CHAPTER

# 3 Location Information Processing

A position location (PL) algorithm provides an estimate of the location or position of a user within a specific frame of reference. It carries out the task of gathering subscriber information in a given coverage area so that a location is estimated by processing this information. The system usually gets information from measurements of the signal transmitted by the node to be located and this information is processed to determine parameters such as the time of arrival (TOA), direction of arrival (DOA), or the received signal strength (RSS), among others, as described in Chapter 2. The location estimate is generally obtained from mappings or geometric relationships that relate such parameters to coordinates, or from the optimal solution to nonlinear and overdetermined systems of equations.

Reconfigurable networks, such as ad hoc and sensor networks, must be aware of these services and applications as well, but their specific characteristics change the traditional PL point of view in a wireless scenario. In these networks, satellite-based positioning, such as GPS, is not a good solution since line-of-sight problems would make indoor positioning practically impossible. In addition to that, the power consumption of satellite-based positioning would reduce node battery life and the use of satellite positioning devices would dramatically increase node cost and size.

Although position location information (PLI) is often provided through GPS, trilateration processes based on range, differential distance, or angular views from known land-fixed reference points present several drawbacks for ad hoc and sensor network environments, becoming more complex as such networks are self-creating, self-organizing, and self-managing [9]. Further, the nodes in these networks are usually power limited such that they may only be able to communicate with their closest neighbors, which may not necessarily correspond to land references.

In many scenarios of interest, such as ad hoc networks, a node of interest (NOI) whose location must be estimated may not have direct connectivity to any or some land references in the network and the direct inference of range or angle of arrival (AOA) from signal measurements may not be possible. In this type of environment, connectivity to a specific NOI may be achieved by the concatenation of multiple links between intermediate devices whose location is also uncertain. In this chapter, we will explore techniques to infer range and/or angle of arrival from previously described multihop links and describe ways to combine the resulting measurements to obtain position estimates of a specific NOI in the network environment.

## 3.1 THE MULTILATERATION PROBLEM

As previously described, the PL problem involves the observation of range or AOA measurements that are obtained by an NOI via signal transmissions to or from reference nodes or land references. Multiple measurements may be obtained from various land references and subsequently combined in an optimal or suboptimal way to obtain an estimate of the position of the NOI. This process of combining the multiple range- or AOA-related observations to obtain a final position estimate is usually referred to as *multilateration*. Whenever the number of observations reduces to three, then the process is referred to as *trilateration*, or *triangulation*, which correspond to classical geometric concepts.

Popular multilateration methods are usually divided into two main groups depending on the way the range- or AOA-related information is processed. Methods that find intersections between areas generated by means of the estimated range and/or AOA from several reference points to the NOI are referred to as *geometric multilateration*. On the other hand, methods that consider the minimization of range and/or AOA measurement errors via some statistical criterion are referred to as *statistical multilateration*.

In the presence of noise or measurement inaccuracies, a range or AOA-related observation between the $i$-th land reference and the NOI may be expressed as

$$r_i = f_i(x, y) + \eta_i, \tag{3.1}$$

where $f_i(x, y)$ is a function that describes the type of measurement (i.e., range- or AOA-related measurements) obtained between the $i$-th land reference and the NOI, $(x, y)$ are the unknown NOI coordinates that we want to estimate, and $\eta_i$ is the measurement error or noise that may be assumed to be a zero mean random variable. Function $f_i(x, y)$ may take the form of the Euclidean distance

$f_i(x,y)=\sqrt{(x-x_i)^2+\left(y-y_i\right)^2}$ in the case of a range-related measurement, or the form $f_i(x,y)=\tan^{-1}\left[\frac{y-y_i}{x-x_i}\right]$ in the case of an AOA-related measurement.

Collecting $N$ measurements arising from various land references in an observation vector $\mathbf{r}=[r_1\ldots r_N]^H$, we obtain

$$\mathbf{r}=\mathbf{f}(x,y)+\boldsymbol{\eta}, \tag{3.2}$$

where $\mathbf{f}(x,y)=[f_1(x,y)\ldots f_N(x,y)]^H$ is a vector that collects the noise-free measurements between every land reference and the NOI (clearly this vector depends on the unknown NOI coordinates $(x,y)$), and $\boldsymbol{\eta}=[\eta_1\ldots\eta_N]^H$ is a noise vector assumed to have a zero mean and covariance matrix $\mathbf{K}=E\{\boldsymbol{\eta}\boldsymbol{\eta}^H\}$. A diagonal covariance matrix $\mathbf{K}$ would imply uncorrelated measurements. The covariance between measurements will usually depend on the geographic separation of the land references. Measurements obtained from closely spaced land references will tend to be correlated due to the similarities of the propagation channel. The diagonal terms of the covariance matrix denote the variance of the noise terms and measure the reliability of the range- or AOA-related measurements.

Based on the observation model presented above, in the following sections we will describe several geometric and statistical multilateration techniques and discuss their performance and implementation issues.

## 3.2 GEOMETRIC MULTILATERATION

Range-based geometric multilateration techniques consist, in general, of the intersection of the areas generated by the estimated distances from several land references to the NOI. Such areas are centered at the land reference locations and extend in a radius defined by the estimated distance to the NOI. The distance between each reference point and the NOI can be estimated through several techniques discussed in Chapter 2 (TOA, AOA, RSS, etc.). On the other hand, AOA-based geometric multilateration consists of the intersection of lines of bearing whose slope corresponds to the AOA measurements between land references and the NOI.

Geometric techniques are commonly based on three range or AOA measurements, reducing the multilateration process to a trilateration one. When three reference points are connected to the NOI, the intersection of the generated areas can provide a good approximation of its position. Whenever more than three range or AOA measurements are available, it is a common practice to combine the results of several trilateration realizations obtained with different

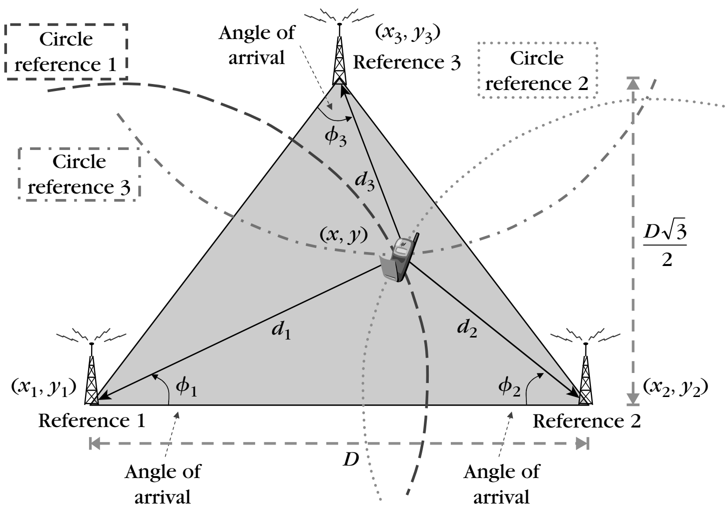



**FIGURE 3.1**

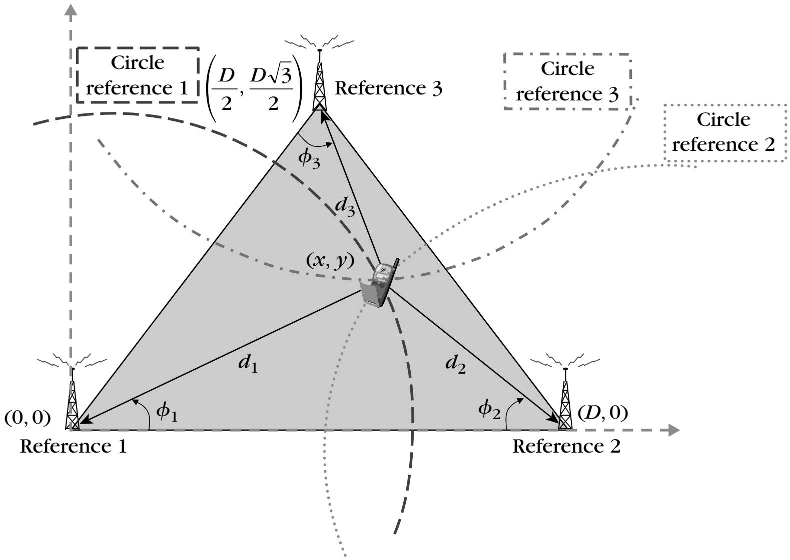
Fundamental multilateration technique. In a noise-free scenario, the point of intersection of circles (if TOA measurements are used), or ellipses (if TDOA measurements are used), centered at the land references with radii equal to the range measurements $d_1$, $d_2$, $d_3$ may be used to estimate the position of the NOI. Likewise, the point of intersection of lines of bearing drawn with slopes corresponding to AOA measurements $\phi_1$, $\phi_2$, $\phi_3$ may also be used to localize the NOI. Estimation accuracy of the position of the NOI may be improved through a combination of intersections of ellipses, circles, and lines of bearing.

combinations of land-reference triplets either by averaging the multiple resulting position estimates or by finding their centroid.

Several geometric multilateration techniques are based on range estimation where a distance from the NOI to a fixed known point in the network is estimated according to measurements of TOA, TDOA, AOA or a combination of several of these. Techniques such as TDOA have been proposed as mobile positioning standards for 3G [1].

Assuming a planar scenario, the basic PL problem treated with geometric trilateration techniques is illustrated in Figure 3.1 for the case of three fixed land references with known position at points $(x_i, y_i)$, $i = 1, 2, 3$, and an NOI whose unknown position $(x, y)$ must be estimated. In this scenario, it is assumed that separation between any pair of land references is fixed to $D$ meters. Estimated ranges or distances $d_i$, $i = 1, 2, 3$, and AOAs $\phi_i$, $i = 1, 2, 3$, can be obtained from signal transmissions between the land references and the NOI, so that in a noise-free scenario, they satisfy the following equations:

$$d_i = \sqrt{(x - x_i)^2 + \left(y - y_i\right)^2} \tag{3.3}$$




$$d_1 = c(t_1 - t_0) = \sqrt{x^2 + y^2}$$

$$d_2 = c(t_2 - t_0) = \sqrt{(D - x)^2 + y^2}$$

$$d_3 = c(t_3 - t_0) = \frac{1}{2}\sqrt{(D - 2x)^2 + \left(D\sqrt{3} - 2y\right)^2}$$

$t_i$: TOA at reference $i$; $c$: speed of light

**FIGURE 3.2**

TOA PL estimation technique.

$$\phi_i = \tan^{-1}\left[\frac{y - y_i}{x - x_i}\right]. \tag{3.4}$$

Without loss of generality, we can assume that the fixed land reference 1 is positioned at the origin of the coordinate system as shown in Figure 3.2. Depending on the parameter used to obtain information and the corresponding processing carried out to estimate distances, the location will be defined geometrically by the intersection of circles, hyperbolas, or lines of bearing as discussed in the following subsections.

