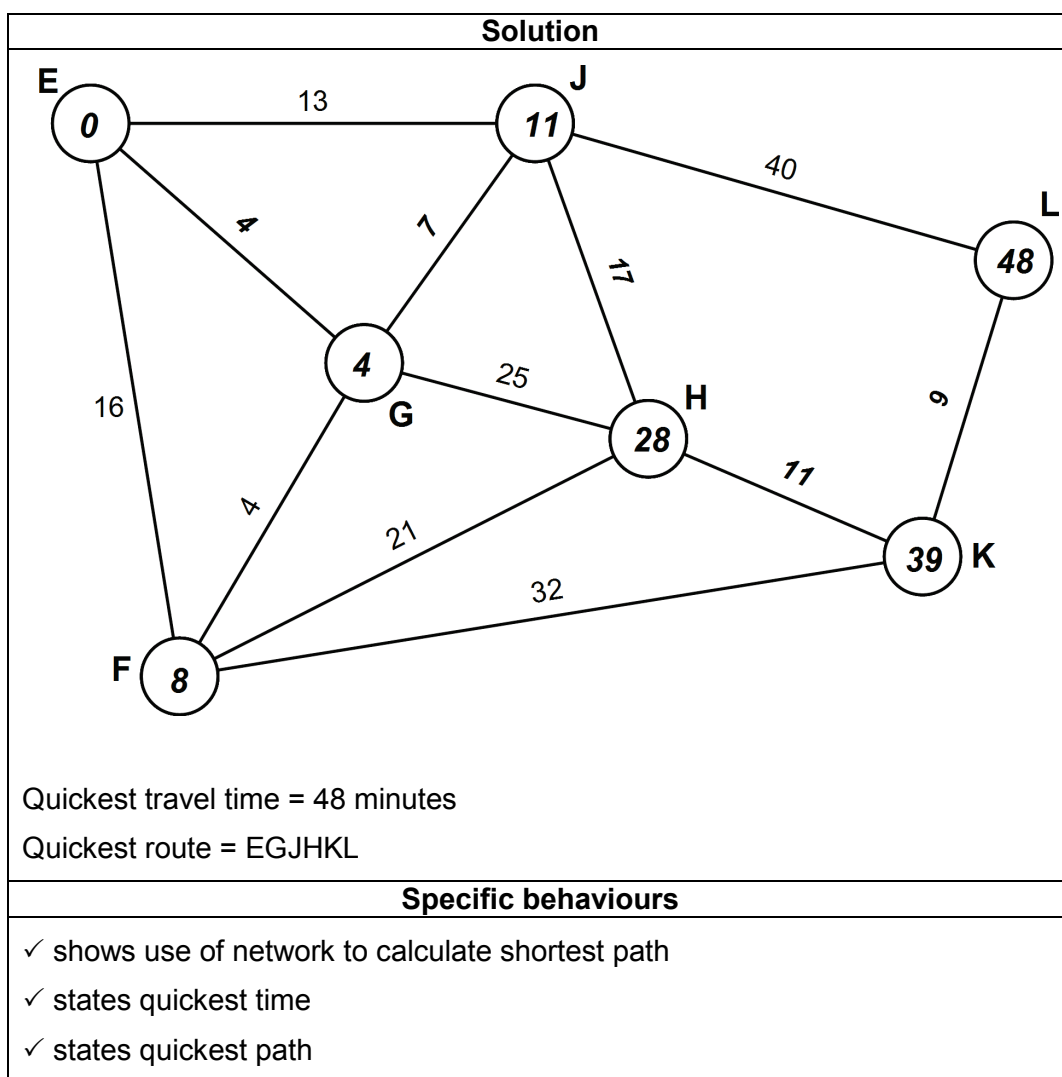
✓ correctly draws and labels all lines

(b)   A second rail network connecting stations E, F, G, H, J, K and L, with travel times between stations given in minutes, is shown below.
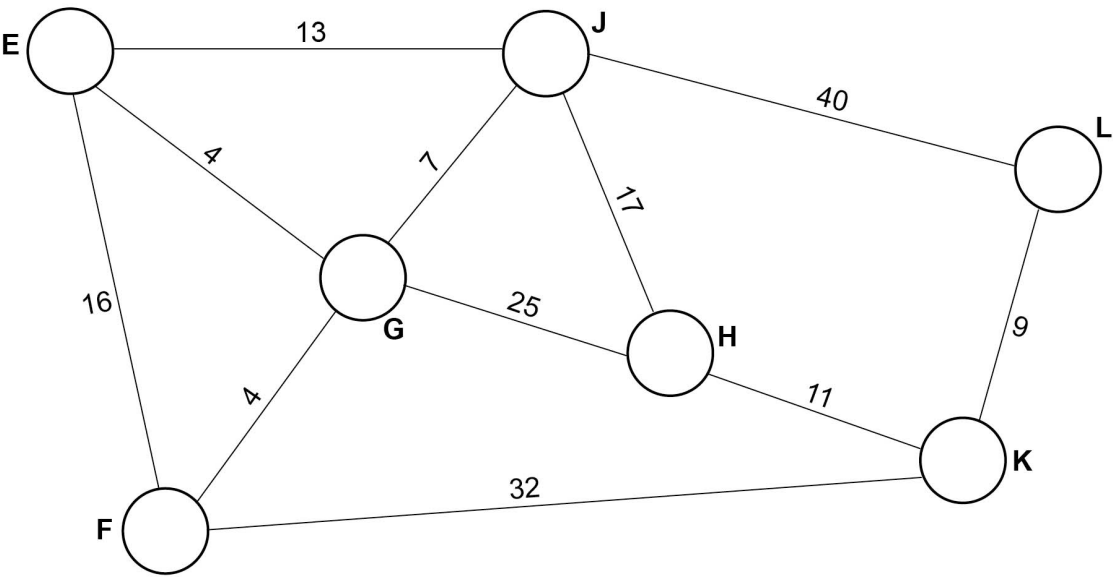
(i)   Explain, with reasons, whether a maintenance worker could travel all lines of this second network without going along any line more than once.   (2 marks)

