MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2021

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Section One: Calculator-free 35% (52 Marks)

Question 1 (5 marks)

Hanai is a successful college basketball player. His coach has warned him that he will lose his scholarship if he scores 54% or below on a weekly assessment. On his first three weekly assessments he scored 84%, 81% and 78% respectively.

Assume Hanai's weekly assessments continue to follow this pattern.

(a) Deduce a rule for the nth term of this sequence. (2 marks)

Solution
$T_n = 84 + (n-1)(-3)$
$\quad = 87 - 3n$
Specific behaviours
✓ correctly identifies an arithmetic sequence
✓ correctly states the rule for the nth term

(b) Determine Hanai's score on his sixth weekly assessment. (1 mark)

Solution
$T_6 = 87 - 3(6)$
$\quad = 69$
Therefore, he gets 69% on his sixth assessment
Specific behaviours
✓ calculates the correct value

(c) Predict when Hanai will lose his scholarship. (2 marks)

Solution
$54 = 87 - 3n$
$3n = 33$
$n = 11$
Therefore, Hanai will lose his scholarship after the 11th weekly assessment
Specific behaviours
✓ substitutes 54 correctly
✓ identifies correct weekly assessment

Question 2 (10 marks)

A construction company uses five different machines and has five employees who operate those machines.

The adjacency matrix below shows each of the five employees (A, B, C, D, E) and the five machines they are trained to operate. These are the only machines they may use.

$$
\text{Machine}
\begin{array}{c}
\\
1 \\
2 \\
3 \\
4 \\
5
\end{array}
\begin{array}{ccccc}
A & B & C & D & E \\
1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0
\end{array}
$$

(a) Draw the adjacency matrix as a bipartite graph. The A1 connection has already been drawn on the graph. (2 marks)

Solution

Specific behaviours
✓ correctly allocates at least 6 connections
✓ correctly allocates all connections

Question 2 (continued)

(b)   Does the bipartite graph in part (a) represent

(i)   a planar graph? (1 mark)
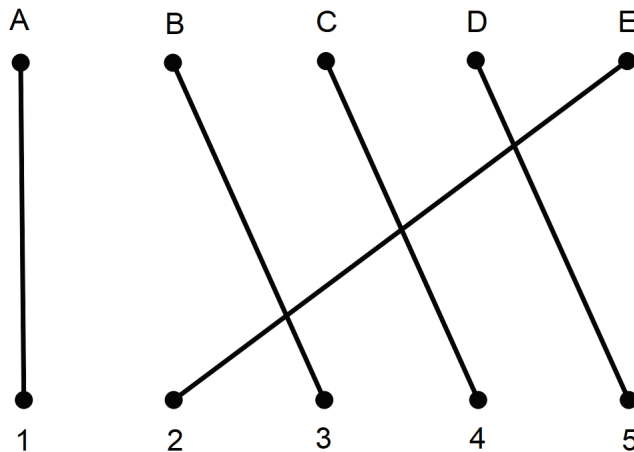
Solution
Yes
Specific behaviours
✓ correct answer

(ii)   a connected graph? (1 mark)

Solution
Yes
Specific behaviours
✓ correct answer

(c)   Explain why the bipartite graph in part (a) is a simple graph. (2 marks)

Solution
A simple graph has no loops or multiple edges.
Specific behaviours
✓ correctly states there are no loops
✓ correctly states there are no multiple edges

(d)   Complete the table below to show how an allocation of all employees to a machine could be achieved. (2 marks)

Employee	A	B	C	D	E
Machine	1	3	4	5	2

Solution
See table above (or graph)
Specific behaviours
✓ states at least 3 correct allocations
✓ all correct allocations

