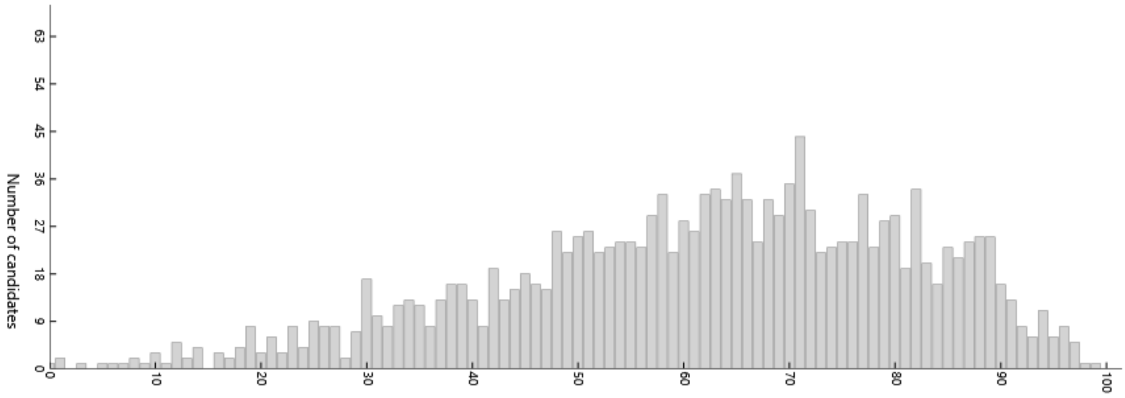
Government of **Western Australia**
School Curriculum and Standards Authority

2018 ATAR course examination report:
Mathematics Specialist

Year	Number who sat	Number of absentees
2018	1546	21
2017	1463	12
2016	1427	17

Examination score distribution–Written



Summary

The examination consisted of Section One: Calculator-free and Section Two: Calculator-assumed.

Attempted by 1546 candidates	Mean 61.73%	Max 99.29%	Min 0.00%

Section means were:			
Section One: Calculator-free	Mean 63.68%		
	Mean 22.29(/35)	Max 35.00	Min 0.00
Section Two: Calculator-assumed	Mean 60.78%		
	Mean 39.51(/65)	Max 64.29	Min 0.00

General comments

The paper was well received, with a common theme in the feedback being that the paper was of an appropriate standard for the Mathematics Specialist cohort. The paper contained a range of questions allowing the typical candidate to show facility with key standard concepts, yet containing elements in each question to discriminate amongst candidates.

The length of the paper was deemed to be appropriate, as evidenced by 96% of candidates attempting Question 20 part (a), as compared to 92% attempting Question 19 part (a).

As with 2017, there was no single question that was highlighted as the ‘challenge’ question, but there were many questions requiring conceptual understanding beyond the standard text type questions.

The distribution of marks in 2018 exhibited a relatively high spread, indicated by the standard deviation of 19.46% compared to the 2017 figure of 17.40% (despite the comparable means of 61.73% and 63.95%). This year there appeared to be a significant number of candidates who were not able to answer many straightforward questions or offer any response to questions.

Many markers reported that it was very pleasing to see a good number of able candidates showing elegant and original solutions to some of the more conceptually demanding questions. This was evident particularly for Question 3 part (c), where a couple of candidates displayed a solution purely with the use of trigonometric identities. Question 11 part (b), Question 17 part (c), Question 19 part (c) and Question 20 part (c) were some notable others where different mathematics ideas were displayed to that in the solutions and marking key. In these cases, the correct use of alternative mathematics was amply rewarded.

However, a striking feature was that many candidates (even those scoring in the 70-80% range), demonstrated a disturbing lack of facility expected of Mathematics Specialist candidates in the performance of algebraic processes and the appropriate use of brackets. This was particularly noticeable in the following questions:

- Question 2 part (a) – solving simultaneous linear equations by algebraic means
- Question 3 part (a) – determining the real part of a complex number
- Question 9 part (a) – simplifying an expression requiring the squaring of a binomial term
- Question10 – showing a result that required the use of brackets
- Question 20 parts (a) and (b) – differentiating/evaluating an expression that required the appropriate use of brackets.

