EXAMEN FINAL ASS JUNIO 2018

Ejercicio 1

a) No son deltas equiespaciadas, no es una señal periódica

b) Su TF no es simétrica, por tanto no es una señal real.

c) Valor de $x[0]$

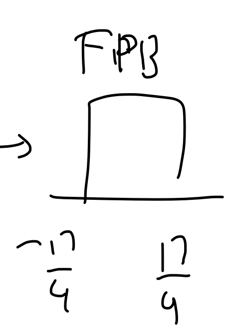
$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})\, \cancel{e^{-j\Omega 0}}^{\,0}\, d\Omega = \frac{1}{2\pi}\cdot 4\cdot\frac{1}{4}\cdot\frac{\pi}{4} = \frac{1}{8}$$

d) $\displaystyle\sum_{n=-\infty}^{\infty} |x[n]|^2$

Por Parseval, sabemos que $E = \displaystyle\sum_{n=-\infty}^{\infty} |X(e^{j\Omega})|^2$

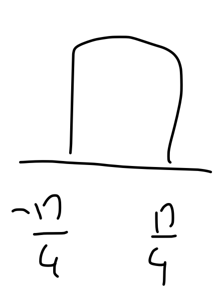
$$E = \frac{1}{2\pi}\int_{0}^{\frac{\pi}{4}} 4\cdot\left|\frac{1}{4}\right|^2 d\Omega = \frac{1}{2\pi}\cdot 4\cdot\left(\frac{1}{2}\right)^2\cdot \Omega\Big|_0^{\pi/4} = \frac{1}{32}$$

e) $x[n] \rightarrow$ FPB $\rightarrow y[n]$

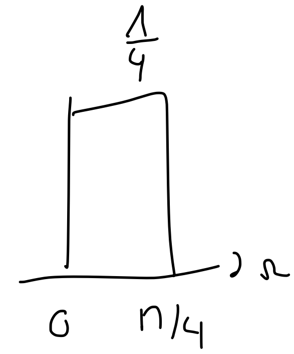


$z[n] = y\left[\frac{n}{2}\right]$

$X(e^{j\Omega}) \rightarrow$

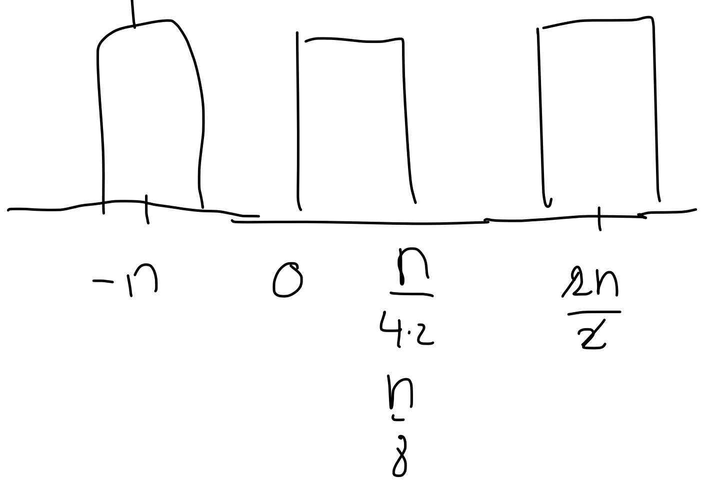


$\rightarrow Y(e^{j\Omega})$



periódica 2π

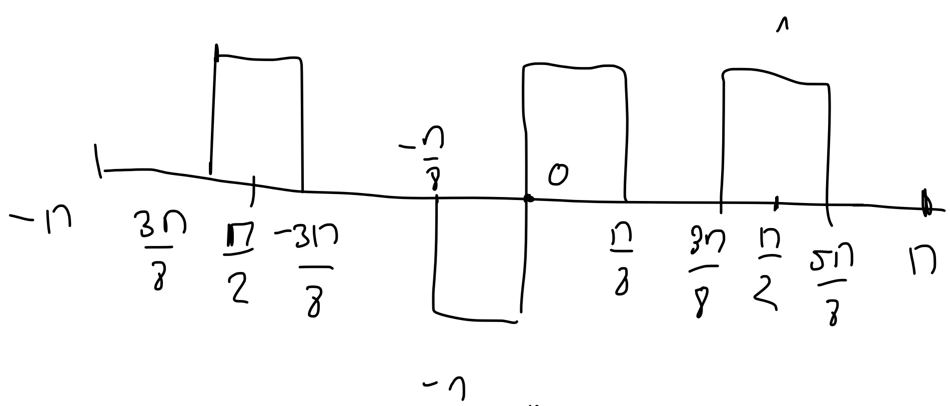
Interpolado:



