FINAL ASS DICIEMBRE 2014

Ejercicio 1

$$x[n] = \cos\left[\frac{2\pi}{3}n\right] + \operatorname{sen}\left[\frac{2\pi}{5}n\right]$$

a) ¿Periodo de $x[n]$?

$$2\pi = \frac{2\pi}{3}N \Rightarrow N = \frac{2\pi}{2\pi/3}$$

$$\frac{2\pi}{\frac{2\pi}{3}} = 3$$

$$2\pi = \frac{2\pi}{5} \Rightarrow N = \frac{2\pi}{2\pi/5} = 5$$

Periodo:

$$\text{m.c.m}\{3,5\} = \boxed{15 = N}$$

b) Obtenga los coeficientes del DSF

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2}\left(e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right)$$

$$\operatorname{sen}\left(\frac{2\pi}{5}n\right) = \frac{1}{2j}\left(e^{j\frac{2\pi}{5}n} - e^{-j\frac{2\pi}{5}n}\right)$$

$$\frac{1}{2}e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n} + \frac{1}{2j}e^{j\frac{2\pi}{5}n} - \frac{1}{2j}e^{-j\frac{2\pi}{5}n} =$$

$$= \frac{1}{2}e^{j\frac{2\pi}{15}n\cdot 5} + \frac{1}{2}e^{-j\frac{2\pi}{15}n\cdot 5} + \frac{1}{2j}e^{j\frac{2\pi}{15}n\cdot 3} - \frac{1}{2j}e^{-j\frac{2\pi}{15}n\cdot 3}$$

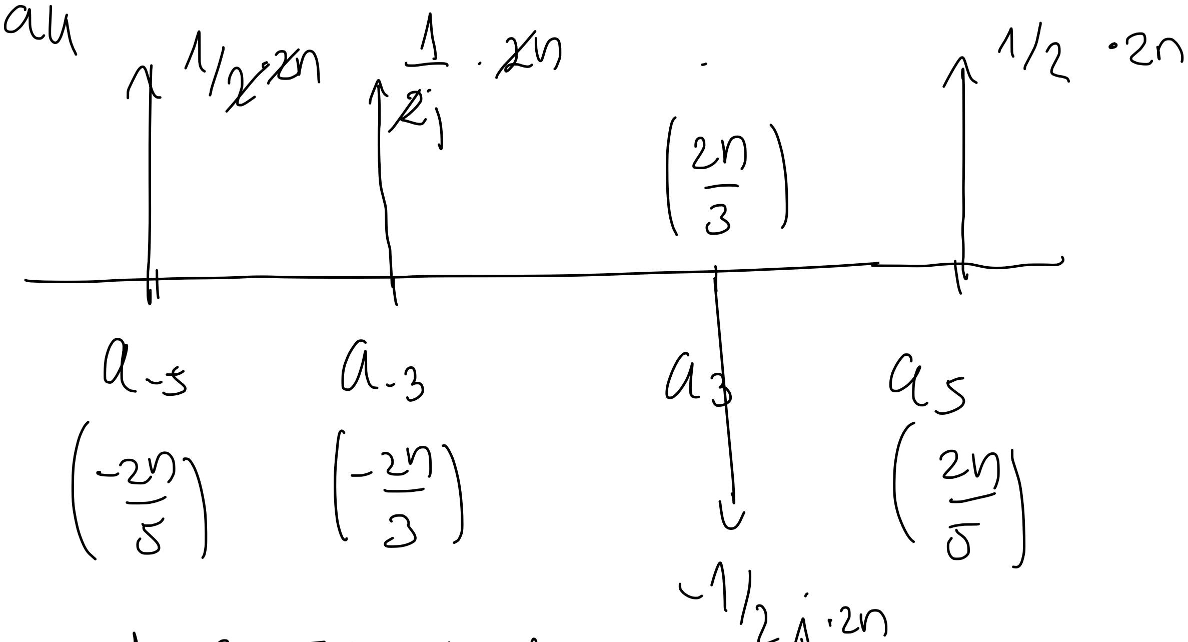
$$\frac{2\pi}{15}x = \frac{2\pi}{3} \Rightarrow x = \frac{\cancel{2\pi}/3}{\cancel{2\pi}/15} = \frac{\cancel{2\pi}\cdot 15}{\cancel{2\pi}\cdot 3} = 5$$

$\left\{ a_5 = \frac{1}{2} ;\ a_{-5} = \frac{1}{2} ;\ a_3 = \frac{1}{2j} ;\ a_{-3} = -\frac{1}{2j} \right.$
Hay 15 a_k, el resto son 0. $\}$

c) Módulo y fase de $X(e^{j\Omega})$

Tengo el DSF, por lo que lo puedo relacionar con la TF para sacar $X(e^{j\Omega})$ haciendo:

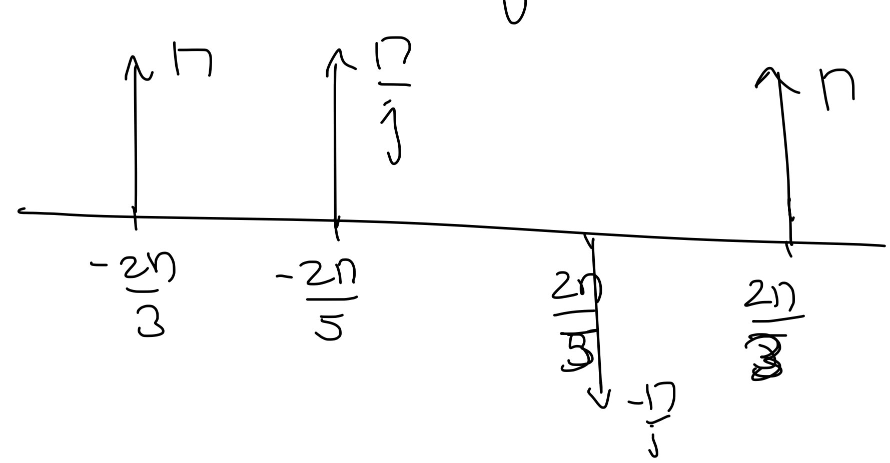
TF $\rightarrow$ $2\pi \cdot a_k$



Si lo hiciera haciendo la TF directamente:

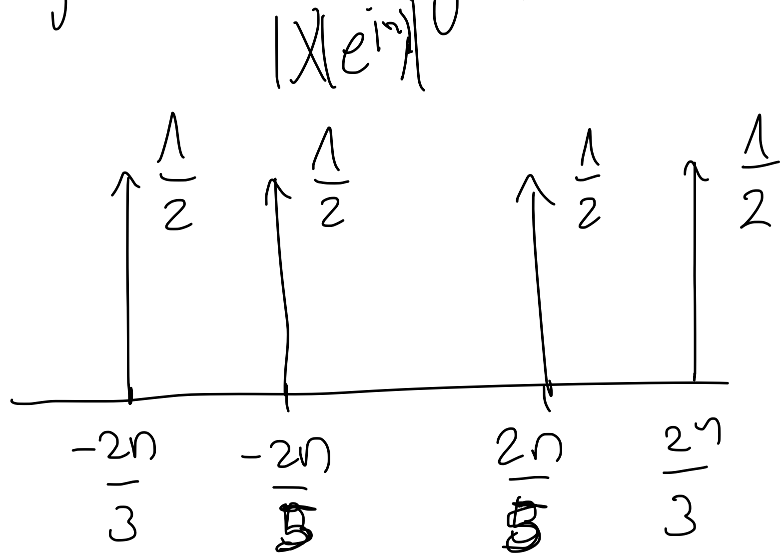
$$TF\left\{ \cos\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{2\pi}{5}n\right) \right\}$$

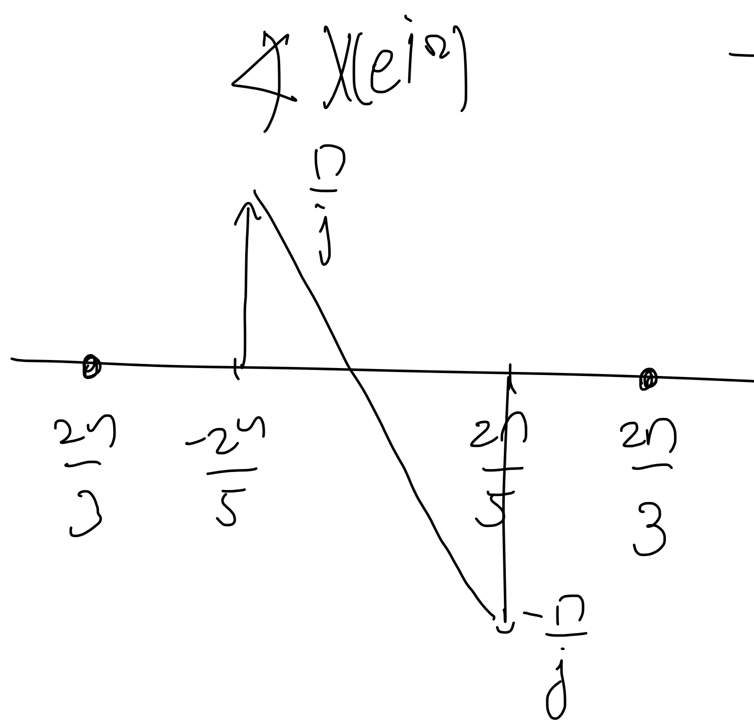
$$\pi\left(\delta\left(\Omega - \frac{2\pi}{3}\right) + \delta\left(\Omega + \frac{2\pi}{3}\right)\right) \qquad \frac{\pi}{j}\left(\delta\left(\Omega - \frac{2\pi}{5}\right) - \delta\left(\Omega + \frac{\pi}{5}\right)\right)$$

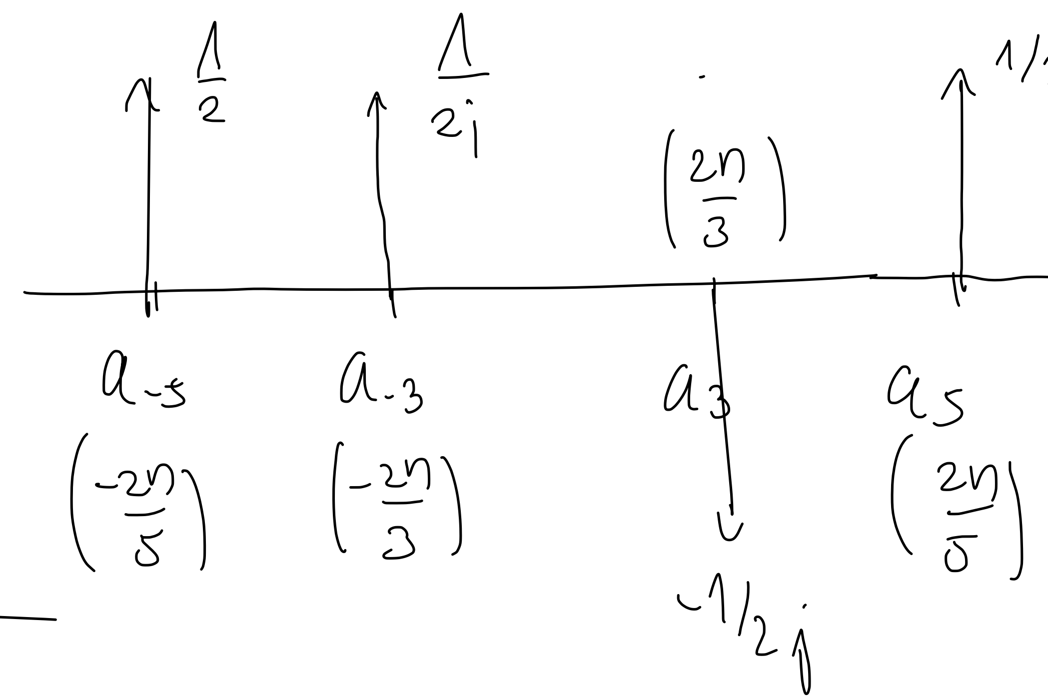


Esas son las dos maneras de sacarlo.

