

Assignment 3

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Exercise 3.1

Exercise 3.2

The project is calculated in Excel, as shown in Figure 1 of the appendix. The expected durations for each activity are displayed in columns B16 through H16. The inverse transformation method must be used as these are expected durations. All activity durations are independent and distributed exponentially. In addition, the duration follows an exponential distribution with a parameter of 1/4 for the rate. The inverse transformation method for exponential distribution is thus required. The calculation in Excel is: `'-ln(rand()) / "lambda"'`. Lambda is the same the rate parameter so this will be 1/4. This function is used to calculate the realizations of activity durations for the seven activities in the cells B24 until H10023. So for example, the calculation made in cell B20 is `'=-(LN(RAND()))/(1/B16)'`. Realizations of finish time activities are calculated in cells I24 until O10023. This is done by adding the duration of the activity with the duration of the former one. So for example, in cell J20 it is `'=I20+C20'`. However, in cell M20 it is `'=MAX(J20;K20)+F20'`. This is because the activity five has to wait for both activity two as well as three to finish. The max makes sure that the longest of those two activities gets chosen as activity five can only start once both previous activities are finished.

a)

We can assume that the central limit theorem applies since the project has to be simulated 10000 times. The expected project finish time will be the average project finish time of those 10000 simulations. That calculation is shown in appendix Table 1 and the result is 12,089 (see appendix Figure 2).

b)

The confidence interval is calculated using the t-test. The use of the t-test is possible as we can assume a normal distribution (as described above) and we don't know anything about the population standard deviation, but do know the sample standard deviation. The confidence interval looks as follows:

$$[\bar{X} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}, \bar{X} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}]$$

Where \bar{X} = expected project finish time (which is the average expected project finish time), n = number of sample values, s = sample standard deviation, and t is the t-distribution.

The calculations are shown in appendix Table 1. These give a confidence interval of [11,994; 12,183] (see appendix Figure 2). This means that we are 95% confident that the mean lies within this interval. So, if we would run this project several times and calculate the project finishing time, the mean would lie in this interval for 95% of the cases.

c)

We start with the following null-hypothesis and alternative hypothesis:

$$H_0 : \text{expected project duration} \leq 12$$

$$H_0 : \text{expected project duration} > 12$$

Then, we can use a t-test. We are going to use a t-test since we concluded in question a) that the central limit theorem applies and the standard deviation is unknown (see slide 22, lecture 2).

The probability that an expected project time is calculated using the test statistic T:

$$T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

Where \bar{X} = expected project finish time, μ = is the average expected project finish time, s = the sample standard deviation, and n = number of sample values. Excel's calculation is shown in appendix Table 1. The T-value is then used to calculate the p-value (see appendix Table 1). The p-value is 0,362 and the confidence interval is [11,883; 12,076]. Since the p-value is not lower than 0,05, we conclude that there is not enough statistical evidence to reject H_0 . This can also be checked by looking at the interval. 12 lies within that interval which also means that we can not reject H_0 .

Exercise 3.3

a) We can simulate the hotel room rent in excel by using multiple rows for random, binomial formulas. We arranged the data in a table (see appendix 3.3a table). With a simulation budget of 100000, we will use $100000 / 9 = 11111$ rows.

When graphed (appendix 3.3b), we can see that the 140 mark is the cost with the highest estimate revenue.

We use a t-test for these calculations. The t-test is done in excel by estimating the mean, std. dev, and t value. From this, a lower- and upperbound confidence interval is calculated. These are also graphed in 3.3b.

b) To execute the steps in "ranking and selection, option 2" we can reuse the sheet created in 3.3a. With a starting budget of 900, and 9 prices, we first simulate $900 / 9 = 100$ rows. These are shown in appendix 3.3c.

Next, we perform pairwise t-test for each price. We can do this in Excel by creating a 9 by 9 table containing the test: $y(\pi) \leq y(\pi') - \beta_{1-\alpha} \frac{\sqrt{s^2(\pi) + s^2(\pi')}}{\sqrt{(N)}}$. The results of these tests are in appendix 3.3d. With this, we can reject the prices 100, 100, 160, 170, 180.

To continue, we execute step 4, which is to perform the rest of the testing budget. This is $9100 / 9 = 1011$ rows. The end result is shown in appendix 3.3e and 3.3f. Now, the maximum revenue is estimated to be with 130.

An important note is that these values are generated by a random number generated, which resets between each simulation. Therefore, these test may show different results. Nonetheless, if simulated enough, the result should weight to a single price point eventually.

