EXAMEN FINAL ASS JUNIO 2019

Ejercicio 1

$$x(n) = 2 + \operatorname{sen}\left(\frac{\pi n}{3}\right) - 2\cos\left(\frac{\pi n}{6}\right)$$

a) Calcula el DSF

$$\operatorname{sen}\left(\frac{\pi n}{3}\right) = \frac{1}{2j}\left(e^{j\frac{\pi n}{3}} - e^{-j\frac{\pi n}{3}}\right) \qquad \frac{2\pi}{\pi/3} \Rightarrow \boxed{N=6}$$

$$-2\cos\left(\frac{\pi n}{6}\right) = \frac{1}{2}\left(e^{j\frac{\pi n}{6}} + e^{-j\frac{\pi n}{6}}\right) \qquad \frac{2\pi}{\frac{\pi}{6}} \Rightarrow \boxed{N=12}$$

m.c.m $\boxed{12}$

$$2e^{0} + \frac{1}{2j}e^{j\frac{\pi n}{3}} - \frac{1}{2j}e^{-j\frac{\pi n}{3}} - 2\frac{1}{2}e^{j\frac{\pi n}{6}} - 2\frac{1}{2}e^{-j\frac{\pi n}{6}}$$

$$\frac{2\pi}{12}x = \frac{\pi}{3}$$

$$x = 2$$

$$2e^{j\frac{2\pi}{12}n\cdot 0} + \frac{1}{2j}e^{j\frac{2\pi}{12}n\cdot 2} - \frac{1}{2j}e^{-j\frac{2\pi}{12}n\cdot 2} - 2\frac{1}{2}e^{j\frac{2\pi}{12}n\cdot 1} - 2\frac{1}{2}e^{-j\frac{2\pi}{12}n\cdot 1}$$

$a_0 = 2$ $\qquad a_1 = -1$

$a_2 = \frac{1}{2j}$ $\qquad a_{-1} = -1$

$a_{-2} = -\frac{1}{2j}$

Habrá 12 a_k, el resto son 0.

b) Calcular Energía y potencia

Es una señal definida en potencia, por ser periódica, como tengo su DSF, por Parseval:

$$P = \sum |a_k|^2 = 2^2 + (-1)^2 + (-1)^2 + \left|\frac{1}{2j}\right|^2 + \left(\frac{-1}{2j}\right)^2 = \frac{11}{2}$$

$E = \infty$

c) $x[n] = x_1[n] + x_2[n]$

La suma de energías no es un operador lineal, por tanto no se puede escribir así

$$E\{x_1\} + E\{x_2\} \neq E x_1^2 + E x_2^2$$

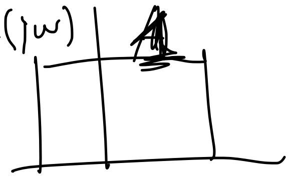
$$E\{x_1\} + E\{x_2\} = \sum |x_1|^2 + |x_2|^2$$

$$\neq$$

$$y[n] = E\{x_1[n] + x_2[n]\} = \sum |x_1 + x_2|^2$$

Ejercicio 2

$X(jw)$ A



$-2\pi \cdot 10^3$ $\quad 2\pi \cdot 10^3$

a) Espectro de $x[n]$ y energía $x[n] = x(10^{-4} n]$

$X(e^{j\Omega})$ $\quad \frac{A}{10^{-4}} = A \cdot 10^4$

$-2\pi \cdot 10^3 \cdot 10^{-4}$ $\quad 2\pi \cdot 10^3 \cdot 10^{-4}$

$-\frac{\pi}{5}$ $\quad \pi/5$

$2\pi \cdot 10^3 \cdot 10^{-4} < \pi$

$2\pi \cdot 10^{-1}$

$\pi/5 < \pi$ ✓ No hay solape.

