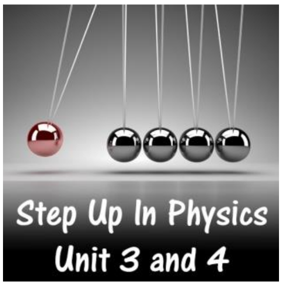
Relativistic Energy and Momentum

Problems Worksheet



1. A 1.00 kN force is applied to the back of a 1.00 tonne car starting at rest, causing it to accelerate at 1.00 ms^{-2}. After 1.00 s the car is moving at 1.00 ms^{-1} and after 10.0 s the car is moving at 10.0 ms^{-1}. Explain why the car will never reach the speed of light even if this force can be applied for 3.00×10^8 s.

2. A proton is accelerated in a linear accelerator up to a speed of 0.999c.
   a. What is the energy of the proton in the rest frame of the proton?

   b. What is the energy of the proton in the rest frame of the linear accelerator?

   c. What is the kinetic energy of the proton measured in the rest frame of the linear accelerator?

3. A double quarter pounder from McDonalds has a total mass of 305 g. According to the nutritional information, a single burger supplies 3570 kJ of energy.
    a. Calculate the rest energy of the burger.

    b. Compare the rest energy with the energy quoted on the nutritional information. Give a justification for the discrepancy between the two values.

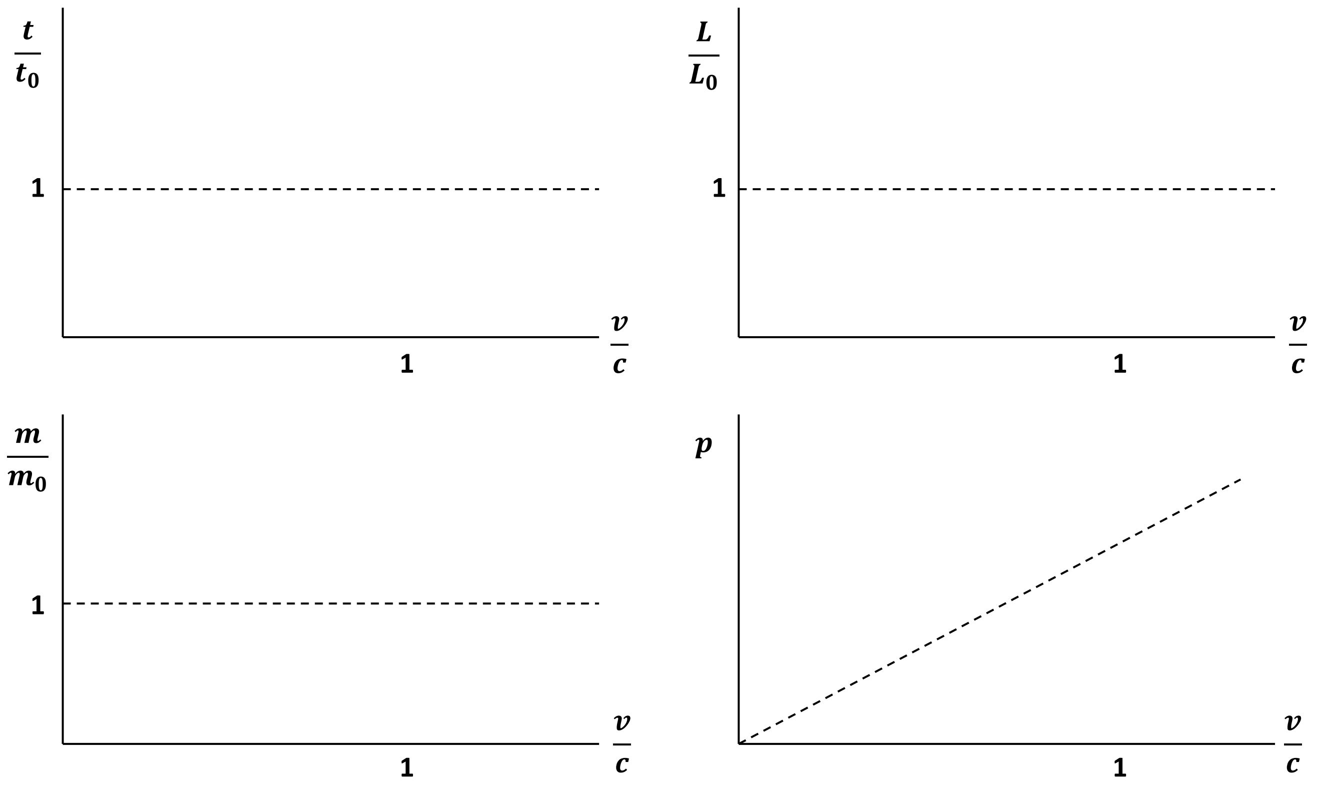
4. A single blueberry's rest mass is, on average, 1.50 g. There is a single blueberry on Tom's spacecraft as it moves at 1.80×10^{8} ms^{-1} past the Earth.
    a. Using Newtonian physics, calculate the classical momentum of this blueberry as observed by Earth.