periódica 2π

Ejercicio 2

a) $X(e^{j\Omega})$

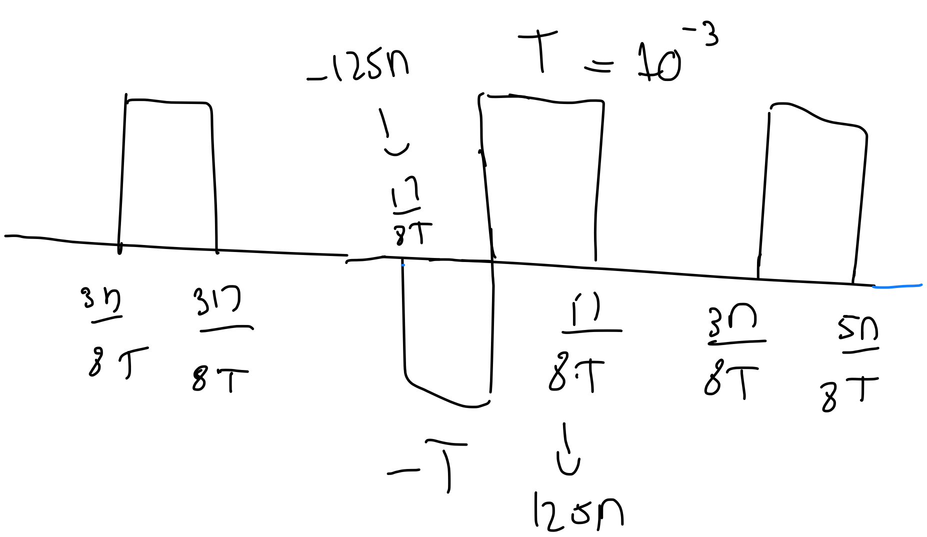


→ Periódica 2π

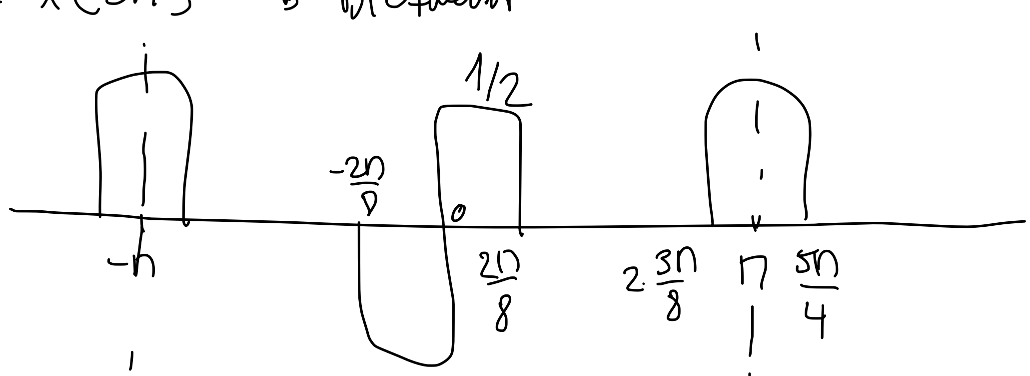
Se ha obtenido con un periodo $T = 0{,}001$ s

