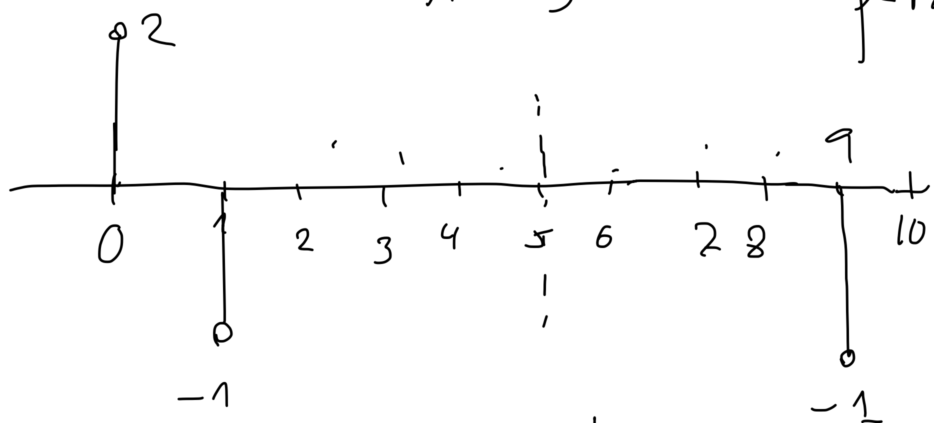
EXAMEN FINAL ASS DICIEMBRE 2016

Ejercicio 1

$X[n]$ periódica.



a) $a_0 = \sum_{n=0}^{9} x[n]\, e^{-j\frac{2\pi}{10}n}\Big|_{n=0} \quad \sum_{n=0}^{9} x[n] = 2+(-1)+(-1) = 0$

b) $\sum_{k=1}^{9} |a_k|^2 \Rightarrow |a_1|^2 + |a_2|^2 + |a_3|^2 \cdots + |a_9|^2$

$$P_m = \frac{1}{N}\sum_{n\langle N\rangle} |x[n]|^2 = \sum_{n\langle N\rangle} |a_k|^2$$

$$\frac{1}{N}\sum_{n=0}^{9} |x[n]|^2 = 2^2 + (-1)^2 + (-1)^2 = \frac{6}{10} = 3/5$$

$$\| $$

$$\sum_{n=0}^{9} |a_n|^2$$

Como sabemos que $a_0 = 0$ $\quad 3/5 - 0 \rightarrow \boxed{\sum_{k=1}^{9} |a_k|^2 = 3/5}$

c) Indica si $a_3 = a_4$

$x(n)$

$a_3 = a_{13} = a_{-7}$

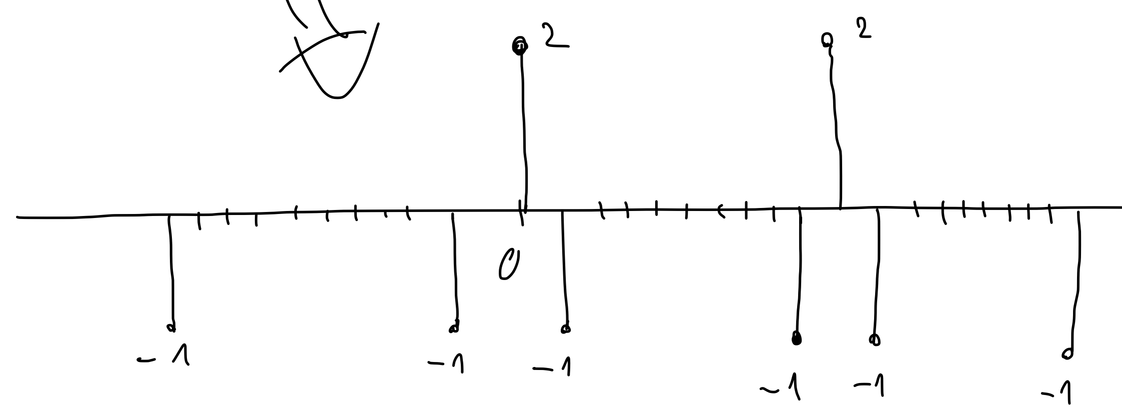
$a_4 = a_{14} = a_{-6}$

Es real y par $\Rightarrow$ a_k real y par.