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \rightarrow$$

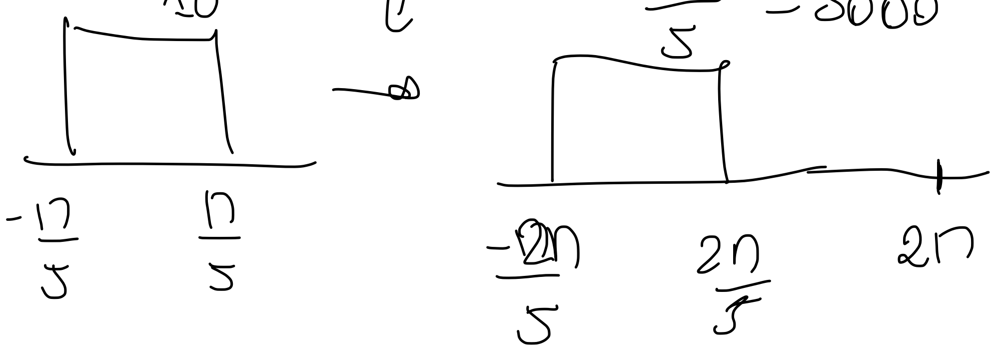
$$\rightarrow \frac{1}{2\pi} \int_{\langle 2\pi \rangle} |X(e^{j\Omega})|^2 \, d\Omega$$

$$\frac{1}{2\pi} \int_{-\pi/5}^{\pi/5} (A 0^4)^2 = 20.000.000$$

$$\frac{1}{2\pi}\left(10^4\right)^2\cdot\frac{2\pi}{5}$$

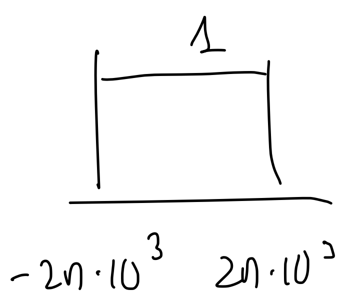
b)

Muestrear → Diezmar.



$$\frac{10^4}{5} = 5000$$

$$\frac{2\pi}{5} < \pi \checkmark$$



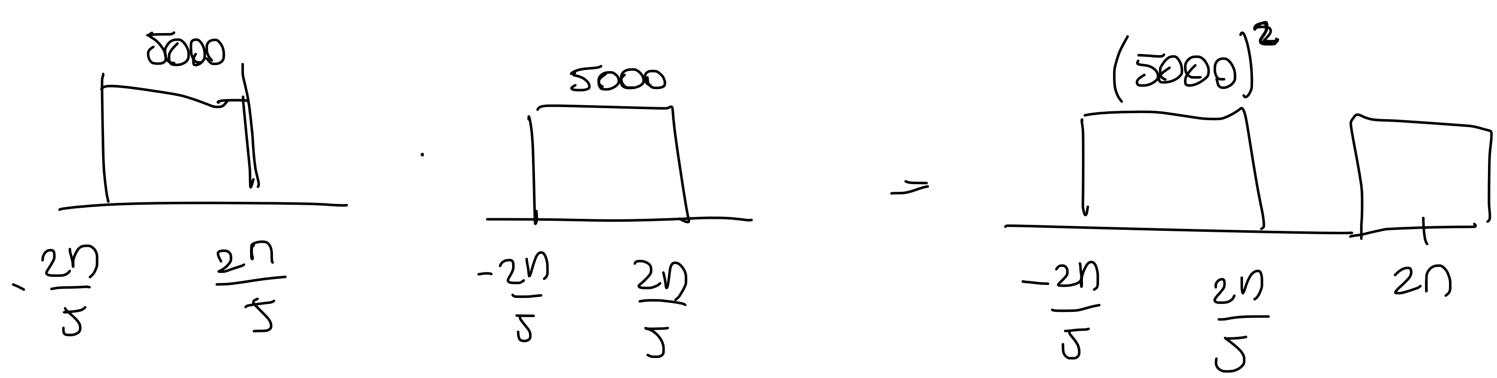
$$2\pi 10^3 X = \frac{2\pi}{5}$$

$$\boxed{X = \frac{2\pi/5}{2\pi\cdot 10^3} = \frac{1}{5\cdot 10^3} = \frac{1}{5000}}$$

