EXAMEN FINAL JUNIO 2016

Ejercicio 1

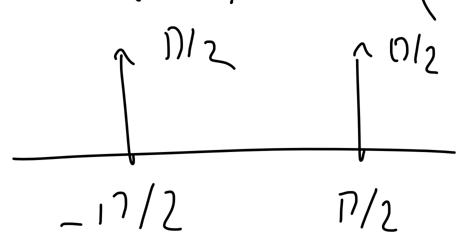
a) $X(e^{j\Omega})$ es real, sólo nos dan una gráfica, no una en 3D ni un par complejo-real o módulo-fase.

b) La TF de una señal real y par es una señal real y par
La TF de una señal real e impar es imaginaria pura. Así que no.

c) Es real y par.

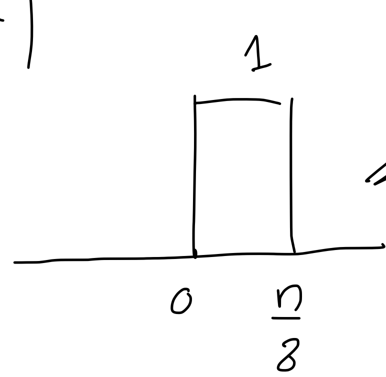
Ejercicio 2

Sé que $TF\{\cos(\Omega)\} = \pi\left(\delta(\Omega+\Omega_0) + \delta(\Omega-\Omega_0)\right)$



$\pi \cdot X = \frac{\pi}{2} \rightarrow X = \frac{1}{2}$

$\frac{1}{2}\cos\left(\frac{\pi}{2}n\right)$



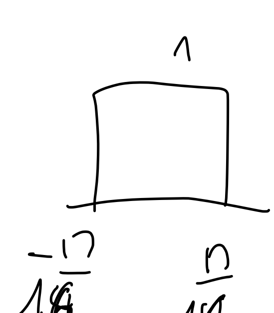
→ Proviene de una sinc:

Sabemos que

$TF\{\text{sinc}\} \rightarrow \sqcap$

Pues:

$\left(\frac{\text{sen}\left(\frac{\pi}{16}n\right)}{\pi \cdot n}\right) \rightarrow$



Desplazo a derecha e izquierda

$\frac{\text{sen}\left(\frac{\pi}{16}n\right)}{\pi \cdot n} e^{j\frac{\pi}{16}n}$

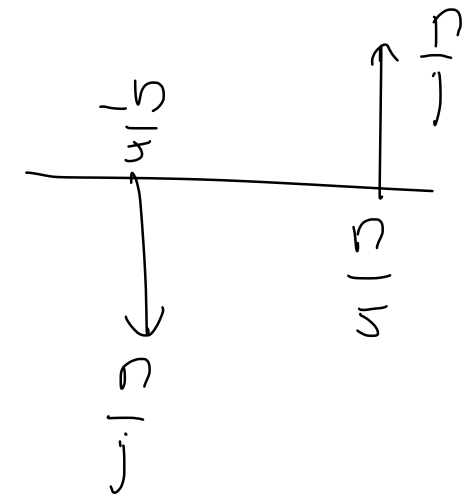
$- \frac{\text{sen}\left(\frac{\pi}{16}n\right)}{\pi \cdot n} e^{-j\frac{\pi}{16}n}$

Solución: $\frac{1}{2}\cos\left(\frac{\pi}{2}n\right) + \operatorname{sen}\left(\frac{\pi/16\, n}{\pi \cdot n}\right)\left(e^{+j\pi/16\, n} - e^{-j\pi/16\, n}\right)$