### 3.2.1 Geometric Multilateration Based on TOA Measurements

To determine the range between the NOI and a land reference, the TOA technique exploits the knowledge of the propagation speed of a wave (acoustic or electromagnetic) in a specific medium such as air or water. In a 2D scenario, three land references are required to perform a trilateration process as illustrated in

Figure 3.2. If the estimated distance between the NOI and the first land reference is denoted as $d_1$, then the mobile will be located on a circle of radius $d_1$ centered at the reference coordinates. Note that the use of a single land reference would yield an NOI position ambiguity equivalent to the circumference of the aforementioned circle. This position ambiguity will be reduced to the area created by the intersection of two circles when another land reference is added to the system. Finally, in a noise-free scenario, a third land reference will allow the intersection of a third circle that will cause the position ambiguity to be reduced to a single point that should correspond to the true position of the NOI.

TOA-based trilateration methods require knowledge of absolute propagation times; thus, clock synchronization between the NOI and the land references is essential to avoid errors in distance computation and location estimates. Assuming that the NOI initiates transmission of a signal at time $t_0$, and that this signal is received at the $i$-th land reference at time $t_i$, the range or distance estimate may be found as

$$d_i = c\,(t_i - t_0)\,, \tag{3.5}$$

where $c$ is the wave propagation speed (equal to the speed of light $c = 3 \times 10^8$ m/s for the case of an electromagnetic wave propagating in the air). Ranges from three land references to the NOI are estimated and are used in the trilateration process, which, considering the scenario shown in Figure 3.2, may be formulated using the following system of nonlinear equations:

$$\begin{aligned} d_1^2 &= x^2 + y^2, \\ d_2^2 &= (x_2 - x)^2 + (y_2 - y)^2 = (D - x)^2 + y^2, \\ d_3^2 &= (x_3 - x)^2 + (y_3 - y)^2 = \frac{1}{2}\,(D - 2x)^2 + \left(D\sqrt{3} - 2y\right)^2. \end{aligned} \tag{3.6}$$

Note that this system of equations consists of three circles centered at each of the reference points. With this system of equations, we can find two different solutions for $(x, y)$ that correspond to the two intersection points of the circles centered at their corresponding land references. The third equation in (3.6) allows us to solve the two-solution ambiguity by selecting the one that is closest to the range estimate given by the circle of radius $d_3$. One of the drawbacks of this method is that the third equation is not used in an optimal way to find the position location.

In a noise-free scenario, the three circles will intersect at exactly one point. Note, however, that in the case of noisy distance measurements, the circles will be displaced and their intersection will yield a polygon with an area that will correspond to a position ambiguity. It is common practice to use the final NOI position estimate as the centroid of this resulting polygon. Further, whenever more than three land references are available, one may obtain the NOI position estimate as the average, or centroid, of the estimated coordinates obtained from all possible trilateration realizations calculated with various triplet combinations of land references.

Although geometric trilateration procedures are simple to implement and rather computationally inexpensive, they are limited in the sense that they do not consider the reduction of noise effects and the optimal combination of multiple (more than three) range measurements. In the following sections, we will discuss basic techniques that help to overcome such problems by using statistical location-estimation methods such as least squares and maximum likelihood (ML), among others.

We can extend the TOA trilateration algorithm to the case of a 3D scenario as shown in Figure 3.3. In this scenario, each *i*-th reference with known position $(x_i, y_i, z_i)$ has a range estimate based on TOA that defines a sphere of radius $R_i$ around reference $i$ within which the NOI must be located. The ambiguity, in this case, is defined not only by two points of intersection but by curves of intersection between two spheres. This ambiguity can be resolved by using the third range estimate obtained from the remaining land reference. The system of equations for this scenario is given by

$$R_i^2 = (X - x_i)^2 + \left(Y - y_i\right)^2 + (Z - z_i)^2, \quad i = 1, 2, 3. \tag{3.7}$$

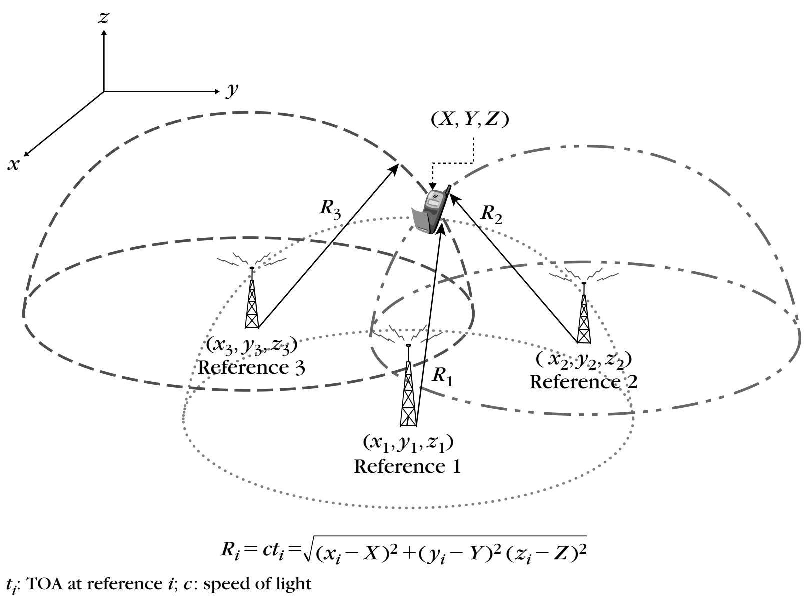



**FIGURE 3.3**

TOA in 3D.

We can see that each of the equations in (3.7) is a sphere of radius $R_i$, $i = 1, 2, 3$, centered at each of the references.

### 3.2.2 Geometric Multilateration Based on AOA Measurements

The AOA technique estimates the position location of an NOI by means of angular direction observations measured with respect to a reference axis using directional antennas or antenna arrays as shown in Figure 3.4. This direction, as it has been explained in Chapter 2, may be calculated through the phase differences of the elements of an antenna array.

At least two AOA measurements from two different references are necessary to provide the position location estimation of a mobile, one less than the number required for the TOA localization case. The system of equations for this scenario is given by

$$\begin{aligned} x &= d_i \cos(\phi_i) + x_i, \\ y &= d_i \sin(\phi_i) + y_i, \quad i = 1,\ 2,\ 3. \end{aligned} \tag{3.8}$$

A single AOA measurement restricts the location of the source along a line in the estimated line of bearing [22]. When multiple AOA measurements are

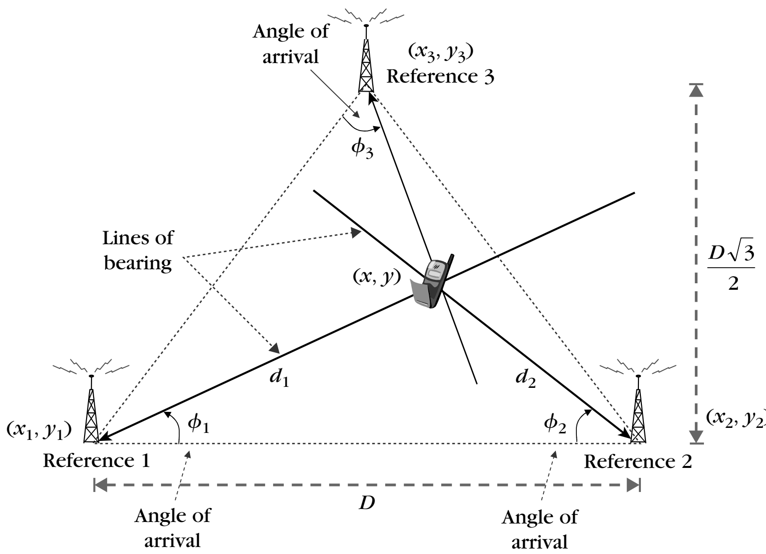



**FIGURE 3.4**

AOA technique.

obtained simultaneously by multiple land reference stations, a triangulation method may be used to form a location estimate of the NOI at the intersection of these lines [23]. In theory, direction-finding systems require only two receiving sensors to locate a mobile user, but in practice, and to improve accuracy, finite angular resolution, multipath, and noise often dictate the need for more than two references.

An advantage of AOA-based techniques is that they do not require clock synchronization; however, accurate angle measurements may require directional antennas, multiple-element antenna arrays, and possibly computationally expensive array processing algorithms.

### 3.2.3 Geometric Multilateration Based on TDOA Measurements

A variant of the TOA technique is the TDOA, which uses differences in the TOAs to locate the mobile. The TDOA involves the intersection of hyperbolas rather than circles as shown in Figure 3.5. The main advantage of TDOA over TOA-based methods is that the former do not require knowledge of the time at which a transmission took place at the NOI (i.e., $t_0$). Therefore strict time synchronization between an NOI and land references is not required. Note, however, that time synchronization among the different land references is still necessary.

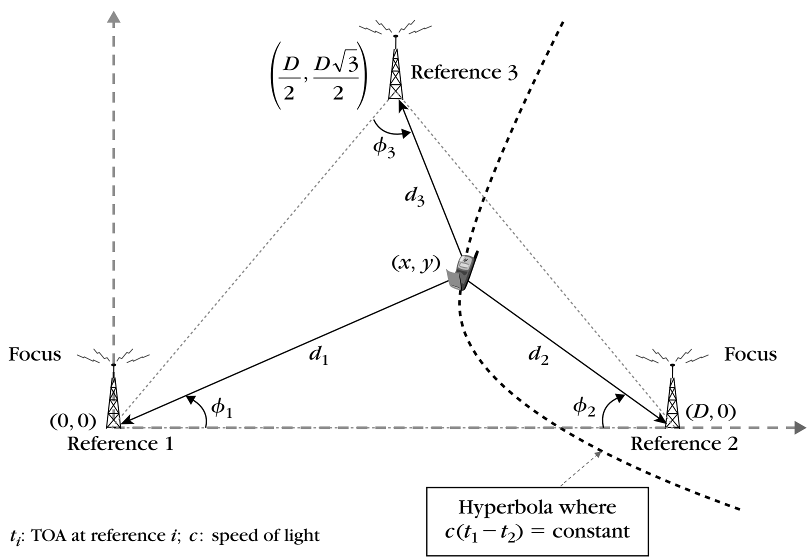



**FIGURE 3.5**

TDOA technique.

The TDOA between land references $i$ and $j$ (i.e., $t_i - t_j$) may be used to obtain the distance difference, which may be written as

$$d_{ij} = d_i - d_j = c\,(t_i - t_0) - c\left(t_j - t_0\right) = c\left(t_i - t_j\right),\ \ i = 1, 2,\ 3, j = 1, 2,\ 3,\ i \neq j. \quad \textbf{(3.9)}$$

Notice that the common time reference $t_0$ (i.e., the time at which the NOI started a transmission) cancels out in this calculation and hence, as it has been stated previously, its knowledge is not necessary. It follows from Equation (3.9) that

$$d_i = d_{ij} + d_j; \quad \textbf{(3.10)}$$

thus, it is possible to calculate the square of the range estimate seen by reference 2 as

$$\begin{aligned} d_2^2 &= (d_{21} + d_1)^2 \\ &= (x_2 - x)^2 + \left(y_2 - y\right)^2 \\ &= x_2^2 - 2x_2x + x^2 + y_2^2 - 2y_2y + y^2 \\ &= x_2^2 - 2x_2x + y_2^2 - 2y_2y + d_1^2, \end{aligned} \quad \textbf{(3.11)}$$

where the last equality follows from the fact that reference 1 is assumed to be at the coordinate system's origin (0, 0), such that

$$d_1^2 = x^2 + y^2. \quad \textbf{(3.12)}$$

We can rearrange terms in (3.12) to obtain

$$\left(d_{21}^2 - x_2^2 - y_2^2\right) + 2d_{21}d_1 = -2x_2x - 2y_2y, \quad \textbf{(3.13)}$$

and we can extend this procedure to obtain another equation as

$$\left(d_{31}^2 - x_3^2 - y_3^2\right) + 2d_{31}d_1 = -2x_3x - 2y_3y. \quad \textbf{(3.14)}$$

Since, except for the NOI coordinates, $(x, y)$, all the terms in the equations derived above are known, we can solve the system of equations to obtain the unknown NOI position.