$$\frac{1}{5000} = K_{\text{decodor}}\cdot T_{\text{muestre}}$$

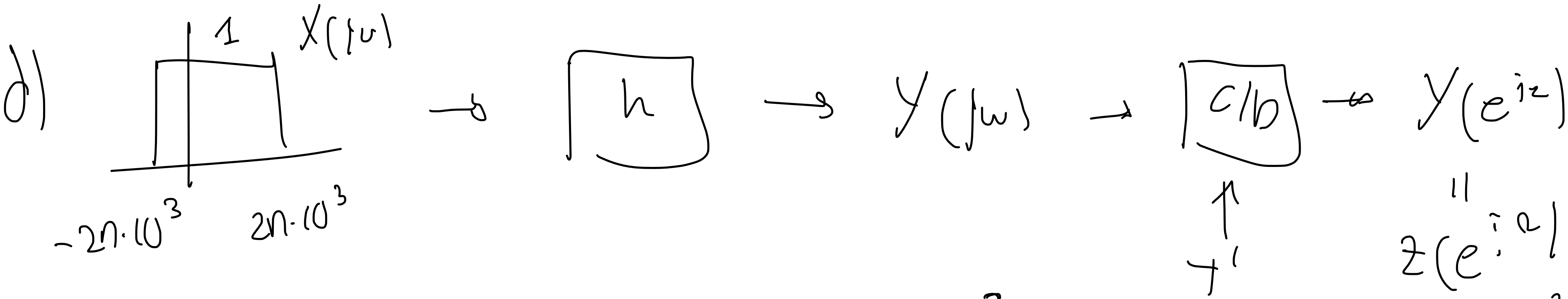
c)

$Z(e^{j\Omega})$



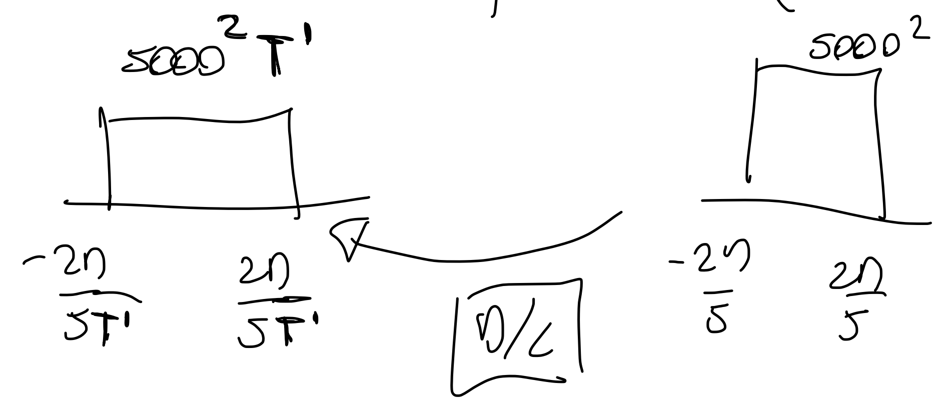
→ per 2π

d)