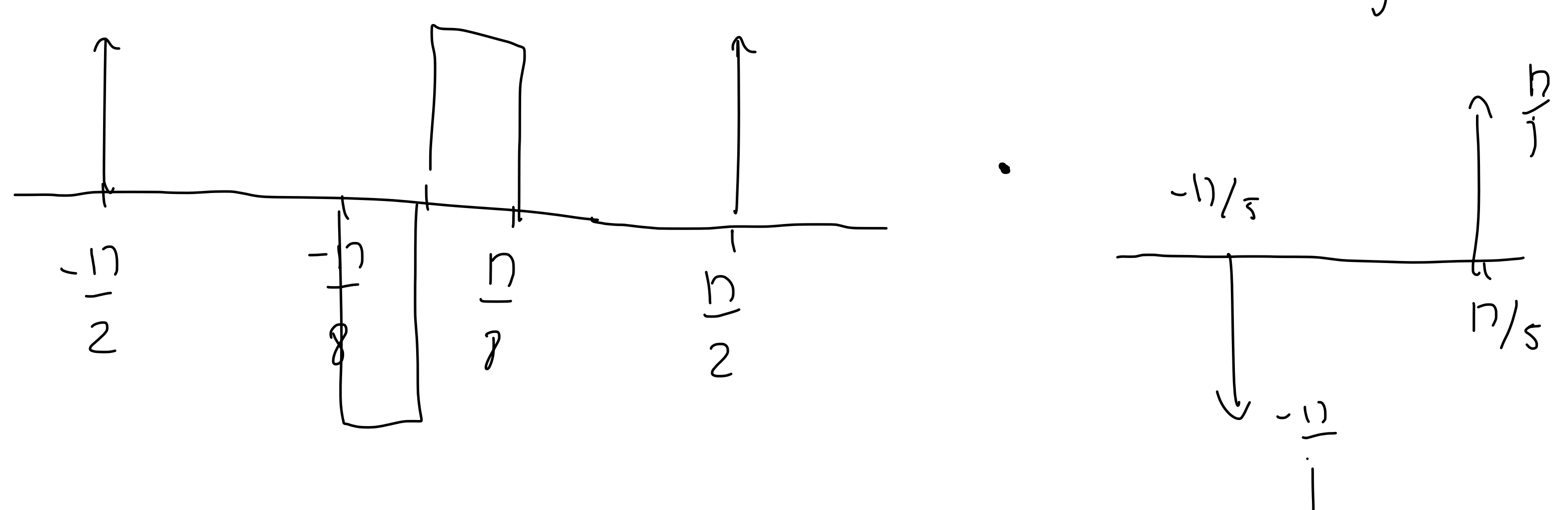
b) $v[n] = \operatorname{sen}\left(\frac{2\pi}{10}n\right)$

$z[n] = x[n] * v[n]$

$V(e^{j\Omega}) \rightarrow \frac{\pi}{j}\left(\delta\left(\Omega + \frac{2\pi}{10}\right) - \delta\left(\Omega - \frac{2\pi}{10}\right)\right)$



Entonces:

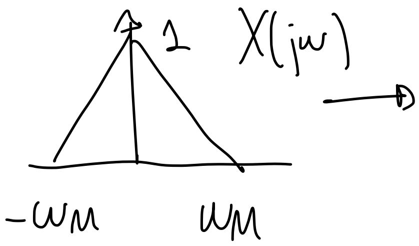


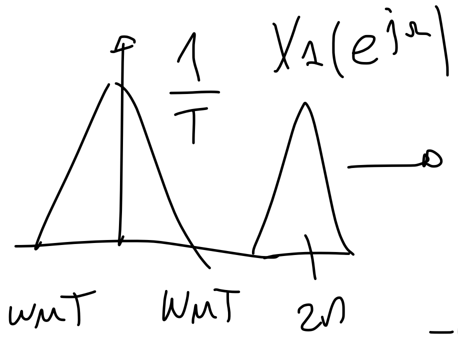
$z[n] = 0$; No coincide en ninguna frecuencia.

c) No es posible, porque en $Y(e^{j\Omega})$ aparecen nuevas frecuencias, que no estaban en $X(e^{j\Omega})$

No sería un LTI.

Ejercicio 3



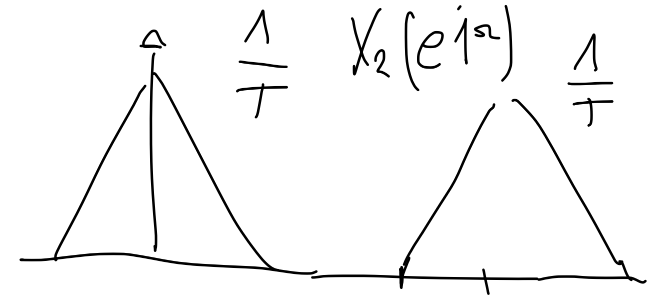


$-\omega_M T \rightarrow -\frac{\pi}{4}$

$\omega_M T \rightarrow 2000\pi \cdot \frac{1}{8000} = \frac{\pi}{4}$

$\frac{\pi}{4} < \pi$ ✓

Periódica 2π



$\frac{-\omega_M T}{P} \rightarrow -\frac{\pi}{16}$

$\frac{\omega_M T}{P} \rightarrow \frac{\frac{\pi}{4}}{4} \rightarrow \frac{\pi}{16}$ $P = 4$