$\Downarrow$

$a_3 = a_{13} = a_{-7}$

$a_4 = a_{14} = a_{-6}$



Por ser pares:

$a_3 = a_{-3} \quad a_7$

No se puede afirmar que sean el mismo

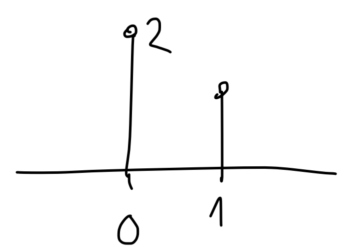
Si hacemos la TF:

$$2 - 2\cos\left(\frac{2\pi}{10}n\right) \left\{ \begin{array}{l} a_3 = \frac{1}{10}\left(2 - 2\cos\left(\frac{\pi}{5}3\right)\right) \\ a_4 = \frac{1}{10}\left(2 - 2\cos\left(\frac{\pi}{5}4\right)\right) \end{array} \right.$$

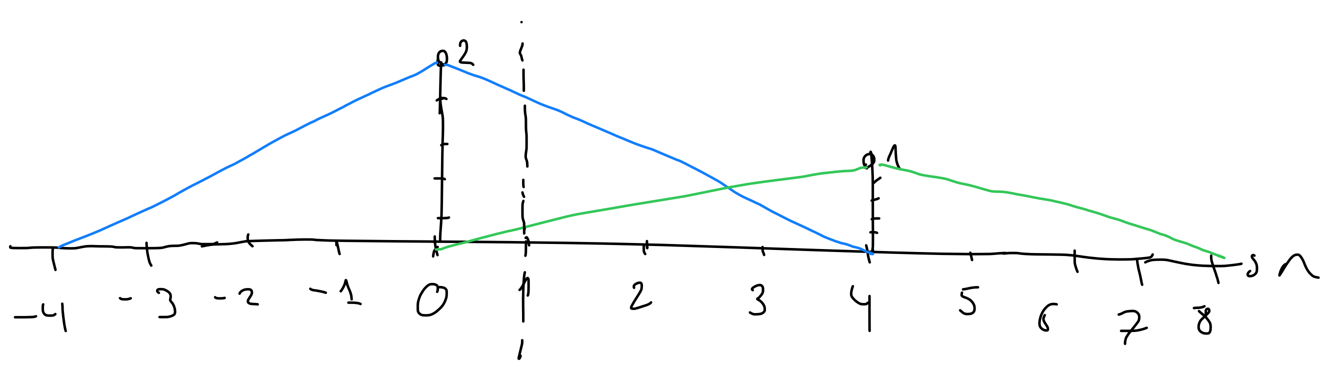
$\neq$ son diferentes

Ejercicio 2

x[n]



a) Interpolador de orden 1: , interpolador factor 4 = T



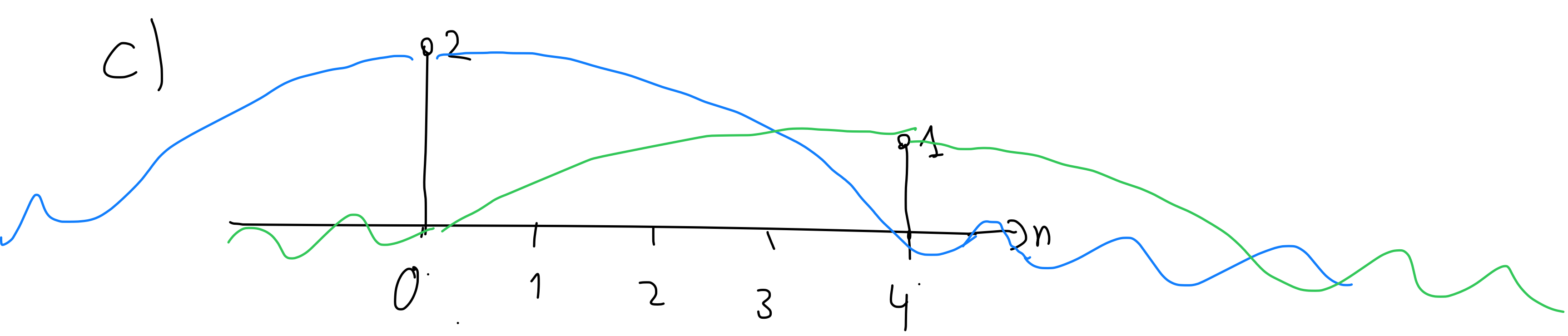
En $y[1] = 0{,}25 \cdot 1 + 0{,}75 \cdot 2 = 7/4$

