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**Note:**

* Follow step 2 if Python 3.7.3 is not installed on the local machine.
* Follow step 3 if MiniSat version 1.14 is not installed on the Linux machine.
* Follow step 4 and 5 if project has not been downloaded to the local machine.
* More information about GUI: [Click link](https://realpython.com/python-gui-tkinter/#building-your-first-python-gui-application-with-tkinter).

## N-queens Problem

The n-queens puzzle is the problem of placing N queens on an N×N chessboard such that no two queens attack each other.

To encode the n-queens (n is a given number) problem into the DIMACS CNF format and solve it by the SAT solver, we need to encode the following constraints

* 1) no two queens on the same row,   
  2) no two queens on the same column, and   
  3) no two queens on the same diagonal.

## Concept

Let denote the propositional letter for the position of the board, which has size : the row, and the column of the board where can be and can be . If is true; there is a queen on ; otherwise, there is no queen

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#### **ROW**

Let us first consider rows. For such a row, there should be at least one true, and at most one true. At least one, since in total, there should be queens. So on every row, there should be at least one queen. Moreover, on every row, there should be at most one queen; otherwise, they can hit each other. That means and are in the same row if and only if .

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Let us start with at least one queen. To express the observation that there is a queen for row, that is easily expressed in the following formula.

At least one of these s should be true. It can be expressed by simply taking the or of all of these propositional letters. Moreover, in shorthand notation, we can write it in this way.

The next is, at most, one of them is true. If there is a sequence of propositional letters, shown below, and at most one may be true, that means if we take two of them, then it is not allowed that both are true.

So if we take two of them, if one is true, then the other one is false; that's expressed in the following formula, for row.

In other words, if we take two of them, at least one should be false. It can be expressed in the following formula, for row.

If we remove the duplicate cases, it will be expressed in the formula not or not for every j less than k or expressed in the following formula, for row.

Moreover, in shorthand notation, we can write it in this way.

#### **COLUMN**

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Next, we consider columns that have exactly the same requirements. The only difference is that the and the are swapped. So, that's expressed in the following formula.

For at least one queen on column,

In shorthand notation,

For at most one queen on column,

In shorthand notation,

So until now, we have requirements for at least one queen on every row, at most one queen on every row, at least one queen on every column, at most one queen on every column. That's expressed in the following formula.

For at least one queen on every row,

For at most one queen on every row,

For at least one queen on every column,

For at most one queen on every column,

#### **DIAGONAL**

Furthermore, now it remains to express the requirements on diagonals. On every diagonal, there should be at most one queen. Let us consider such a diagonal.

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We can see that two elements are on the same diagonal and if they have the same sum. In this case, the Orange color, the sum is By Moreover, it is not allowed that two of these variables are both true by following constraints that no two queens on the same diagonal. So, that's expressed in the following formula.

In the other direction, we have something similar. Nevertheless, now the requirement is not the sum but expresses the difference.

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and are on such a diagonal if and only if So, that's expressed in the following formula.

So for all in such a way that is different from . However, they are on the same diagonal, expressed by or . We have the requirement that it's not allowed that both are true. So not or not should hold. More, we can do this for all . Nevertheless, then we have many things double. We may restrict to the cases where is less than . Here, we have a big formula.

We let and run over all the values from 1 to N and and run on all the values in such a way that they corresponding two positions are on the same diagonal.

#### **OVERALL**

Now we have collected all ingredients of the total formula, and here is the total formula presented.

## 

## Pseudocode

To encode the problem into the CNF form, first, we create two beside functions to deal with, at least one queen and at most one queen requirements. Both are shown below.

#### **Function 1**

Function 1 will produce a clauses of conjunction of all element in the *propositions* then add to the *cnf* which is the list of clauses.

|  |  |
| --- | --- |
| **Function 1:** atLeastOneQueen | |
|  | **input** : *propositions* = A list of proposition letters.  *cnf* = A list of clauses. |
|  | **output** : *cnf* = A list of clauses. |
| 1 | Append *cnf* with *propositions* |
| 2 | **return** *cnf* |

The *propositions* is the list of proposition letters that receive from *cnf* *function*. It cloud be the list of proposition letters in any row or any column. For example,

|  |  |  |
| --- | --- | --- |
|  | Any row | Any column |
| Example Table | |  |  |  |  |  | | --- | --- | --- | --- | --- | | A | B | C | D | E | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | A |  |  |  |  | | B |  |  |  |  | | C |  |  |  |  | | D |  |  |  |  | | E |  |  |  |  | |
| *propositions* |  | |
| clauses appended to *cnf* |  | |
| Meaning |  | |

#### **Function 2**

Function 2 will produce a clauses of each pair conjunction of negation in the propositions then add to the *cnf* which is the list of clauses.