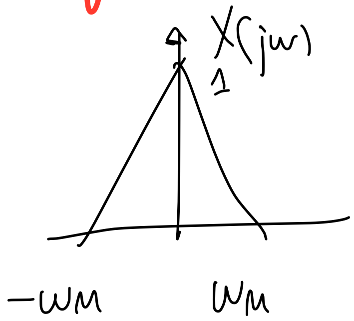
Me pide módulo y fase así que:



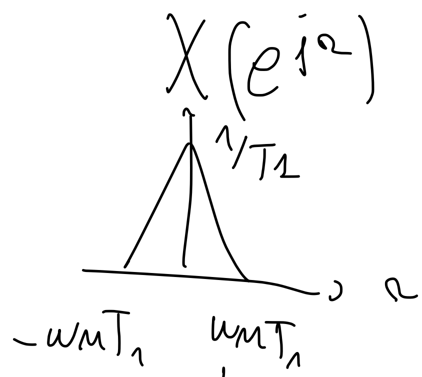




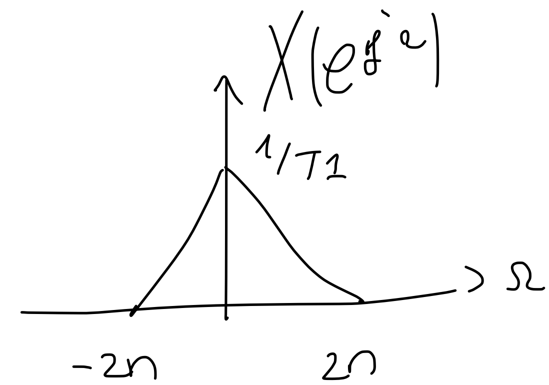
Ejercicio 2



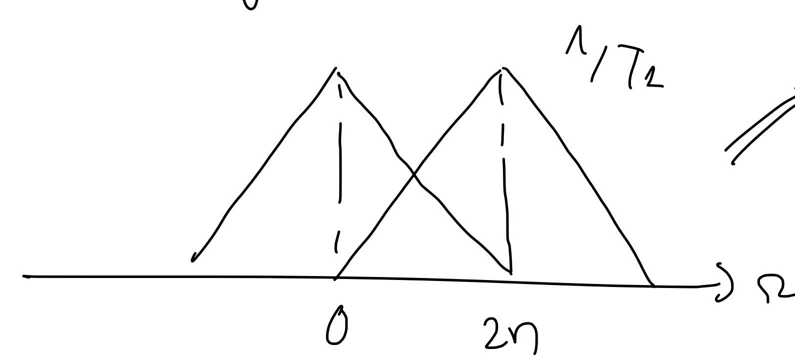
a) TF de g(t)



$\frac{2\pi}{T_1}\cdot T_1$

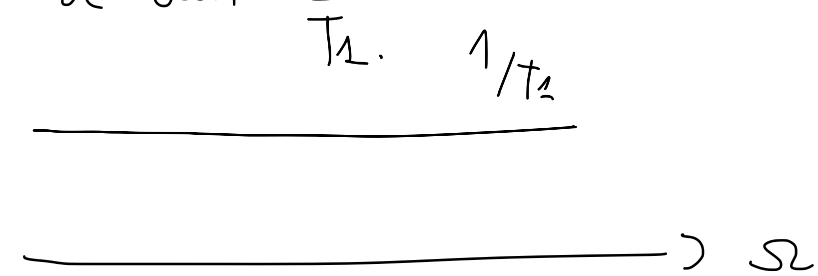


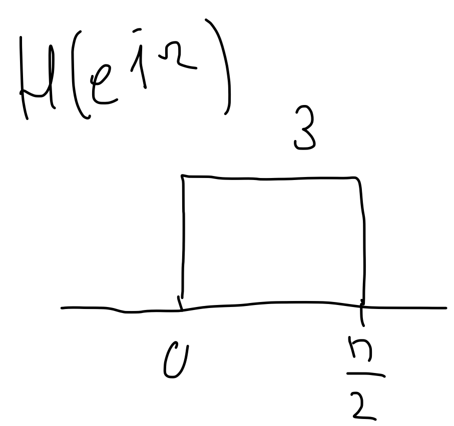
No cumple el teorema de Nyquist, hay solapamiento en el muestreo

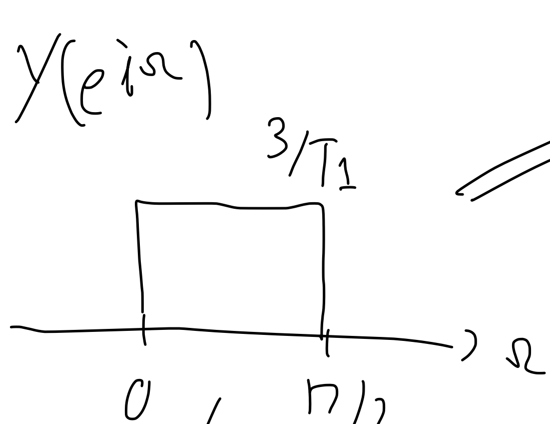


Periódica 2π

Va justo a la mitad, es una de valor $\frac{1}{T_1}$.