En $y[2] = 0{,}5 \cdot 1 + 0{,}5 \cdot 2 = 3/2$

En $y[8] = 0$

b) $y[7] = 0{,}25 \cdot 1 = 1/4$

c)



$$2\,\frac{\operatorname{sen}\left(n\left(\frac{p}{T_s}\right)\right)}{n\left(\frac{p}{T_s}\right)} + \frac{\operatorname{sen}\left(n\left(\frac{p - T_s}{T_s}\right)\right)}{n\left(\frac{p - T_s}{T_s}\right)}$$

$T_s \to 4$, $\to 4$, $T_s \to 4$, $T_s \to 4$

2. $$\frac{\operatorname{sen}\left(\pi\left(\frac{n}{4}\right)\right)}{\pi\left(\frac{n}{4}\right)} + \frac{\operatorname{sen}\left(\pi\left(\frac{n-4}{4}\right)\right)}{\pi\left(\frac{n-4}{4}\right)}$$

$y[1] = 2{,}1$

$y[2] = 1{,}9$

$y[8] \triangleq 0$

En $y[7] = 0{,}04$

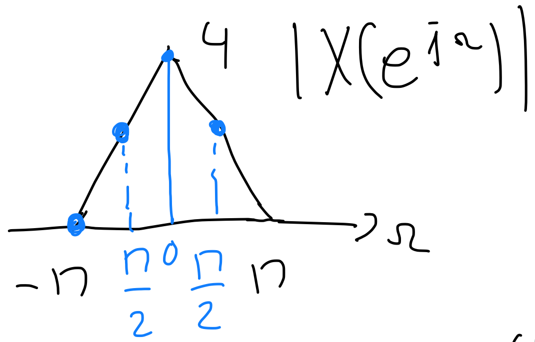
No haría falta, la sinc se hace 0 en los múltiplos de T.

Las dos sinc están separadas T así que no es 0.

