EXAMEN ASS ENERO 2022

Ejercicio P2.1

$x(t) = \cos(1000\pi t)$

$f_s = 5kHz \longrightarrow T = \frac{1}{5000}$ ↓ tiempo de una muestra

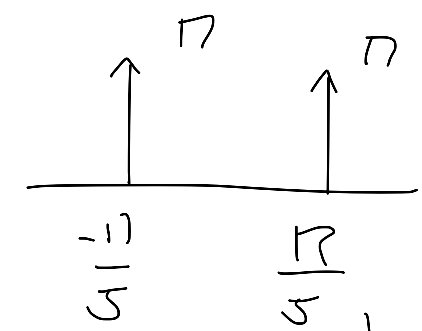
$t = 0{,}002$ seg

$x(t) = \cos(1000\pi t)$ $\quad \frac{0{,}002}{1/5000} = 10$ muestras.

⇓

$$\frac{1}{2}\left(e^{j1000\pi t} + e^{-j1000\pi t}\right)$$

a)



So: 10 muestras.

$$x[n] = \cos\left(\frac{\pi}{5}n\right)$$

$$\frac{1}{2}\left(e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n}\right)$$

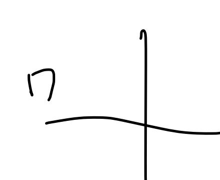
b) $$X_{10}[k] = \sum_{k=0}^{9} \cos\left(\frac{\pi}{5}n\right) e^{-j\frac{2\pi}{N}n\cdot k} =$$

$$= \cos(0)\cdot e^{0} + \cos\left(\frac{\pi}{5}\right) e^{-j\frac{2\pi}{10}k} + \cos\left(\frac{2\pi}{5}\right) e^{-j\frac{2\pi}{10}2k} +$$

$$+ \cos\left(\frac{3\pi}{5}\right) e^{-j\frac{2\pi}{10}\cdot 3k} + \cos\left(\frac{4\pi}{5}\right) e^{-j\frac{2\pi}{10}\cdot 4k} + \cos\left(\frac{5\pi}{5}\right) e^{-j\frac{2\pi}{10}\cdot 5k} +$$

$$+ \cos\left(\frac{6\pi}{5}\right) e^{-j\frac{2\pi}{10}\cdot 6k} + \cos\left(\frac{7\pi}{5}\right) e^{-j\frac{2\pi}{10}7k} + \cos\left(\frac{8\pi}{5}\right) e^{-j\frac{2\pi}{10}8k} +$$

$$+ \cos\left(\frac{9\pi}{5}\right) e^{-j\frac{2\pi}{10}9k}$$



$$= 1 + 0{,}8\, e^{-i\frac{\pi}{5}k} + 0{,}3 e^{-i\frac{2\pi}{5}k} - 0{,}3\, e^{-i\frac{3\pi}{5}k} - 0{,}8\, e^{-i\frac{4\pi}{5}k} - 1 e^{-i\pi k}$$

$$-0{,}8\, e^{-i\frac{6\pi}{5}k} - 0{,}3\, e^{-i\frac{7\pi}{5}k} + 0{,}3 e^{-i\frac{8\pi}{5}k} + 0{,}8 \cdot e^{-i\frac{9\pi}{5}k} - (-1)^k$$

$$1 + 0{,}8\left(e^{-i\frac{\pi}{5}k} + e^{\cancel{-i\frac{9\pi}{5}k}\; +i\frac{\pi}{5}}\right)$$

$$1 + 0{,}8 \cdot 2 \cdot \cos\left(\frac{\pi}{5}k\right) + 0{,}3 \cdot 2 \cdot \cos\left(\frac{2\pi}{5}k\right) - 0{,}3 \cdot 2 \cos\left(\frac{3\pi}{5}k\right) - 0{,}8 \cdot 2 \cos\left(\frac{4\pi}{5}k\right) - (-1)^k$$