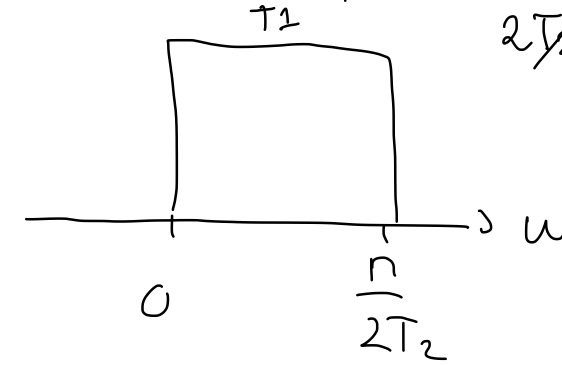






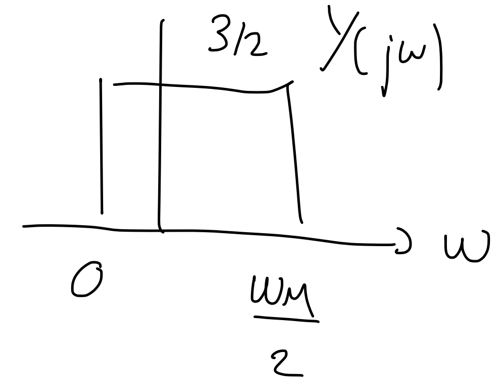
↳ Periódica 2π

$\frac{3}{T_1}T_2 = \frac{3\cancel{T_2}}{2\cancel{T_2}} = 3/2$



No periódica

↳ $\frac{\pi}{\cancel{2}\frac{T_1}{\cancel{2}}}$ $\quad T_1 = \frac{2\pi}{w_M} \quad \frac{\cancel{\pi}}{\frac{2\cancel{\pi}}{w_M}}$



b) Indique si x(t) e y(t) son señales reales o complejas.

$TF\{x(t)\} = X(j\omega)$

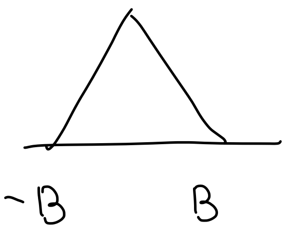
↳ señal real y par

Por lo tanto tiene que venir de una señal real y par.

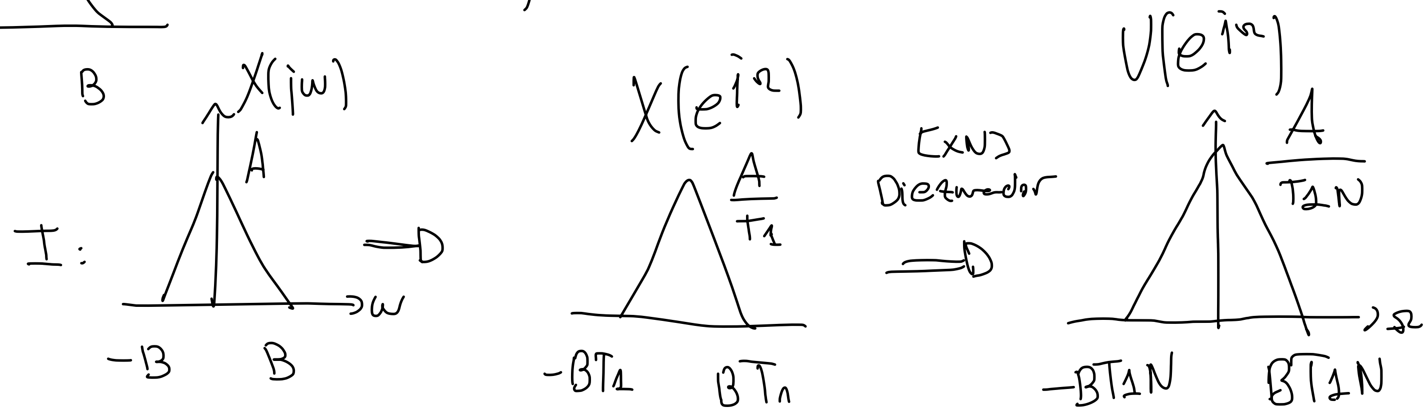
El módulo de la $TF\{y(t)\} = Y(j\omega)$ no es par

por tanto proviene de una señal compleja.

Ejercicio 3



a) En el esquema I $V[n] \to$ muestrear $x(t)$ utilizando un periodo $T_1 N$



↳ Periódica 2π ↳ Periódica 2π

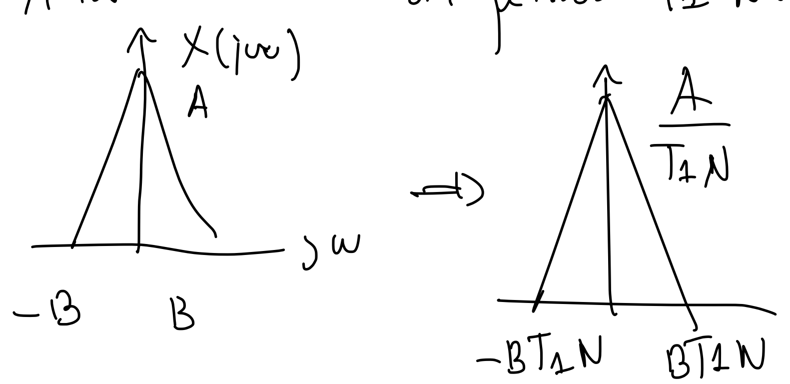
tma Nyquist: $BT_1 < \pi$

⇓

Debe cumplirse:

$BT_1 N < \pi$

• Ahora muestreo con periodo $T_1 \cdot N$:



↳ Periódica 2π

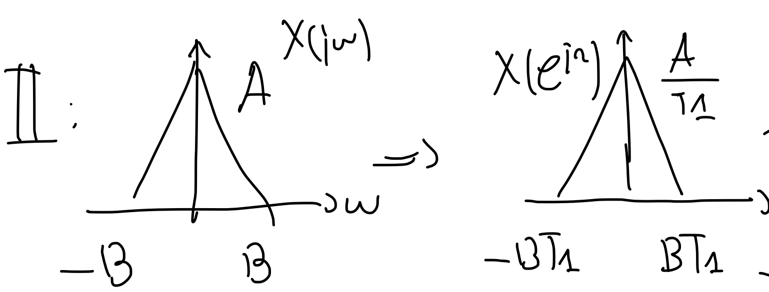
Debe cumplirse:

$BT_1 N < \pi$

VERDADERO

b) $\frac{2\pi}{T_1} > 2B \to$ Esquema II

$V[n]$ = muestreo de $x(t)$ con periodo $\frac{T_1}{L}$



→ Per. 2π

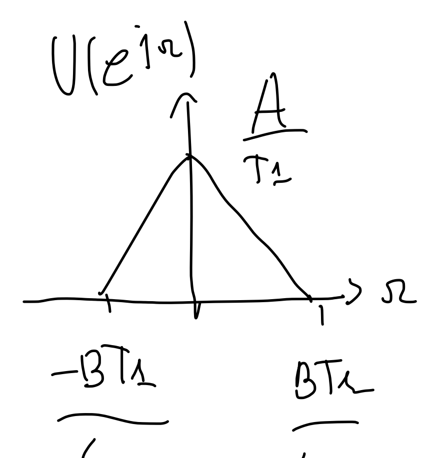
Interpolador $\left(\frac{n}{L}\right)$

→ $BT_1 < \pi$

Sé que $\frac{2\pi}{T_1} > 2B$

$\frac{\cancel{2}\pi}{\cancel{2}} > BT_1$

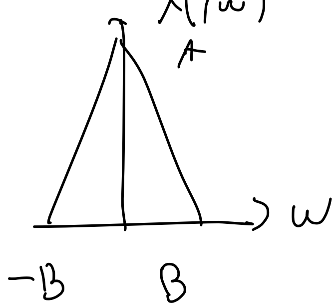
Se cumple



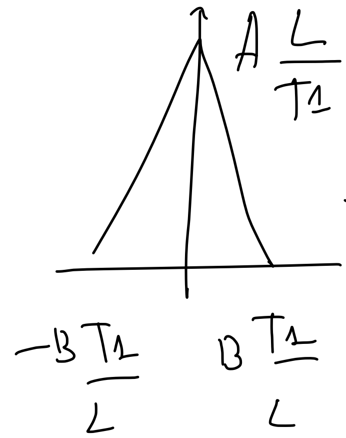
↳ Periódica $\frac{2\pi}{L}$

No hay solape por ser interpolador

• Ahora muestreo con período $\frac{T_1}{L}$



$\Longrightarrow$



$\Longrightarrow$ Periódica 2π

Sé que $\frac{\cancel{2}\pi}{T_1} > \cancel{2}B$

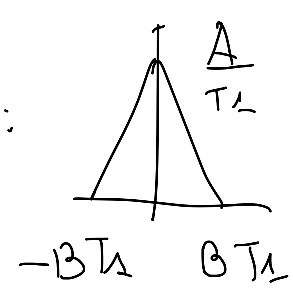
no debe ampliarse $\frac{BT_1}{L} < \pi$

$BT_1 < \pi$

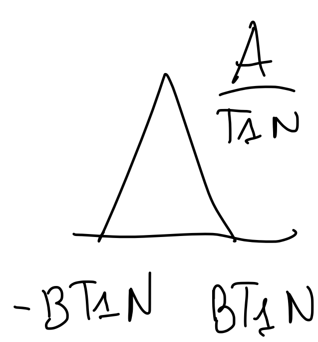
FALSO No coincide la amplitud.

c) Si $\frac{2\pi}{T_1} > 2B$, $L = N$, $T_1 = T_2$, la salida para $I = II$

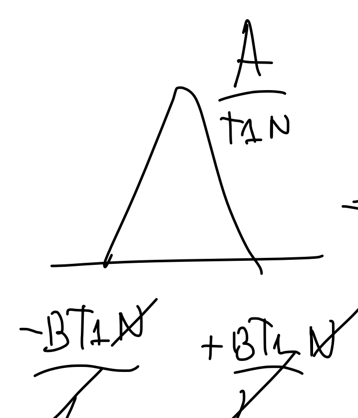
I:



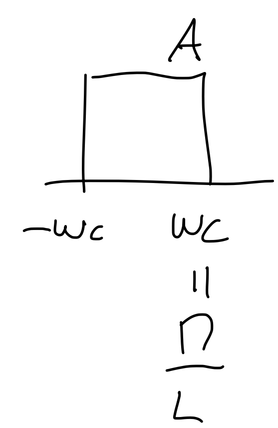
$\Longrightarrow$



$\Longrightarrow$



$\Longrightarrow$



Como son iguales no hay solape

$\frac{2\pi}{T_1} > 2B$

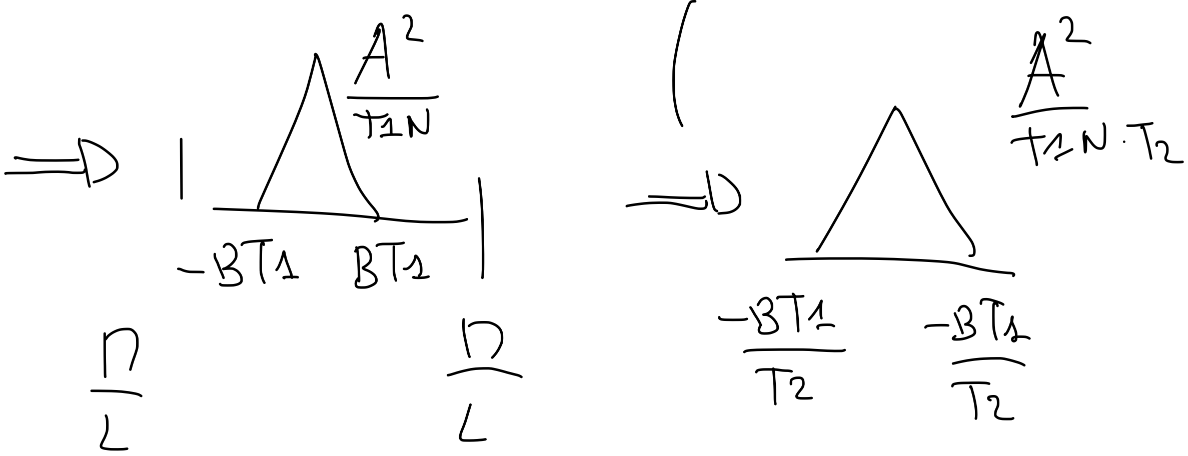
¿$BT_1 < \frac{\pi}{L}$? no depende de L

$\cancel{2}\pi > \cancel{2}BT_1$

$BT_1 < \pi$.