(e) The company foreman decides to allocate Employee E to Machine 4. Explain what effect this will have to the allocation of the remaining employees to the machines.
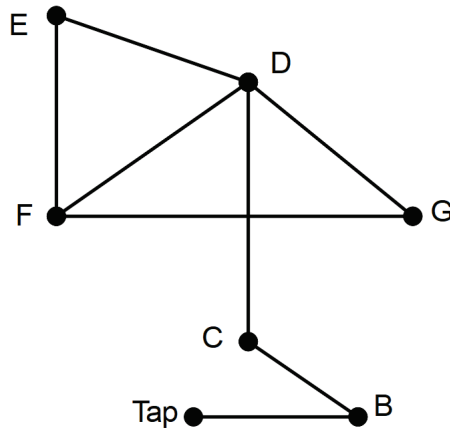
(2 marks)

Solution
$E4 \Rightarrow C5 \Rightarrow D1 \Rightarrow A3 \Rightarrow B$ cannot connect to Machine 2
or
$E4 \Rightarrow C5 \Rightarrow D1 \Rightarrow B3 \Rightarrow A$ cannot connect to Machine 2
or
$E4 \Rightarrow C5 \Rightarrow A1 \Rightarrow B3 \Rightarrow D$ cannot connect to Machine 2
or
$E4 \Rightarrow D5 \Rightarrow A1 \Rightarrow B3 \Rightarrow C$ cannot connect to Machine 2
Specific behaviours
✓ correctly states that one employee will always not be allocated
✓ correctly states it is not possible to allocate Machine 2 to an employee

Question 3 **(6 marks)**

The graph below shows the current network of reticulation pipes in Tarik's garden.

(a) Using Euler's formula, stating the number of vertices, edges and faces, show that the graph is planar. (2 marks)

Solution
$f = 3$, $v = 7$ and $e = 8$
Using Euler's formula $v + f - e = 2$
L.H.S = $7 + 3 - 8 = 2$
Therefore, the graph is planar.
Specific behaviours
✓ correctly states the number of vertices and faces
✓ correctly states the number of edges and verifies Euler's formula

The water needs to travel from the tap and through all pipes.

(b) List a possible route for the water. (1 mark)

Solution
Possibilities:
Tap-B-C-D-G-F-D-E-F
Tap-B-C-D-G-F-E-D-F
Tap-B-C-D-F-E-D-G-F
Tap-B-C-D-E-F-D-G-F
Tap-B-C-D-F-G-D-E-F
Tap-B-C-D-E-F-G-D-F
Specific behaviours
✓ correctly states a possible route

(c) What is the mathematical term for the route listed in part (b)? (1 mark)

Solution
semi-Eulerian trail
Specific behaviours
✓ states correct term

(d) Tarik would like to increase the water pressure by removing one edge (pipe).

(i) Identify any edge that **cannot** be removed. (1 mark)

Solution
He cannot remove:
Tap-B **or**
B-C **or**
C-D
Specific behaviours
✓ states a correct edge

(ii) What is the name given to the type of edge identified in part (d)(i)? (1 mark)

Solution
A bridge
Specific behaviours
✓ states correct name

Question 4 **(9 marks)**

A public opinion survey was conducted on the statement 'our overwhelming dependence on computers is a good thing', with partial results being shown in the table below.

Opinion

		Agree	Disagree	Undecided	Total
Age	**20–39 years**	40	28	**12**	80
	40–59 years	38	**42**	20	100
	60–79 years	20	**12**	18	**50**
	Total	**98**	**82**	**50**	230

(a) Complete the table above. (3 marks)

Solution
See table above
Specific behaviours
✓ determines at least 3 correct entries
✓ determines at least 5 correct entries
✓ determines correctly all entries

(b) Identify the response variable. (1 mark)

Solution
Opinion
Specific behaviours
✓ correct answer

(c) Use the template below to construct a percentaged two-way frequency table showing **either** column or row percentages as appropriate, to investigate if there is an association between age and opinion. (4 marks)