$k = 0$, $k = 1$ ... $k = 9$

c)

$$k = \frac{f_c N}{f_s} \qquad k f_s = f_c N$$

$$f_c = \frac{k \cdot f_s}{N} = \frac{9 \cdot 5000}{10} = \underline{4500 \text{ Hz}}$$

$$5000 - 4500 = \boxed{500 \text{ Hz}}$$ (Mitad superior

$$\underbrace{1\ 2\ 3\ 4\ 5}_{\text{normal}} \quad \underbrace{6\ 7\ 8\ 9\ 10}_{\text{rota}}$$

d)

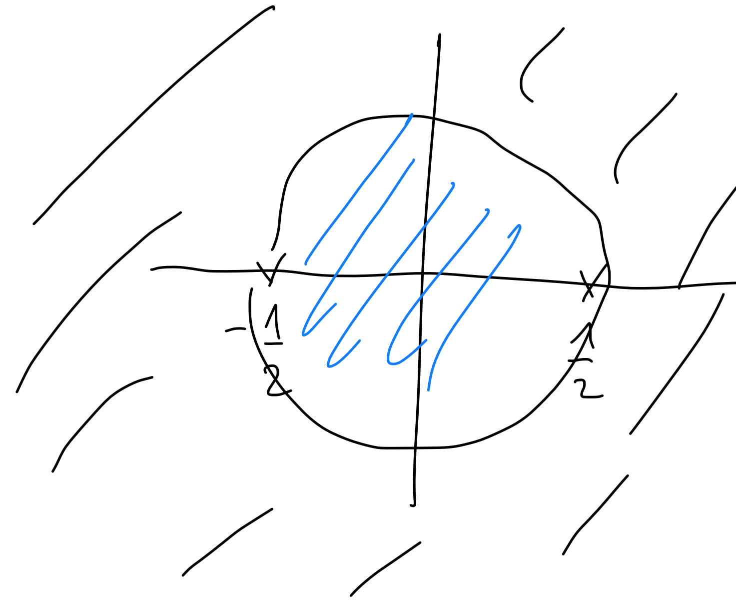
$$X_{10}[k] = X\left(e^{j\Omega}\right)\Big|_{\Omega = \frac{2\pi}{10}k} \qquad \text{Si}$$

Ejercicio P2.2

a) Ceros $\boxed{z=0}$ $\boxed{z=1}$

Polos $\boxed{z=\frac{1}{2}}$ $\boxed{z=-\frac{1}{2}}$

doble



b) Dentro no causal (no incluye al cero (sino sería anti)

Fuera causal (convergencia a 0, incluye el ∞)

c) Fuera estable

No lo es (no incluye la unidad)

c)3

$$H(z) \; \frac{z^2-z}{\left(1-\frac{1}{2}z^{-1}\right)^2\left(1+\frac{1}{2}z^{-1}\right)} \cdot \frac{z^{-3}}{z^{-3}} = \frac{z^{-1}-z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)^2\left(1+\frac{1}{2}z^{-1}\right)}$$

$$\left(z-\frac{1}{2}\right)\left(\cancel{z}-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)$$

$$\cancel{z}\,\cancel{\tfrac{1}{2}} \left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)$$

$$\frac{z^{-1}-z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)^2\left(1+\frac{1}{2}z^{-1}\right)} = \frac{A}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1-\frac{1}{2}z^{-1}\right)^2} + \frac{C}{\left(1+\frac{1}{2}z^{-1}\right)}$$

$$z^{-1}-z^{-2} = A\cdot\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right) + B\left(1+\frac{1}{2}z^{-1}\right) + C\left(1-\frac{1}{2}z^{-1}\right)^2$$

$z=\frac{1}{2}$ ~~an~~

anula el $-\frac{1}{2}$

$$z^{-1}-z^{-2} = B\left(1+\frac{1}{2}z^{-1}\right)$$

$$\left(\frac{1}{2}\right)^{-1}-\left(\frac{1}{2}\right)^{-2} = B\left(1+\frac{1}{2}\left(\frac{1}{2}\right)^{-1}\right)$$

$$2-4 = B\left(1+1\right)$$

$$-2 = 2B \Rightarrow \boxed{B=-1}$$

$z=-\frac{1}{2}$

$$z^{-1}-z^{-2} = C\left(1-\frac{1}{2}z^{-1}\right)^2$$

$$\left(-\frac{1}{2}\right)^{-1}-\left(-\frac{1}{2}\right)^{-2} = C\left(\underbrace{1-\frac{1}{2}\left(-\frac{1}{2}\right)^{-1}}_{1\,-\,-1}\right)^2$$