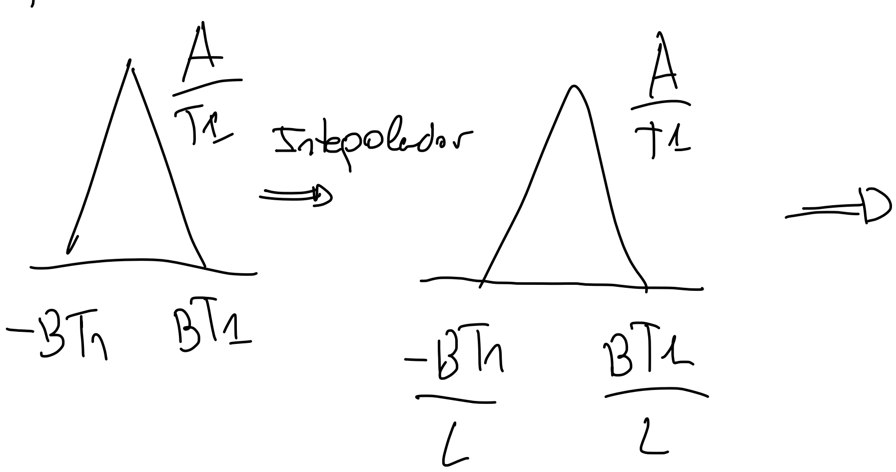
Si L hace que $\frac{\pi}{L} > \pi$ hay solape en el filtro

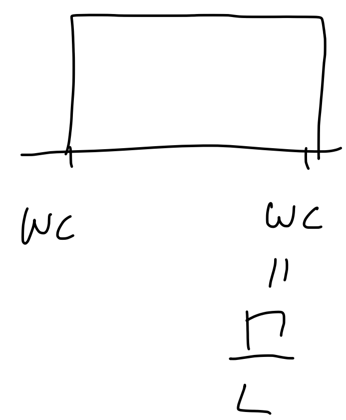
Si no solapa.



FALSO

Esquema III





$¿\frac{BT_1}{2} < \frac{\Pi}{L}?$

$BT_1 < \Pi$

$\frac{\cancel{2}\Pi}{T_1} > \cancel{2}B$

$\Pi > BT_1$

Se cumple.

Por teoría sabemos que si tenemos L segundo de N y $L \geq N$ (nuestro caso) no se pierde información

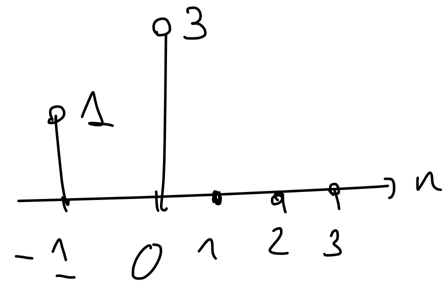
En III NUNCA SE PIERDE INFO

En I SE PIERDE INFO SI EL VALOR DE L HACE QUE $w_c < BT_1$ $\quad \frac{\Pi}{L} < BT_1$

Ejercicio 4

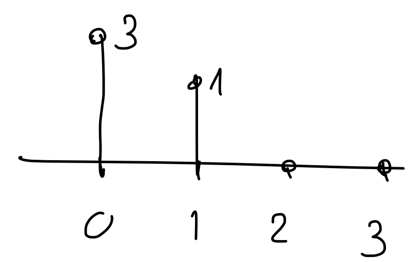
Calcular la DFT:

a) $x_1[n] = \delta[n+1] + 3\delta[n]$

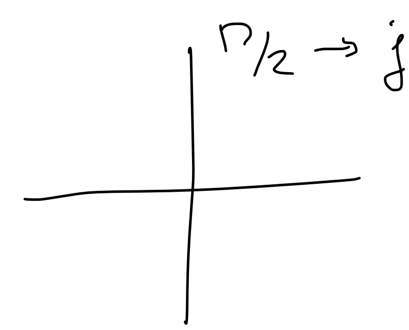


$$X_4[k] = \sum_{n=0}^{3} 3\, e^{-j\frac{\cancel{2}\pi}{\cancel{4}2}nk} = \cancel{3\, e^{-j\frac{\pi}{2}\cdot 0 \cdot k}} \to 1 = \boxed{3} \quad k=0,1,2,3$$

b) $x_2[n] = 3\delta[n] + \delta[n-1]$

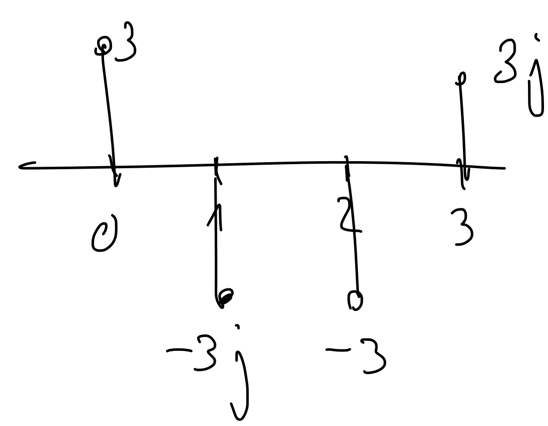


$$X_4[k] = \sum_{n=0}^{3} \left(3\delta[n] + \delta[n-1]\right) e^{-j\frac{2\pi}{4}nk} =$$



$$= 3\cdot 1 + \underbrace{e^{-j\frac{\cancel{2}\pi}{\cancel{4}2}k}}_{(-j)^k} = \boxed{3+(-j)^k}$$