⇓ $X(jw)$

$T = 10^{-3}$



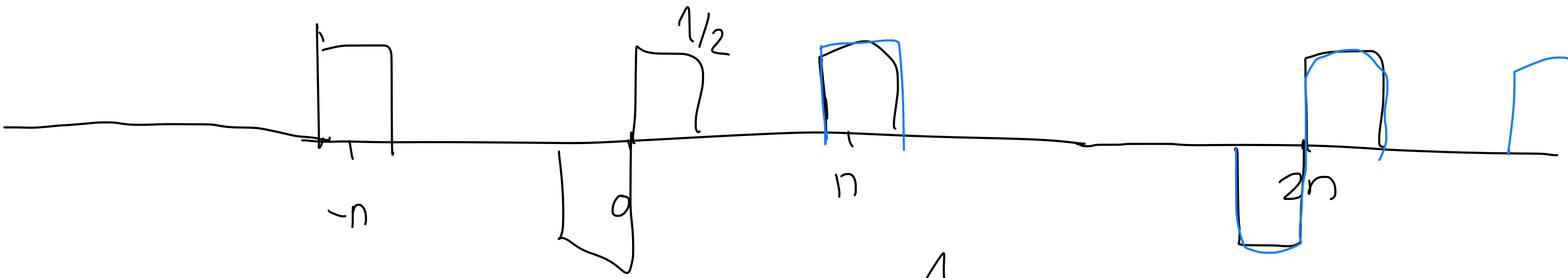
b) $g[n] = x[2n]$ → Diezmador



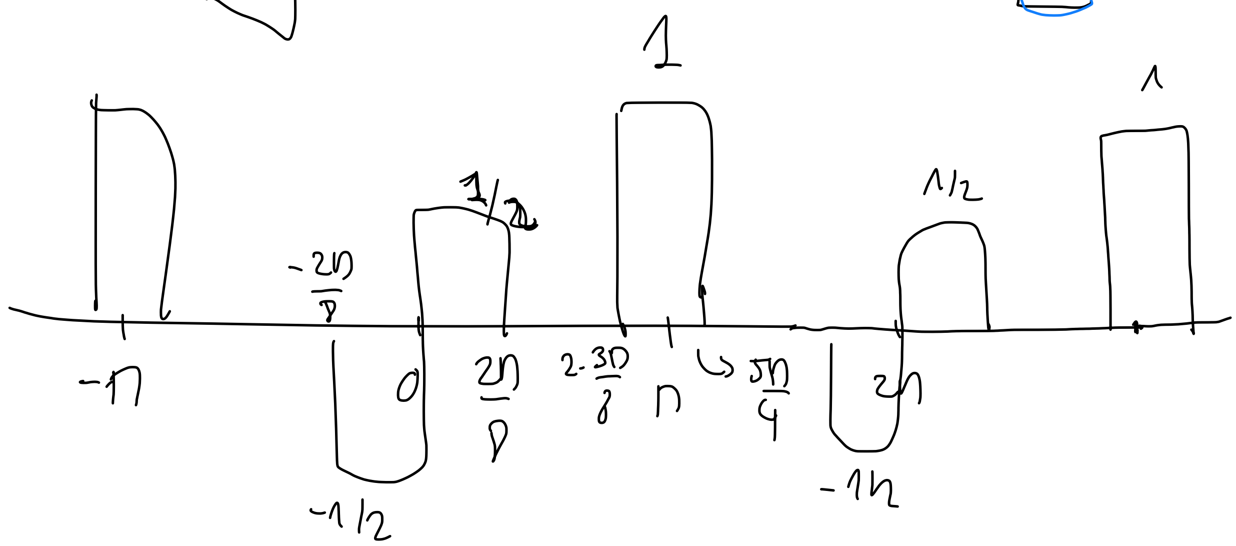
→ Periódica 2π

Esto crea un solapamiento espectral

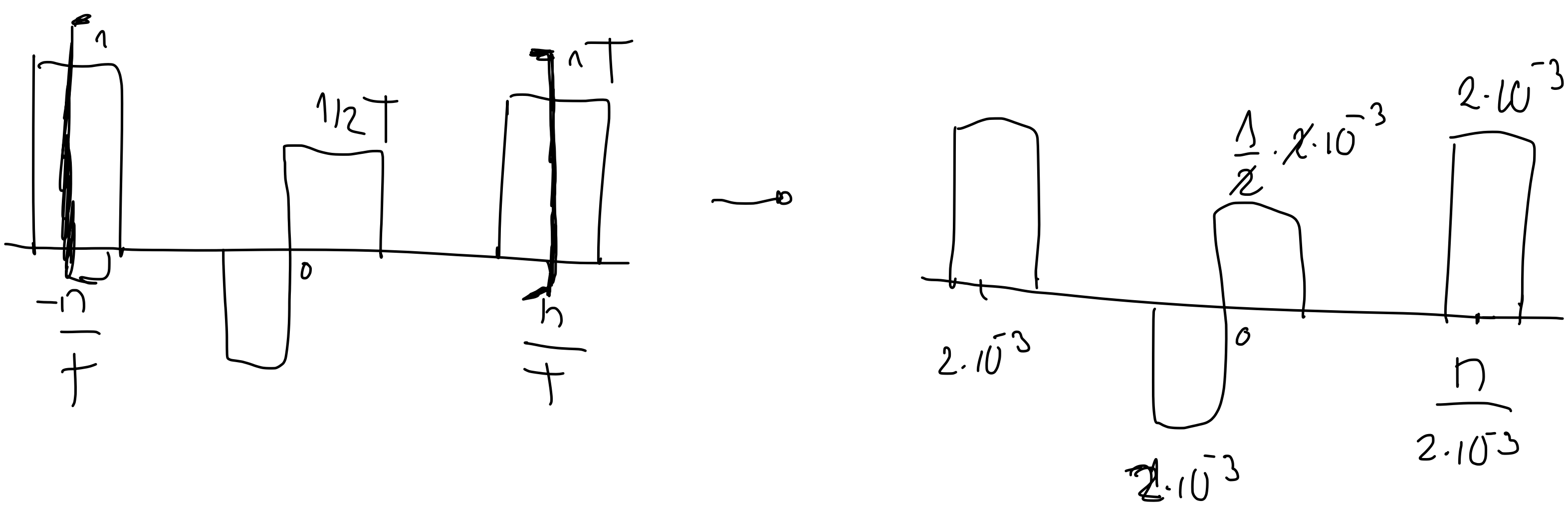
5π / 4π



Periódica 2π



c) $T = 2ms$



Ejercicio 3

a) Determinar las DFT siguientes:

$$X_1[k] = \sum_{n=0}^{2} x[n] e^{-j\frac{2\pi}{3} n \cdot k} = 1 \cdot e^0 + 3 e^{-j\frac{2\pi}{3}k} + 3 e^{-j\frac{4\pi}{3}k} \Rightarrow$$

$$2\pi - \frac{4\pi}{3} = \frac{2\pi}{3}$$