$$-6 = C\left(2\right)^2 \qquad \boxed{C=\frac{-3}{2}}$$

$z=1$

$$\left(1\right)^{-1}-\left(1\right)^{-2}=A\left(1+\overset{3/2}{\cancel{\tfrac{1}{2}(1)^{-1}}}\right)\left(1-\overset{1/2}{\cancel{\tfrac{1}{2}(1)^{-1}}}\right)+$$

$$+B\left(1+\overset{3/2}{\cancel{\tfrac{1}{2}(1)^{-1}}}\right)+C\left(1-\overset{1/4}{\cancel{\tfrac{1}{2}(1)^{-1}}}\right)^{2}$$

$$0=A\frac{3}{4}-\frac{3}{2}-\frac{3}{8}$$

$$0=\frac{3}{4}A-\frac{15}{8}\to\boxed{A=\frac{15/8}{3/4}=\frac{5}{2}}$$

$$\frac{\frac{5}{2}}{\left(1-\frac{1}{2}z^{-1}\right)}+\frac{-1}{\left(1-\frac{1}{2}z^{-1}\right)^{2}}+\frac{-3/2}{\left(1+\frac{1}{2}z^{-1}\right)}$$

★

$na^{n}u[n]$	$\frac{az^{-1}}{\left(1-az^{-1}\right)^{2}}$

$$|z|>\frac{1}{2}:\quad\frac{5}{2}\left(\frac{1}{2}\right)^{n}u(n)-\frac{1}{2}(n+1)\left(\frac{1}{2}\right)^{n+1}u[n+1]-\frac{3}{2}\left(-\frac{1}{2}\right)^{n}u[n]$$

★ $\frac{\frac{1}{2}\;1\;z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^{2}}z^{-1}$ $\to$ multiplicar por z es $n+1$

$\frac{\cancel{1}\,\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^{2}}\cdot 2\cdot\frac{1}{2}\to 2\cdot\frac{1}{2}\,na^{n}u(n)$

$(n+1)a^{n+1}u[n+1]$

$$|z|<\frac{1}{2}\quad-\frac{5}{2}\left(\frac{1}{2}\right)^{n}u[-n-1]+2(n+1)\left(\frac{1}{2}\right)^{n+1}u[-n-\cancel{1}\cancel{+1}]+\frac{3}{2}\left(-\frac{1}{2}\right)^{n}u[-n-1]$$

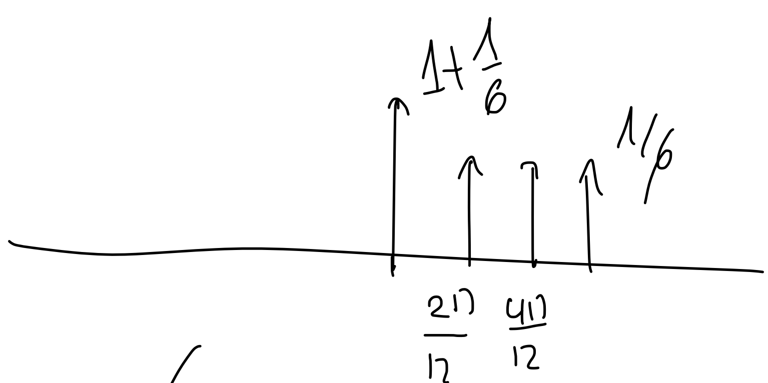
No es FIR, tiene valores pasados en la salida