Another variation of TOA-based techniques is the time sum of arrival (TSOA) of the propagating signal from an NOI to two land references to produce a range sum measurement. The range sum estimate defines an ellipsoid with foci at two receivers, and when multiple range sum measurements are obtained, the position location estimate of the user occurs at the intersection of the ellipsoids as shown in Figure 3.6. Consequently, range sum position location systems are also known as TSOA or elliptical position location systems. When the transmitted power at the mobile is known, the receiver can measure the signal strength

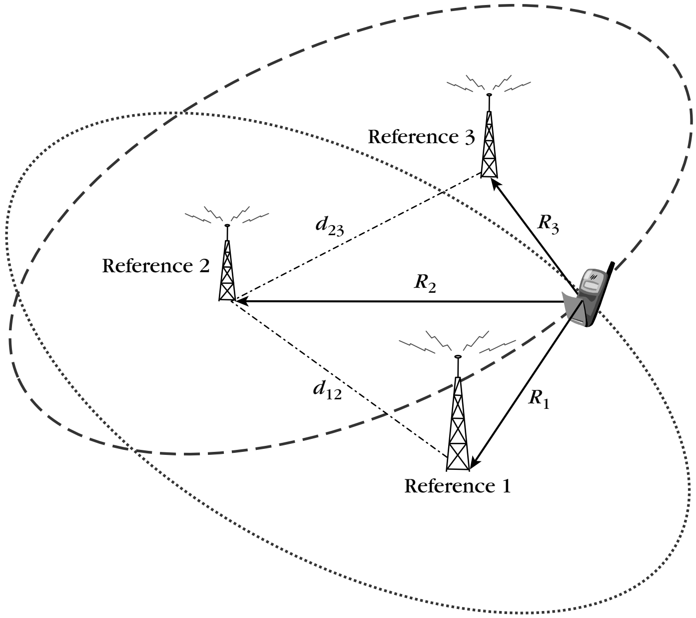



**FIGURE 3.6**

Technique using ellipsoids.

and provide a distance estimation between the mobile and the receiver using mathematical models of path loss that depend on distance.

## 3.3 STATISTICAL MULTILATERATION

In previous sections, we showed that geometric multilateration techniques rely on the assumption of noise-free measurements, and thus are strongly affected by noisy range or AOA observations. Position estimation ambiguities caused by noise effects can only be conciliated through heuristic methods such as finding the centroid of the resulting areas of uncertainty. Further, these techniques do not provide an optimal way to combine more than three observations that may well be needed in order to improve accuracy in noisy scenarios.

In what follows, we will present some statistical multilateration procedures that allow the optimal combination of three or more range- or AOA-related measurements in order to obtain a position estimate of an NOI while minimizing the effects of observation noise. Even measurements arising from different types of observations such as TOA, TDOA, and AOA may be combined into a single set of equations that may be solved optimally in order to reduce the effects of noise.

Consider the observation of $N$ noisy measurements collected in the vectorial observation model described in Equation (3.2) and rewritten here for convenience:

$$\mathbf{r} = \mathbf{f}(x, y) + \boldsymbol{\eta}. \tag{3.15}$$

The problem at hand is to combine all $N$ observations in an optimal way such that the noise effects are minimized. In the following, we will describe popular optimal statistical multilateration procedures commonly used for solving the PL problem.

### 3.3.1 Least-Squares Multilateration

Let us start by defining vector $\mathbf{q}$ as a bidimensional column vector containing the NOI unknown coordinates $(x, y)$ such that $\mathbf{q} = [x\,y]^H$. Then, given a set of $N$ noisy observations $\mathbf{r} = \mathbf{f}(\mathbf{q}) + \boldsymbol{\eta}$, the least-squares (LS) optimization problem may be posed as follows:

$$\hat{\mathbf{q}} = \arg\min_{\mathbf{q}} [\mathbf{r} - \mathbf{f}(\mathbf{q})]^H [\mathbf{r} - \mathbf{f}(\mathbf{q})]. \tag{3.16}$$

Whenever the noise covariance matrix $\mathbf{K}$ is known, a variant of the LS problem called the weighted LS (WLS) problem [23] may be posed as

$$\hat{\mathbf{q}} = \arg\min_{\mathbf{q}} [\mathbf{r} - \mathbf{f}(\mathbf{q})]^H \mathbf{K}^{-1} [\mathbf{r} - \mathbf{f}(\mathbf{q})], \tag{3.17}$$

where the cost function is defined as

$$J = [\mathbf{r} - \mathbf{f}(\mathbf{q})]^H \mathbf{K}^{-1} [\mathbf{r} - \mathbf{f}(\mathbf{q})]. \tag{3.18}$$

In this last problem, the inverse covariance matrix $\mathbf{K}^{-1}$ acts as a weighting factor that emphasizes measurements containing the smallest noise variances.

Clearly, problems described in Equations 3.16 and 3.17 will be nonlinear since $f_i(\mathbf{q})$, $i = 1, \ldots, N$ may take the form of the nonlinear Euclidean distance or line-of-bearing angle equations as they appear in Equations (3.3), and (3.4), respectively. These nonlinear LS problems may be solved by numerous numeric minimization techniques such as the interior-reflective Newton algorithm described in Coleman and Li [2]. Solution of the nonlinear LS problem may pose several problems, such as high computational loads and, more importantly, convergence issues caused by local minimum points. To avoid these issues, a linearization procedure, based on a first-order Taylor series expansion of function vector $\mathbf{f}(\mathbf{q})$ around a known position vector $\mathbf{q_0} = [x_0\,y_0]^H$, may be applied

[4, 25]. The result of this procedure will be a linearized LS (LLS) problem whose solution is well known and straightforward to obtain [23]. Let us then present the derivation of the LLS algorithm.

It is easy to show that the first-order Taylor series expansion of the function vector $\mathbf{f}(\mathbf{q})$ around the known point $\mathbf{q_0} = [x_0 y_0]^H$ (it will be assumed that the value $\mathbf{q_0}$ is sufficiently close to the true position vector $\mathbf{q}$ so that the series expansion is an accurate approximation) is given by

$$\mathbf{f}_l(\mathbf{q}) = \mathbf{f}(\mathbf{q_0}) + \mathbf{D}(\mathbf{q} - \mathbf{q_0}), \tag{3.19}$$

where subindex $l$ in term $\mathbf{f}_l(\mathbf{q})$ stands for the *linearized* approximation, and $\mathbf{D}$ is a $N \times 2$ matrix given by

$$\mathbf{D} = \begin{bmatrix} \frac{\partial f_1(\mathbf{q})}{\partial x}\Big|_{\mathbf{q}=\mathbf{q_0}} & \frac{\partial f_1(\mathbf{q})}{\partial y}\Big|_{\mathbf{q}=\mathbf{q_0}} \\ \vdots & \vdots \\ \frac{\partial f_N(\mathbf{q})}{\partial x}\Big|_{\mathbf{q}=\mathbf{q_0}} & \frac{\partial f_N(\mathbf{q})}{\partial y}\Big|_{\mathbf{q}=\mathbf{q_0}} \end{bmatrix}. \tag{3.20}$$

For instance, if all functions in $\mathbf{f}(\mathbf{q})$ correspond to Euclidean distances between land references with known positions $(x_i, y_i)$, $i = 1 \ldots N$, and a NOI, terms in matrix $D$ become

$$\frac{\partial f_i(\mathbf{q})}{\partial x}\Big|_{\mathbf{q}=\mathbf{q_0}} = \frac{x_0 - x_i}{\left[(x_i - x_0)^2 + (y_i - y_0)^2\right]^{1/2}}, \qquad i = 1 \ldots N, \tag{3.21}$$

and

$$\frac{\partial f_i(\mathbf{q})}{\partial y}\Big|_{\mathbf{q}=\mathbf{q_0}} = \frac{y_0 - y_i}{\left[(x_i - x_0)^2 + (y_i - y_0)^2\right]^{1/2}}, \qquad i = 1 \ldots N. \tag{3.22}$$

Substituting the linearized function vector given in Equation (3.19) into the cost function in Equation (3.18), and redefining the observation vector as

$$\mathbf{r}' = \mathbf{r} - \mathbf{f}(\mathbf{q_0}) + \mathbf{D}\mathbf{q_0}, \tag{3.23}$$

the cost function becomes

$$J = (\mathbf{r}' - \mathbf{D}\mathbf{q})^H \mathbf{K}^{-1} (\mathbf{r}' - \mathbf{D}\mathbf{q}), \tag{3.24}$$

which is now clearly linear.

Minimizing (3.24) with respect to $\mathbf{q}$ yields the linearized WLS estimator, which is given by [23] as

$$\hat{\mathbf{q}} = (\mathbf{D}^H \mathbf{K}^{-1} \mathbf{D})^{-1} \mathbf{D}^H \mathbf{K}^{-1} \mathbf{r}', \tag{3.25}$$

where the hat superscript denotes an estimated quantity.

Expanding $\mathbf{r}'$ as in (3.23) and rearranging terms, the estimator can be rewritten as

$$\hat{\mathbf{q}} = \mathbf{q_0} + (\mathbf{D}^H \mathbf{K}^{-1} \mathbf{D})^{-1} \mathbf{D}^H \mathbf{K}^{-1} [\mathbf{r} - \mathbf{f}(\mathbf{q_0})]. \tag{3.26}$$

This last equation suggests the possibility of finding the WLS estimate $\hat{\mathbf{q}}$ with a recursion that uses, at each step, the actual estimate of $\hat{\mathbf{q}}$ as the value of $\hat{\mathbf{q}}_{\mathbf{0}}$. This recursion is given as

$$\hat{\mathbf{q}}(n+1) = \hat{\mathbf{q}}(n) + (\mathbf{D}^H\mathbf{K}^{-1}\mathbf{D})^{-1}\mathbf{D}^H\mathbf{K}^{-1}[\mathbf{r} - \mathbf{f}(\hat{\mathbf{q}}(n))]. \tag{3.27}$$

It has been observed that this recursion converges even when the initial $\hat{\mathbf{q}}(0)$ value is far from the true position vector $\mathbf{q}$ [17].