Appendix

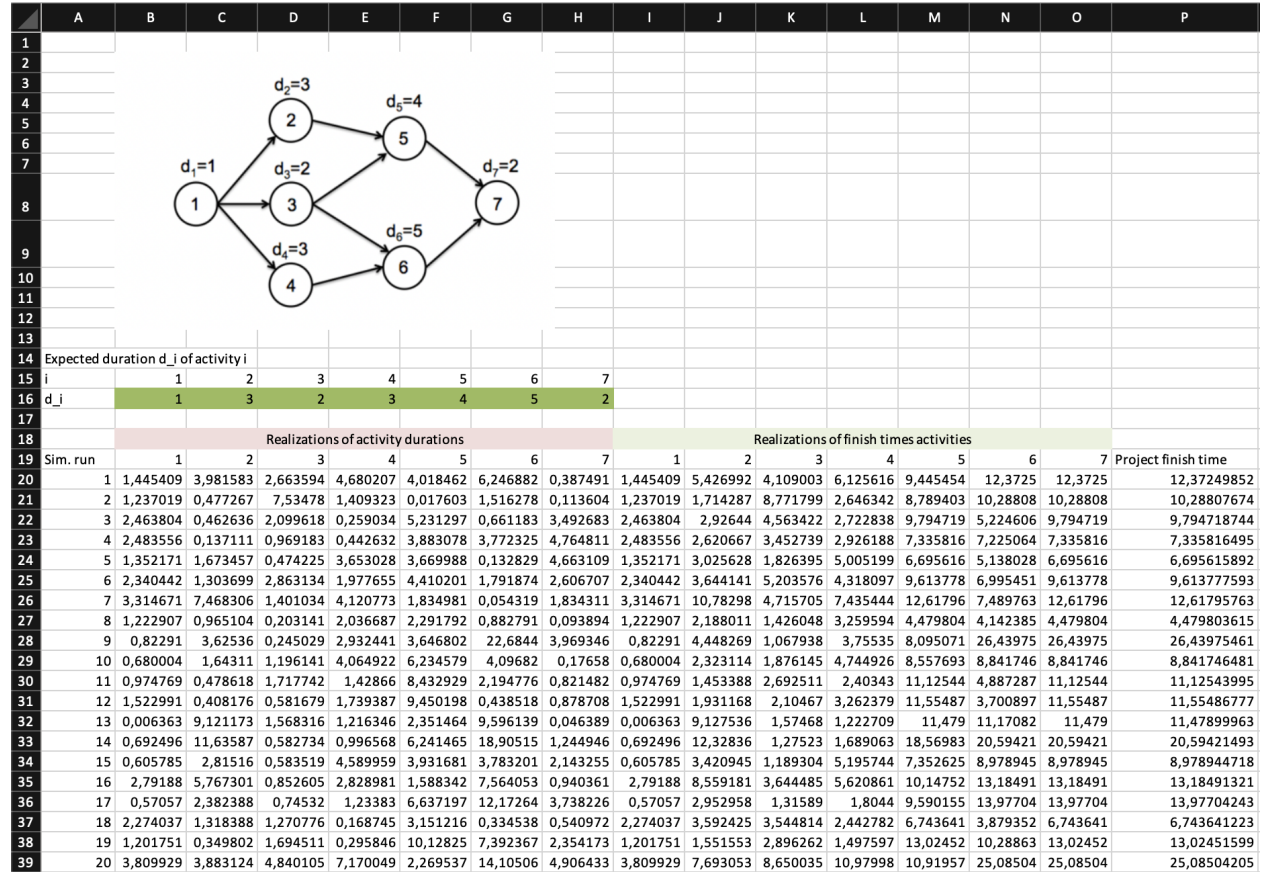


Figure 1: Screenshot 3.2a

Table 1: Formulas used

Name	Calculation
Sample average	=AVERAGE(Q23:Q10022)
N	=COUNT(Q23:Q10022)
95 percentile of t-dist with n-1	=T.INV(0,95;U28-1)
Sample standard deviation (s)	=STDEV.S(Q23:Q1122)
Left CI bound	=U25-U29*U30/SQRT(U28)
Right CI bound	=U25+U29*U30/SQRT(U28)
Test statistic T	=(12-T39)/(T44/SQRT(T42))
P-value	=TDIST(T48;(T42-1);1)

Sample average		12,08881
Confidence interval (CI) calculation		
n		10000
95 percentile of t-dist with n-1		1,645006
Sample standard deviation (s)		5,733531
Left CI bound		11,99449
Right CI bound		12,18313