|  |  |
| --- | --- |
| **Function 2:** atMostOneQueen | |
|  | **input** : *propositions* = A list of proposition letters.  *cnf* = A list of clauses. |
|  | **output** : *cnf* = A list of clauses. |
| 1 | **for** *k := 0* **to** *propositions length* **do** |
| 2 | **for** *j := 0* **to** *k* **do** |
| 3 | Create an empty list *temp* |
| 4 | Append *temp* with -*propositions[j]* |
| 5 | Append *temp* with -*propositions[k]* |
| 6 | Append *cnf* with *temp* |
| 7 | **end for** |
| 8 | **end for** |
| 9 | **return** *cnf* |

The *propositions* is the list of proposition letters that receive from *cnf* *function*. It cloud be the list of proposition letters in any row, any column or any diagonal. For example,

|  |  |  |  |
| --- | --- | --- | --- |
|  | Any row | Any column | Any diagonal |
| Example Table | |  |  |  |  | | --- | --- | --- | --- | | A | B | C | D | |  |  |  |  | |  |  |  |  | |  |  |  |  | | |  |  |  |  | | --- | --- | --- | --- | | A |  |  |  | | B |  |  |  | | C |  |  |  | | D |  |  |  | | |  |  |  |  | | --- | --- | --- | --- | | A |  |  |  | |  | B |  |  | |  |  | C |  | |  |  |  | D | |
| *propositions* |  | | |
| clauses appended to *cnf* | , , , , , | | |
| Meaning |  | | |

#### **Function 3**

Function 3 will create the *input.cnf* file and send it to Linux machine to run SAT Solver which is MiniSat version 1.14. Then download the result which is *output.txt* from the Linux machine to the local machine. And return the solution in the *output.txt*.

|  |  |
| --- | --- |
| **Function 3:** cnf | |
|  | **input** : *N* = The size of the problem  *show\_all* = Boolean to show all solutions, if true the function will return all possible solutions, otherwise will return only one solution. |
|  | **output** : *solutions* = A list of solutions. |
| 1 | Create an empty 2D list *board* |
| 2 | Assign the initial value of the board with running number from 1 to N\*N |
| 3 | Create an empty list to collect solutions. *solutions* |
| 4 | Create an empty list to collect clauses. c*nf* |
| 5 | **for** *i := 0* **to** *N* **do** |
| 6 | Create an empty list to collect all proposition letters in the *ith* row. *propositions\_row* |
| 7 | Create an empty list to collect all proposition letters in the *jth* column. *propositions\_col* |
| 8 | **for** *j := 0* **to** *N* **do** |
| 9 | Append *propositions\_row* with *board[i][j]* |
| 10 | Append *propositions\_col* with *board[j][i]* |
| 11 | **end for** |
| 12 | **Call** *atLeastOneQueen(propositions\_row, cnf)* |
| 13 | **Call** *atMostOneQueen(propositions\_row, cnf)* |
| 14 | **Call** *atLeastOneQueen(propositions\_col, cnf)* |
| 15 | **Call** *atMostOneQueen(propositions\_col, cnf)* |
| 16 | **end for** |
| 17 | **for** *i := 0* **and** *N-1* **do** |
| 18 | **for** *j := 0* **to** *N* **do** |
| 19 | Create an empty list to collect all proposition letters in the first direction of each diagonal. *propositions1* |
| 20 | Create an empty list to collect all proposition letters in the second direction of each diagonal. *propositions2* |
| 21 | **for** *i' := 0* **to** *N* **do** |
| 22 | **for** *j' := 0* **to** *N* **do** |
| 23 | **if** *i* **equal to** *N-1* **and** *j* **equal to** *0* **then** |
| 24 | Do nothing |
| 25 | **else if** *i* **equal to** *N-1* **and** *j* **equal to** *N-1* **then** |
| 26 | Do nothing |
| 27 | **else then** |
| 28 | **if** *i* **equal to** *i'* **and** *j* **equal to** *j'* **then** |
| 29 | Append *propositions1* with *board[i'][j']* |
| 30 | Append *propositions2* with *board[i'][j']* |
| 31 | **else if** *i-j* **equal to** *i'-j'* **then** |
| 32 | Append *propositions1* with *board[i'][j']* |
| 33 | **else if** *i+j* **equal to** *i'+j'* **then** |
| 34 | Append *propositions2* with *board[i'][j']* |
| 35 | **end if** |
| 36 | **end if** |
| 37 | **end for** |
| 38 | **end for** |
| 39 | **Call** *atMostOneQueen(propositions1, cnf)* |
| 40 | **Call** *atMostOneQueen(propositions2, cnf)* |
| 41 | **end for** |
| 42 | **end for** |
| 43 | Convert a list *cnf* to a *input.cnf* file |
| 44 | Send the input.cnf file to Linux Machine to run SAT Solver which is MiniSat version 1.14 |
| 45 | Download output.txt which is the solution of the problem to the local machine |
| 46 | Append *solutions* with the solution from output.txt file |
| 47 | **if** *show\_all* **equal to** *True* **then** |
| 48 | **repeat** |
| 49 | Append *cnf* with the solution from output.txt file |
| 50 | Convert a list *cnf* to a *input.cnf* file |
| 51 | Send the input.cnf file to Linux Machine to run SAT Solver which is MiniSat version 1.14 |
| 52 | Download output.txt which is the solution of the problem to the local machine |
| 53 | Append *solutions* with the solution from output.txt file |
| 54 | **until** the solution from output.txt file is unsatisfiable |
| 55 | **end if** |
| 56 | **return** *solutions* |

There are 3 main sections in this function,

**The first section**: line 5-16

This part will create the list of proposition letter for each row and each column and send it to Function 1 and Function 2 to make clauses and append it to *cnf* variable which is a list of clauses. This part will math with the formula below.