$$\Rightarrow 1 + 3\left(e^{-j\frac{2\pi}{3}k} + e^{j\frac{2\pi}{3}k}\right) \qquad \cos(\Omega) = \frac{1}{2}(e^- + e^+)$$

$$1 + 3 \cdot 2 \cos\left(\frac{2\pi}{3}k\right)$$

$$\left[1 + 6\cos\left(\frac{2\pi}{3}k\right) = X_1[k]\right]$$

$$X_2[k] = \sum_{n=0}^{2} x[n] e^{-j\frac{2\pi}{6} n \cdot k} = 1 \cdot e^0 + 3 e^{-j\frac{\pi}{3}k} + 3 e^{-j\frac{4\pi}{6}k}$$

$$-j\frac{2\pi}{3}k$$

$$2\pi - \frac{2\pi}{3} =$$

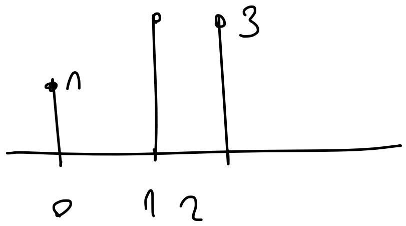
$$1 + 3\left(e^{-j\frac{\pi}{3}k} + e^{-j\frac{2\pi}{3}k}\right)$$