Ejercicio P1.1

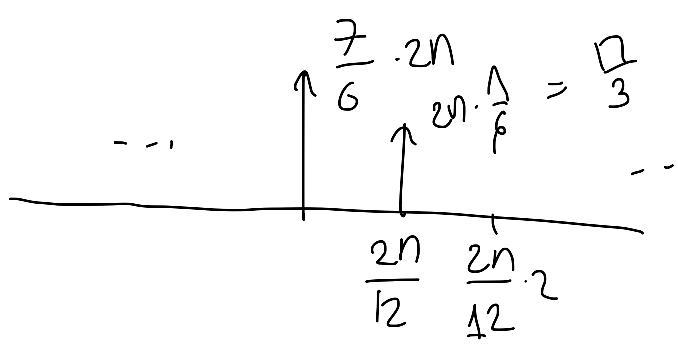
pdf

a)

ampl $\frac{1}{12} \cdot 2 = \frac{1}{6}$

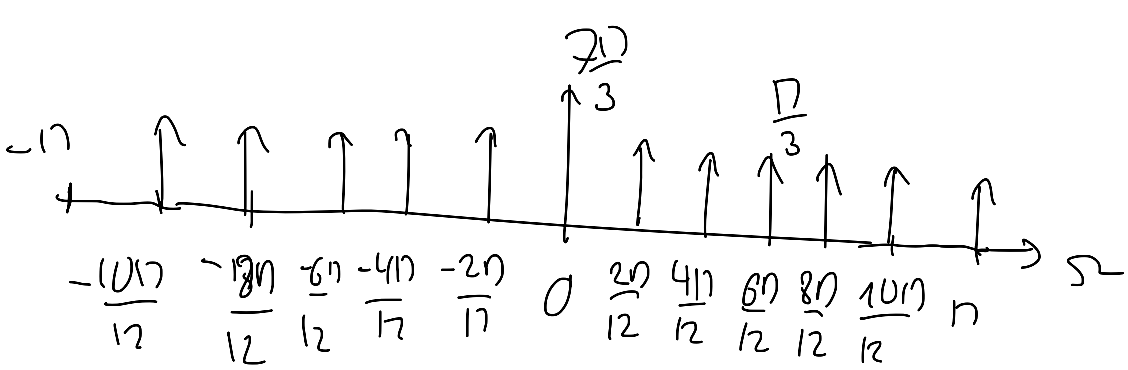


$TF = 2\pi a_k$



periódica 2π.

$$TF: \delta[\Omega] + \sum_{n=0}^{11} \delta\left[\Omega - \frac{\pi}{\cancel{6}\,3} n\right) \longrightarrow \text{periódica } 2\pi$$



b) Definida en Potencia $\Rightarrow$ $\boxed{E=\infty}$

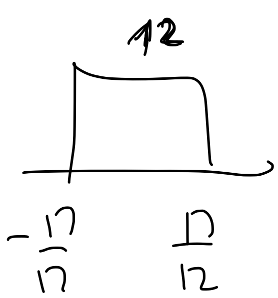
$$P = \sum |a_k|^2 = \frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 \longrightarrow \frac{1}{12}\left((1+2)^2 + 1^2 \cdot 11\right) = \frac{5}{3}$$