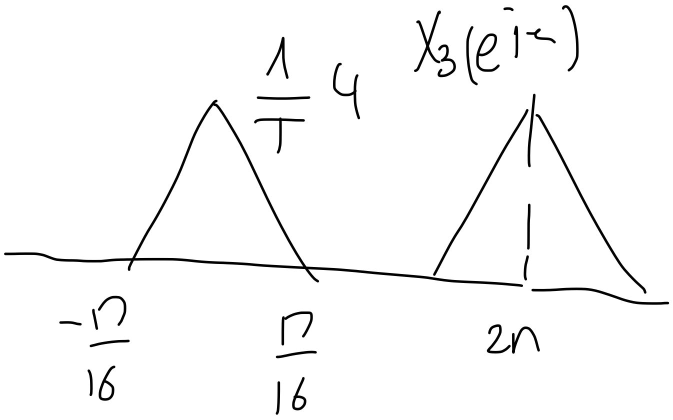
$\frac{2\pi}{P} = \frac{2\pi}{\cancel{4}\,2}$

no periódica $\frac{\pi}{2}$

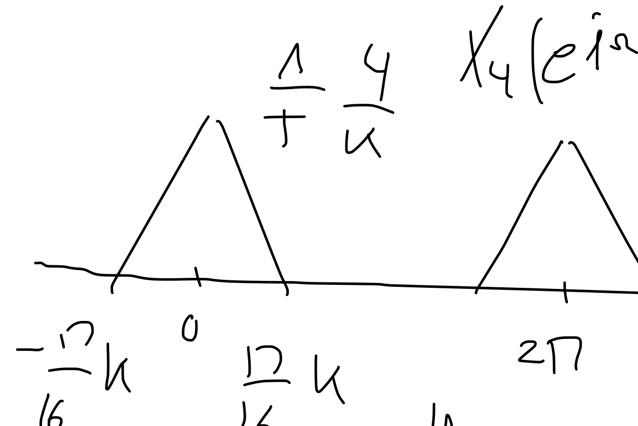
Al ser un interpolador no introduce solape

$\frac{\pi}{2} - \frac{\pi}{16} = \frac{7\pi}{16}$

Entra una sola réplica por cuadrado en



Periódica 2π

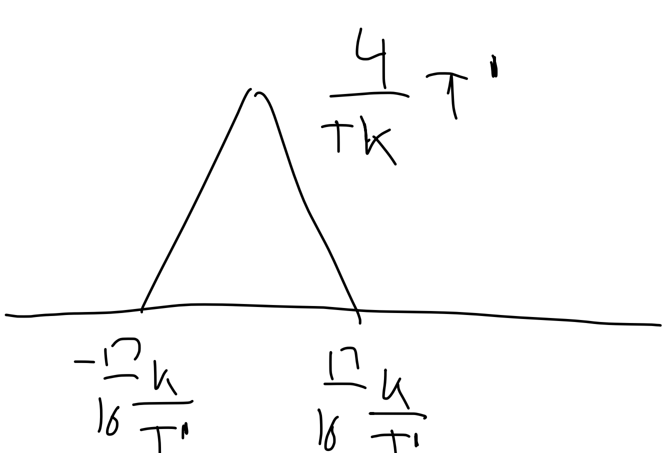


$\Downarrow$

$\left[\frac{\pi}{16}K < \pi\right]$

$\Downarrow$

Periódica 2π



a) $X_1(e^{j\Omega})\Big|_{\Omega=\pi/2} = 0$

b) $X_2(e^{j\Omega})\Big|_{\Omega=\frac{\pi}{2}} = \frac{1}{T} = \underline{\underline{8000}}$