$X_3[k] \rightarrow$ Desplazar $X_1[n]$ 1 a la derecha $\longleftrightarrow$ DFT $\times\, e^{-j\frac{2\pi}{3} 1 k}$

$$X_3[k] = \left(1 + 6\cos\left(\frac{2\pi}{3}k\right)\right) e^{-j\frac{2\pi}{3}k}$$

$$X_4[k] = \left(1 + 3\left(e^{-j\frac{\pi}{3}k} + e^{-j\frac{2\pi}{3}k}\right)\right) e^{-j\frac{2\pi}{3} \cdot 2k}$$

b)



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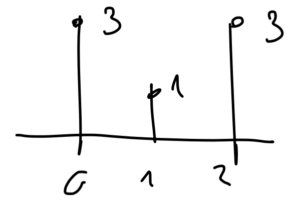

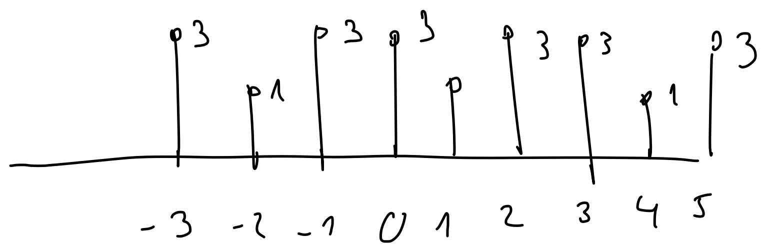

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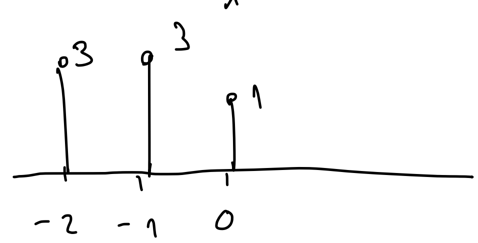

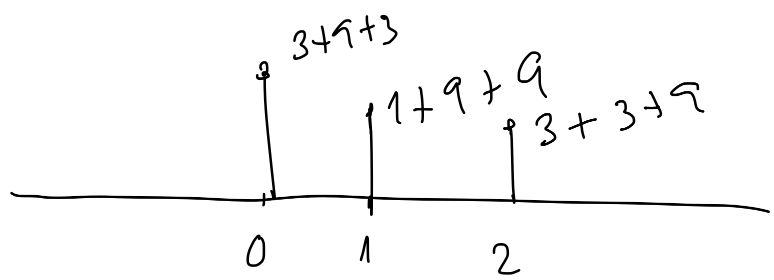

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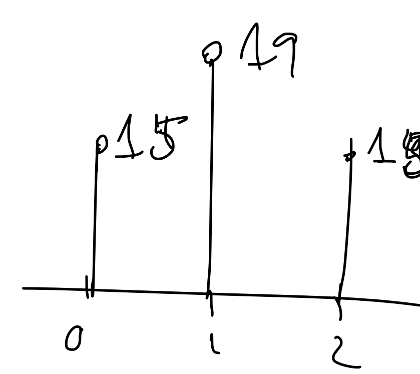


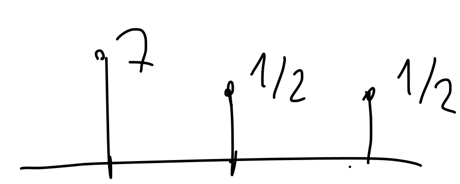
c) $x_1(n)$ (3) $x_3(n)$ $\cancel{\times}$ DFT_1, DFT_2 $\rightarrow \otimes \rightarrow$ $\boxed{DFT^{-1}}$

$$1 + 6\cos\left(\frac{2\pi}{3}k\right)$$

$k=0 \rightarrow 7$

$k=1 \rightarrow -1/2 + 1$

$k=2 \rightarrow -1/2 + 1$



$$1 + 6\cos\left(\frac{2\pi}{3}k\right) e^{-j\frac{2\pi}{3}k}$$

$k=0 \rightarrow 7$

$k=1 \rightarrow$

Ejercicio 4

$$Y(z)\left(1 - 2z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{2}z^{-3}\right) = X(z)\left(1 - \frac{1}{2}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{2}z^{-1}}{1 - 2z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{2}z^{-3}}$$

Ceros: $1 - \frac{1}{2}z^{-1} = 0 \Rightarrow 1 = \frac{1}{2z} \Rightarrow \boxed{z = \frac{1}{2}}$

Polos

Soluc.

