MATHEMATICS

2A/2B

Calculator-free

WACE Examination 2011

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Question 1   **(7 marks)**

(a)   Evaluate:

$15 - 12 \div 2 \times 3 + 5$ (2 marks)

Solution
$15 - 12 \div 2 \times 3 + 5$
$= 15 - 18 + 5$
$= 2$
Specific Behaviours
✓ applies the rule of order for division and multiplication correctly
✓ carries through calculation correctly

(b)   Expand and simplify:

$(2x - 3)(x + 5)$ (2 marks)

Solution
$(2x - 3)(x + 5)$
$= 2x^2 - 3x + 10x - 15$
$= 2x^2 + 7x - 15$
Specific Behaviours
✓ expands the binomial correctly
✓ adds like terms correctly

(c)   Estimating a calculation can be made easier by first rounding the numbers. One way to round is to round to the **leading** digit, for example, 534 rounds to 500, 39 rounds to 40 and 19 345 rounds to 20 000.

   (i)   Round 189 to the leading digit.   (1 mark)

Solution
200
Specific Behaviours
✓ rounds correctly

   (ii)   Hence, estimate the value of the calculation 189×42 by using the method of rounding to the leading digit.   (1 mark)

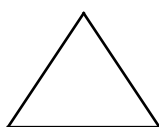| Solution |
|---|
| 189×42 |
| $\approx 200 \times 40$ |
| $= 8000$ |
| **Specific Behaviours** |
| ✓ applies specified rounding method to both factors to estimate the value correctly |

(d)   Indonesia has a population of 275 million. Write this number in scientific notation.(1 mark)

Solution
$275\,000\,000 = 2.75 \times 10^{8}$
Specific Behaviours
✓ expresses the number in scientific notation correctly

Question 2   **(10 marks)**
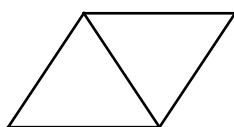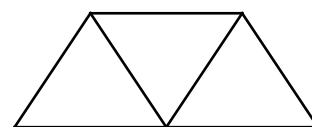
A sequence of shapes is made of matches to form triangles, as shown below.
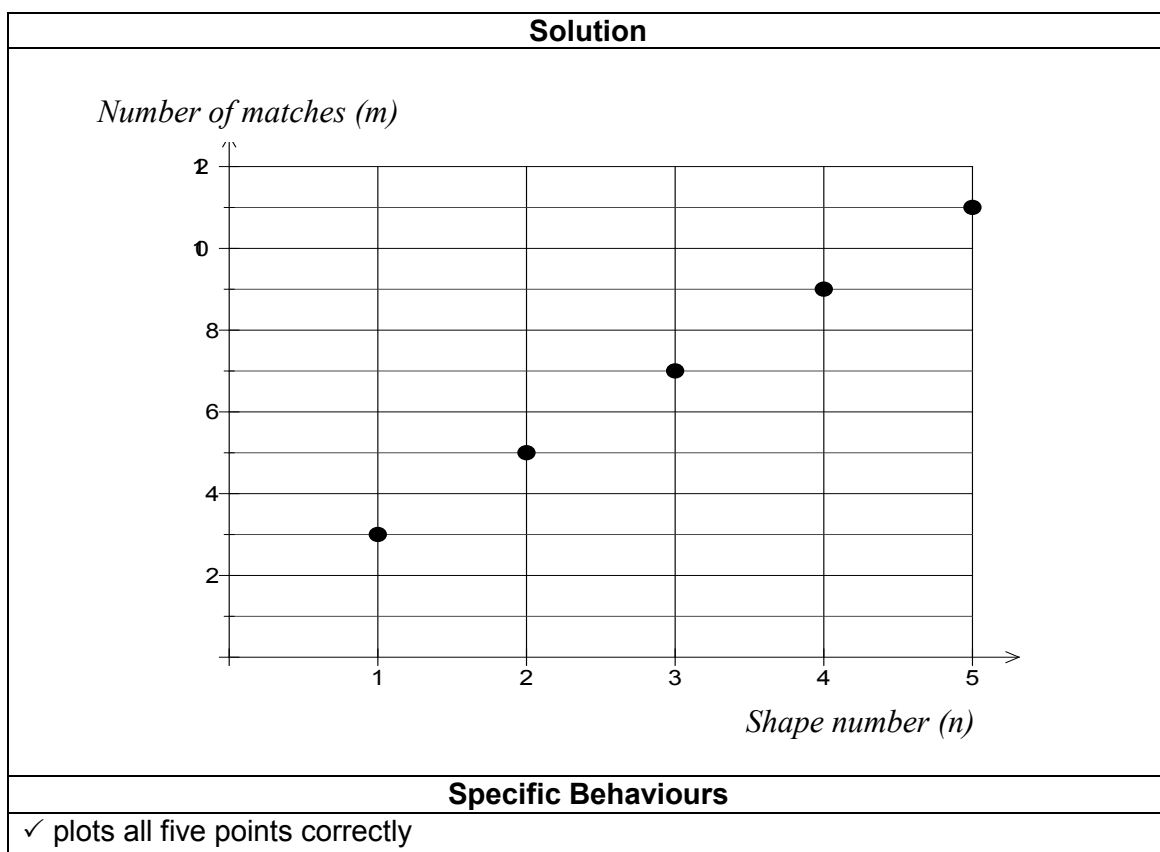