c) $X_3(e^{j\Omega})\Big|_{\Omega=\frac{\pi}{2}} = 0$ //

d) $\frac{\pi}{16} \cdot k\, 16000 = \pi k\, 1000$

que sea $= \omega_M = 2000\pi$

$1000\cancel{\pi}k = 2000\cancel{\pi}$ $\boxed{k = \frac{2000}{1000} = 2}$

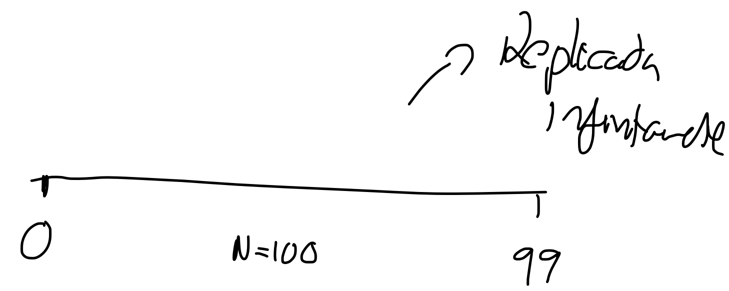
También vale:

$\frac{4T'}{Tk} = 1 \quad \Rightarrow \quad \frac{4 \cdot 8000}{16000\, k} \Rightarrow \boxed{k=2}$

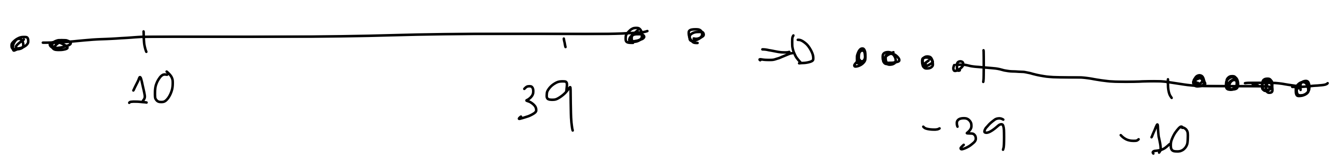
Ejercicio 4

a) $y[n] = x_1[n] \circledcirc_{100} x_2[n]$

$x_2[n]$



$x_1[n]$



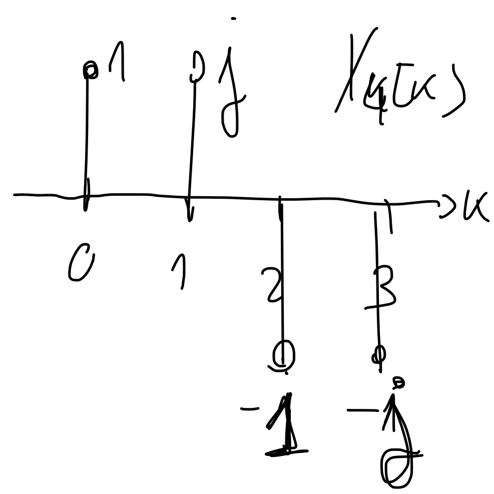
¿Cuando es igual la convolución lineal?

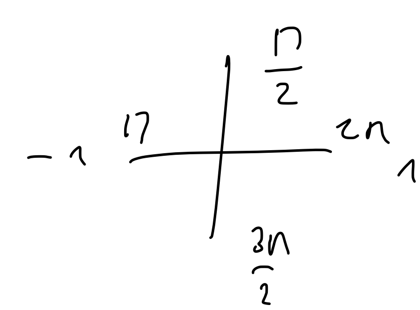
Cuando deje de afectar el solape anterior

Desde que el -39 coincide con el 0 hasta que el -10 coincide con el 99

$$39 \leq N \leq 99$$

b)





Pide la DFT inversa:

$$\frac{1}{4} \sum_{k=0}^{3} X_4[k] \cdot e^{+j \frac{2\pi}{4} n \cdot k}$$

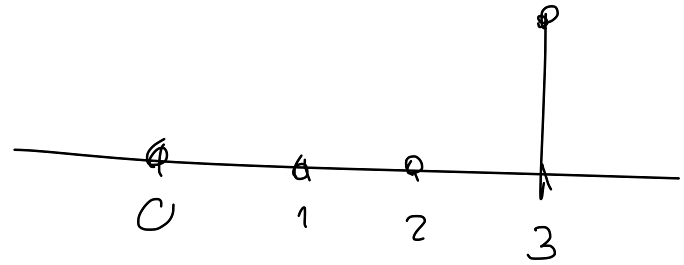
$$\frac{1}{4}\left(1 \cdot e^{0} + j e^{j \frac{2\pi}{4} n} + (-j) e^{j \frac{2\pi}{4} 2 n} + (-1) e^{j \frac{2\pi}{4} 3 n} \right)$$