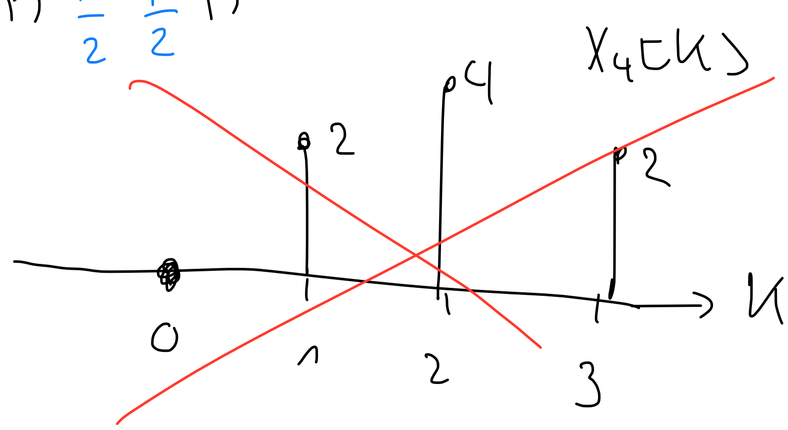
Ejercicio 3

a) DFT de 4 puntos

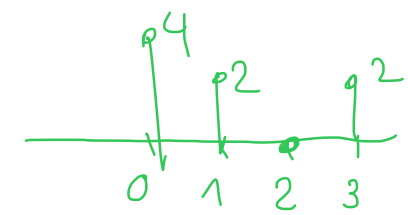
Muestreo con $N=4$



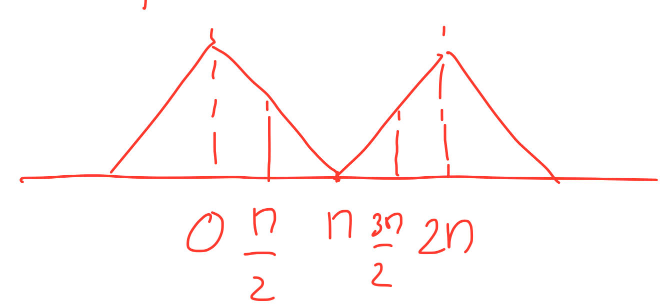
Muestrear cada $\frac{\pi}{2}$



¡Cuidado! Hay que empezar en 0 y hacerlo coincidir con 0.



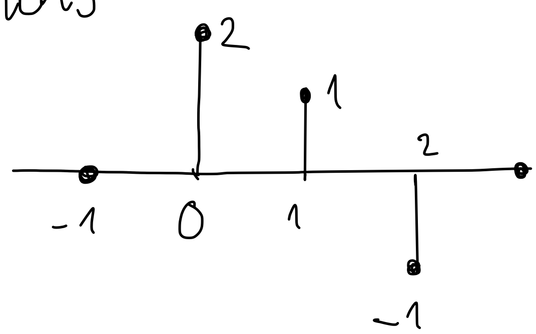
Esto es porque la DFT coincide con el muestreo de la señal justo en ese punto.



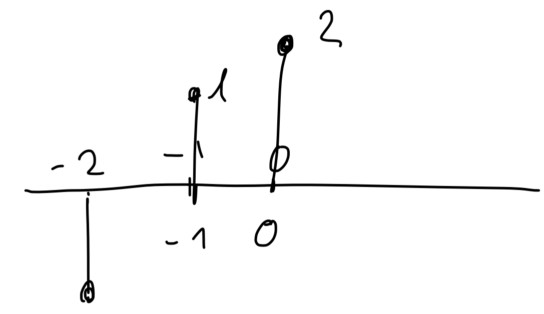
b) $x[n]$

c) $z[n] = x[n] \; 4 \; h[n]$

$h[n]$



$\Rightarrow$



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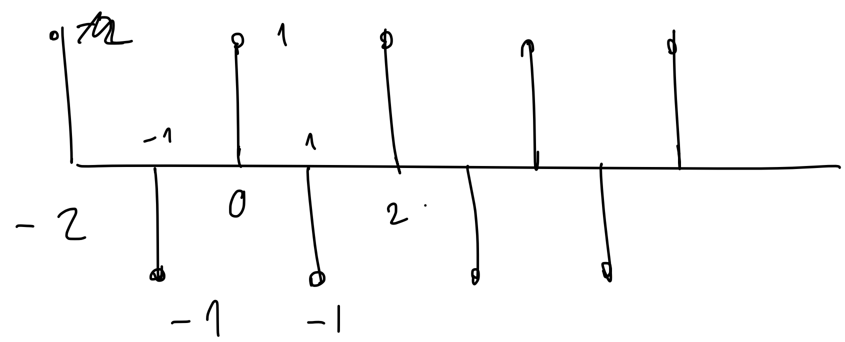
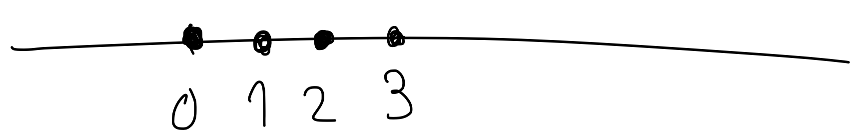




d) $x[n] \rightarrow \boxed{LTI} \rightarrow h[n]$

$\downarrow$

$h[n]$

$N \geq$ long $x[n]$ + long $y[n] - 1$

$N \geq 3+4-1 \Rightarrow \boxed{N \geq 6}$

Ejercicio 4

a) $Y(z) = H_1 X(z) - \left(Y(z) H_2 + Y(z) H_3\right) H_1$

$$Y(z)\left(1 + H_1 H_2 + H_1 H_3\right) = X(z) H_1$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{H_1}{1 + H_1 H_2 + H_1 H_3}$$