For Question 9(a), the ability to expand an expression such as $\left(\sqrt{3}\tan u+1\right)^2$ proved problematic. At least 30.5% of the cohort did not perform this task correctly. Given that 30.5% of the cohort scored zero out of two for this task, it is estimated that this actual number could have been as high as 38%, since 20.8% scored one mark out of two. Those who scored one mark often failed to expand correctly but then scored the mark by correctly applying the identity $\sec^2 u = 1+\tan^2 u$ to an incorrect expansion. In viewing and reconciling scripts, it is estimated that approximately a third of those who scored one mark out of two could not expand correctly.

Advice for candidates

- Write legibly so that markers can discern the digits and ideas in a solution.
- Show the relevant mathematics that was used if a CAS solution is employed, e.g. indicate the mathematical equation that is being solved and refrain from writing calculator-speak type statements.
- In questions requiring explanation or reasoning, refrain from using the word 'it' as usually it is not clear what concept is being referred to, e.g. 'it' is normally distributed. Be specific.
- Understand the correct use of brackets.
- Use mathematics notation correctly.
- Provide working for all questions worth two or more marks.

Advice for teachers

- Continue to provide opportunities for students to explain aspects of the course.
- Stress the importance of using brackets appropriately to assist students in simplifying expressions.
- Provide opportunities to display conceptual knowledge of concepts often required outside of typical text type questions.
- Improve the mathematics notation used by candidates, specifically in the vectors section of the course. In the 2018 paper, there were no marks allocated for the correct use of notation.

Comments on specific sections and questions

Section One: Calculator-free (35 Marks)

In the calculator-free section the following areas were handled well:

- domain for the function composition $g\left(f\left(x\right)\right)$ in Question 1 (b)
- simultaneous solution of linear equations in Question 2 (a)
- using DeMoivre's theorem in Question 3 (b) for complex numbers
- identifying functions constants from the graph of a rational function in Question 4
- evaluation of the definite integral using a trigonometric substitution in Question 5.

Candidates performed relatively poorly in the following areas:

- explaining whether a system of equations will have a unique solution or not and stating the geometric significance in Question 2 (b)
- using the vector interpretation for a complex number in Question 3 (b) (ii)
- solving a relatively simple complex equation in Question 7 (a)
- explaining how $\underset{\sim}{c}.\left(\underset{\sim}{a}\times\underset{\sim}{b}\right)$ will give the volume of a parallelopiped in Question 8 (b)
- algebraic expansion of a binomial involving a trigonometric expression in Question 9 (a).

Question 1 attempted by 1543 candidates Mean 3.45(/5) Max 5 Min 0
Part (a) was a straightforward question on function composition and candidates obtained a good start to the paper. Part (b) was done well generally, with most candidates obtaining the requirement $x \geq 3$ but some unfortunately writing $x > 3$ not realising that $\sqrt{0}$ is mathematically acceptable. The additional condition that the overall denominator cannot be zero was omitted by many candidates, hence not obtaining the restriction $x \neq 7$.
In part (c), candidates generally stated that the statement was not true but were not able to offer a coherent response as to why it was incorrect. Many mistakenly stated that $x = -1$ was not part of the domain of f, which while being true, was not relevant. Some spoke about domains and ranges without specifying which function $\left(f \text{ or } f^{-1}\right)$ was being mentioned. This question exposed a gap in understanding in the relationship between the range of a function and the domain of its inverse.

Question 2 attempted by 1542 candidates Mean 3.31(/6) Max 6 Min 0
Part (a) was done well as permitted by the marking key. Many errors occurred often by candidates wanting to obtain coefficients of 1 in a row, which often induced errors with fractional coefficients. Teachers should emphasise that this is not necessary but should encourage students to write down what their row operation was so a marker can follow their solution. Part (b) was not handled well, yet this is quite a standard question for this section of the course. Some candidates wanted to perform their own row reduction, not realising that this had already been done. Many candidates did not clearly state which values of k corresponded to the unique solution or no solution. Some incorrectly stated that $k = -1$ yielded infinitely many solutions. In general, the poor standard of written expression did not help markers in awarding marks.