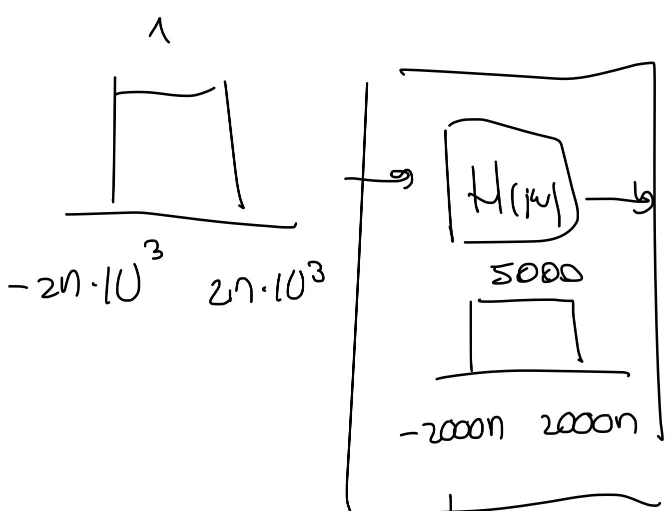


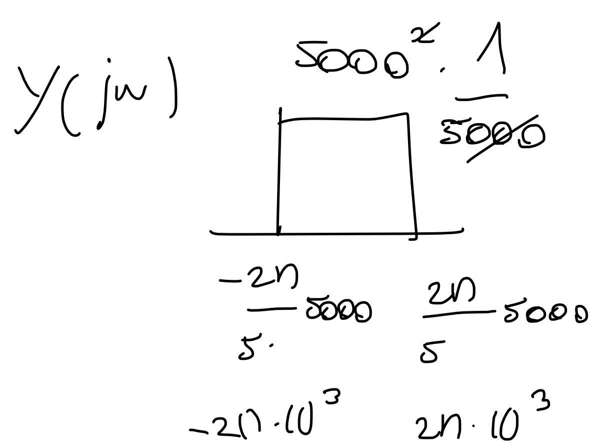
Σύγκριση με $\cancel{2\pi}\cdot 10^3 = \frac{\cancel{2\pi}}{5T'}$ 1

$$\boxed{T' = \frac{1}{5\cdot 10^3} = \frac{1}{5000}}$$



$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{5000^2 T'}{1}$$





TF^{-1}

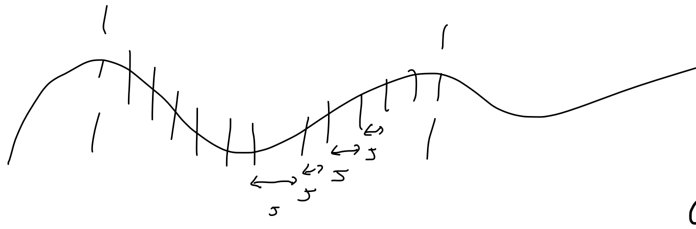
$$\frac{5000\,\mathrm{sen}(2000\pi n)}{\pi n}$$

Ejercicio 3

a)

2^n

máximo 5kHz



$5 \cdot N = 12000 \Rightarrow N_{veces} = \frac{12000}{5}$

↓

nº muestras

$12000 \frac{\text{muestras}}{\text{seg}} = 12\,kHz = f_{muestreo}$

$\frac{12000}{5} = 2{,}4\,kHz \; ; \; \frac{2400 \text{ muestras}}{\text{seg}}$

$2^{11} = 2048$ ✗ NO

$2^{12} = 4096 \longrightarrow$ longitud DFT

b) $k = \frac{f_c N}{f_s} = \frac{4600 \cdot 4096}{12000} = 1570{,}13$

$4096 - 1570{,}13 = 2525{,}27$

Ejercicio 4

$$w[n] = x[n] * h_1[n] + w[n] * h_2[n] * h_1[n]$$

$$y[n] = w[n] * h_3(n) + w[n] * h_4[n]$$

$$\left.\begin{aligned} W(z) &= X(z) H_1(z) + W(z) H_1(z) H_2(z) \\ Y(z) &= W(z) H_3(z) + W(z) H_4(z) \end{aligned}\right\}$$

$$Y(z) = W(z) \left(H_3(z) + H_4(z) \right)$$

$$W(z) \left(1 - H_1(z) H_2(z) \right) = X(z) H_1(z)$$

$$W(z) = \frac{X(z) H_1(z)}{1 - H_1 H_2}$$

$$Y(z) = \frac{X(z) H_1(z)}{1 - H_1(z) H_2(z)} \left(H_3(z) + H_4(z) \right)$$

$$\frac{Y(z)}{X(z)} = \frac{H_1(z)}{1 - H_1(z) H_2(z)} \left(H_3(z) + H_4(z) \right)$$

$H_1(z) = z^{-1}$ $\qquad H_3(z) = 2z^{-2}$

$H_2(z) = 9z^{-1}$ $\qquad H_4(z) = 3z^{-1}$

$$H(z) = \frac{z^{-1}}{1-9z^{-2}}\left(2z^{-2}+3z^{-1}\right) = \frac{2z^{-3}+3z^{-2}}{1-9z^{-2}}$$