Para $k = 0, 1, 2, 3$

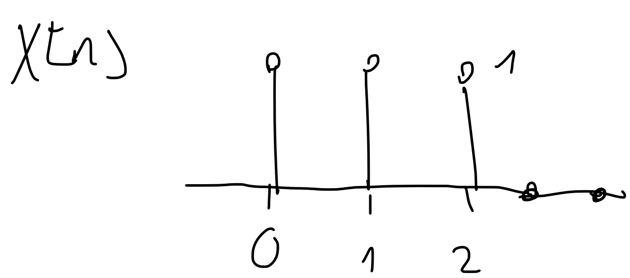


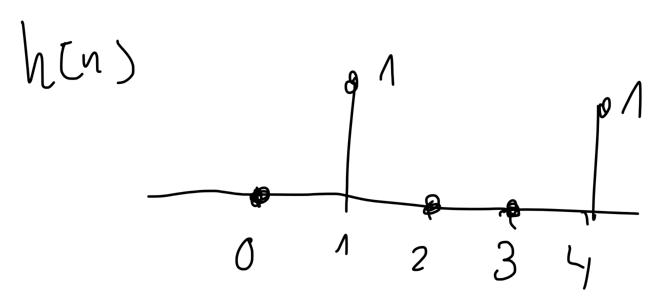
Ejercicio 5

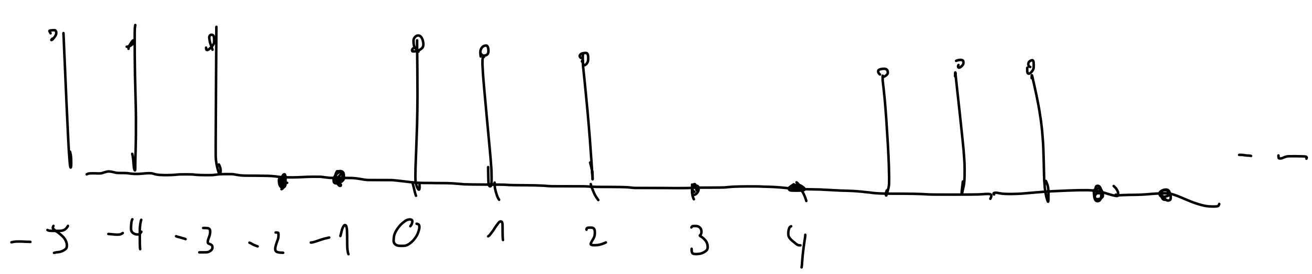
$$y[n] = x[n] \, (N) \, h[n]$$

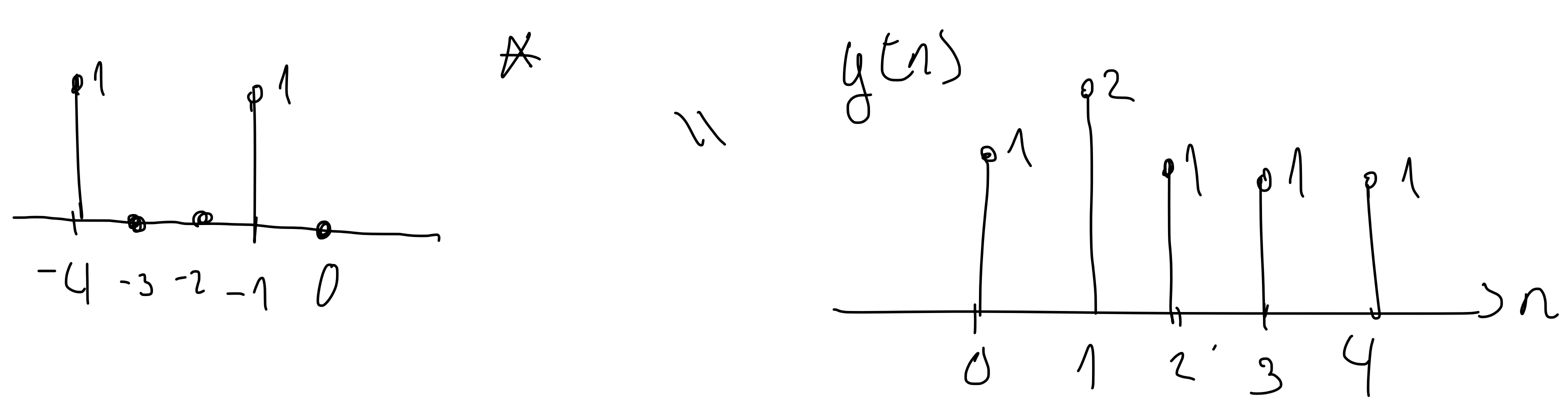
b) Sí, por propiedades sabemos que hacer la DFT de una señal, hacer la de otra, multiplicar las DFT y hacer su inversa es igual que convolucionar en N puntos las originales.

$$y[n] = x[n] \, (5) \, h[n]$$









Ejercicio 6

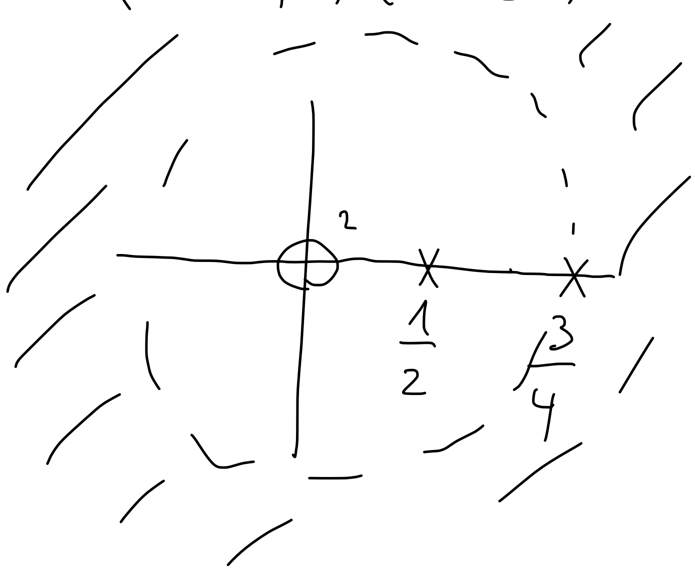
$$H(z) = \frac{1/8}{\left(1+\frac{3}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}$$

a) Representar polos y ceros.