Opinion

		Agree	Disagree	Undecided	Total
Age	**20–39 years**	**50**	**35**	**15**	**100**
	40–59 years	**38**	**42**	**20**	**100**
	60–79 years	**40**	**24**	**36**	**100**

| Solution |
|---|
| See table above |
| **Specific behaviours** |
| ✓ determines row percentages are required |
| ✓ determines at least 3 correct entries |
| ✓ determines at least 6 correct entries |
| ✓ determines correctly all entries |

(d)   State an association that can be observed from the percentaged two-way frequency
        table. (1 mark)

Solution
As age increases, the percentage undecided increases.
Specific behaviours
✓ gives correct association

Question 5   **(8 marks)**

The network below shows the relative distances, in hundreds of metres, between wi-fi hotspots around a university campus.
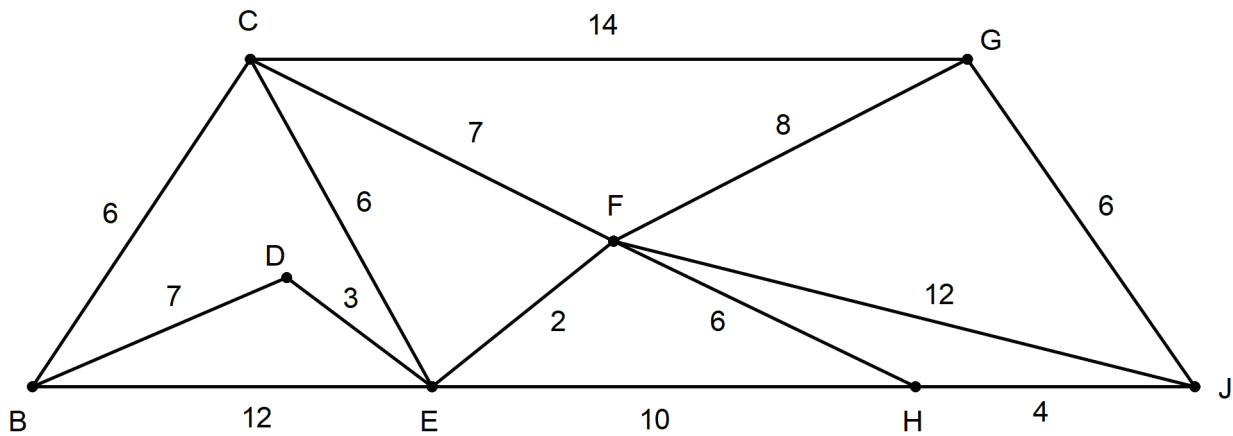
The control room for this system of hotspots is located at B. A problem has been reported with the hotspot at J.

(a)   A technician is sent from the control room to solve the problem at J. To get there as quickly as possible, she wants to use the shortest path, travelling from hotspot to hotspot. Determine the required path and its length.   (3 marks)

Solution
BDEFHJ $\Rightarrow$ 22
$\therefore$ 2200 metres
Specific behaviours
✓ identifies correct path
✓ path = 22
✓ path length 2200 metres

(b)   A second technician is sent from the control room to J to help resolve the problem. The technician uses an open path travelling from hotspot to hotspot. If the technician travels 2700 metres, determine the path he used.   (1 mark)

Solution
BCFGJ
Specific behaviours
✓ identifies correct path

(c)   After repairs have been made, all hotspots need to be checked. A technician is sent from the control room, travelling to all hotspots once only, and finishing back at the control room.