Shape Number 1   *Shape Number 2*   *Shape Number 3*

The table of results is shown below:

Shape Number (n)	1	2	3	4	5
Number of matches (m)	3	5	7	9	11

(a)   Plot the data from above on the axes below.   (1 mark)

Solution
Specific Behaviours
✓ plots all five points correctly

(b)   Can these points be joined? Explain your answer in the context of the situation.
                                                                                    (1 mark)

Solution
No, as you can only have whole numbers for Shape Numbers. For example, shape Number 1.5 does not make sense in this context.
Specific Behaviours
✓ answers 'no' and gives appropriate reason

(c)   Write a rule linking m and n, where m and n are as defined in the table.   (2 marks)

Solution
$m = 2n + 1$
Specific Behaviours
✓ identifies gradient ✓ determines the correct rule in terms of m and n

(d)   Determine the number of matches required for Shape Number 12.   (1 mark)

Solution
$m = 2n + 1$ $\quad = 2(12) + 1$ $\quad = 25$
Specific Behaviours
✓ determines the correct value, based on response in (c) or correctly continues the pattern

(e)   Justify that the point (20, 41) lies on the line that would pass through the points plotted in (a).   (1 mark)

Solution
$m = 2n + 1$ $\quad = 2(20) + 1$ $\quad = 41$ therefore the point lies on the line (or not, based on response in(c))
Specific Behaviours
✓ shows when $n = 20$, $m = 41$ or consistent with response in (c)

(f)   Madeline has 50 matches. What is the biggest Shape Number she would be able to make?   (2 marks)

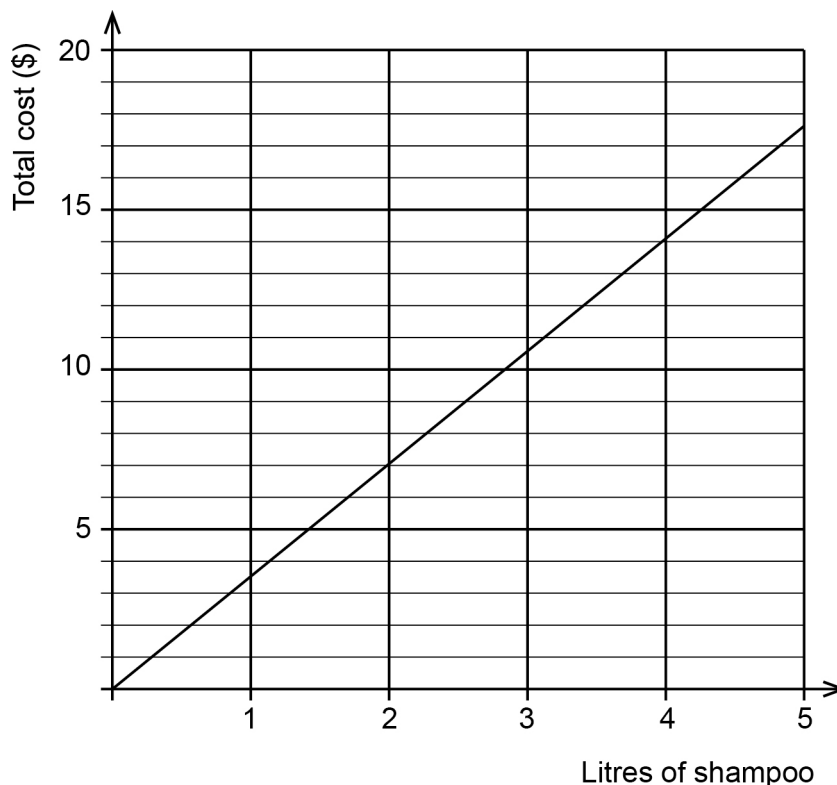| Solution |
|---|
| $2n + 1 = 50$ |
| $\quad 2n = 49$ |
| $\quad\ n = 24.5$ |
| therefore the biggest shape number she could make is Shape Number 24. |
| **Specific Behaviours** |
| ✓ solves for n, correctly, based on response in (c) or uses the pattern to deternmine n correctly |
| ✓ rounds the answer down |
| Note: Full marks for answer of 24 without working |