Figure 2: Screenshot 3.2a2

Sample average		11,97934
Confidence interval (CI) calculation		
n		10000
95 percentile of t-dist with n-1		1,645006
Sample standard deviation (s)		5,859297
Left CI bound		11,88295
Right CI bound		12,07572
Test statistic T		0,35262
P-value		0,36219

Figure 3: Screenshot 3.2c

	A	B	C	D	E	F	G	H	I	J
1	Optimal hotel prices: 3.3a									
2										
3	Prices	100	110	120	130	140	150	160	170	180
4										
5	Mean	1135,82936	1172,13212	1199,568	1210,86851	1213,82054	1203,48303	1172,17172	1135,28665	1086,01026
6	Stddev	222,344583	246,604774	265,545568	291,38632	308,611378	327,973513	344,333622	359,678607	367,260498
7	N	11111	11111	11111	11111	11111	11111	11111	11111	11111
8	t	1,96017753	1,96017753	1,96017753	1,96017753	1,96017753	1,96017753	1,96017753	1,96017753	1,96017753
9	Lowerbound 95% CI	1131,69465	1167,54627	1194,62992	1205,4499	1208,08161	1197,38405	1165,7685	1128,59808	1079,18069
10	Upperbound 95% CI	1139,96407	1176,71798	1204,50607	1216,28712	1219,55947	1209,58202	1178,57494	1141,97523	1092,83983
11										
12		Price								
13	Simulation run	100	110	120	130	140	150	160	170	180
14	1	700	1320	1440	1690	1820	1350	1280	1020	1620
15	2	1000	1210	1560	1170	840	1050	1120	1700	1620
16	3	900	1210	960	910	840	1050	960	1530	900
17	4	1200	880	1200	1430	1260	1950	800	1360	900
18	5	1200	1100	1320	1690	1260	900	1440	680	1440
19	6	1000	990	960	910	1120	1650	960	850	360
20	7	1300	1540	1080	910	1400	1800	1760	1360	900
21	8	900	1210	840	780	980	1350	1280	1020	1620
22	9	1100	1320	1320	910	1680	1500	800	510	1440
23	10	1500	990	1200	1170	1540	1050	1280	850	720
24	11	700	770	1920	1170	1120	1350	800	1530	1260
25	12	1100	1100	1440	1690	1400	1500	1120	340	540
26	13	1100	1100	1200	1560	1540	1500	960	1190	540
27	14	1500	880	1200	1430	980	1350	1280	1700	1260
28	15	1100	1320	1320	1170	560	1200	1760	1360	900
29	16	1000	1210	1440	1170	420	1350	1280	1190	1260
30	17	1200	1210	1200	1300	840	1500	1440	850	1260
31	18	1000	1420	1000	1170	1120	1050	1280	850	1000