Finally, it is easy to show, after some algebraic manipulations, that the bias and covariance of the WLS estimator are given respectively by

$$\mathbf{b} = E\{\hat{\mathbf{q}}\} - \mathbf{q} = (\mathbf{D}^H\mathbf{K}^{-1}\mathbf{D})^{-1}\mathbf{D}^H\mathbf{K}^{-1}[\mathbf{f}(\mathbf{q}) - \mathbf{f}_l(\mathbf{q})], \tag{3.28}$$

and

$$\mathbf{C} = E\{[\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\}][\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\}]^H\} = (\mathbf{D}^H\mathbf{K}^{-1}\mathbf{D})^{-1}, \tag{3.29}$$

where $\mathbf{f}_l(\mathbf{q})$ is the linearized function vector given in (3.19). Clearly, the bias is nonzero due to the linearization error $\mathbf{f}(\mathbf{q}) - \mathbf{f}_l(\mathbf{q})$. Also, the covariance of the estimator is dependent on both the covariance matrix of the distance observations $\mathbf{K}$ and the linearization matrix $\mathbf{D}$. Note that the covariance expression $(\mathbf{D}^H\mathbf{K}^{-1}\mathbf{D})^{-1}$ appears in the recursion in Equation (3.27). This allows us to compute both the position estimate and its covariance simultaneously.

Up to this point no assumptions have been made about the probability distribution of noise vector $\boldsymbol{\eta}$. Note, however, that if this vector is Gaussian distributed, then the estimator given by (3.25) corresponds to the maximum likelihood estimator.

### 3.3.2 LS Multilateration with Uncertain Reference Node Positions

Up to this point, we have assumed perfect knowledge of the position coordinates of every land reference in the system. This condition will hardly be met in a real system due to measurement and mapping errors. If not taken into consideration, land reference position uncertainties may strongly degrade performance of the PL techniques.

A modified LWLS position estimation algorithm may be used to account for land reference position uncertainties. This algorithm was presented in Kovavisaruch and Ho [9], and is summarized in this section.

Let us denote the set of true position coordinates for the $i$-th land reference as $\mathbf{q}_i = [x_i\, y_i]^H$, $i = 1 \ldots N$ (vector $\mathbf{q} = [\mathbf{x}\, \mathbf{y}]^{\mathbf{H}}$ without any subindex still corresponds to the unknown position coordinates of the NOI), and the corresponding set of uncertain land reference position observations as

$$\tilde{\mathbf{q}}_i = \mathbf{q}_i + \boldsymbol{\Delta}_i = [\tilde{x}_i\, \tilde{y}_i]^H = [x_i + \Delta_{x_i}\, y_i + \Delta_{y_i}]^H,\ i = 1 \ldots N, \tag{3.30}$$

where $\Delta_{x_i}$ and $\Delta_{y_i}$ are coordinate error terms that may be grouped into vector $\boldsymbol{\Delta}_i = [\Delta_{x_i}\ \Delta_{y_i}]^H$, $i = 1 \ldots N$. We may then define a $3N$-dimensional extended vector of observations as

$$\tilde{\mathbf{s}} = [\mathbf{r}^H\ \tilde{\mathbf{q}}_1^H \ldots \tilde{\mathbf{q}}_N^H]^H, \tag{3.31}$$

where $\mathbf{r}$ corresponds to the $N$ noisy range- or AOA-related observations as described in Equation (3.15), and $\tilde{\mathbf{q}}_i$, $i = 1 \ldots N$ are the available erroneous land reference position measurements. Equation (3.31) may be written in extended form as

$$\begin{bmatrix} \mathbf{r} \\ \tilde{\mathbf{q}}_1 \\ \vdots \\ \tilde{\mathbf{q}}_N \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{q}) \\ \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_N \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{\Delta}_1 \\ \vdots \\ \boldsymbol{\Delta}_N \end{bmatrix}, \tag{3.32}$$

or simply

$$\tilde{\mathbf{s}} = \mathbf{s} + \mathbf{n}, \tag{3.33}$$

where $\mathbf{s} = [\mathbf{f}(\mathbf{q})^H\ \mathbf{q}_1^H \ldots \mathbf{q}_N^H]^H$, and $\mathbf{n} = [\boldsymbol{\eta}^H\ \boldsymbol{\Delta}_1^H \ldots \boldsymbol{\Delta}_N^H]$.

We may assume that the extended noise vector $\mathbf{n}$ has zero mean and that the range- or AOA-related observation noise vector $\boldsymbol{\eta}$ and the land reference position error vectors $\boldsymbol{\Delta}_i$, $i = 1 \ldots N$ are statistically independent. Thus, the extended noise vector covariance matrix may be written as

$$\mathbf{Q} = E\{\mathbf{n}\mathbf{n}^H\} \begin{bmatrix} \mathbf{K} & \mathbf{0}^H \\ \mathbf{0} & \mathbf{K}_\Delta \end{bmatrix}, \tag{3.34}$$

where $\mathbf{0}$ corresponds to a $(2N \times N)$ matrix of zeros, $\mathbf{K}$ is the $(N \times N)$-dimensional covariance matrix of noise vector $\boldsymbol{\eta}$ as described in Section 3.1, and $\mathbf{K}_\Delta$ is the $(2N \times 2N)$-dimensional covariance matrix of the overall vector of land reference position errors $[\boldsymbol{\Delta}_1^H \ldots \boldsymbol{\Delta}_N^H]^H$.

If we treat the true NOI, and land reference coordinates as unknown parameters and group them in vector $\boldsymbol{\theta}$ such that $\boldsymbol{\theta} = [x\ y\ x_1\ y_1 \ldots x_N\ y_N]^H$, then we may pose the following WLS problem to estimate this parameter vector as follows:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} [\tilde{\mathbf{s}} - \mathbf{s}(\boldsymbol{\theta})]^H \mathbf{Q}^{-1} [\tilde{\mathbf{s}} - \mathbf{s}(\boldsymbol{\theta})], \tag{3.35}$$

where we have now emphasized the dependence of the noiseless observation vector on parameter $\boldsymbol{\theta}$. Again, we note that this is a nonlinear least-squares problem whose solution may involve large computational loads and convergence problems due to local minima. A linearized WLS problem may, however, be derived following the same Taylor series expansion procedure used in Section 3.3.1. Let us then approximate the noiseless observation vector $\mathbf{s}$ using its first-order Taylor series expansion around a known parameter vector $\boldsymbol{\theta}_0$ so that

$$s_l(\boldsymbol{\theta}) = s(\boldsymbol{\theta}_0) + \mathbf{D}(\boldsymbol{\theta} - \boldsymbol{\theta}_0), \tag{3.36}$$

where now the linearization matrix $\mathbf{D}$ corresponds to

$$\mathbf{D} = \begin{bmatrix} \frac{\partial \mathbf{f}(\mathbf{q})}{\partial \mathbf{q}} & \frac{\partial \mathbf{f}(\mathbf{q})}{\partial \mathbf{q}_1} & \cdots & \frac{\partial \mathbf{f}(\mathbf{q})}{\partial \mathbf{q}_N} \\ \frac{\partial \mathbf{q}_1}{\partial \mathbf{q}} & \frac{\partial \mathbf{q}_1}{\partial \mathbf{q}_1} & \cdots & \frac{\partial \mathbf{q}_1}{\partial \mathbf{q}_N} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{q}_N}{\partial \mathbf{q}} & \frac{\partial \mathbf{q}_N}{\partial \mathbf{q}_1} & \cdots & \frac{\partial \mathbf{q}_N}{\partial \mathbf{q}_N} \end{bmatrix} \Bigg|_{\boldsymbol{\theta} = \boldsymbol{\theta}_0}, \tag{3.37}$$

and where we note that $\frac{\partial \mathbf{q}_i}{\partial \mathbf{q}} = [\mathbf{0}]_{2 \times 2}$, $\frac{\partial \mathbf{q}_i}{\partial \mathbf{q}_j} = [\mathbf{0}]_{2 \times 2}$ for $i \neq j$, and $\frac{\partial \mathbf{q}_i}{\partial \mathbf{q}_i} = [\mathbf{I}]_{2 \times 2}$. Finally, following the same procedure as in Section 3.3.1, the estimate of vector $\boldsymbol{\theta}$ may be found recursively as follows:

$$\hat{\boldsymbol{\theta}}(n+1) = \hat{\boldsymbol{\theta}}(n) + (\mathbf{D}^H \mathbf{Q}^{-1} \mathbf{D})^{-1} \mathbf{D}^H \mathbf{Q}^{-1} [\tilde{\mathbf{s}} - \mathbf{s}(\hat{\boldsymbol{\theta}}(n))]. \tag{3.38}$$

Note that apart from estimating the unknown NOI coordinates, this algorithm also estimates the land reference positions that were originally known with uncertainty. The overall effect of this procedure is to obtain better NOI position estimates while also obtaining corrections for the erroneous land reference coordinates.

### 3.3.3 Hybrid Location Estimation Systems

The WLS criterion does not pose any condition on the type of equations that may be used in the minimization problem described in Equation (3.17). This means that we may combine different types of measurements to obtain position estimates. For instance, a subset of the $N$ equations may correspond to AOA observations, while another subset may correspond to range observations obtained from TOA measurements, and another subset may correspond to range difference observations obtained from TDOA measurements. Location estimation systems that combine different types of observations are commonly referred to as *hybrid* localization systems.

## 3.4 LOCATION ESTIMATION IN MULTIHOP SCENARIOS

As stated in the previous section, geographic and statistical multilateration techniques rely on the estimation of range between a node with unknown location and a set of land reference nodes whose location is fixed and known. These techniques assume that direct signal transmissions between the land references and the NOI can be established at any time so that TOA, TDOA, AOA, or RSS observations are available for the estimation of range and/or line of bearing. GPS and cellular localization systems are examples of schemes that apply direct multilateration to estimate the position of a device or mobile phone, respectively. Algorithms that relay on direct connectivity between land references and an NOI are usually referred to as single-hop schemes.

In the case of ad hoc and mobile sensor-network scenarios, a mobile node whose location must be estimated may not have enough transmission power to establish a direct connection to any or some land references in the network, and the direct multilateration techniques treated in the previous sections of this chapter will not be applicable. In this type of environments, two nodes that are not within the maximum transmission radius of each other communicate by means of a concatenation of multiple links between intermediate neighboring nodes. Since a multilateration process cannot be applied directly in an ad hoc environment due to the lack of direct connectivity of nodes to well-located land references, multihop algorithms are needed.

Iterative algorithms are a popular type of multihop position-estimation scheme [13, 15]. In these schemes, any type of useful data, such as land reference coordinates and/or measurements of proximity, range, and AOA between network nodes, is propagated iteratively throughout one or more sectors of the network so that, after a number of iterations, an NOI that was initially away from the neighborhood of any land reference nodes may receive this information and use it to estimate its position. During the iteration process, intermediate nodes may also be able to estimate their positions and may act as newly acquired anchor nodes if desired. Depending on the size of the network sector and the number of intermediate nodes involved in the data propagation process, iterative algorithms may be classified as localized or nonlocalized [13].

In a typical nonlocalized iterative algorithm, a node that is in the vicinity (within its transmission reachability radius) of more than one anchor node may be able to estimate its position using multilateration techniques based on (possibly combined) measurements of proximity, range, and AOA. When this node accomplishes its position estimation procedure, it may now act as a new anchor node (with reduced position accuracy) and assist its neighboring nodes in their own position estimation processes. Newly generated anchors are iteratively propagated through the network until all nodes locate themselves or until sufficient information reaches an NOI so that it can be localized. In order to request the location of an NOI at a specific time when this node is not in the neighborhood of a sufficient number of anchor nodes, several intermediate nodes may need to be located throughout the network in order to propagate the necessary anchors into the NOI's reachability region. The number of necessary intermediate nodes that need to be involved in the iterative process in order to reach an NOI will depend on the network density, the nodes' reachability radii, and the position and density of original land references. Depending on these parameters, and on the position estimation algorithm itself, the region occupied by the intermediate nodes may expand a large sector or even the complete network area. Nonlocalized methods are highly sensitive to anchor densities and can get stuck in places where these densities are sparse. Further, error propagation of newly created anchor nodes may become an issue in large network scenarios.