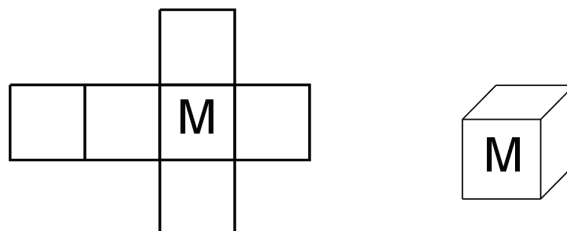| Solution |
|---|
| The network has four odd vertices (E, F, H and K) therefore the network is not traversable, hence the maintenance worker could not travel all lines of this network without going along any line more than once. |
| **Specific behaviours** |
| ✓ determines there are more than two odd vertices |
| ✓ concludes whether the network is traversable based on the number of odd vertices determined |

(ii)   What is the quickest travel time between stations E and L, and what route gives this quickest time?   **(3 marks)**

Solution

Quickest travel time = 48 minutes

Quickest route = EGJHKL

Specific behaviours

✓ shows use of network to calculate shortest path

✓ states quickest time

✓ states quickest path

Question 11 (10 marks)

A six-sided die has one side marked with the letter 'M'. The other five sides are each randomly labelled with one of the letters M, N and P. For example, if there was a one-half chance of rolling an 'M' then one possibility is the die was labelled M, M, M, N, N, P. The net of the original die with just the one permanent letter 'M' is drawn below.

(a) If the die is labelled with 3 Ms, 2 Ns and 1 P, what is the probability of rolling an N?

(1 mark)

Solution
$\dfrac{2}{6} = \dfrac{1}{3}$ (Accept either)
Specific behaviours
✓ correctly states the probability

(b) Label the net of the die to show there is a two-thirds chance of rolling an N. (1 mark)
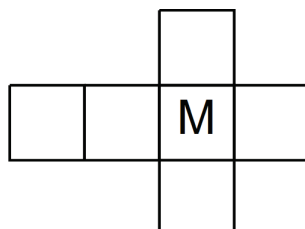
Solution
i.e. 4 Ns, 1 P and 1 M.
Specific behaviours
✓ correctly labels the net to include 4 Ns

(c)   Label the net of the die to show there is a one-third chance of rolling a P and a greater
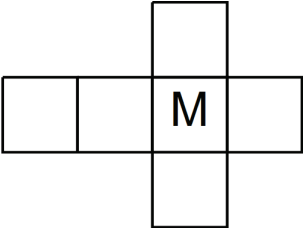   chance of rolling an N than an M. (2 marks)

	M	
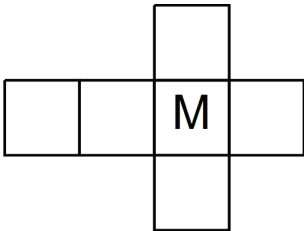
Solution
P N N M P N i.e. Any solution where there are 2 Ps and 3 Ns.
Specific behaviours
✓ labels two sides with Ps
✓ labels three sides with Ns

(d)   The die is rolled 12 times. The results are below.

Letter	Frequency
M	1
N	7
P	4

(i)   Based on the results in the table above, what would be the most likely labelling of the die?   (1 mark)

Solution
N N M P (with P above M and N below M)
Specific behaviours
✓ correctly labels the net with 1M, 2Ps and 3Ns

(ii)   Is it possible that the die was labelled M, M, M, M, N, P? Give a reason to support your answer.   (1 mark)
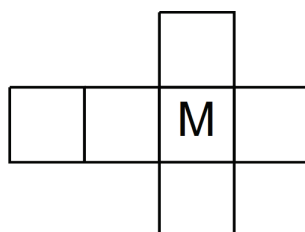
Solution
It is possible, as M, N and P appear on the die.
Specific behaviours
✓ gives a reason linked to the possibility

(e) The same die from Part (d) is rolled 1200 times, with the results below:

Letter	Frequency
M	405
N	600
P	195

What would be the most likely labelling of the die? (2 marks)

Solution
Specific behaviours
✓ correctly labels the net with 3 Ns
✓ correctly labels the net with 1 P

(f) In which labelling of the die, from Parts (d) and (e), would you be more confident? Why? (2 marks)

Solution
Answer from Part (e) as there has been many more trials.
Specific behaviours
✓ correctly states Part (e)
✓ gives valid reason

Question 12 **(10 marks)**

The Brown Taxi company charge a flag fall of $3 and a rate of $2 per kilometre travelled. They use the formula: $F = 3 + 2d$, where d is the distance travelled in kilometres, to calculate the fare charged F (in dollars). For example, a taxi ride of 5 km would cost $13.