Question 3 attempted by 1525 candidates Mean 5.00(/9) Max 9 Min 0
Part (a) was a relatively straightforward task for most candidates. Most errors tended to arise from an inability to expand a product involving real and imaginary parts, i.e. many candidates could not correctly perform the algebra. In part (b)(i), most candidates understood that the polar form was required for $2i$ and then apply DeMoivre's Theorem. Those who decided to write everything in terms of the Cartesian form $a + bi$ were either not familiar with polar form or did not read the instruction in the question. In part (c)(ii), this question posed the first conceptual challenge on the paper. Those candidates who decided to draw an Argand

diagram using the idea of a complex number as a vector were invariably awarded one mark. There were many attempts that rather 'robotically' used the Cartesian form $z = r\left(\cos\theta\right) + i\left(r\sin\theta\right)$ that did not lead anywhere (except for some outstanding candidates who could then use trigonometric identities). Those who wrote an expression $\theta = \tan^{-1}\left(\dfrac{r\sin\theta}{r + r\cos\theta}\right)$ were not awarded any marks as it did not indicate the required conceptual understanding. There were some excellent diagrams and reasoning provided by many candidates, which meant this question discriminated well with 12.4% of the cohort achieving full marks for this part.

Question 4 attempted by 1526 candidates Mean 3.29(/4) Max 4 Min 0
Candidates performed well in this question in recognising the role played by the vertical asymptotes, horizontal intercepts and a horizontal asymptote. Many decided to justify their value of k by using an ordered pair or by observing the horizontal asymptote $y = 2$.

Question 5 attempted by 1529 candidates Mean 3.42(/4) Max 4 Min 0
The evaluation of the definite integral using the substitution was done well, with many candidates realising the significance of their answer! One marker observed that many candidates had to write down 1009×2 on paper to calculate the denominator 2018 rather than do this task mentally.

Question 6 attempted by 1537 candidates Mean 4.12(/5) Max 5 Min 0
Part (a) with the partial fractions was done well, with various techniques displayed, including an amusing reference to the 'Cover-up Rule'! In part (b), many candidates did not see that a factor of one-half was required to relate the given integrand with the expression from part (a). This was generally handled well, albeit with many candidates not using natural logarithms of an absolute value and not using a constant of integration.

Question 7 attempted by 1530 candidates Mean 4.33(/6) Max 6 Min 0
In part (a), the solving of a complex equation using polar form is a standard item for this cohort. For reasons unknown, the major hurdle was to be to be able to express -1 as $cis\pi$. Virtually all candidates knew that the roots had an angular separation of $\dfrac{2\pi}{3}$ but had trouble expressing the other solutions correctly in terms of the first solution. In part (b), most candidates were successful, with many using long division and, pleasingly, many doing this by inspection or with the reverse use of distributive multiplication. Part (c) was supposed to be a straightforward task, although candidates had to work harder here for two marks. Many did not consider the polar solutions from part (a) or could not solve the quadratic from part (b) correctly. Incorrect exact values for sines and cosines of $\dfrac{\pi}{3}$ were also evident.

Question 8 attempted by 1510 candidates Mean 2.89(/5) Max 5 Min 0
Part (a) was done well generally, although with the cross-product formula available on the formula sheet, it is difficult to see how many candidates could not score full marks here. In part (b) it was anticipated that candidates may struggle. Only 6% of candidates could achieve a full two marks to satisfactorily explain why $\underset{\sim}{c}.\left(\underset{\sim}{a} \times \underset{\sim}{b}\right)$ would represent the volume. The most common response was to state that 'the cross product is the area of the base and $\underset{\sim}{c}$ is the height'. Very few candidates correctly referred to angle ϕ using θ in its place. Part (c) was straightforward in using vectors to perform a dot product, with most candidates doing this successfully.

Question 9 attempted by 1486 candidates Mean 3.07(/7) Max 7 Min 0
As outlined earlier in this report, the performance on part (a) was very poor and affected candidates' efforts in part (b). It cannot always be assumed that candidates will be provided with scaffolding by being told what expression should be obtained. In part (b) it was felt that segmenting the task of evaluating the definite integral into two parts would help, but candidates' poor algebra skills prevented a higher level of success in this question. Even those who obtained the correct expression $3\sec^2 u$ then had trouble coping with the power $\frac{3}{2}$ in the denominator. It was disappointing that only 15.5% of candidates achieved the full five marks for this question part.

Section Two: Calculator-assumed (65 Marks)
In the calculator-assumed section the following areas were handled well:

- recognition of the locus of a complex number in Question 11 (a)
- understanding of the distribution of the sample mean being normal in Question 12 (a)
- sketching the graph of the function $y = \dfrac{1}{f(x)}$ in Question 14 (a)
- determining the area trapped between a curve and a line and the resultant volume in Question 15
- implicit differentiation in Question 20 (a).