$$h_1[n] = \left(\frac{1}{2}\right)^n u[n] = \frac{1}{1 - \frac{1}{2} z^{-1}}$$

$$h_2[n] = \left(\frac{3}{32}\right)^n u[n] = \frac{1}{1 - \frac{3}{32} z^{-1}}$$

$$h_3[n] = -\delta[n] = -1$$

$$\frac{\frac{1}{1 - \frac{1}{2} z^{-1}}}{1 + \frac{1}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{3}{32} z^{-1}\right)} + \frac{-1}{\left(1 - \frac{1}{2} z^{-1}\right)}} = \frac{\frac{1}{1 - 1/2\, z^{-1}}}{\frac{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{3}{32} z^{-1}\right) + 1 - \left(1 - \frac{3}{32} z^{-1}\right)}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{3}{32} z^{-1}\right)}}$$

$$\left(1 - \frac{1}{2} z^{-1}\right) \cdot \left(1 - \frac{3}{32} z^{-1}\right) = 1 - \frac{3}{32} z^{-1} - \frac{1}{2} z^{-1} + \frac{3}{64} z^{-2}$$

$$1 - \cancel{\frac{3}{32} z^{-1}} - \frac{1}{2} z^{-1} + \frac{3}{64} z^{-2} + \cancel{1} - \cancel{1} + \cancel{\frac{3}{32} z^{-1}} = 1 - \frac{1}{2} z^{-1} + \frac{3}{64} z^{-2}$$

$$\frac{\dfrac{1}{1-\frac{1}{2}z^{-1}}}{\dfrac{1-\frac{1}{2}z^{-1}+\frac{3}{64}z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{3}{32}z^{-1}\right)}} = \frac{\cancel{\left(1-\frac{1}{2}z^{-1}\right)}\left(1-\frac{3}{32}z^{-1}\right)}{\cancel{\left(1-\frac{1}{2}z^{-1}\right)}\left(1-\frac{1}{2}z^{-1}+\frac{3}{64}z^{-2}\right)}$$

$$= \boxed{\frac{1-\frac{3}{32}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{3}{64}z^{-2}}}$$

Ceros:

$$1-\frac{3}{32}z^{-1}=0$$

$$1=\frac{3}{32z} \Rightarrow \boxed{z=\frac{3}{32}}$$

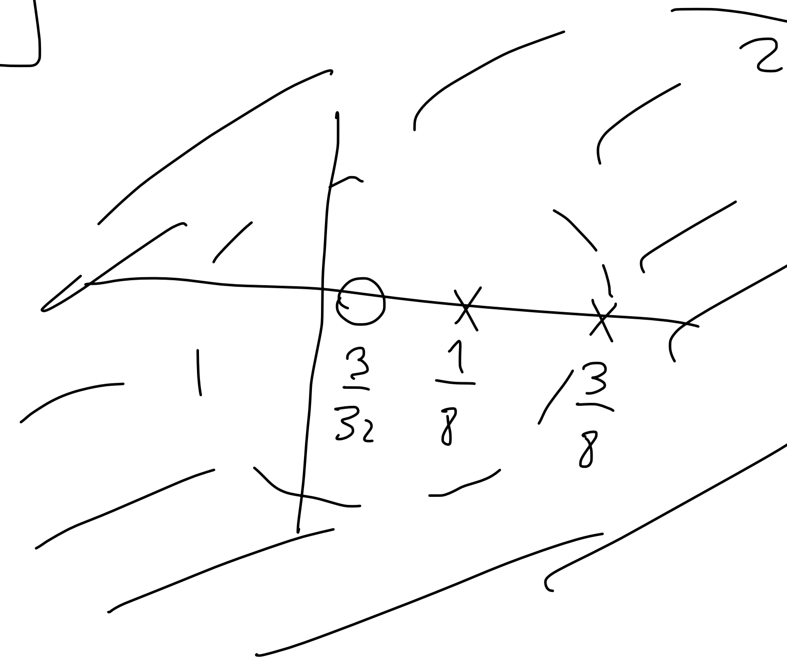
Polos:

$$\frac{\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2-4\cdot 1\cdot\frac{3}{64}}}{2\cdot 1}$$

$$\frac{\frac{1}{2}+\frac{1}{4}}{2} = \boxed{\frac{3}{8}=z}$$

$$\frac{\frac{1}{2}-\frac{1}{4}}{2} = \boxed{\frac{1}{8}=z}$$

ROC: $\boxed{|z|>\frac{3}{8}}$



b)

$$\frac{1-\frac{3}{32}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{3}{64}z^{-2}} = \frac{Y(z)}{X(z)}$$

$$X(z)\left(1-\frac{3}{32}z^{-1}\right) = Y(z)\left(1-\frac{1}{2}z^{-1}+\frac{3}{64}z^{-2}\right)$$

$$\boxed{x[n]-\frac{3}{32}x[n-1] = y[n]-\frac{1}{2}y[n-1]+\frac{3}{64}y[n-2]}$$

c) Respuesta al impulso del sistema.

$$\frac{1-3/32\, z^{-1}}{\left(1-\frac{3}{8}z^{-1}\right)\left(1-\frac{1}{8}z^{-1}\right)} = \frac{A}{\left(1-\frac{3}{8}z^{-1}\right)} + \frac{B}{\left(1-\frac{1}{8}z^{-1}\right)}$$

$$1-\frac{3}{32}z^{-1} = A\left(1-\frac{1}{8}z^{-1}\right)+B\left(1-\frac{3}{8}z^{-1}\right)$$

Si $z=\frac{3}{8}$; $1-\frac{3}{32}\frac{8}{3} = A\left(1-\frac{1}{8}\frac{8}{3}\right) \Rightarrow A = \frac{1/3}{61/64} = \boxed{9/8}$

Si $z=\frac{1}{8}$; $1-\frac{3}{32}8 = B\left(1-\frac{3}{8}\cdot 8\right) \Rightarrow \boxed{B=-1/8}$

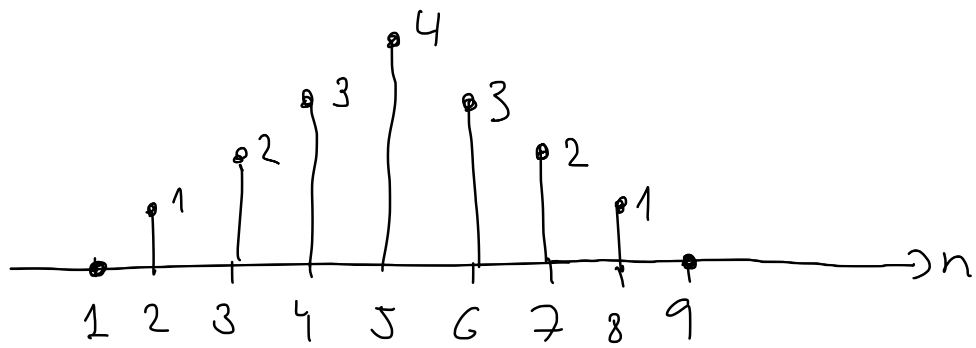
$$\frac{9/8}{\left(1-\frac{3}{8}z^{-1}\right)}-\frac{\frac{1}{8}}{\left(1-\frac{1}{8}z^{-1}\right)}$$

$$\downarrow$$

$$\left[\frac{9}{8}\left(\frac{3}{8}\right)^{n}u(n)-\frac{1}{8}\left(\frac{1}{8}\right)^{n}u(n)\right]$$

Ejercicio 5

a) ¿Fase lineal? Tiene que tener simetría en sus coeficientes.



$\Leftrightarrow$ Tiene fase lineal.

Retardo de grupo:

b) El punto de simetría está en $n=5$ $\left(\underbrace{H(e^{j\Omega})}_{\text{módulo}} \cdot e^{\underbrace{-j5\Omega}_{\text{fase}}}\right)$ $\Rightarrow$ Desplazada

$$\sphericalangle\{H(e^{j\Omega})\} = -5\Omega$$

$$\tau(\Omega) = -\frac{d}{d\Omega}(-5\Omega) = 5$$