Localized iterative methods, on the other hand, do not require the NOI to be at one-hop proximity to anchor nodes. The only requirement is that the NOI has connectivity, via multihop routes, to three or more land references. Hence, the only intermediate nodes involved in the NOI position estimation process are the nodes that forms those routes. If shortest-distance routing is used in the network, then it is clear that this type of algorithm will only involve intermediate nodes located in limited network regions that lie in the shortest paths between land references and the NOI. It is important to note that these algorithms will not necessarily require that intermediate nodes become anchors (i.e., there is no need to estimate their coordinates) as was the case in nonlocalized techniques. For these reasons, localized algorithms considerably reduce computational load, power consumption, communications traffic, and the time needed to obtain a position estimate of a specific single NOI.

Some nonlocalized methods are based on multilateration of propagated anchor nodes and on the solution of computationally intensive global nonlinear optimization problems to reduce error propagation effects [3, 20, 21]. Other methods may involve obtaining pairwise range estimates between every node in the network in order to solve an ML problem that estimates the position of all network nodes at once [14]. Simpler methods may involve purely geometric range-free solutions such as the centroid localization algorithm [1] where the NOI estimates its position as the centroid of neighboring anchor nodes, or the approximate point-in-triangulation (APIT) algorithm [6] where anchor nodes are generated (and then propagated through the network until the NOI is reached) by calculating the maximum overlap of triangular regions formed by triplets of neighboring anchor nodes. Other simple nonlocalized iterative position estimation methods called Euclidean and DV-bearing (DV stands for distance vector) are presented in Niculescu and Nath [11, 12]. These schemes require an NOI to be in the proximity of at least two neighboring nodes that know their distance to a land reference. The Euclidean method is based on ranging while the DV-bearing method is based on ranging and AOA measurements. APIT, Euclidean, and DV-bearing algorithms do not take into consideration error propagation of newly created beacons, and may be considerably affected by shadowing and small-scale fading, as has been posed in Zhou et al. [5] and He et al. [6].

A well-known, localized iterative position estimation scheme is the ad hoc positioning system (APS) algorithm [11]. APS has two variations called DV-hop and DV-distance. DV-hop uses mere connectivity to infer range estimates of an NOI to several land references. It employs two stages, a calibration stage in which the average size of a hop is estimated by a land reference and distributed to nearby nodes, and a second stage in which an NOI measures the number of hops required to reach different land references via the shortest possible paths. Land references know their exact coordinates and the exact coordinates of all other land references in the network so they are able to estimate average hop sizes by communicating constantly with each other and measuring the number of hops required for a message to travel the distance between them. Distance in hops from an NOI to a land reference is then multiplied by the estimated size of a hop to produce a range estimate. If the NOI is able to obtain range estimates to three or more land references, then it can perform multilateration to find its position estimate. A variation of DV-hop is the DV-distance algorithm where the NOI uses measured distances between nodes at each hop in a shortest-path connection to a land reference and adds these distances together to obtain an overestimate of its range to that land reference. This overestimation is corrected by applying a correcting factor whose value is continuously being broadcasted by the land references in the network. Similar to the DV-hop case, the correcting factor is estimated by the land references in the network while they communicate with each other and compare resultant hop-distance sums to actual geographic distances.

Disadvantages of these relatively simple methods are that they require a uniform geographic distribution of nodes, nodes with similar transmission power and antenna gains, and a large node density in order for hop sizes and number of hops per unit distance to be fairly constant throughout the network. These methods do not take into consideration measurement error statistics, and their performance may degrade considerably in the presence of estimation noise. These algorithms are very sensitive to channel variations due to the fact that the hop-size estimation becomes less precise in the presence of irregular radio reception patterns. Examples of this behavior have been presented in [6]. Finally, because DV-hop–based algorithms use flood-based correction factor propagation, they generate large communications overhead and power consumption.

To overcome the limitations of the DV-hop and DV-distance schemes, a localized iterative technique based on a dead-reckoning–like scheme has been proposed in Hernandez et al. [7]. In this scheme, error accumulation effects are accounted for by modeling of the statistical behavior of ranging and AOA noisy measurements at each hop. As in DV-hop and DV-distance, this algorithm does not require the NOI to be at a one-hop proximity from at least three anchors. Instead, the requirement consists of at least three land references being able to establish a link with any NOI via multiple hops.

Multihop algorithms may also be classified as centralized or distributed [7, 13]. This category division has direct impact on the applicability of the algorithm and on the efficiency of the positioning system. Centralized algorithms collect measurements of the entire network at a central node that optimizes for the position estimation of the NOI. Besides gathering measurements of the entire network in a single main powerful central node that must deal with large and complex data structures, these algorithms may face the problem of communication bottleneck and higher energy drain at and near the central node caused by the large amount of traffic associated with large networks.

Distributed algorithms require no specialized central node because nodes share information with their neighbors in an iterative fashion. These algorithms rely on the propagation of anchor nodes in the network until the NOI ends up being surrounded by them. These algorithms have the advantage of load balancing because they do not depend on a single central node that represents a single failure point, where, if this node fails, the entire chain breaks down and no localization can be achieved. As they iteratively share information among themselves, nodes must be capable of computing data and handling the necessary calculations.

In the following sections, we will describe some of the aforementioned multihop localization techniques. Most of them rely on the multilateration problem described in Section 3.1, but some use another type of heuristic to obtain position estimates.

### 3.4.1 Centroid Algorithm

Position estimation algorithms that do not use any type of signal measurement to infer range or AOA information between a land reference and an NOI are usually referred to as *range-free* schemes. The centroid algorithm, proposed in Bulusu et al. [1], falls into this category.

Consider an NOI that is in the vicinity of $N$ anchor nodes with coordinates $(x_i, y_i)$, $i = 1 \ldots N$. The anchor nodes communicate and transmit their coordinate points to the NOI. After receiving the anchor coordinates, the NOI simply estimates its location as the centroid of those points. The centroid of all anchor coordinates is calculated as follows:

$$(\hat{x}, \hat{y}) = \left( \frac{1}{N} \sum_{i=1}^{N} x_i,\ \frac{1}{N} \sum_{i=1}^{N} y_i \right). \tag{3.39}$$

The centroid algorithm is a nonlocalized distributed scheme where an NOI needs to be in the vicinity of $N$ anchor nodes. Iterative anchor node propagation may be necessary if, initially, no anchor nodes exist in the vicinity of the NOI. Note that the centroid algorithm may also be applicable to single-hop scenarios, but the accuracy will degrade as the reachability radii of the network devices increases.

Although this algorithm may suffer from limited accuracy, it has the advantage of being very simple and easy to implement.

### 3.4.2 Approximate Point-in-Triangulation Algorithm

APIT [6] is a nonlocalized iterative algorithm that uses beacon transmissions from anchor nodes. It employs an area-based approach to perform location estimation by isolating the environment into triangular regions between beaconing nodes as shown in Figure 3.7. A node's presence inside or outside these triangular regions allows this node to narrow down the area in which it can potentially reside. By using combinations of anchor positions, the diameter of the estimated area in which a node resides can be reduced to provide accurate location estimates.

The theoretical method used to narrow down the possible area in which a node resides is called the point-in-triangulation (PIT) test. In this test, a node chooses three anchors from all audible anchors and tests whether it is inside the triangle formed by connecting these three anchors. APIT repeats this PIT test with different audible anchor triplets until all combinations are exhausted or the required accuracy has been achieved. In the final step, APIT calculates the center of gravity (COG) of the intersection of all the triangles in which a node resides to determine its estimated position. Then the APIT algorithm may be summarized in four steps: (1) beacon exchange, (2) PIT testing, (3) APIT aggregation, and (4) COG calculation.

The critical component of the APIT scheme is to determine whether an NOI is inside a triangle formed by three anchor nodes. Theoretically, if an NOI is inside a triangle formed by anchor nodes $A$, $B$, $C$, as shown in Figure 3.8,

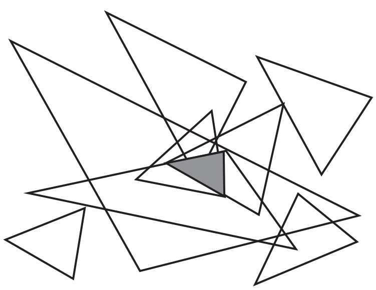


**FIGURE 3.7**

APIT is based on triangular intersections.

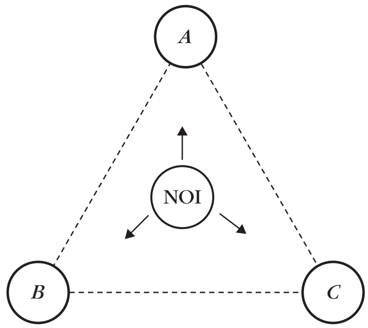



**FIGURE 3.8**

PIT test.

when this node is shifted in any direction, the new position must be nearer to (or further from) at least one anchor $A$, $B$, or $C$. On the other hand, if the NOI is outside the $ABC$ triangle, when the NOI is shifted, there must exist a direction in which the position of this NOI is further or closer to all three anchors $A$, $B$, $C$ simultaneously. Although theoretically correct, in practice it is impossible to recognize the directions of departure of the NOI without its actual movement. Further, exhaustively testing all possible directions in which the NOI might depart or approach nodes $A$, $B$, and $C$ simultaneously is impossible. For these reasons, an approximate PIT test must be implemented in practice.

The approximate PIT test relies on the assumption that in a certain propagation direction, defined to be within a narrow angle from the transmitting anchor, the received signal strength decreases monotonically with distance. Approximate PIT uses neighbor information, exchanged via beaconing, to emulate the node movement required in the theoretical test. Referring to Figure 3.9, if no neighbor of the NOI is further from/closer to all three anchors $A$, $B$, $C$ simultaneously, the NOI assumes that it is inside the triangle $ABC$. Otherwise, the NOI assumes that it resides outside this triangle. Although this is a powerful test, it may suffer from errors in cases when the NOI resides near the edge of the triangle, or when the NOI is placed in the very improbable position where the simultaneous further from/closer to condition in relation to all three anchors $A$, $B$, $C$ does not hold.

Once an individual PIT test is finished, APIT aggregates the results (inside/outside decisions) by means of the grid shown in Figure 3.10. The network area is divided into small square regions on a grid and this grid array is used to represent the maximum area in which a node will likely reside. For each PIT

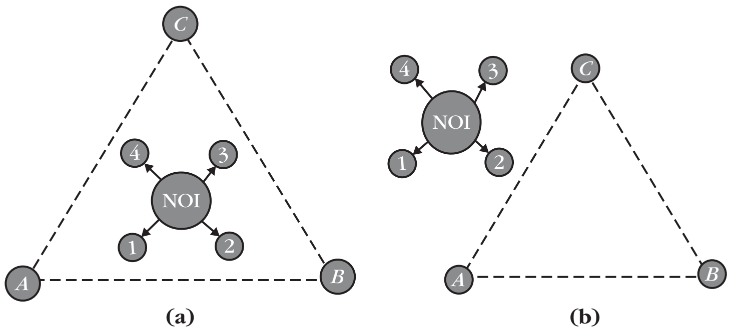



**FIGURE 3.9**

Approximate PIT test. (a) Inside case. (b) Outside case.