Candidates performed relatively poorly in the following areas:

- determining the acceleration of a particle where $v = f(x)$ in Question 13 (a)
- rejecting the notion that we can know with certainty which confidence interval will contain an unknown population mean in Question 16 (d)
- determining the equation of a sphere given that a plane is a tangent to the sphere in Question 17 (c)
- deriving the vector expression for $\underset{\sim}{r}(t)$ from the acceleration vector for a projectile. far too many candidates could not fluently and convincingly show how the integration constants were used to develop $\underset{\sim}{r}(t)$ in Question 19 (a)
- determining the cartesian equation from the vector equation in Question 19 (b) was also poorly done.

Question 10 attempted by 1488 candidates Mean 2.66(/4) Max 4 Min 0
This question was answered quite well, with the use of the polar form well recognised. What was not clearly indicated was the role played by the parity of the cosine and sine functions. With many candidates not using brackets clearly, many were not able to convince the markers that they knew or had demonstrated that $\sin(-x) = -\sin x$. Many candidates scored three marks out of four as a result. Some candidates attempted to prove this using mathematical induction, usually without success.

Question 11 attempted by 1513 candidates Mean 4.66(/8) Max 8 Min 0
Part (a) was done well generally. Many candidates thought the radius was 2 units and others did not write an inequality. Part (b) proved challenging for many candidates, with many making no attempt. In the situation of the locus being a ray, this should suggest that an argument needs to be specified. There were some novel attempts to define the locus in terms of points equidistant from two given points, but then these candidates needed to restrict the full line to a ray, without stating $\text{Re}(z) < 0$. In part (c) it was pleasing that most candidates recognised that the maximum occurred from a tangent to the circle. A major error made by some candidates was to assume the point of tangency could be read from the diagram. Minor errors were made with calculations of side lengths or applying an incorrect trigonometric ratio in a right triangle.

Question 12 attempted by 1515 candidates Mean 8.33(/11) Max 11 Min 0
Part (a) was done well. Candidates pleasingly stated that the distribution would be normally distributed. Part (b) required the straightforward use of the normal distribution but with the adjusted standard deviation for the sample mean. In part (c), most candidates recognised that the answer would not change. Candidates needed to refer to the distribution of the 'sample mean' still being normally distributed rather than making vague comments such as 'it' is normally distributed. Part (d) was answered well. Markers reported that more than half the cohort used their CAS calculator to answer this, using $solve\left(normCDF\left(25,\infty,\frac{20}{\sqrt{n}},20\right)=0.03,n\right)$. Pleasingly most candidates knew that they needed to conclude with an integer solution. Candidates should still be encouraged to write the mathematics that leads to the use of the CAS calculator, i.e. $1.88079... = \frac{25-20}{\frac{20}{\sqrt{n}}}$.

Question 13 attempted by 1541 candidates Mean 2.53(/5) Max 5 Min 0
There was quite a disappointing performance in part (a). It appeared that many in the cohort were unprepared for rectilinear motion where the velocity was given as a function of position. This was the first time that this section of the current Mathematics Specialist course was examined. Part (b) was also very disappointing. Many candidates wanted to use their answer from part (a), which was essentially irrelevant. Those candidates who got as far as to write $x^2 = 4t+4$ needed to pay attention to the requirement that a function was required for their final answer.

Question 14 attempted by 1534 candidates Mean 5.20(/9) Max 9 Min 0
Sketching the reciprocal of a given function in part (a) was handled quite well, with candidates obtaining the vertical asymptotes and key coordinates. The 'hole' at $(0,-1)$ was missed by many. Sketching $y = f\left(-|x|\right)$ in part (b) was a more challenging question yet the marking key permitted those who had symmetry about $x = 0$ to score one mark. The challenge was to include the point $(0,2)$ since $f\left(-|0|\right) = f(0) = 2$ which was overlooked by almost all candidates. In part (c), candidates should interpret this question from a graphical perspective and not attempt to do this algebraically as per the intent of the syllabus. The sketch of $y = \left|f(x)-1\right|$ could be sketched as with $y = 1$ and then read off the x coordinates.

Question 15 attempted by 1540 candidates Mean 6.79(/8) Max 8 Min 0
Finding the equation of a tangent to a curve in part (a) was a routine question and done well generally. Determining the area between the tangent and the curve in part (b) was done well and a standard question for this course. Determining the volume of the solid generated in part (c) was also done quite well. Some candidates forgot the factor of π or opted to square the difference of the functions.