	1	-2	1/4	-1/2
2		+2	0	1/2
	1	0	1/4	0

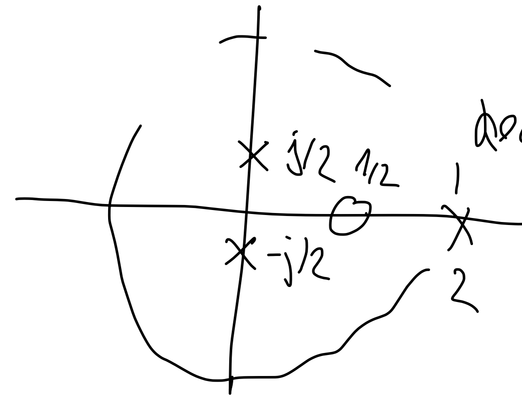
$$1 + \frac{1}{4}z^{-2} = 0$$

$$\frac{1}{4z^2} = -1 \Rightarrow z^2 = -\frac{1}{4}$$

$\sqrt{\frac{1}{4}}\, j = \frac{1}{2} j$

$\sqrt{\frac{1}{4}}\, |-j| = -\frac{j}{2}$

Polos $\boxed{z = 2}$ $\boxed{z = -\frac{j}{2}}$ $\boxed{z = \frac{j}{2}}$



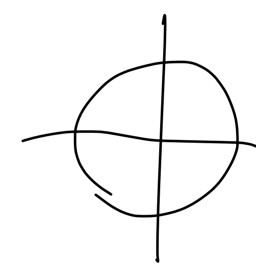
Causal:

ROC: $|z| > 2$

c) ¿Es estable?

Es causal pero sus polos no $< |1|$, no es estable

d) $G(z) = \dfrac{1 - 2z^{-1}}{1 - \frac{1}{2}z^{-1}}$; ROC : $|z| > \dfrac{1}{2}$



$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 - 2z^{-1})(1 - \frac{j}{2}z^{-1})(1 + \frac{j}{2}z^{-1})}$$

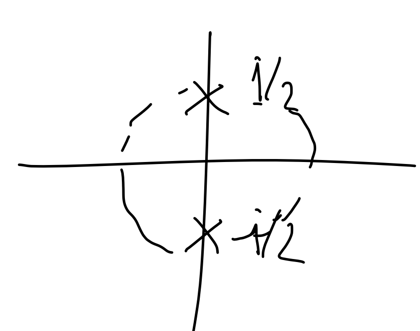
$$G(z) \cdot H(z) = \frac{\cancel{(1 - 2z^{-1})}\cancel{(1 - \frac{1}{2}z^{-1})}}{\cancel{(1 - \frac{1}{2}z^{-1})}\cancel{(1 - 2z^{-1})}(1 - \frac{j}{2}z^{-1})(1 + \frac{j}{2}z^{-1})} = \frac{1}{(1 - \frac{j}{2}z^{-1})(1 + \frac{j}{2}z^{-1})}$$

Ceros no hay

Polos: $1 - \frac{j}{2}z^{-1} = 0$

$1 = \dfrac{j}{2z} \Rightarrow \boxed{z = \dfrac{j}{2}}$

$-1 = \dfrac{j}{2z} \Rightarrow \boxed{z = \dfrac{-j}{2}}$



Se ha ido el polo de $|z| = 2$ entonces

ROC $\boxed{|z| > \dfrac{1}{2}}$

e)

$$Q(z) = \frac{1}{\left(1-\frac{j}{2}z^{-1}\right)\left(1+\frac{j}{2}z^{-1}\right)} = \frac{A}{\left(1-\frac{j}{2}z^{-1}\right)} + \frac{B}{\left(1+\frac{j}{2}z^{-1}\right)}$$