    b. Using special relativity, calculate the relativistic momentum of the blueberry as observed by Earth.

    c. Using special relativity, calculate the relativistic momentum of the blueberry as observed by Tom on the spacecraft.

d. The blueberry is weighed on Earth and the measurement confirms the blueberry has a 1.50 g mass. Describe why the classical momentum does not agree with the relativistic momentum even though the mass and velocity have now been measured.

5. The dashed lines on the graphs below show the classical trends of time, length, mass and momentum as a function of velocity. On each graph, make a sketch of the trends of these variables as predicted by special relativity. The vertical axes are not well scaled so only an approximate curve is required.



6. Einstein's theory of special relativity led to a connection between mass and energy that revolutionised the understanding of matter within nuclear reactions.
   a. Define the concept of a rest mass.

b. Relativistic energy is the sum of the rest energy of an object and its kinetic energy. Show that the kinetic energy of a massive object is given by $E_K = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) m_0 c^2$.

7. Calculate the velocity required for an object to have 2.50 times more energy compared to when it is at rest.

8. Uranium-235 is an important isotope used in nuclear reactors. A single uranium-235 nucleus has a 3.90 × 10^{-25} kg mass. One of the possible outcomes of neutron induced fission of uranium-235 in nuclear reactors can produce bromine, lanthanum and three neutrons. This fission reaction results in a loss of 2.12 × 10^{-28} kg for each uranium that splits into the daughter nuclei. The heat produced by this fission reaction is used to turn water into steam and power a turbine.
   a. What is the source of the heat energy produced during a fission reaction?

   b. Calculate the heat energy produced by the fission of 8.64 × 10^{32} uranium-256 nuclei into barium and lanthanum. You may consider the energy to be measured in the rest frame of the reaction.

9. Synchrotrons use magnetic fields to keep very high speed electrons (up to 0.999c) revolving in a circle. The magnetic force acting on the electrons supplies the centripetal force. The magnetic flux density required to keep electrons moving at a speed v within a radius r is given by the following formula:

$$F_C = F_M$$
$$\frac{mv^2}{r} = qvB$$
$$\therefore B = \frac{mv}{rq}$$

At low speeds, the synchrotron is capable of keeping electrons within the circular path. As the speed increases researchers find that the magnetic flux density needs to be many times greater than that predicted by the formula above.
   a. Explain why the magnetic flux density needs to be increased at a greater rate than that predicted by the formula above as v increases.

b. Derive a formula that will accurately predict the required magnetic flux density as a function of r and v.

c. How much stronger is the required magnetic flux density compared to that predicted by the formula above for the fastest electrons in the synchrotron?

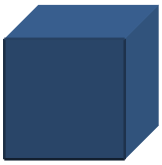
10. Pair production is the creation of matter from energy. A photon, moving near an atomic nucleus, is converted into an electron-positron pair. A positron has the same rest mass as an electron but an opposite charge.
    a. Calculate the rest energy of an electron.

    b. Show that the minimum frequency of a photon capable of electron-positron pair production is 2.47×10^{20} Hz.

c. Where does the excess energy of the photon end up when the frequency of the photon exceeds the value given in part (b)?

d. Both the electron and positron move off with the same speed (high fraction of the speed of light). Calculate this speed when the pair production occurs from a 2.89×10^{20} Hz photon.

11. The density of a cube can be calculated from knowledge of its mass and volume using $\rho = \frac{m}{V}$. When at rest, a cube has a 1.00 m^3 volume and a 12.0 kg mass. Calculate the density of the cube as observed by an observer moving at 0.60 c relative to the cube, parallel with one of its edges.



12. Ant Man and Yellow Jacket have the same 90.0 kg rest mass. The Wasp is flying at 0.85c relative to the ground as she watches the two fight. The Wasp observes Ant Man as stationary while Yellow Jacket jumps towards him at 0.56c.

    a. Calculate the momentum of Ant Man as observed from the ground.

    b. Calculate the momentum of Yellow Jacket as observed from the ground. The ground based observer sees Ant Man and Yellow Jacket moving towards each other.

13. How much energy, in terms of its rest energy, must a particle have if its momentum as defined by special relativity is three times larger than what its momentum would be if calculated using the classical view of momentum?