(a) Matthew catches a Brown Taxi from Perth to home, a distance of 25 km. Calculate the fare Matthew was charged. (1 mark)

Solution
$F = 3 + 2(25)$ $\quad = \$53$
Specific behaviours
✓ correctly evaluates formula or correctly calculates fare

(b) Peta is charged $41 for catching a Brown Taxi home. What distance did she travel in the taxi? (2 marks)

Solution
$41 = 3 + 2d$ $\quad d = 19$ km
Specific behaviours
✓ correctly substitutes into formula
✓ correctly solves for distance

The Orange Taxi company charge a flag fall of $6 and $1.50 per kilometre travelled.

(c) Write a formula for the fare charged by the Orange Taxi company in terms of d (the distance travelled). (1 mark)

$F =$

Solution
$F = 6 + 1.5d$
Specific behaviours
✓ correctly states the formula

(d)   Madeline is deciding whether to catch a Brown Taxi or Orange Taxi to her home. She
       wants to choose the cheaper company. She needs to travel 10 km. Which company
       should she choose? Justify your answer with calculations.   (2 marks)

Solution
$\text{Orange} = 6 + 1.5(10)$ $\text{Brown} = 3 + 2(10)$ $\qquad\quad = \$21$ $\qquad\qquad = \$23$
Therefore choose Orange Taxi
Specific behaviours
✓ correctly calculates cost of travelling 10 km in both Orange and Brown Taxis
✓ states 'choose Orange Taxi'

(e)   Determine the distance travelled that will result in the fares for the Orange and Brown
       Taxis being equal.   (2 marks)

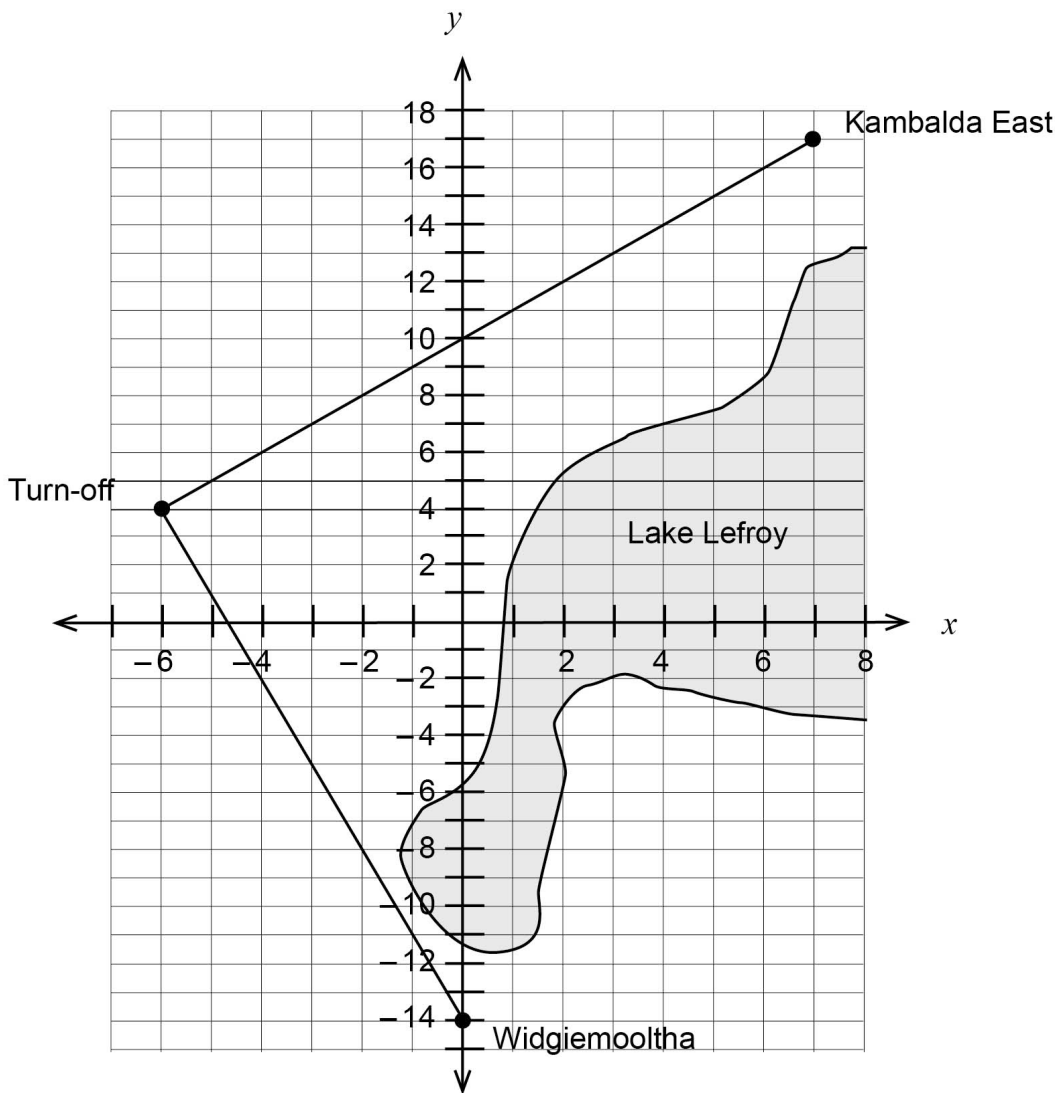| Solution |
|---|
| $3 + 2d = 6 + 1.5d$
$0.5d = 3$ or   Draw graphs and find point of intersection
$d = 6$ or   uses 'guess and check' |
| Therefore 6 km |
| **Specific behaviours** |
| ✓ sets up linear equation to solve or graphs the functions or correctly uses 'guess and check' |
| ✓ solves linear equation or reads point of intersection |