$X[0] = 1$

$X[1] = 0$

$X[2] = \frac{1}{4}$

$$1 = \left(1+\frac{j}{2}z^{-1}\right)A + \left(1-\frac{j}{2}z^{-1}\right)B$$

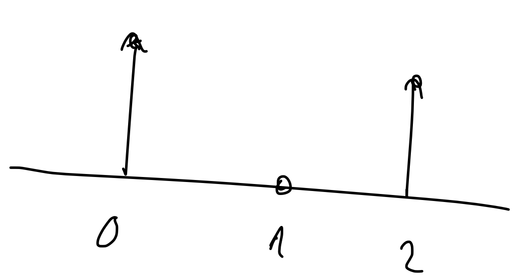
Si $z = \frac{j}{2} \Rightarrow 1 = \left(1 + \frac{\cancel{j}\,\cancel{2}}{\cancel{2}\,j}\right)A \Rightarrow 1 = 2A \Rightarrow \boxed{A = \frac{1}{2}}$

Si $z = -\frac{j}{2} \Rightarrow 1 = B\left(1 + \frac{j}{2}\frac{2}{j}\right) \Rightarrow 1 = 2B \Rightarrow \boxed{B = \frac{1}{2}}$

$$\frac{1/2}{\left(1-\frac{j}{2}z^{-1}\right)} + \frac{1/2}{\left(1+\frac{j}{2}z^{-1}\right)}$$

$$h[n] = \frac{1}{2}\left(\frac{j}{2}\right)^n u[n] + \frac{1}{2}\left(-\frac{j}{2}\right)^n u[n]$$

$$*$$

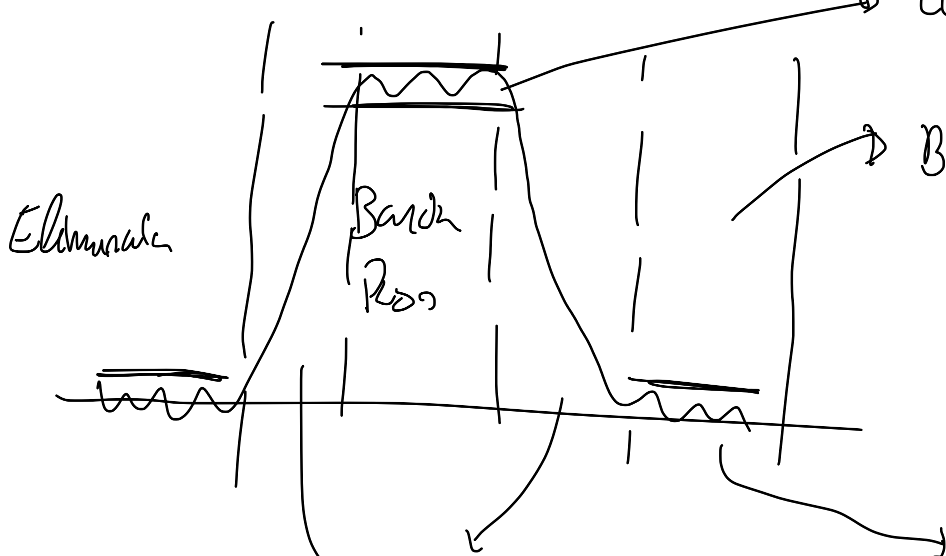


$$h[n] + 0 + \frac{1}{4}h[n-2]$$

$$\frac{1}{2}\left(\frac{j}{2}\right)^n u[n] + \frac{1}{2}\left(-\frac{j}{2}\right)^n u[n] + \frac{1}{2}\frac{1}{4}\left(\frac{j}{2}\right)^{n-2} u[n-2] + \frac{1}{4}\frac{1}{2}\left(-\frac{j}{2}\right)^{n-2} u[n-2]$$

Ejercicio 5

a) En función de la respuesta a frecuencia deseada se diseña un tipo de filtro u otro



Cuanto más recto más fiel a la señal.

Banda eliminada

Filtro paso banda

Banda de transición

Cuanto más plana menos ruido tendrán.