$(+j)^n \qquad (-1)^n \qquad \frac{6\pi}{4} = \frac{3\pi}{2} \qquad -j$

$$\frac{1}{4}\left| 1 + j(+j)^n + (-1)\cdot(-1)^n + (-j)\cdot(-j)^n \right| \quad \text{Para } n = 0, 1, 2, 3$$

$-(j^2) = -1 \qquad j(j^2) = -1 \qquad j^3 \qquad -j \qquad -1$

$n=0 \to 0$
$n=1 \to 0$
$n=2 \to 0$
$n=3 \to 1$



Ejercicio 5

a) $y[n] = x[n] - \frac{K}{4}x[n-1] - \frac{K}{3}y[n-1]$

$$Y(z)\left(1+\frac{K}{3}z^{-1}\right) = X(z)\left(1-\frac{K}{4}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{K}{4}z^{-1}}{1+\frac{K}{3}z^{-1}}$$

Ceros:

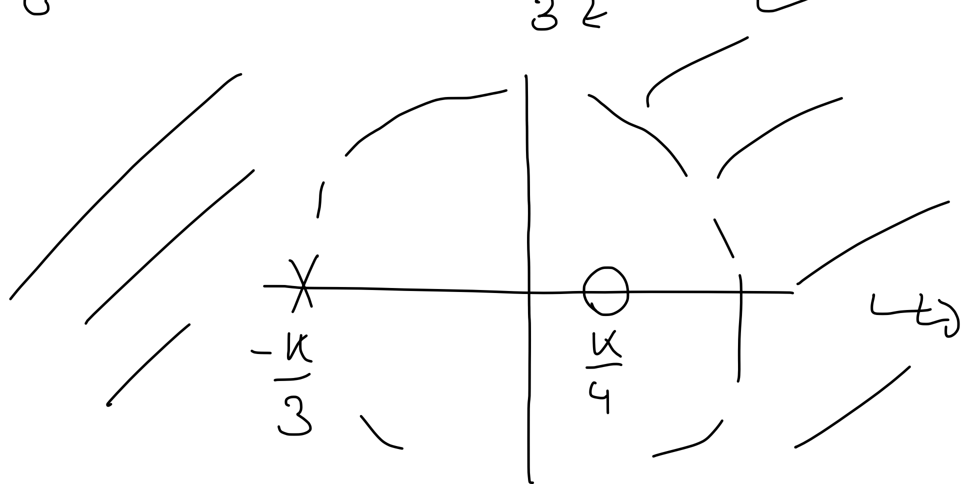
$$1-\frac{K}{4}z^{-1}=0 \Rightarrow 1=\frac{K}{4z} \Rightarrow \boxed{z=\frac{K}{4}}$$

Polos

$$1+\frac{K}{3}z^{-1}=0 \Rightarrow 1=\frac{-K}{3z} \Rightarrow \boxed{z=\frac{-K}{3}}$$

ROC:

$$\boxed{|z| > \left|\frac{K}{3}\right|}$$



↳ CAUSAL

b) Como es causal, $-\frac{k}{3} < 1$

$$-\frac{k}{3} < 1$$

$$-k < 3 \quad \boxed{k < -3}$$

c) $y[n]$ si $k=1$

y $x[n] = \left(\frac{2}{3}\right)^n \quad \forall n$

$$H(z) = \frac{1 - \frac{k}{4} z^{-1}}{1 + \frac{k}{3} z^{-1}}$$

$$x[n] \longrightarrow H(z) \longrightarrow y[n]$$

$$y[n] = \left(\frac{2}{3}\right)^n \cdot H(z)\Big|_{z=\frac{2}{3}} = \left(\frac{2}{3}\right)^n \cdot \left(\frac{1 - \frac{1}{4\left(\frac{2}{3}\right)}}{1 + \frac{1}{3\left(\frac{2}{3}\right)}}\right)$$

$$\frac{5}{12}$$

$$\boxed{y[n] = \left(\frac{2}{3}\right)^n \frac{5}{12}}$$