(f)   Emily wants to give her mother some advice about which taxi company to choose so
       that her fare is always the cheapest. What advice should Emily give her mother? (2 marks)

Solution
Brown Taxis are cheaper up to 6 km. At 6 km they are equal in price. More than 6 km and Orange Taxis are cheaper.
Specific behaviours
✓ states Brown Taxis are cheaper for less than 6 km
✓ states Orange Taxis are cheaper for more than 6 km

Question 13   **(5 marks)**
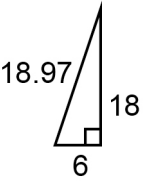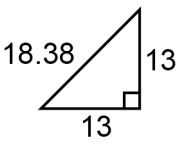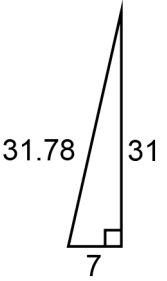
When Lake Lefroy is dry, Jaime drives home directly in a straight line from Widgiemooltha to Kambalda East. When Lake Lefroy has water in it, she has to drive via the turn-off shown on the map below. One unit on the map represents 1 km.

(a)   State the coordinates of the three points plotted, that is, Widgiemooltha, the turn-off and Kambalda East.   (1 mark)

Solution
Widgiemooltha (0, –14), turn-off(–6, 4), Kambalda East (7, 17)
Specific behaviours
✓ correctly states coordinates of all three points

(b) How much further does Jaime have to drive when Lake Lefroy has water in it compared
with when it is dry? (4 marks)

Solution
$WT = \sqrt{18^2 + 6^2}$ $= 18.97$ km $TK = \sqrt{13^2 + 13^2}$ $= 18.38$ km $WK = \sqrt{31^2 + 7^2}$ $= 31.78$ km $WT + TK = 37.35$ km   Therefore Jaime travels 5.57 km further
Specific behaviours
✓✓✓ correctly calculates the distance for each leg, *WT, TK* and *WK*
✓ correctly calculates the difference

Question 14 **(11 marks)**

A survey was conducted in two suburbs, Deakin and Curtin, regarding the number of devices in each household with internet access. Twenty-five households in each suburb were randomly selected.

Number of devices in household with internet access	Deakin frequency	Curtin frequency
0	3	1
1	3	2
2	4	2
3	3	17
4	4	2
5	3	0
6	2	0
7	3	0
8	0	1

(a) State **one (1)** way in which these data may have been collected. (1 mark)