For at least one queen on every row, line: 5-12 and 16

For at most one queen on every row, line: 5-11, 13 and 16

For at least one queen on every column, line: 5-11, 14 and 16

For at most one queen on every column, line: 5-11 and 15-16

**The second section**: line 17-42

This part will create the list of proposition letter for each diagonal which have 2 directions and send it to Function 1 and Function 2. This part will math with the formula below.

For at most one queen on every diagonal,

**The last section**: line 43-56

This part will convert *cnf* to the *input.cnf* file and send it to Linux machine to run SAT Solver which is MiniSat\_version 1.14. Then download the result which is output.txt from the Linux machine to the local machine. And return the solution in the *output.txt*.

## Install Python to Local machine

1. Go to this [link](https://www.python.org/downloads/release/python-373/).
2. Download Python 3.7.3 for your Operating System (e.g., Mac OS X, Windows)
3. Install Python 3.7.3 by following this [link](https://realpython.com/installing-python/).

## 

## Install SAT Solver to Linux machine

1. Connect to TTUnet VPN by following this [link](https://www.askit.ttu.edu/portal/app/portlets/results/viewsolution.jsp?guest=0&solutionid=181128172622147&hypermediatext=null).
2. Connect to the Linux server which name is *willow.cs.ttu.edu* by following this [link](https://www.linuxbabe.com/linux-server/ssh-windows). Note: *willow.cs.ttu.edu* is not a link. It is a machine name.
3. Download and install SAT Solver which is MiniSat version 1.14 to this Linux machine, type:

*$ wget http://minisat.se/downloads/MiniSat\_v1.14\_linux  
$ chmod u+x MiniSat\_v1.14\_linux*

### More about MiniSat version 1.14

* Carefully read this [link](https://dwheeler.com/essays/minisat-user-guide.html) for instructions on how to invoke MiniSat, how to save a solution, and how to get all solutions. You will not need to use any of MiniSat’s optional flags.
* To run the MiniSat
  + Require input file in a simplified "DIMACS CNF" format, follow steps below to create the simple one to test, [More info](https://linuxize.com/post/how-to-use-nano-text-editor/).
    - To create input.cnf file, type:
    - *$ nano input.cnf*
    - Paste the text below to the terminal,
    - *p cnf 5 4  
      1 -5 4 0  
      -1 5 3 4 0  
      -3 -4 0  
      -1 -2 3 -4 -5 0*
    - To save the file, press *Ctrl+o*.
    - To exit nano press *Ctrl+x*.
  + To get the solution, type:
  + *$ ./MiniSat\_v1.14\_linux input.cnf output.txt*
  + To read the solution, type:
  + *$ cat output.txt*

The output.txt should show as below,

* + *SAT  
    1 -2 -3 4 5 0*

## Download project to Local machine

### First method (clone repository)

1. Open Command Prompt or Terminal. [More information](https://www.groovypost.com/howto/open-command-window-terminal-window-specific-folder-windows-mac-linux/).
2. Change the current working directory to the location where you want to download the project.
3. Clone repository:
   * + *$ git clone https://github.com/rinriko/N-Queens.git*

**Note:** If Git is not installed, please complete the following [instructions](https://git-scm.com/book/en/v2/Getting-Started-Installing-Git). Or follow the [second method](#second-method-download-project) instructions.

### Second method (download project)

1. Go to <https://github.com/rinriko/N-Queens>.
2. Above the list of files, click ↓ Code.
3. Click "Download ZIP".
4. Save the zip file into a convenient location on your PC and start working on it.
5. Extract the ZIP file.

## Set up project

### From the first method

1. Continue use Command Prompt or Terminal, type:
   * *$ cd N-Queens*
2. In order to install the project requirements, type:
   * *$ make setup*

### From the second method

1. Open Command Prompt or Terminal. [More info](https://www.groovypost.com/howto/open-command-window-terminal-window-specific-folder-windows-mac-linux/).
2. Change the current working directory to the location where you have the cloned directory. [More info](https://www.earthdatascience.org/courses/intro-to-earth-data-science/open-reproducible-science/bash/bash-commands-to-manage-directories-files/#:~:text=Often%2C%20you%20may%20want%20to,to%20check%20the%20new%20path).
3. In order to install the project requirements, type:

*$ make setup*

## Run project

Inside the N-Queens directory, use the Command Prompt or Terminal and type:

*$ make run*

## Help

Inside the N-Queens directory, use the Command Prompt or Terminal and type:

$ make help