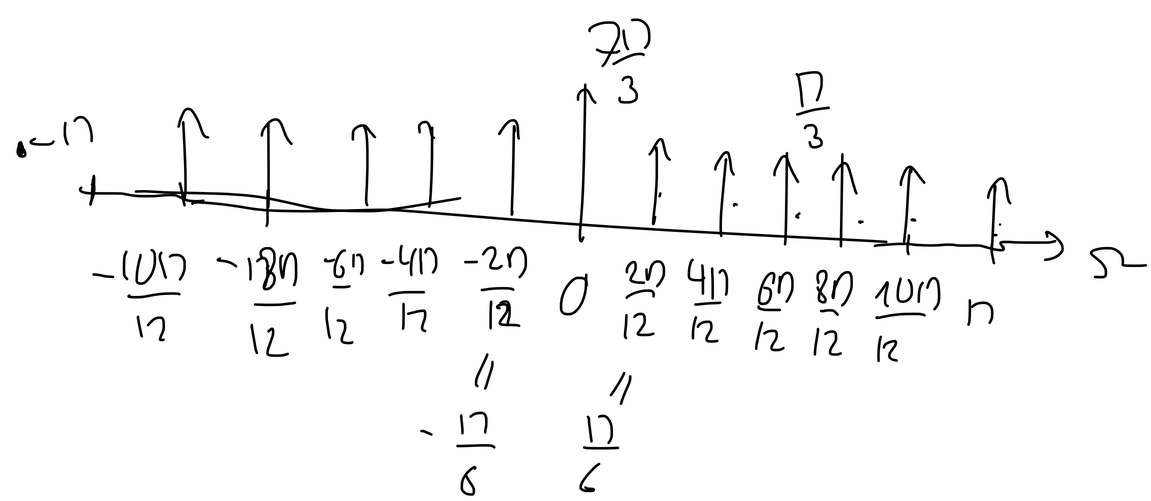
//

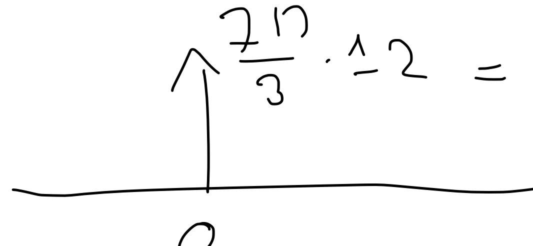
$$\left(1+\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 \cdot 11 = 5/3$$

c) $h[n] = \dfrac{\operatorname{sen}\left(\frac{\pi}{12} n\right)}{\frac{\pi}{12} n}$ $\qquad y[n] = x[n] * h[n]$

$\Downarrow$

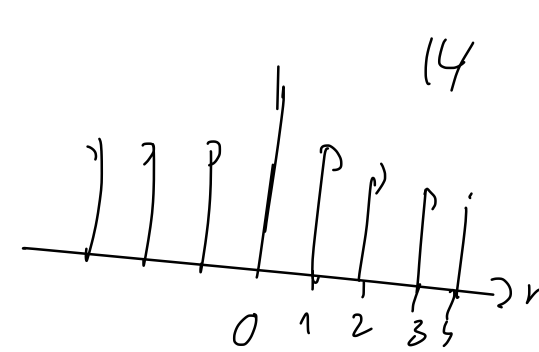






$$\frac{7\pi}{3} \cdot 12 = \underline{\underline{28\pi}}$$

$$y[n] \xrightarrow{TF^{-1}} \frac{28\cancel{\pi}}{2\cancel{\pi}} = \boxed{\underline{\underline{14}} = y[n]}$$