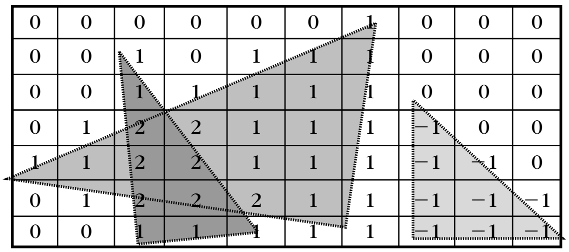



**FIGURE 3.10**

Aggregation procedure.

*inside* decision, the values of the grid regions over which the corresponding triangles reside are incremented. On the other hand, for an *outside* decision, the grid area is decremented. After all triangular regions have been computed, the resulting information is used to find the maximum overlapping area (i.e., the grid squares with the largest counters equal to two in the example figure), which is then used to calculate the center of gravity for position estimation.

Although simple to implement, APIT may suffer from irregular node transmission patterns, and from large- and small-scale fading that may prevent received signal strength from reducing monotonically with distance, in which case PIT test failure rates will increase. Further, APIT accuracy will be strongly dependent on the grid size used in the aggregation step of the algorithm. Finally, APIT is very sensitive to the network's anchor node density. Very few anchor nodes will not allow enough triangular regions to overlap and, in this scenario, the accuracy of the algorithm will decrease.

### 3.4.3 Ad Hoc Positioning System Algorithms

As the name implies, ad hoc positioning system (APS) algorithms were specifically designed for ad hoc scenarios. They consist of relatively simple multihop schemes that obtain node position estimates based on indirect measurements of distance [11] or AOA [12] between anchor nodes and NOIs. In this section, we will describe four APS modalities called Euclidean, DV-bearing, DV-hop, and DV-distance. Here, the acronym *DV* stands for distance vector, which in turn refers to the class of algorithms that rely on distance observations based on calculations of the magnitude of vectorial sums obtained from range and/or AOA measurements at each hop of a multihop link connecting an anchor node to an NOI.

#### *Euclidean APS*

This is a nonlocalized iterative positioning scheme that propagates Euclidean distance estimates to a land reference via neighboring nodes. (Figure 3.11). Suppose that NOI *A* has at least two neighbors *B* and *C*, which have already obtained estimates of their Euclidean distance to the anchor node *L*. Further suppose that node *A* has also measured estimates for distances *AB*, *AC*, and that nodes *B* and *C*, which are also assumed to be neighbors, have communicated their distance *BC* to *A*. In this scenario, the lengths of all sides of the quadrilateral *ABCL*, and one of its diagonals *BC*, are known. This allows NOI *A* to compute the second diagonal *AL*, which corresponds to its distance to the land reference. Clearly, this scheme allows node *A* to infer its distance to land reference *L* by means of distance knowledge that was previously acquired by neighboring nodes *B* and *C*, possibly through the same propagation mechanism with the help of other neighboring nodes. In fact, node *A* may now be available to assist other nodes in their calculation of range to that same land reference *L*.

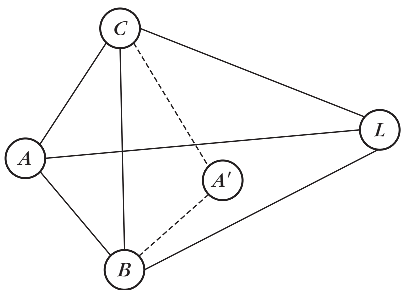



**FIGURE 3.11**

Euclidean APS scheme.

The previously described propagation mechanism may be applied by every node in the network to find their range to $N \geqslant 3$ land references. Once this is done, one of the multilateration techniques as described in Section 3.1 may be used to obtain final node position estimates.

Note in Figure 3.11 that there is a possibility that node $A$ lies to the right of the $BC$ line; this possibility has been depicted here using node $A'$. In this case, the distance to node $L$ will be different. The choice between the two possibilities should be made locally by node $A$, either by comparison to other neighbors that already have an estimate of range to $L$, or by examining the relation with other common neighbors of $B$ and $C$. Node $A$ may have to delay its decision until enough neighboring nodes with estimated Euclidean distances to $L$ are available to render the comparisons reliable.

### *Distance Vector–Bearing APS*

DV-bearing is similar to the Euclidean method in the sense that bearing information available at neighboring nodes is propagated iteratively to other nodes that have no AOA information available to estimate their positions using a multilateration scheme. Referring to Figure 3.12, suppose that NOI $A$ knows its bearings to immediate neighbors $B$ and $C$. These bearings have been denoted by angles $\hat{b}$ and $\hat{c}$, respectively. Note that the angle measurement is done with respect to a reference axis established arbitrarily by node $A$. Nodes $B$ and $C$, in turn, know their bearings to a distant land reference node $L$. The problem is for $A$ to find its bearing to $L$ (denoted by the dashed arrow in Figure 3.12). If $B$ and $C$ are neighbors, then $A$ has the possibility to find all the angles in triangles $ABC$ and $BCL$. This would allow $A$ to find the angle $\widehat{LAC}$, which yields the bearing of $A$ with respect to $L$ as $\hat{c} + \widehat{LAC}$. In this sense, neighbors $B$ and $C$ have propagated their bearing knowledge, allowing $A$ to find its bearing to $L$. Now $A$ may

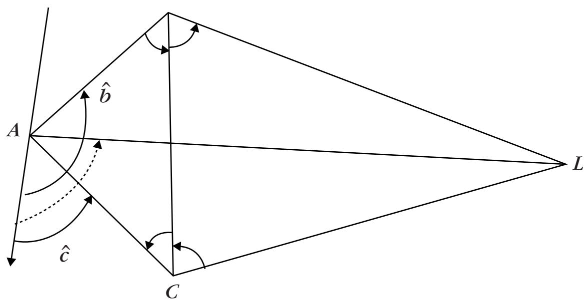



**FIGURE 3.12**

DV-bearing scheme.

assist other closest neighbors in acquiring their bearings to $L$ as well. In the end, when every node in the network has been able to estimate at least two bearings with respect to two land reference nodes, AOA-based multilateration techniques such as those treated in Section 3.1 may be used to obtain their final position estimates.

### *Distance Vector–Hop APS*

DV-hop is a very simple localized multihop localization scheme that translates the number of hops in a multihop link into distance in meters by means of a correcting factor that is propagated through the network by the anchor nodes. This method has two stages, calibration and multilateration.

Suppose that $N$ anchor nodes are available and that these nodes have perfect knowledge of their own position and of the position of all other anchor nodes in the network such that they may calculate all the true Euclidean distances between each other. Then, if the $i$-th and $j$-th anchor nodes establish a communication path via a multihop link, they will be able to relate the number of hops $M_{ij}$ in the route to the actual distance $d_{ij}$ between them to find a correcting factor that corresponds to the number of meters per hop as

$$\mathrm{CHOP}_{ij} = \frac{d_{ij}}{M_{ij}} = \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{M_{ij}}. \tag{3.40}$$

If every anchor node in the network establishes a multihop link with all other anchor nodes (or a subset of them), an overall average correction factor may be obtained and broadcast to all the nodes in the network to complete the calibration stage.

Now consider an NOI that establishes communication with an anchor node through an $M$-hop multihop link. The NOI will then be able to estimate its range to the anchor node by simply multiplying the latest broadcasted average correcting factor by the number of hops in the route. Finally, if the NOI is able to establish multihop communication links with $N \geqslant 3$ anchor nodes, then a multilateration problem may be solved using one of the range-based geometric or statistical multilateration methods treated in Section 3.1.

DV-hop has the advantage of being quite simple and robust to distance estimation errors since it does not depend on TOA, TDOA, or RSS measurements. On the other hand, as has been shown in He et al. [6] and Perez [16], performance of the algorithm degrades in low node-density scenarios and in the presence of nonuniform node distributed networks. These situations will cause nonuniform hop sizes at each multihop route and a nonhomogeneous number of meters per hop at various network regions. The accuracy of the correcting factor calculation also degrades as the reachability radii of the network nodes increases, since fewer hops are required to complete a link, and thus fewer observations are available to accurately infer the hop size in meters. Another disadvantage of this algorithm is the increase in communication overhead caused by the anchor node–to–anchor node connections required to calculate correcting factors, and by the necessary broadcast of updated correcting factors to all nodes in the network.

Although DV-hop can be seen as a localized scheme where an NOI estimates its position via the multihop connection to $N$ land references, this algorithm may also be used as a nonlocalized scheme where every node in the network estimates its position [6]. In this setting, one anchor node broadcasts a beacon to be flooded throughout the network containing the anchor's position with a hop-count parameter initialized to one. Each receiving node maintains the minimum counter value per anchor of all beacons it receives and ignores those beacons with higher hop-count values. Beacons are flooded outward with hop-count values incremented at every intermediate hop. With this, all nodes in the network, including all other anchors, get the shortest distance, in hops, to every anchor. In the end, all nodes have a table that contains the set of hop-count values obtained from each of the available anchors, the anchor true coordinates, and the average correcting factor corresponding to the average hop length in meters. With this information, every node in the network may convert the hop counts to actual distances and apply a multilateration technique among those described in Section 3.1.

### *Distance Vector–Distance APS*

DV-distance is a variation of the DV-hop scheme in which hop counts are substituted by actual distance estimation between two nodes in an intermediate link. $M$ distance measurements $d_i$, $i = 1 \ldots M$, are collected in an $M$-hop multihop link between the $i$-th and $j$-th anchor nodes and added together to obtain an estimate of the total range between them such that $\hat{d}_{ij} = \sum_{i=1}^{M} d_i$. If the multihop route does not consist of a straight line connecting the two anchor nodes, then the addition of these $M$ distances will yield an *overestimate* of the true range, that is, $\hat{d}_{ij} \geqslant d_{ij}$. However, since the true range between the anchor nodes is known, a correcting factor may be calculated as follows:

$$\mathrm{CDIST}_{ij} = \frac{\hat{d}_{ij}}{d_{ij}}. \tag{3.41}$$

The multilateration stage of the DV-distance algorithm is exactly the same as that corresponding to DV-hop. An NOI communicates with $N \geqslant 3$ anchor nodes via multihop links and obtains range observations based on the correcting factor. Then it performs multilateration with the resulting range measurements to obtain a final position estimate of the NOI.

DV-distance requires a uniform geographic distribution of nodes, nodes with similar transmission power and antenna gains, and a large node density in order for hop sizes to be fairly constant throughout the network. This method does not take into consideration measurement error statistics, and its performance may degrade considerably in the presence of estimation noise and shadowing since it relies on actual distance estimates that may be obtained through TOA or RSS observations. This algorithm is very sensitive to channel variations due to the fact that the correcting factor estimation becomes less precise in the presence of irregular radio reception patterns. Examples of this behavior are presented in [6].