(i)   State the name given to this type of path.   (2 marks)

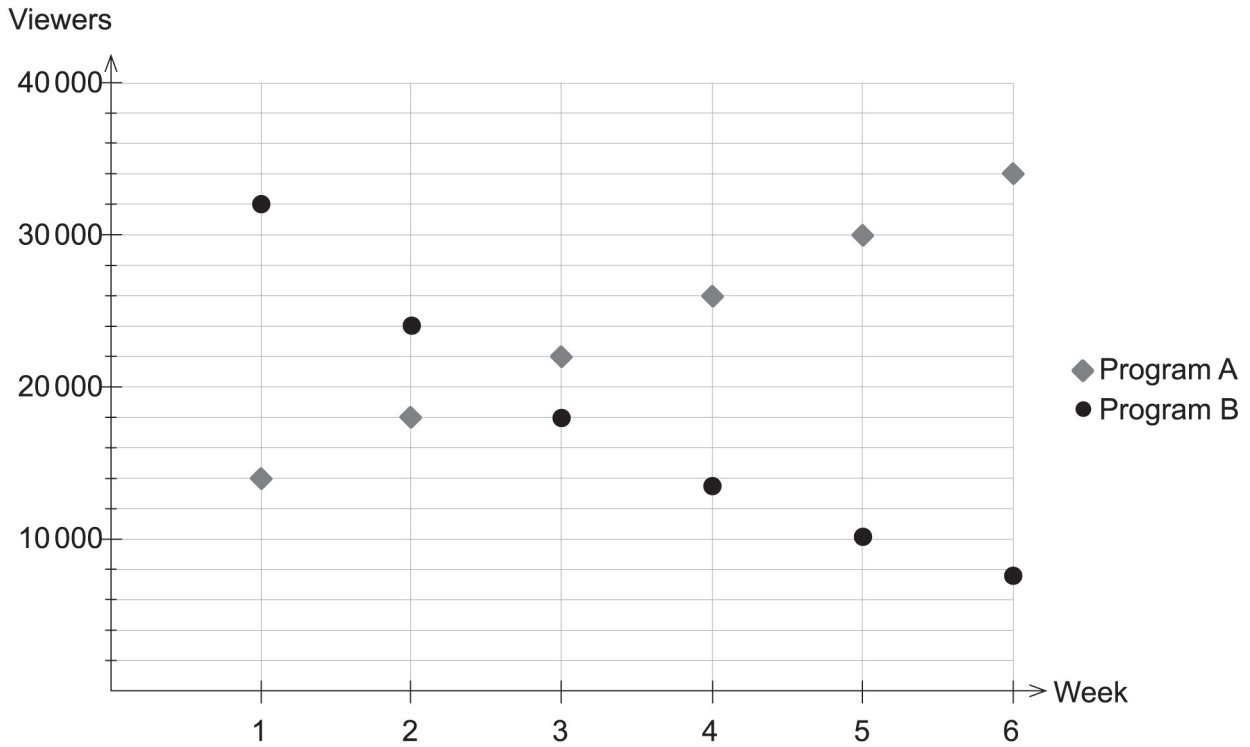| Solution |
|---|
| Hamiltonian cycle |
| **Specific behaviours** |
| ✓ correctly states it is a cycle |
| ✓ correctly states it is Hamiltonian |

(ii)   Determine the length of the shortest path possible and state the path used.

(2 marks)

Solution
BCGJHFEDB $\Rightarrow$ 4800 metres
Specific behaviours
✓ correctly gives the shortest closed path ✓ states correct length

Question 6 **(7 marks)**

A television network programmer was analysing the number of viewers for two children's programs over a period of several weeks, to decide which program should be given the better time slot. The viewing numbers, displayed on the graph below, formed an arithmetic sequence and a geometric sequence.

(a) Write a recursive rule for the arithmetic sequence. (2 marks)

Solution
$T_{n+1} = T_n + 4000, T_1 = 14\ 000$
Specific behaviours
✓ states a correct recursive rule
✓ states correct first term

(b) Using the first two data points, deduce a rule for the nth term of the geometric sequence.