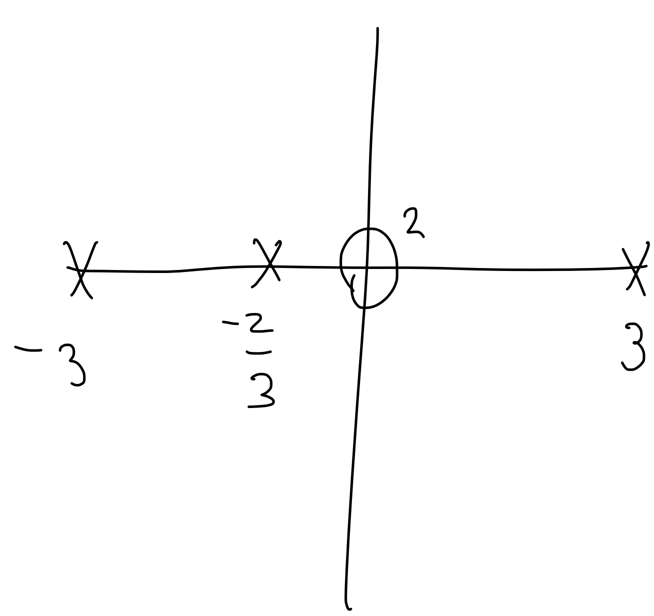
Polos

$$1-9z^{-2}=0 \qquad 1=\frac{9}{z^2} \Rightarrow z^2 = 9 \Rightarrow \boxed{z=\pm 3}$$

Ceros

$$z^{-2}\left(2z^{-1}+3\right) \qquad \boxed{\text{Cero doble a } 0}$$

$$\hookrightarrow \frac{2}{z} = -3 \qquad \boxed{z=\frac{-2}{3}}$$



Posibles:

$|z| > 3$

$|z| < 3$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-3} + 3z^{-2}}{1 - 9z^{-2}}$$

$$Y(z)\left(1 - 9z^{-2}\right) = \left(2z^{-3} + 3z^{-2}\right)X(z)$$

$$y[n] - 9y[n-2] = 2x[n-3] + 3x[n-2]$$

$$\boxed{y[n] = 3x[n-2] + 2x[n-3] + 9y[n-2]}$$

d) si $|z| > 3$ es causal, si no, no lo es.

e) $H(z) = \dfrac{2z^{-3} + 3z^{-2}}{1 - 9z^{-2}}$

$$\begin{array}{l|l} 2z^{-3} + 3z^{-2} & -9z^{-2} + 1 \\ \hline -2z^{-3} \qquad\qquad + \frac{2}{9}z^{-1} & -\frac{2}{9}z^{-1} - \cancel{\frac{3}{9}}\ \frac{1}{3} \\ \hline 3z^{-2} + \frac{2}{9}z^{-1} & \\ -3z^{-2} \qquad + \frac{3}{9} & \\ \hline \end{array}$$

$$\frac{2z^{-3}}{-9z^{-2}} = \frac{-2}{9}z^{-1}$$

$$9 \cdot \frac{2}{9} =$$

$$\frac{3\cancel{z^{-2}}}{-9\cancel{z^{-2}}} = \frac{-3}{9}$$

$$+\frac{3}{\cancel{9}} \quad +\cancel{9}$$

$$\underbrace{\frac{2}{9}z^{-1} + \frac{1}{3}}$$

$$-\frac{2}{9}z^{-1} - \frac{1}{3} + \frac{2/9\, z^{-1} + \frac{1}{3}}{1 - 9z^{-2}}$$

$$1 - 9z^{-2} = 0$$

$$1 = \frac{9}{z^2} \rightarrow \boxed{z = \pm 3}$$

$$\frac{2/9\, z^{-1} + \frac{1}{3}}{(1 - 9z^{-2})} = \frac{A}{(1 + 3z^{-1})} + \frac{B}{(1 - 3z^{-1})}$$

$$2/9\, z^{-1} + \frac{1}{3} = A(1 - 3z^{-1}) + B(1 + 3z^{-1})$$

Si $z = 3$

$A = 0$

$$\frac{2}{9}\frac{1}{3} + \frac{1}{3} = B\underbrace{\left(1 + 3 \cdot \frac{1}{3}\right)}_{2}$$