### 3.4.4 Dead-Reckoning

Dead-reckoning (DR) is a method that has been historically used for navigation. It consists of the estimation of location based on consecutive distance and direction of travel estimates departing from the last known position or fix [19]. In the multihop scenario, DR consists of a localized iterative localization technique that does not require the NOI to be at a one-hop proximity from at least three anchors. Instead, the requirement is that at least three land references are able to establish a link with any NOI via multiple hops. The method relies solely on multiple independent observations coming from multihop paths originating at distinct land references and ending at the NOI.

DR-based localization is similar to the DV-distance algorithm except that in the former, instead of using distance-correcting factors, true range and AOA estimates are obtained at every hop that forms a path from the land reference to the NOI. From now on we will refer to the multihop path that connects a land reference to an NOI as a dead-reckoning path (DRP). Effectively, under noiseless conditions, one DRP will be enough to estimate the position of an NOI if range and AOA measurements are available at each hop. However, under more realistic scenarios where estimates of range and AOA contain errors, multilateration may be used with as many DRPs as can be established by various land references in the network to reduce the effects of range and AOA estimation inaccuracies as well as of error accumulation that arises from the vectorial sum of noisy range and AOA pairs.

The advantages of DR-based multihop localization schemes are that they are robust to nonuniform node positions and unequal node transmission powers and antenna gains, do not require very large node densities to obtain accurate results, avoid the need for the land references to be in constant communication with each other in order to estimate correction factors and the need to broadcast these factors continuously to the entire network. These last points allow a considerable reduction in communication overhead and power consumption in the network.

Analogous to the DR navigational method, location of an NOI may be estimated based on the vectorial sum of range and AOA pairs measured at each of the various multiple hops that link a fixed land reference to that node. Figure 3.13 shows an $M$-hop DRP that originates at the land reference and ends at the NOI. Figure 3.14 shows a close-up of the $i$-th hop in this DRP. At each hop a pair of nodes communicates and estimates their range $\rho_i$ and relative angle $\alpha_i$ (measured with respect to the $x$-axis), $i = 1, \ldots, M$. If no measurement errors exist, the vectorial sum of range-angle pairs over all hops yields a resultant vector with magnitude $d$ and angle $\theta$ that uniquely defines the position of the

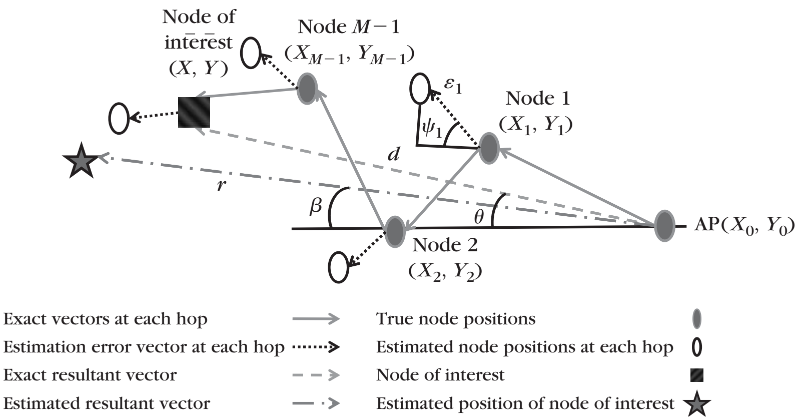



**FIGURE 3.13**

Dead-reckoning scheme for estimating position of an NOI.

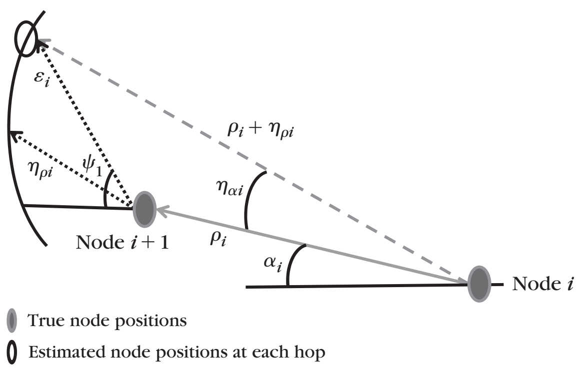



**FIGURE 3.14**

Scheme for the $i$-th hop conforming to the DRP.

NOI. Due to range and AOA measurement errors at each hop, denoted by $\eta_{\rho,i}$ and $\eta_{\alpha,i}$, $i=1,\ldots,M$ in Figure 3.14, the resultant vector will differ from the true position vector by $|d-r|$ meters in magnitude and by $\theta-\beta$ degrees in direction.

Let us envision a set of nodes with TOA and AOA estimation capabilities. AOA estimation schemes are usually considered costly, computationally expensive, and bulky (since they may require large antenna arrays). Note, however, that technology development trends in wireless networks favor the use of higher-frequency bands that will allow the implementation of smaller antenna arrays on the network nodes. Further, recent developments propose accurate and computationally simple AOA estimation schemes that require small antenna arrays with a reduced number of elements [8, 10, 18, 22]. Further, it has been shown in Perez [16] that the mean-square error performance of the DR localization scheme remains fairly insensitive to AOA errors as large as 12 degrees. This means that this algorithm does not require very-high-resolution AOA estimators to perform accurately.

Consider a network with computationally limited nodes where joint TOA/AOA estimation at each hop becomes prohibitive. In this scenario, signal measurements necessary for the estimation of TOA/AOA pairs at each hop may be acquired by the receiving node and propagated through the DRP. Upon the completion of the $M$-hop DRP, the complete set of signal measurements may be transmitted back to a computationally powerful land reference (possibly through the same DRP route) that may use these data to calculate the $M$ corresponding TOA/AOA pairs. Depending on the TOA/AOA estimation algorithm, large data sets acquired at each hop may be compressed to a sufficient statistic without any loss of estimation accuracy. This will allow a significant reduction of overhead traffic.

DR schemes require a single DRP originating at a single land reference to obtain a node position estimate described by the resultant range-angle pair $(r, \beta)$. However, if the network environment contains more than three land references, the resulting vector angle $\beta$ could be ignored and the consequent direction ambiguity avoided by observing the estimated resultant vector magnitudes $r_i$, $i=1,\ldots,N$ obtained from DRPs originating at $N \geqslant 3$ different land references. Then optimal WLS estimates of position could be obtained by multilateration. With this approach the localization scheme profits from multiple independent range observations to improve localization performance via a statistical optimization method, and it allows for the improvement of poor position estimates that may arise from single DRPs conformed by nodes with low-cost, small, and low-complexity antenna array systems with low-resolution AOA estimation capabilities.

Thus, consider the estimation of a specific NOI with unknown coordinates $(x, y)$ using a single DRP between a land reference with known coordinates $(x_0, y_0)$ and that node (see Figures 3.13 and 3.14). The intermediate nodes that link the NOI with the land reference conform to the $M$ hops of the DRP. The coordinates of these intermediate nodes will be denoted as $(x_i, y_i)$, $i = 1, \ldots, M-1$ and $x_M = x$, $y_M = y$. We define the magnitude of the location vector observation as

$$||\hat{\mathbf{p}}|| = ||\mathbf{p} + \mathbf{e}|| = ||\sum_{i=1}^{M}(\boldsymbol{\gamma}_i + \boldsymbol{v}_i)||, \tag{3.42}$$

where $\mathbf{p}$, $\hat{\mathbf{p}}$, and $\boldsymbol{\gamma}_i$ are vectors described by the magnitude-angle pairs $(d, \theta)$, $(r, \beta)$, and $(\rho_i, \alpha_i)$, respectively, and $\rho_i = [(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2]^{1/2}$, $\alpha_i = \tan^{-1}[(y_i - y_{i-1})/(x_i - x_{i-1})]$, $i = 1, \ldots, M$. Also, vector $\boldsymbol{v}_i$, which is described by the magnitude-angle pair $(\varepsilon_i, \psi_i)$, $i = 1, \ldots, M$, corresponds to the measurement error vector at the $i$-th hop of the DRP (see Figures 3.13 and 3.14).

Following a simple trigonometric calculation, it can be shown that at the $i$-th hop

$$\varepsilon_i = \{2\rho_i[1 - \cos(\eta_{\alpha,i})][\rho_i + \eta_{\rho,i}] + \eta_{\rho,i}^2\}^{1/2}, \tag{3.43}$$

and

$$\psi_i = \pi + \alpha_i - \cos^{-1}\left[\frac{\rho_i - (\rho_i + \eta_{\rho,i})\cos(\eta_{\alpha,i})}{\varepsilon_i}\right]. \tag{3.44}$$

Hence, the error vector at the $i$-th hop, $\boldsymbol{v}_i$, depends on the ranging and AOA estimation errors $\eta_{\rho,i}$ and $\eta_{\alpha,i}$, and on the range $\rho_i$ and angle $\alpha_i$ between the nodes conforming to the $i$-th hop.

In this treatment we may assume that a routing algorithm exists that chooses the shortest path between a land reference and the NOI congruent with two basic limitations—node density $D$ and node coverage range $R$. It may also be assumed that nodes remain quasistatic in the time interval spanning the propagation and processing delay at each hop so that a DRP, with corresponding TOA/AOA estimation pairs, is established in a scenario where the positions of the NOI and intermediate nodes remain fairly constant during the multihop routing process. The quasistatic node assumption is reasonable, for example, in mobile portable computing network scenarios where nodes (users) tend to stay at a fixed position for long periods of time.

Routing angle, $\alpha_i$, may be modeled as a random variable symmetrically distributed around $\theta$ degrees with standard deviation $\sigma_{\text{spread}}$ also denoted as routing angular spread. Parameter $\sigma_{\text{spread}}$ describes the average angular amount by which each hop on the actual routing sequence deviates from being collinear to the direct resultant range vector defined by the pair $(d, \theta)$ as shown in

Figure 3.13. Its value will be directly determined by the network's node density $D$ and the node coverage range $R$.

Ranging estimation errors $\eta_{\rho,i}$ $(i=1,\ldots,M)$ that arise from TOA, TDOA, or RSS measurement inaccuracies may be modeled as independent zero mean random variables with variance $\sigma^2_{\eta_\rho}$. In a similar manner, angular errors $\eta_{\alpha,i}$ $(i=1,\ldots,M)$ will depend on the optimality of the AOA estimators in the presence of additive noise and other channel impairments. These errors may also be assumed independent, with zero mean and variance $\sigma^2_{\eta_\alpha}$. In the end, the overall vectorial sum for a DRP will yield a resulting vector whose magnitude $||\hat{\mathbf{p}}||$ will consist of the sum of the true distance between the land reference and the NOI, and an error term as given in Equation (3.15) with functions $f_i(x,y)$ corresponding to Euclidean distances between the land references and the NOI.

Consider $N \geqslant 3$ DRP position vector estimates. If we ignore the estimated resultant vector angle $\beta$ at each of the $N$ DRPs, the position of an NOI may be estimated by applying a multilateration algorithm based on $N$ observations that contain true distance magnitudes corrupted by additive noise.