Figure 4: Screenshot 3.3a

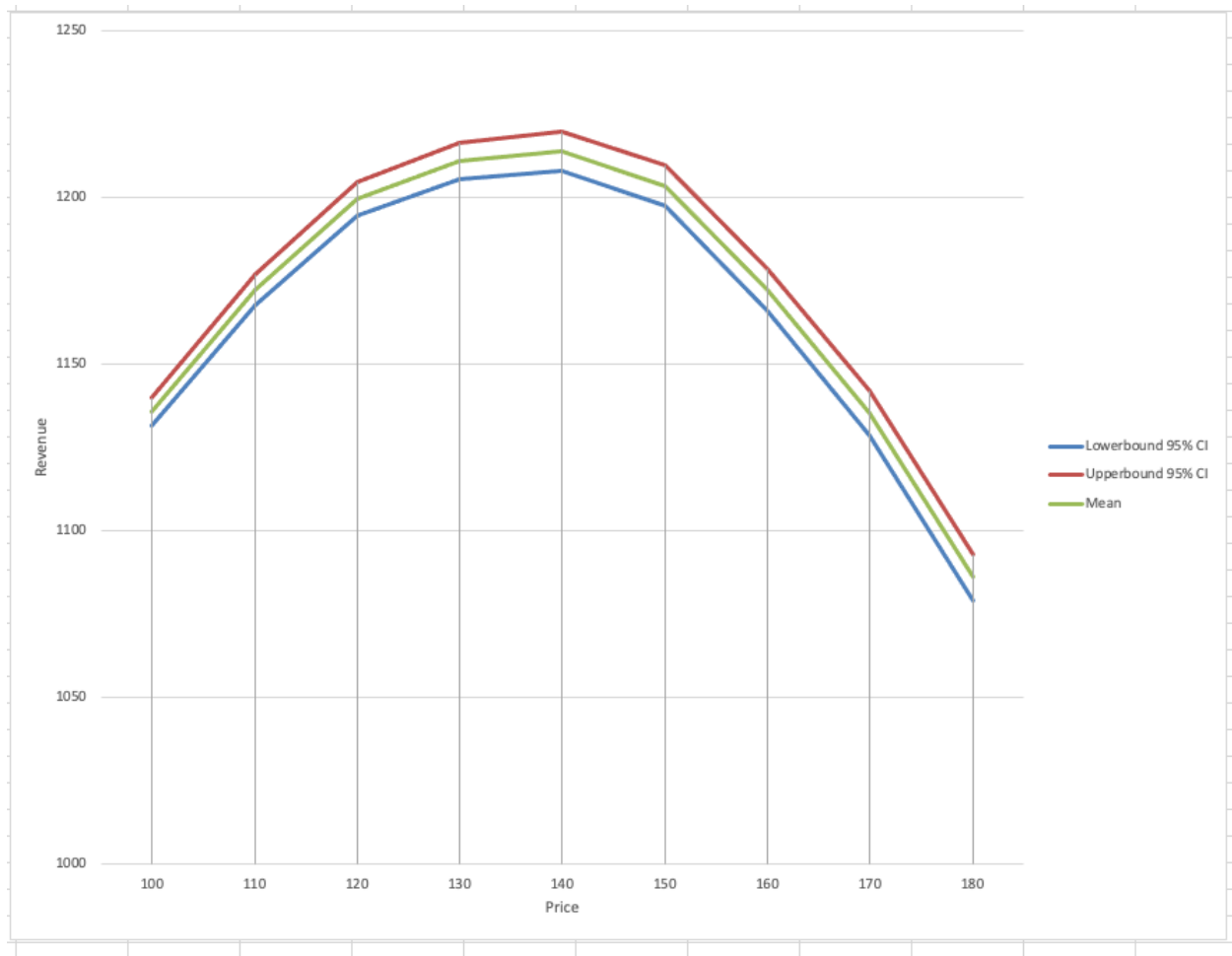


Figure 5: Screenshot 3.3b

	A	B	C	D	E	F	G	H	I	J
1	Optimal hotel prices: 3.3b									
2										
3	Prices	100	110	120	130	140	150	160	170	180
4										
5	Mean	1119	1171,5	1183,2	1280,5	1257,2	1198,5	1166,4	1098,2	1058,4
6	Stddev	235,999914	220,762567	260,918657	297,580733	300,451109	358,97265	315,356204	376,98533	384,851329
7	N	100	100	100	100	100	100	100	100	100
8	t	1,98421695	1,98421695	1,98421695	1,98421695	1,98421695	1,98421695	1,98421695	1,98421695	1,98421695
9	Lowerbound 95% CI	1072,1725	1127,69592	1131,42808	1221,45353	1197,58398	1127,27204	1103,82649	1023,39793	982,037147
10	Upperbound 95% CI	1165,8275	1215,30408	1234,97192	1339,54647	1316,81602	1269,72796	1228,97351	1173,00207	1134,76285
11										
12		Price								
13	Simulation run	100	110	120	130	140	150	160	170	180
14	1	1000	1210	1080	1170	1120	1500	960	1530	1260
15	2	900	1210	960	910	840	1200	960	680	1440
16	3	1200	1210	1320	1820	1120	1050	1120	1360	900
17	4	1500	1100	720	1170	840	750	1120	850	900
18	5	1100	1320	1320	1300	980	750	1280	1020	720
19	6	1600	1210	1320	1170	1260	1050	1120	1190	540
20	7	900	1210	1680	1430	1400	1500	1440	1020	1260
21	8	1400	990	1320	1430	1120	1350	960	850	1800
22	9	800	1320	1560	1430	980	0	800	1360	1800
23	10	1100	880	1200	910	1400	1500	800	340	1260
24	11	1000	1430	960	1170	1400	1650	1760	1020	720
25	12	1000	880	840	1300	1260	1500	1120	1870	1260
26	13	1300	1100	1440	1430	980	900	1120	1870	1260
27	14	1500	1320	960	1040	1120	900	320	850	900
28	15	1000	990	1200	1170	1120	1350	1920	1190	1440
29	16	900	1100	720	1560	1400	1500	1440	680	1440
30	17	1100	1430	1200	1040	1400	1200	1600	1700	360
31	18	1100	1100	1080	780	1120	1100	1120	850	1620