$$\frac{11}{27}$$

$$\boxed{B = \frac{11/27}{2} = \frac{11}{54}}$$

Si $z = -3$

$B = 0$

$$\frac{2}{9}\left(-\frac{1}{3}\right) + \frac{1}{3} = A\left(1 + 3 \cdot \frac{1}{3}\right)$$

$$\frac{7}{27} = A\,2$$

$$\boxed{A = \frac{7/27}{2} = \frac{7}{54}}$$

$$-\frac{2}{9}z^{-1} - \frac{1}{3} + \frac{7/54}{(1+3z^{-1})} + \frac{11/54}{(1-3z^{-1})}$$

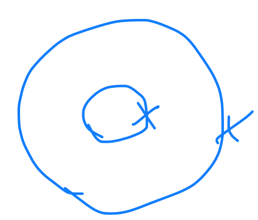
Si $|z| > 3 \rightarrow$ a derechas

$$-\frac{2}{9}\delta[n-1] - \frac{1}{3}\delta[n] + \frac{7}{54}\cdot(-3)^n u[n] + \frac{11}{54}(3)^n u[n]$$

Si $|z| < 3 \rightarrow$ a izquierdas.

$$\underbrace{-\frac{2}{9}\delta[n-1] - \frac{1}{3}\delta[n]}_{\text{no cambian.}} - \frac{7}{54}(-3)^n u[-n-1] - \frac{11}{54}(3)^n u[-n-1]$$

Si hubiera sido bilateral, la del círculo de fuera es como la de derechas, la del centro, como si fuera a izquierdas.



f) ¿Estable?

Como la de fuera es causal, todos los polos deberían estar dentro de la ROC, no lo están.

Si no lo es, los polos debería incluir el círculo unidad (si la TF existe)

Ejercicio 5

Es causal y estable pq su ROC abarca todo el plano.

No es de fase mínima pq no todos los polos y ceros están dentro del círculo unidad.

Fase lineal $\Rightarrow$ simetría sí

$$H(z) = \frac{\left(1-\frac{4}{3}e^{-j\frac{3\pi}{4}}z^{-1}\right)\left(1-\frac{3}{4}e^{-j\frac{3\pi}{4}}z^{-1}\right)\left(1+z^{-2}\right)\left(1+\frac{4}{3}e^{-j\frac{3\pi}{4}}\right)\left(1+\frac{3}{4}e^{-j\frac{3\pi}{4}}z^{-1}\right)}{z^{-5}}$$

5 polos

FIR $\Rightarrow$ Un polo a cero

$$(a+b)(a-b) = a^2 \cancel{-ab} + \cancel{ab} - b^2$$

$$H(\Omega) = \frac{\left(e^{j\Omega 2} - \cancel{\left(\frac{4}{3}\right)^2}^{\frac{16}{9}} \cancel{e^{-j\frac{3\pi}{4}2}}^{j}\right)\left(e^{j\Omega}+1\right)\left(e^{j\Omega 2} - \cancel{\left(\frac{3}{4}\right)^2}^{\frac{9}{16}} \cancel{e^{-j\frac{3\pi}{4}2}}^{j}\right)}{e^{j\Omega}}$$

$$\frac{6\pi}{4} = \frac{3\pi}{2} \quad -j$$

$$H(z) = \frac{\left(e^{j2\Omega} - \frac{16}{6}j\right)\left(e^{j\Omega}+1\right)\left(e^{j2\Omega} - \frac{9}{16}j\right)}{e^{j\Omega}}$$

$$H(0) = \frac{\left(1-\frac{16}{6}j\right)(1+1)\left(1-\frac{9}{16}j\right)}{1} \; ; \; H\left(\frac{\pi}{2}\right) = \frac{\left(-1-\frac{16}{6}j\right)(-1+1)\left(-1-\frac{9}{6}j\right)}{-1}$$

$$H(\pi) = \frac{\left(1-\frac{16}{6}j\right)(1+1)\left(e^{j2\Omega}-\frac{9}{16}j\right)}{1}$$