Define a vector containing the true coordinates of the NOI as $\mathbf{q}=[x\,y]^H$, and let $N \geqslant 3$ land references, with coordinates $\{x_{LR,i}, y_{LR,i}\}, i=1,\ldots N$, be available in the network, and the $i$-th land reference establish a link to the NOI via $M_i$ hops. The $N$ resultant range observations are collected in a vector to obtain

$$\mathbf{r}=\mathbf{d}(\mathbf{q})+\boldsymbol{\eta}, \tag{3.45}$$

where $\mathbf{r}=[r_1\,r_2\ldots r_N]^H$, $\mathbf{d}(\mathbf{q})=[d_1(\mathbf{q})\,d_2(\mathbf{q})\ldots d_N(\mathbf{q})]^H$, $\boldsymbol{\eta}=[\eta_1\,\eta_2\ldots\eta_N]^H$, and $d_i(\mathbf{q})=[(x_{LR,i}-x)^2+(y_{LR,i}-y)^2]^{\frac{1}{2}}$ is the true Euclidean distance between the unknown node position and the $i$-th land reference. Noise samples $\eta_i$ and $\eta_j$ $(i \neq j)$ may be assumed independent; however, they are not identically distributed unless all DRPs have the same number of hops. The independence assumption will be reasonable whenever the node density and reachability radius are large enough, and the measured resultant distances come from DRPs originating at sufficiently separated land references such that no route shares common hops.

The last step in the DR algorithm consists of applying the LS or WLS multilateration process, described in Section 3.3.1, to the $N$-dimensional vector of noisy distances given in Equation (3.45).

## 3.5 PERFORMANCE ASSESSMENT OF LOCATION ESTIMATION SYSTEMS

In this section, we present some performance measures that describe the accuracy of a location estimation scheme based on the probability distribution of range- or AOA-related observations.

### 3.5.1 Cramer Rao Bound

The problem that has been treated in this chapter is that of estimating the NOI coordinates vector $\mathbf{q}$ given a set of noisy observations collected in vector $\mathbf{r}$ as expressed in Equation (3.15).

The Cramer Rao bound (CRB) provides a theoretical lower limit to the error covariance matrix $\mathbf{C}$ of any unbiased estimator of coordinate vector $\mathbf{q}$ such that

$$\mathbf{C} = E\{[\hat{\mathbf{q}} - \mathbf{q}][\hat{\mathbf{q}} - \mathbf{q}]^H\} \geqslant \mathbf{J}^{-1}, \tag{3.46}$$

where $\mathbf{J}$ corresponds to the Fisher information matrix given by

$$\mathbf{J} = -E\left\{\frac{\partial}{\partial \boldsymbol{\theta}}\left(\frac{\partial}{\partial \boldsymbol{\theta}} \ln p_{\mathbf{q}}(\mathbf{r})\right)^H\right\}. \tag{3.47}$$

Here, $p_{\mathbf{q}}(\mathbf{r})$ corresponds to the joint probability density distribution of observation vector $\mathbf{r}$.

If the observation noise in Equation (3.15) is Gaussian with zero mean and covariance matrix $\mathbf{K}$, then the observation vector $\mathbf{r}$ is also Gaussian with mean vector equal to $\mathbf{f}(\mathbf{q})$ and with the same covariance matrix $\mathbf{K}$. In this case, the CRB for position estimate vector $\hat{\mathbf{q}}$ is easily calculated as in Scharf [23]:

$$\mathrm{CRB} = \mathbf{J}^{-1} = \frac{\partial \mathbf{f}(\mathbf{q})^{\mathbf{H}}}{\partial \mathbf{q}} \mathbf{K}^{-1} \frac{\partial \mathbf{f}(\mathbf{q})}{\partial \mathbf{q}}. \tag{3.48}$$

### 3.5.2 Circular Error Probability

The circular error probability (CEP) [25] is a simple measure of accuracy defined as the radius of the circle that has its center at the mean and contains half the realizations of a random vector of coordinate estimates. It is a measure of uncertainty in the location estimator $\hat{\mathbf{q}}$ relative to its mean $E\{\hat{\mathbf{q}}\}$. If the location estimator is unbiased, the CEP is a measure of the estimator uncertainty relative to the true NOI position. If the magnitude of the bias vector is bounded by $B$, then with a probability of one-half, a particular estimate is within a distance of $B + CEP$ from the true position. This concept is illustrated in Figure 3.15.

From its definition, the CEP may be derived by solving the following equation:

$$\frac{1}{2} = \int\int_R p_{\hat{\mathbf{q}}}(\boldsymbol{\zeta}) d\zeta_1 d\zeta_2, \tag{3.49}$$

where $p_{\hat{\mathbf{q}}}(\boldsymbol{\zeta})$ is the probability density function of vector estimate $\hat{\mathbf{q}}$, and the integration region is defined as $R = \{\boldsymbol{\zeta} : |\boldsymbol{\zeta} - E\{\hat{\mathbf{q}}\}|\} \leqslant \mathrm{CEP}$. Most of the time, a closed-form expression of Equation (3.49) is difficult to find and numerical integration must be performed. However, the following approximation, which is accurate to within 10%, is often used [25]:

$$\mathrm{CEP} \approx 0.75\sqrt{E\{(\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\})^H(\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\})\}} = 0.75\sqrt{\lambda_1 + \lambda_2} = 0.75\sqrt{\sigma_1^2 + \sigma_2^2}. \tag{3.50}$$

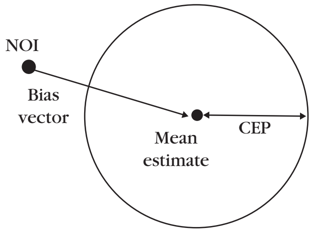



**FIGURE 3.15**

Geometry of the CEP definition.

Here $\lambda_1$ and $\lambda_2$ are the eigenvalues of the estimator covariance matrix, which is given by

$$E\{(\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\})^H (\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\})\} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_1^2 \end{bmatrix}. \tag{3.51}$$

### 3.5.3 Geometric Dilution of Precision

The geometric dilution of precision (GDOP) [24, 25] provides a measure of the effect of the geometric configuration of the land references on the location estimate. It is defined as the ratio of the root mean-square position error to the root mean-square ranging error. Then, for an unbiased estimator, the GDOP is given as

$$\text{GDOP} = \frac{1}{\sigma_r} \sqrt{E\{(\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\})^H (\hat{\mathbf{q}} - E\{\hat{\mathbf{q}}\})\}}, \tag{3.52}$$

where $\sigma_r$ may be seen as the fundamental ranging or AOA estimation error.

The GDOP indicates how much the fundamental ranging error is magnified by the geometric relation among the NOI and the land references. Comparing Equations (3.50) and (3.52), we observe that

$$\text{CEP} \approx (0.75\sigma_r)\text{GDOP}. \tag{3.53}$$

## REFERENCES

[1] N. Bulusu, J. Heidemann, D. Estrin, GPS-less low cost outdoor localization for very small devices, IEEE Personal Communications Magazine, 7 (5) (2000) 28–34.

[2] T.F. Coleman, Y. Li, An interior, trust region approach for nonlinear minimization subject to bounds, SIAM Journal on Optimization, 6 (1996) 418–445.

[3] L. Doherty, K.S.J. Pister, L. El Ghaoui, Convex position estimation in wireless sensor networks, in: Proceedings of INFOCOM, 3 (2001) 1655–1663.

[4] W.H. Foy, Position-location solutions by Taylor-series estimation, IEEE Transactions on Aerospace and Electronic Systems, 12 (2) (1976) 187–194.

[5] G. Zhou, T. He, S. Krishnamurthy, J.A. Stankovic, Impact of radio irregularity on wireless sensor networks, in: Proceedings of the 2nd International Conference on Mobile Systems, Applications, and Services, June (2004) 125–138.

[6] T. He, C. Huang, B. Blum, J. Stankovic, T. Abdelzaher, Range-free localization schemes in large scale sensor networks, in: Proceedings of the 9th International Conference on Mobile Computing and Networking, September (2003) 81–95.

[7] O. Hernandez, F. Bouchereau, D. Muñoz, Maximum likelihood position estimation in ad-hoc networks using a dead reckoning approach, IEEE Transactions on Wireless Communications, 7 (5) Part I (2008) 1572–1584.

[8] J. Yli-Hietanen, K. Kalliojarvi, J. Astola, Low-complexity angle of arrival estimation of wideband signals using small arrays, in: Proceedings of the 8th IEEE Signal Processing Workshop on Statistical Signal and Array Processing, June (1996) 109–112.

[9] L. Kovavisaruch, K.C. Ho, Modified Taylor-series method for source and receiver localization using TDOA measurements with erroneous receiver positions, in: Proceedings of the IEEE International Symposium on Circuits and Systems, 3 (2005) 2295–2298.

[10] W. Menzel, A. Gronau, V. Maiyappan, W. Mayer, Small-aperture, high-resolution beam-scanning antenna array using nonlinear signal processing, European Conference on Wireless Technology, October (2005) 435–438.

[11] D. Niculescu, B. Nath, Ad hoc positioning system (APS), IEEE Global Telecommunications Conference, 5 (2001) 2926–2931.

[12] D. Niculescu, B. Nath, Ad hoc positioning system (APS) using AOA, in: Proceedings IEEE INFOCOM, 3 (2003) 1734–1743.

[13] D. Niculescu, Positioning in ad hoc sensor networks, IEEE Network, 18 (4) (2004) 24–29.

[14] N. Patwari, A.O. Hero III, M. Perkins, N.S. Correal, R.J. O'Dea, Relative location estimation in wireless sensor networks, IEEE Transactions on Signal Processing, 51 (8) (2003) 2137–2148.

[15] N. Partwari, J.N. Ash, S. Kyperountas, A.O. Hero III, R.L. Moses, S.D. Correal, Locating the nodes: cooperative localization in wireless sensor networks, IEEE Signal Processing Magazine, 22 (4) (2005) 54–69.

[16] V. Perez, Position estimation using dead reckoning and other localized algorithms in wireless networks, M.S. Thesis Dissertation, August 2008, ITESM, Monterrey, Mexico.

[17] R.A. Poisel, Electronic Warfare Target Location Methods, Artech House, 2005.

[18] Z. Popovic, C. Walsh, P. Matyas, C. Dietlein, D.Z. Anderson, High-resolution small-aperture angle of arrival detection using nonlinear analog processing, IEEE International Microwave Symposium Digest, 3 (2004) 1749–1752.

[19] C. Randell, C. Djiallis, H. Muller, Personal position measurement using dead reckoning, in: Proceedings of the 7th IEEE International Symposium on Wearable Computers, October (2003) 166–173.

[20] A. Savvides, C.C. Han, M.B. Srivastava, Dynamic fine-grained localization in ad-hoc networks of sensors, in: Proceedings of the 7th Annual International Conference on Mobile Computing and Networking, July (2001) 166–179.

[21] A. Savvides, H. Park, M.B. Srivastava, The *n*-hop multilateration primitive for node localization problems, Mobile Networks and Applications, 8 (4) (2003) 443–451.

[22] H.G. Schantz, Smart antennas for spatial RAKE UWB systems. IEEE Antennas and Propagation Society International Symposium, 3 (2004) 2524–2527.

[23] L.L Scharf, Statistical Signal Processing—Detection, Estimation, and Time Series Analysis, 2nd edition, Addison-Wesley, 1991.

[24] M.A. Spirito, On the accuracy of cellular mobile station location estimation, IEEE Transactions on Vehicular Technology, 50 (3) (2001) 674–685.

[25] D.J. Torrieri, Statistical theory of passive location systems, IEEE Transactions on Aerospace and Electronic Systems, AES-20 (2) (1984) 183–198.