Figure 6: Screenshot 3.3c

	L	M	N	O	P	Q	R	S	T	U
	t-test reject if TRUE									
		100	110	120	130	140	150	160	170	180
100		FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
110		FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE
120		FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
130		FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
140		FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
150		FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
160		FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE
170		FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
180		FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE
a		0,05								
a*		0,00639115								
b		2,50152958								

	D	E	F	G
0	120	130	140	150
6	1200,70737	1222,10947	1215,14105	1204,35789
3	271,115835	287,2448	308,923401	327,330215
0	2375	2375	2375	2375
5	1,96096376	1,96096376	1,96096376	1,96096376
4	1189,79817	1210,55128	1202,71055	1191,18674
6	1211,61656	1233,66767	1227,57155	1217,52905

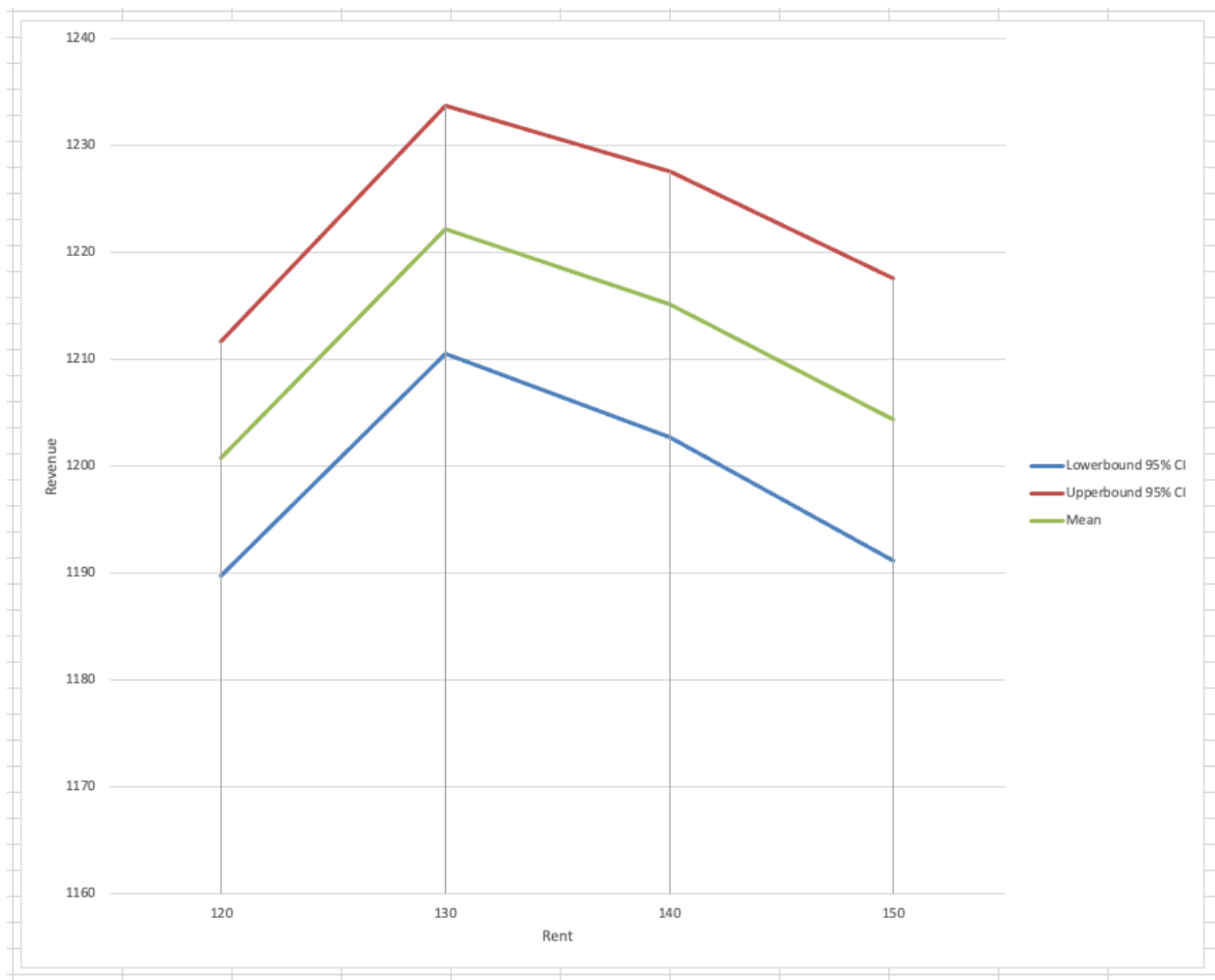


Figure 7: Screenshot 3.3f