(2 marks)

Solution
Using 32 000 and 24 000
Ratio = $\dfrac{24\ 000}{32\ 000} = \dfrac{3}{4} = 0.75$
Therefore $T_n = 32\ 000\left(\dfrac{3}{4}\right)^{n-1} = 32\ 000(0.75)^{n-1}$
Specific behaviours
✓ determines correct ratio
✓ states correct n^{th} term

(c) Explain which program should be given the better time slot. (2 marks)

Solution
Program A because viewer numbers are increasing
Specific behaviours
✓ states correct program
✓ explains why Program A should be given the better time slot

(d) Determine the number of viewers for the more successful program in Week 8. (1 mark)

Solution
$T_8 = 34\ 000 + 8\ 000 = 42\ 000$
Therefore, the number of viewers is 42 000
Specific behaviours
✓ correctly states number of viewers

Question 7 (7 marks)

The ages in years, and salaries in thousands of dollars ($'000), of eight employees at a company are shown below. The equation of the least-squares line for these data is $y = 0.2x + 38$.

x	35	37	41	43	45	47	53	55
y	42	44	47	50	52	51	49	45

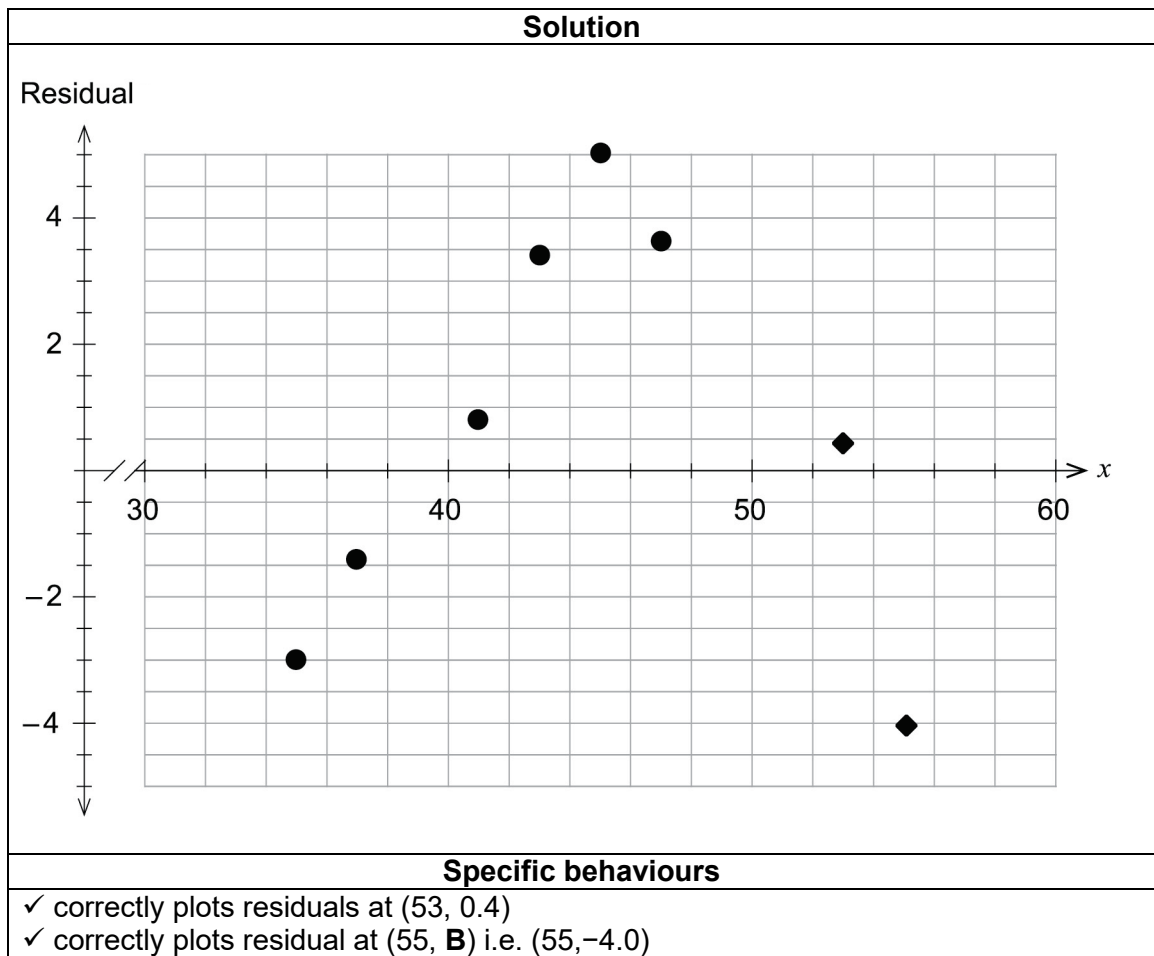The table below shows the predicted y-values, obtained from the equation of the least-squares line, and the corresponding residuals.