(g)   Write a recursive rule for the original table of results.   (2 marks)

Solution
$T_{n+1} = T_n + 2$ where $T_1 = 3$
Specific Behaviours
✓ determines the correct recursive part of the rule
✓ states T_1

Question 3   **(3 marks)**

Cate works as an apprentice hairdresser. She has been asked to order shampoo from a bulk supplier. Her employer gives her the graph below which shows the total cost ($) of ordering a given quantity of shampoo (in litres).

(a)   Determine the gradient of the line.   (1 mark)

Solution
$\frac{7}{2} = 3.5$
Specific Behaviours
✓ calculates the gradient of the straight line correctly

(b)   What does the value of the gradient indicate in this context?   (1 mark)

Solution
The price of shampoo is $3.50 per litre
Specific Behaviours
✓ interprets the gradient in cost per litre

(c)   Cate has to order 12 litres of shampoo. What will this cost?   (1 mark)

Solution
$12 \times 3.5 = \$42$
Specific Behaviours
✓ calculates cost correctly

Question 4 **(6 marks)**

Solve the following equations and inequalities **algebraically**. Show all workings.

(a) $5(n + 2) = 2n + 1$ (2 marks)

Solution
$5(n+2) = 2n+1$ $5n+10 = 2n+1$ $3n = -9$ $n = -3$
Specific Behaviours
✓ expands brackets correctly   ✓ collects like terms correctly and states correct solution

(b) $2k - 6 \leq 15$ (2 marks)

Solution
$2k - 6 \leq 15$ $2k \leq 21$ $k \leq 10.5$
Specific Behaviours
✓ isolates k correctly   ✓ uses the correct inequality sign

(c) $2^m + 7 = 23$ (2 marks)

Solution
$2^m + 7 = 23$ $2^m = 16$ $2^m = 2^4$ $m = 4$
Specific Behaviours
✓ isolates 2^m correctly   ✓ correctly determines m

Question 5   **(10 marks)**

(a)   An antique dealer paid $200 for an old ceramic jug. He decides to sell the jug with at
        least 60% profit. What is his minimum selling price?   (2 marks)

Solution
$200 \times 1.6 = \$320$ or $200 + 0.6 \times 200 = \$320$
Specific Behaviours
✓ identifies 60% profit as a 60% increase or calculates 60% of $200
✓ calculates selling price correctly

(b)   The profit from a family business was shared by three brothers, Paul, Greg and John, in
        the ratio of 3 : 2 : 1 respectively. If Paul's share was $24 000, what was the total profit?
                                                                              (3 marks)

Solution
$P : G : J$
$3 : 2 : 1$
24 000 : 16 000 : 8 000
Total profit   = 24 000 + 16 000 + 8 000   = \$48 000
or Paul receives $\dfrac{3}{6}$ shares of the profit, therefore
Total profit $= 2 \times 24\ 000$ $= \$48\ 000$
Specific Behaviours
✓ recognises unit share or number of shares
✓ calculates the profit for Greg and John correctly or sees Paul's share as half
✓ calculates total profit correctly

(c) The median price of a house in a Perth suburb at the end of 2008 was $300 000. Due to the Global Financial Crisis, the median price dropped by 10% during 2009. During 2010, the market had regained strength and the median house price rose by 10%.