Question 16 attempted by 1516 candidates Mean 5.20(/9) Max 9 Min 0
Part (a) was done well except for some candidates using the interval width rather than the half-width. Part (b) was satisfactory in terms of the mark achieved out of three, but it was concerning to note that many candidates did not understand the difference between the sample standard deviation and the standard deviation of the sample mean. Consequently, for part (b), in many scripts the conclusion was to give the value for $\frac{s}{\sqrt{400}}$ rather than simply state the value for s. Another notable error was to find the incorrect critical z score for a cumulative 0.99 probability rather than 0.995. Aside from these errors, the use of notation suggested that candidates do not distinguish between sample and population values μ versus $\bar{x}$ and σ versus s. In part (c)(i) explanations were clear. This represented a marginal improvement on a similar question in 2017. However, part (c)(ii) was not done well, with candidates not able to state that due to the random sampling, confidence intervals will not necessarily overlap. In part (d), with the specific wording of the question, candidates were almost led to believe that one of the intervals contained the unknown value μ. There were some good answers provided but they were part of a small minority. This type of question is one that many candidates are not exposed to very often and so it was not surprising that many were lured into nominating one of the confidence intervals that contains μ. Candidates needed to have the courage to reject the suggestion implied in the question and state that we cannot be certain which confidence interval contains μ.

Question 17 attempted by 1529 candidates Mean 3.70(/7) Max 7 Min 0
Part (a) was done well, yet some candidates did not make a comment about the significance of this cross product. Markers were instructed to be tolerant in accepting the vocabulary offered. In part (b), obtaining the equation for the plane was handled quite well overall. While poor notation was not penalised, many candidates attempted to reverse engineer the result. In some cases, candidates did not show how the left-hand side was obtained. Part (c) appeared to be a challenging question to many candidates, with many incorrectly using the normal vector as the radius. It is not certain what led to such a disappointing response for this question. There were some elegant solutions using the dot product of a vector with the unit normal in order to determine the radius, but these were rare.

Question 18 attempted by 1530 candidates Mean 8.63(/13) Max 13 Min 0
Part (a) was handled well, with almost all candidates recognising the 60 as a derivative value and hence substituting $N = 100$ to deduce k. Part (b), using the technique of increments, was answered well generally. The main error was in using $\Delta t = 0.15$ hrs. Part (c) seemed to cause more problems than expected. The algebraic manipulation to separate the variables and obtain the correct expression for the integral seemed an issue. Lack of brackets exposed many candidates to write incorrect expressions such as $\frac{1}{1600}\ln N - \ln(1600 - N) = kt + c$. The constant of integration that led to the factor $\frac{1}{15}$ tended to be fudged by some. In part (d) many candidates set the derivative to zero, not realising that they were maximizing the derivative. Some successfully solved when the second derivative was zero using their CAS calculator well. It was pleasing to see many candidates conclude with either a clock time or number of minutes.

Question 19 attempted by 1450 candidates Mean 3.01(/9) Max 9 Min 0
Projectile motion was another section of the course that was examined for the first time in 2019. In part (a), candidates tended to reverse engineer their answers. The use of integration constants was essential and candidates had to explicitly state the values for $\underset{\sim}{v}(0)$ and $\underset{\sim}{r}(0)$ to show the development for $\underset{\sim}{r}(t)$. Generally, vector notation continues to be very poor for this level of candidate. In part (b), finding the cartesian equation proved to be difficult for candidates. One marker suggested that the cognitive load was too high for many with the variables w, θ, t, x, y amongst the vectors. In part (c), candidates needed to have a sound grasp of the problem to score well. This question discriminated well, and there were some excellent CAS calculator solutions. It is worthy to note that one solution (provided by the ultimate subject exhibition winner) solved the trigonometric equation that resulted in setting the derivative of the range function to zero. This quadratic equation in $\cos\theta$ led to $\cos\theta = 0.7$ which gives $\theta = 45.57..^o$.

Question 20 attempted by 1495 candidates Mean 5.57(/8) Max 8 Min 0
Part (a), requiring the use of implicit differentiation, exhibited in general a good set of responses, despite some candidates wanting to expand using their CAS calculator or try to express $\frac{dy}{dx}$ as the subject, despite being told expressly not to do so. Lack of the use of brackets again plagued many answers. Some candidates took the cube root of the original equation to make the differentiation easier. Part (b) was generally answered well, with markers allowing for follow through from part (a). In part (c), candidates were generally able to recognise the need to use $\frac{dy}{dx} = 0$ but a large number were unable to apply this correctly or then pair this equation with the original equation to solve simultaneously.