x	y	Predicted y-value	Residual
35	42	49.0	−3.0
37	44	45.4	−1.4
41	47	46.2	0.8
43	50	46.6	3.4
45	52	47.0	5.0
47	51	47.4	3.6
53	49	48.6	0.4
55	45	**A**	**B**

(a) Determine the value of **A** and **B**. (2 marks)

Solution
$y = 0.2(55) + 38$
$\quad = 11 + 38$
$\quad = 49$
Therefore, **A** = 49
B = 45 − 49 = −4.0
Specific behaviours
✓ correctly calculates the value of **A**
✓ correctly calculates the value of **B**

(b)   Plot the last two residuals on the graph below.   (2 marks)

Solution

Residual

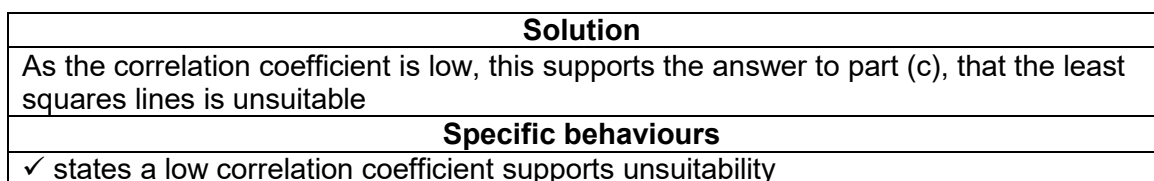| **Specific behaviours** |
| --- |
| ✓ correctly plots residuals at (53, 0.4) |
| ✓ correctly plots residual at (55, **B**) i.e. (55,−4.0) |

(c)   Justify, using the residual plot in part (b), whether the least-squares line is a good model for these data.   (2 marks)

Solution
The residual plot shows a pattern, therefore, this suggests that the least-squares line is unsuitable
Specific behaviours
✓ correctly states the model is unsuitable
✓ correctly states the residual plot shows a pattern
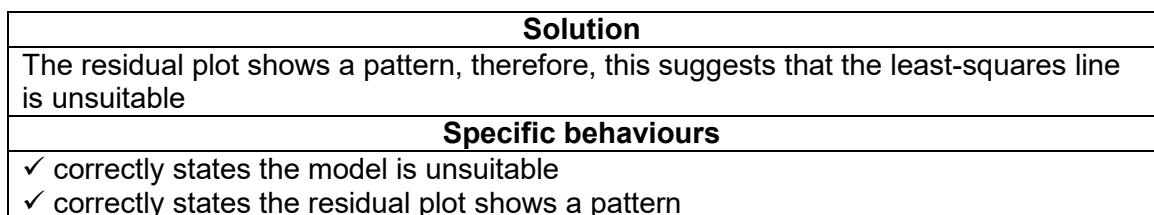
The calculated correlation coefficient for these data is 0.42.

(d)   Describe how this supports your response in part (c).   (1 mark)

Solution
As the correlation coefficient is low, this supports the answer to part (c), that the least squares lines is unsuitable
Specific behaviours
✓ states a low correlation coefficient supports unsuitability