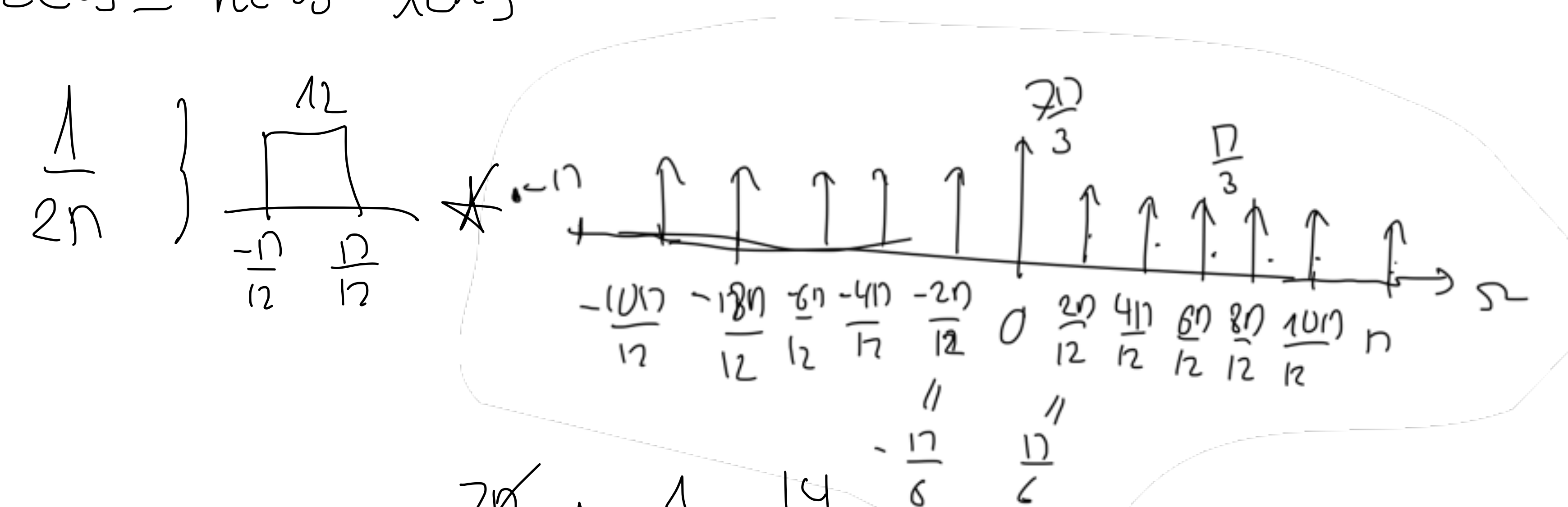


Es una de periodicidad 1

d) $E = \infty$

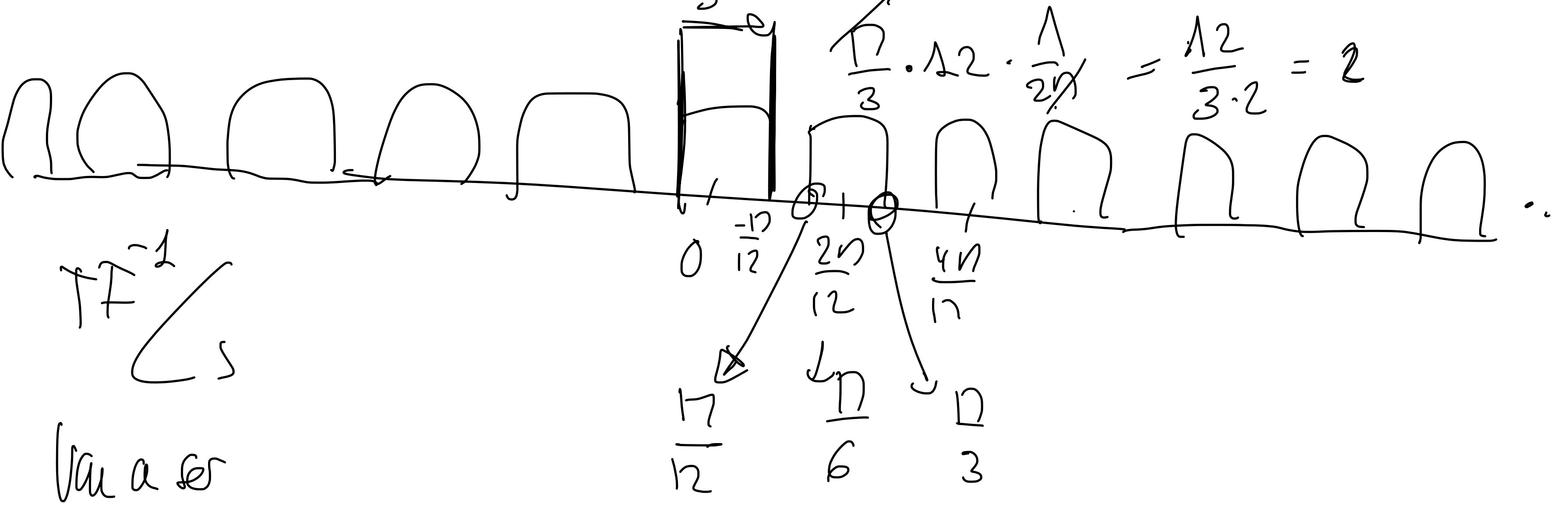
$$P = \frac{1}{1} 14^2 = \underline{\underline{196}}\ W$$

e) $z[n] = h[n] \cdot x[n]$



$$\frac{7\pi}{3} \cdot 12 \cdot \frac{1}{2\pi} = 14$$

$$\frac{\pi}{3} \cdot 12 \cdot \frac{1}{2\pi} = \frac{12}{3 \cdot 2} = 2$$



TF^{-1} Σ

Van a ser sincs

$$\frac{\operatorname{sen}\left(\frac{\pi}{12} n\right)}{\frac{\pi}{12} n} + \sum_{k=-5}^{6} \frac{1}{6} \cdot \frac{\operatorname{sen}\left(\frac{\pi}{12}\left(n - \frac{\pi}{6} k\right)\right)}{\frac{\pi}{12}\left(n - \frac{\pi}{6} k\right)}$$