What was the median house price at the end of 2010? (3 marks)

Solution
End of 2009 price $= 300\ 000 \times 0.9 = \$270\ 000$
End of 2010 price $= 270\ 000 \times 1.1 = \$297\ 000$
Or
End of 2009 price $= \$300\ 000 - 10\%$ of $\$300\ 000$
$\quad\quad\quad\quad\quad\quad = 300\ 000 - 30\ 000$
$\quad\quad\quad\quad\quad\quad = \$270\ 000$
End of 2010 price $= \$270\ 000 + 10\%$ of $\$270\ 000$
$\quad\quad\quad\quad\quad\quad = 270\ 000 + 27\ 000$
$\quad\quad\quad\quad\quad\quad = \$297\ 000$
Specific Behaviours
✓ calculates 10% decrease from $300 000 correctly
✓ calculates a 10% increase correctly
✓ uses $270 000 to calculate the percentage increase correctly

(d) Sonya is at the supermarket to purchase muesli. She notices it is packaged in two sizes: 500 g and 275 g. The larger size costs $5 and the smaller size $2.45. Which is the better buy? (Assume that she doesn't mind what size she buys). Justify your answer.
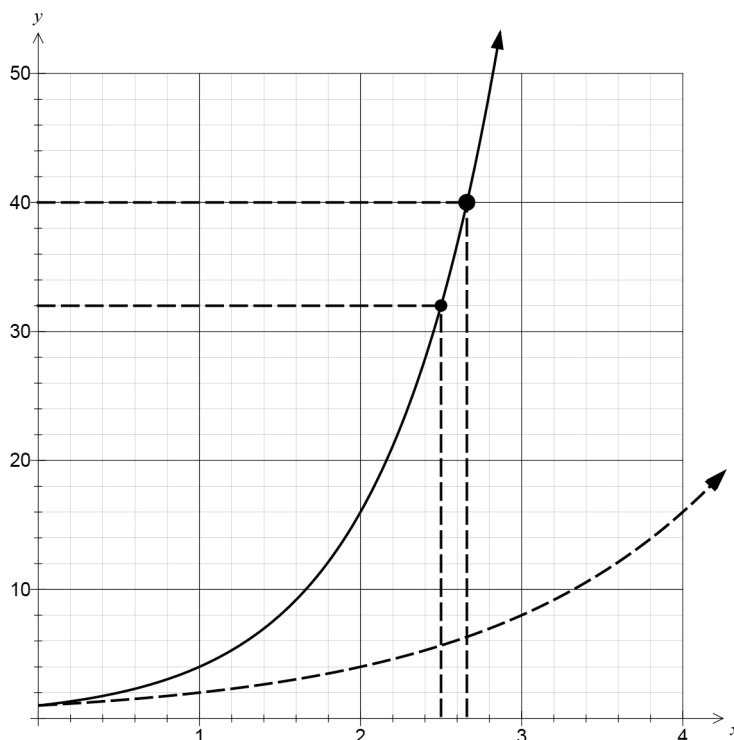
(2 marks)

Solution
Buying two smaller muesli packets
Weight = 2 × 275 g = 550 g
Price = 2 × 2.45 = $4.90
Therefore get more muesli for less money if buying the 275 g packet
275 g packet best buy
Specific Behaviours
✓ compares weight and price correctly
✓ concludes 275 g packet as best buy

Question 6   **(4 marks)**

On the axes below, the graph of $y = 4^x$ is plotted.

(a)   Show clear use of the graph to determine the value of $4^{2.5}$.   (1 mark)

Solution
$4^{2.5} = 32$
Accept 31 – 33
Specific Behaviours
✓ reads value from graph correctly

(b)   Show clear use of the graph to determine the value of x if $4^x = 40$. Give your answer to
       two (2) decimal places.   (1 mark)

Solution
$x \approx 2.66$
Accept $2.61 - 2.70$
Specific Behaviours
✓ reads value from graph correctly

(c)   On the same set of axes above sketch the graph $y = 2^x$.   (2 marks)

Solution
see graph
Specific Behaviours
✓ plots (0,1) and at least two (2) other points correctly on the graph ✓ draws a smooth curve through the plotted points