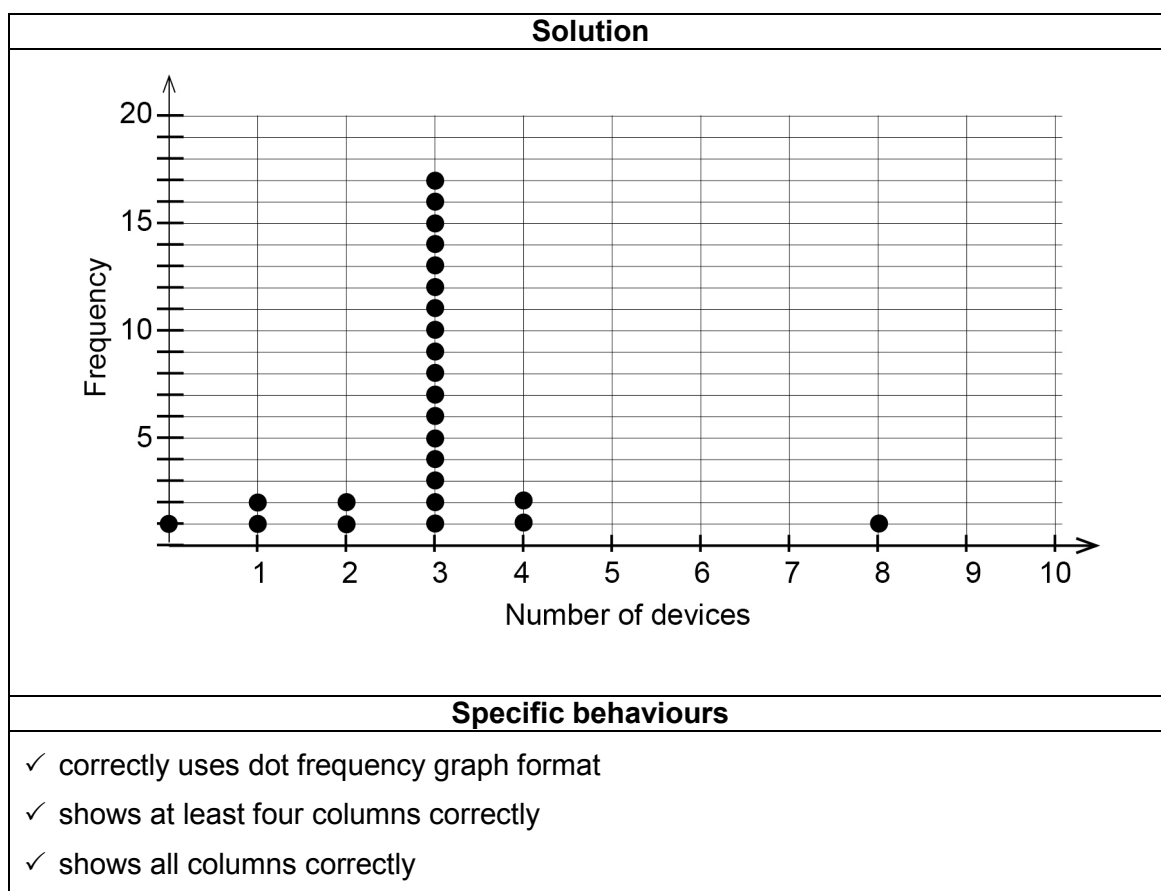| Solution |
|---|
| Households randomly selected from the two suburbs by: assigning a number for each and randomly selecting a number; randomly listing all household addresses and choosing every 50th etc. |
| **Specific behaviours** |
| ✓ correctly describes method that relates to the random nature the households were selected |

(b) Determine the mean, mode, median and range of the number of devices with internet access for the suburb of Curtin. (4 marks)
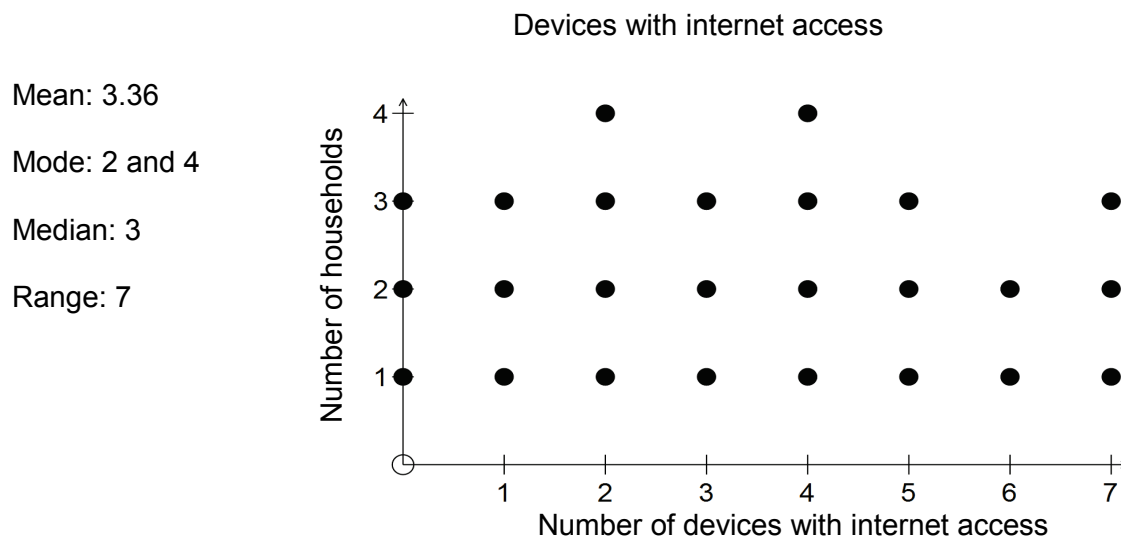
Solution
Mean = 2.92
Mode = 3
Median = 3
Range = 8
Specific behaviours
✓✓✓✓ correctly states each statistic

(c) Draw a dot frequency graph representing these data for the suburb of Curtin on the grid below. (3 marks)

Solution

Specific behaviours

✓ correctly uses dot frequency graph format

✓ shows at least four columns correctly

✓ shows all columns correctly

Some statistics for the suburb of Deakin are shown below. A dot frequency graph is also shown.