$$\frac{1/8\, z^{2}}{\left(z-\frac{3}{4}\right)\left(z-\frac{1}{2}\right)}$$

Ceros: $\boxed{z=0}$ (doble

Polos: $\boxed{z=\frac{3}{4}}$ $\boxed{z=\frac{1}{2}}$



Como es CAUSAL
Incluye el ∞.

ROC: $|z| > \frac{3}{4}$

b) Respuesta al impulso del sistema.
Descomposición en fracciones simples:

$$\frac{1/8}{\left(1+\frac{3}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} = \frac{A}{\left(1+\frac{3}{4}z^{-2}\right)} + \frac{B}{\left(1+\frac{1}{2}z^{-1}\right)}$$

$$1/8 = \left(1+\frac{1}{2}z^{-1}\right)A + \left(1+\frac{3}{4}z^{-1}\right)B$$

$\hookrightarrow 1+\frac{3}{4z} = 0 \Rightarrow \frac{3}{4z} = -1 \Rightarrow 3 = -4z$ $\boxed{z = -3/4}$

$\hookrightarrow 1+\frac{1}{2z} = 0 \Rightarrow \frac{1}{2z} = -1 \Rightarrow -2z = 1 \Rightarrow \Rightarrow z = \frac{-1}{2}$

Si $z = -\frac{1}{2} \Rightarrow \frac{1}{8} = \left(1 + \frac{3}{4 \cdot \left(-\frac{1}{2}\right)}\right) B \Rightarrow \boxed{B = -1/4}$

Si $z = -3/4 \Rightarrow \frac{1}{8} = \left(1 + \frac{1}{2 \cdot \left(-3/4\right)}\right) A \Rightarrow \boxed{A = 3/8}$.

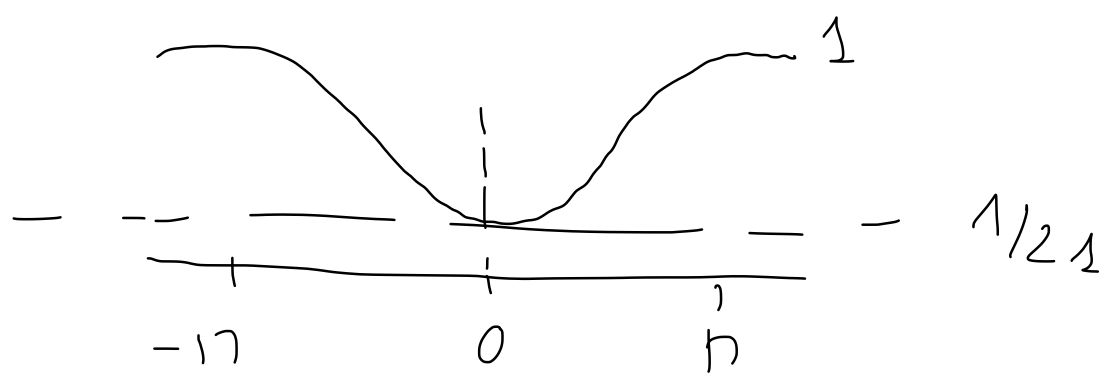
$$\frac{3/8}{\left(1 + \frac{3}{4} z^{-1}\right)} - \frac{1/4}{\left(1 + \frac{1}{2} z^{-1}\right)} \Rightarrow \frac{3}{8} \cdot \left(-\frac{3}{4}\right)^n u[n] - \frac{1}{4}\left(-\frac{1}{2}\right)^n u[n]$$

c) Dibujar $H(e^{j\Omega})$

$$H(z) = \frac{1/8\, z^2}{\left(z - \frac{3}{4}\right)\left(z - \frac{1}{2}\right)}\Bigg|_{z = e^{j\Omega}} = \frac{1/8}{\left(1 + \frac{3}{4} z^{-1}\right)\left(1 + \frac{1}{2} z^{-1}\right)}\Bigg|_{z = e^{j\Omega}}$$

$$\frac{1/8}{\left(1 + \frac{3}{4} e^{-j\Omega}\right)\left(1 + \frac{1}{2} e^{-j\Omega}\right)}\Bigg|_{\Omega = \pi} = \frac{1/8}{\left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{2}\right)} = 1.$$

$$\frac{1/8}{\left(1 + \frac{3}{4} e^{-j\Omega}\right)\left(1 + \frac{1}{2} e^{-j\Omega}\right)}\Bigg|_{\Omega = 0} = \frac{1/8}{\left(1 + \frac{3}{4}\right)\left(1 + \frac{1}{2}\right)} = \frac{1}{21}$$



FILTRO PASO ALTO

d) Obtenga la ecuación en diferencias y dibuje el sistema.

$$\frac{1/8}{\left(1+\frac{3}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} = H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{1}{8}X(z) = \left(1+\frac{1}{2}z^{-1}+\frac{3}{4}z^{-1}+\frac{3}{8}z^{-2}\right)Y(z)$$

$$\frac{1}{8}x[n] = y[n]+\frac{1}{2}y[n-1]+\frac{3}{4}y[n-1]+\frac{3}{8}y[n-2]$$

$$y[n] = \frac{1}{8}x[n]\underbrace{-\frac{1}{2}y[n-1]-\frac{3}{4}y[n-1]}_{-\frac{5}{4}y[n-1]}-\frac{3}{8}y[n-2]$$