Devices with internet access

Mean: 3.36

Mode: 2 and 4

Median: 3

Range: 7

(d)   Compare the number of devices with internet access for the two suburbs by describing **two (2)** ways in which these data sets differ.   (2 marks)

Solution
The most noticeable difference is in the dot plots: The data from Deakin is uniformly distributed whereas the data from Curtin has the score 3 occurring 68% of the time. Their means are close but Deakin's is larger. Deakin has two modes whereas Curtin only has one mode. Curtin has a larger range.
Specific behaviours
✓✓ One mark each for stating any two of: The data from Deakin evenly distributed while the data from Curtin is not;   The data from Curtin has a larger range;   The data from Deakin has two modes whereas the data from Curtin only has one mode;   The data from Curtin has a smaller mean than the data from Deakin.

(e)   Write down **one (1)** advantage of using the median, rather than the mean, to describe the 'average' of a data set.   (1 mark)

Solution
The mean can be affected by outliers making it appear smaller or larger than expected. The median is less influenced by outliers   The median is simpler to find than the mean.
Specific behaviours
✓ states an advantage of using the median

Question 15 **(5 marks)**

Boolham High School has students enrolled in Years 8, 9, 10, 11 and 12. One-fifth of the students are in Year 8, 23% of the students are in Year 9, 11% of the students are in Year 10 and the probability that a randomly chosen student is in Year 11 is 0.26. Within each year group the number of boys is equal to the number of girls.

(a) What is the probability that a randomly chosen student at Boolham High School is in Year 12? (1 mark)

Solution
$1-(0.2+0.23+0.11+0.26)=0.2$
Specific behaviours
✓ correctly calculates the probability

(b) There are 1200 students enrolled at the school. How many of these students are enrolled in Year 11? (1 mark)

Solution
$0.26\times1200=312$
Specific behaviours
✓ correctly calculates the expected value

(c) One student is to be randomly chosen to represent the school at an Arbor Day ceremony. From which year group is the student most likely to be chosen? (1 mark)

Solution
Year 11
Specific behaviours
✓ correctly states the most likely year group

(d)   For the result of choosing the student required in Part (c), rank the following events in decreasing order from most likely to least likely:

A: the student is a girl

B: the student is a Year 12 boy

C: the student is not from Year 8

D: the student is not from Year 11

E: the student is from either Year 9 or Year 10.   (2 marks)

Solution
A: the student is a girl (0.5) B: the student is a Year 12 boy (0.1) C: the student is not from Year 8 (0.8) D: the student is not from Year 11 (0.74) E: the student is from either Year 9 or Year 10 (0.34) C, D, A, E, B
Specific behaviours
✓ correctly has C as first and B as last
✓ correctly orders all five events

Question 16 **(6 marks)**

Scientists were trying to increase the population of a rare species of fish, so they placed 50 fish in a small lake and monitored the population monthly. They discovered that the population of the fish, P, increased according to the rule:

$P = 50 \times 1.15^t$ where t was the time in months after the fish were placed in the lake.

(a) What was the population of the fish after 18 months? (1 mark)

Solution
$P = 50 \times 1.15^{18}$ $\quad = 618.8$ $\quad = 618$ fish   Accept 619 or 620, as the formula is modelling the real situation.
Specific behaviours
✓ correctly calculates the population

(b) By what percentage did the population of the fish increase each month? (1 mark)

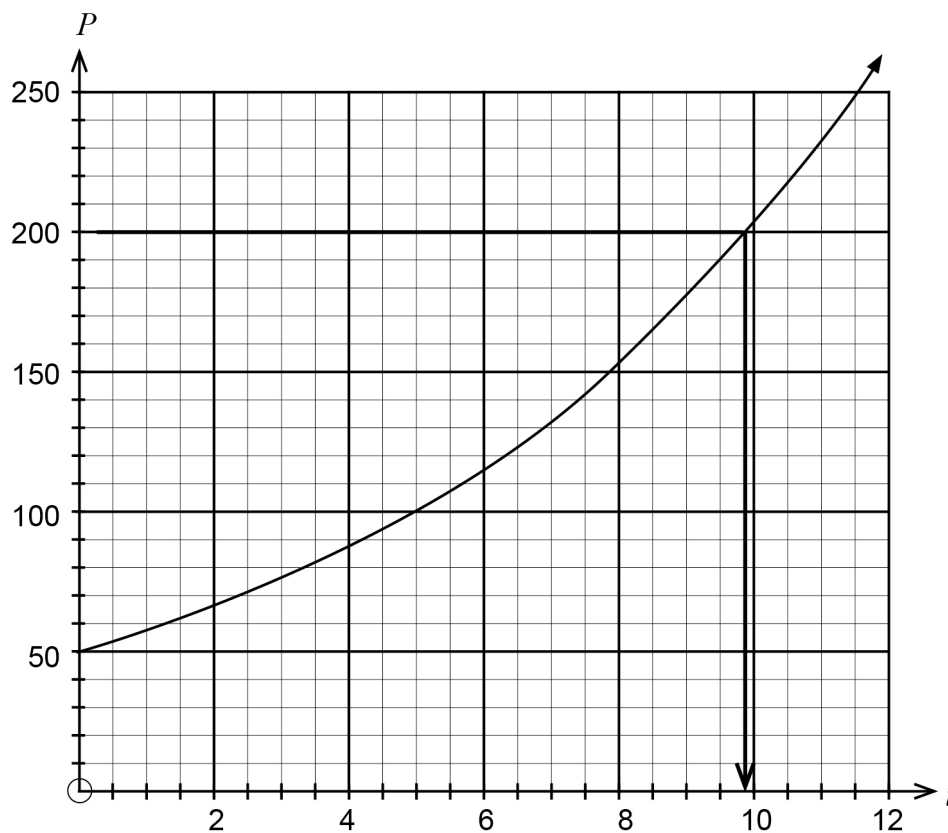| Solution |
|---|
| 15% |
| **Specific behaviours** |
| ✓ correctly states the rate of monthly change |

(c) Complete the recursive rule representing the growth in population of the fish.

$P_{n+1} =$ (2 marks)

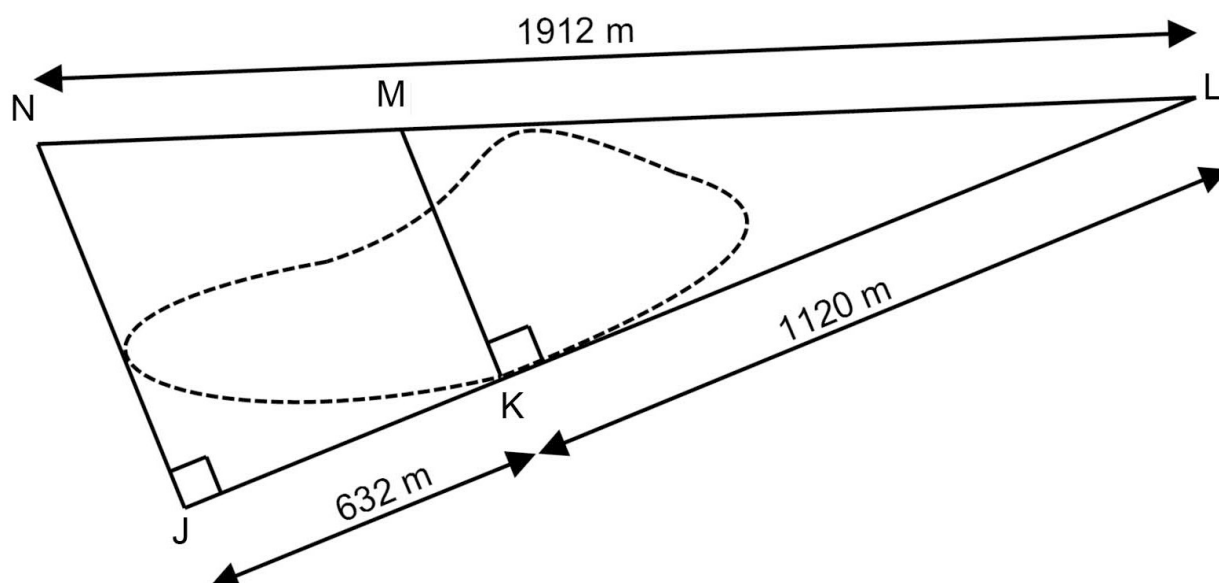| Solution |
|---|
| $P_{n+1} = 1.15 \times P_n, \qquad P_0 = 50$ |
| **Specific behaviours** |
| ✓ correctly states the initial population
✓ correctly states the recursive rule |

(d)   A graph of the fish population is shown below. Use the graph to determine during which month the population of the fish first exceeded 200. Indicate clearly your working on the graph and state your solution.   (2 marks)

Solution
First exceeds in 10th month
Specific behaviours
✓ uses graph to determine time when first exceeds 200
✓ correctly states the 10th month

Question 17   **(8 marks)**

The diagram above shows an area of forest bounded by roads JKL, JN and NML. It is known that JK = 632 m, KL = 1120 m, NL = 1912 m and that angles LJN and LKM are both 90°.

(a)   Determine the length of road JN.   (2 marks)

Solution
$\overline{JN}\,^2 = 1912^2 - (632 + 1120)^2$ $= 765.66$ m $= 766$ m
Specific behaviours
✓ correctly shows use of Pythagoras' Theorem   ✓ correctly states the length of the road

(b)   Use trigonometry to determine the size of angle JLN.   (3 marks)
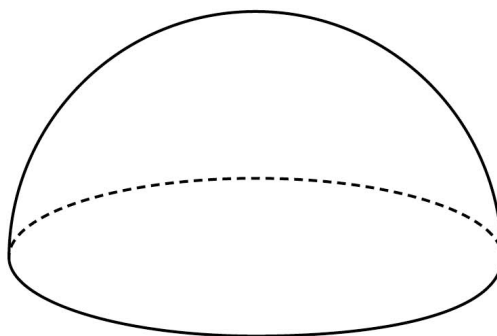
Solution
$\cos\theta = \left(\dfrac{JL}{LN}\right)$ $\sin\theta = \left(\dfrac{JN}{LN}\right)$ $\tan\theta = \left(\dfrac{JN}{JL}\right)$ $\cos\theta = \left(\dfrac{1752}{1912}\right)$ or $\sin\theta = \left(\dfrac{765.66}{1912}\right)$ or $\tan\theta = \left(\dfrac{765.66}{1752}\right)$ $\theta = 23.6°$ $\theta = 23.6°$ $\theta = 23.6°$
Specific behaviours
✓ states the trigonometric ratio equation correctly   ✓ substitutes values into the equation   ✓ correctly calculates the size of the angle

(c) Determine, to the nearest metre, the distance between K and M. (3 marks)

Solution
$\tan 23.6 = \left(\dfrac{KM}{1120}\right)$ $\qquad KM = 1120 \times \tan 23.6$ **or** $\dfrac{\overline{KM}}{1120} = \dfrac{765.66}{1752}$ $\qquad KM = 489.47$ $\overline{KM} = 489.47$ $\qquad\quad = 489$ m $= 489$ m
Specific behaviours
✓ states a correct trigonometric ratio to solve for KM or uses proportion
✓ correctly reorganises ratio to find KM
✓ correctly calculates the length of KM to the nearest metre

Question 18 **(7 marks)**

A solid metal paperweight is in the shape of a hemisphere (half of a sphere) with radius 3 cm.

(a) Determine the surface area of the paperweight. (4 marks)

Solution
Surface area $= 2\pi r^2 + \pi r^2$ $\qquad\qquad = 3 \times \pi \times 3^2$ $\qquad\qquad = 27\pi$ $\qquad\qquad = 84.8$ cm^2
Specific behaviours
✓ shows use of correct formula for the curved surface of the hemisphere
✓ includes area of circular base in surface area
✓ shows correct substitution into formula
✓ correctly evaluates the surface area

(b)   Another solid paperweight, in the shape of a cylinder with radius 3 cm, is made from 60 cm^3 of glass. What is the height of this paperweight?   (3 marks)

Solution
$\text{Volume} = \pi r^2 h$ $60 = \pi \times 3^2 \times h$ $h = \dfrac{60}{9\pi}$ $ = 2.12 \text{ cm}$
Specific behaviours
✓ shows use of correct formula   ✓ substitutes into formula correctly   ✓ correctly evaluates the required height

Question 19 **(8 marks)**

Consider the first four complete rows of the number pattern given below.

$$1^2 + 2^2 + 2^2 \quad = \quad 9 \quad = \quad 3^2 \qquad \text{Row 1}$$

$$2^2 + 3^2 + 6^2 \quad = \quad 49 \quad = \quad 7^2 \qquad \text{Row 2}$$

$$3^2 + 4^2 + 12^2 \quad = \quad 169 \quad = \quad 13^2 \qquad \text{Row 3}$$

$$4^2 + 5^2 + 20^2 \quad = \quad 441 \quad = \quad 21^2 \qquad \text{Row 4}$$

$$5^2 + 6^2 + 30^2 \quad = \quad \underline{\quad} \quad = \quad \underline{\quad} \qquad \text{Row 5}$$

$$\underline{\qquad\qquad} \quad = \quad \underline{\quad} \quad = \quad \underline{\quad} \qquad \text{Row 6}$$

(a)   Complete Row 5. (1 mark)

Solution
$5^2 + 6^2 + 30^2 \quad = \quad 961 \quad = \quad 31^2 \qquad$ Row 5
Specific behaviours
✓ correctly completes Row 5

(b)   Extend the pattern by completing Row 6. (2 marks)

Solution
$6^2 + 7^2 + 42^2 \quad = \quad 1849 \quad = \quad 43^2 \qquad$ Row 6
Specific behaviours
✓ correctly completes left hand expression in Row 6
✓ correctly completes middle term and right hand term in Row 6

(c) Lochie says he calculated Row 20 to have an answer in the middle column of 177 001. Tayla does not know the numbers used by Lochie in his calculation but she knows Lochie's answer is not a possible result. How might Tayla have used her calculator to work out that Lochie's answer was not possible? (1 mark)

Solution
Tayla uses the square root of the number to show it is not a perfect square
Specific behaviours
✓ correctly reasons Lochie's answer is not a perfect square

(d) The left column of the pattern above consists of three terms. For example in Row 1 the three terms are 1^2, 2^2 and 2^2. The bases for these three terms are 1, 2 and 2. Within each row a relationship exists between the bases of the first two terms and the base of the third term. What is this relationship? (2 marks)

Solution
Base 1 × Base 2 = Base 3
Specific behaviours
✓ provides an example to demonstrate relationship
✓ correctly generalises relationship

(e) Write Row 25 of the above number pattern. (2 marks)

Solution
$25^2 + 26^2 + 650^2$ = $423\,801$ = 651^2 Row 25
Specific behaviours
✓ correctly completes the left side of the equation
✓ correctly completes all of Row 25