EXAMEN FINAL ASS ENERO 2021

Ejercicio 1

$$x[n] = 2\cos\left(\frac{\pi n}{2}\right) \quad X(e^{j\Omega}) = 2\pi \cdot \left(\delta\left(\Omega+\frac{\pi n}{2}\right)+\delta\left(\Omega-\frac{\pi n}{2}\right)\right) \rightarrow \text{Per } 2\pi$$

a) $y[n]$? SLI-A?

$$h_A[n] = \left(\frac{1}{2}\right)^n u[n] \Rightarrow H(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} \rightarrow \text{Per } 2\pi.$$

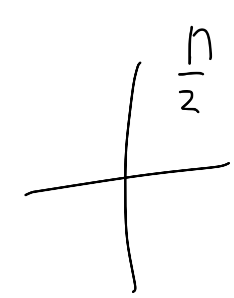
También puedo escribir el coseno como:

$$2\cos\left(\frac{\pi n}{2}\right) = \frac{1}{\cancel{2}}\cancel{2}\left(e^{j\frac{\pi n}{2}}+e^{-j\frac{\pi n}{2}}\right)$$

$$e^{j\frac{\pi n}{2}} \rightarrow H(e^{j\Omega})\Big|_{\Omega=\frac{\pi}{2}} \Rightarrow y[n] = x[n]\cdot H(e^{j\Omega})\Big|_{\Omega=\Omega_1} \;?$$

$$y_1[n] = e^{j\frac{\pi n}{2}} \cdot \frac{1}{1-\frac{1}{2}e^{-j\frac{\pi n}{2}}}$$

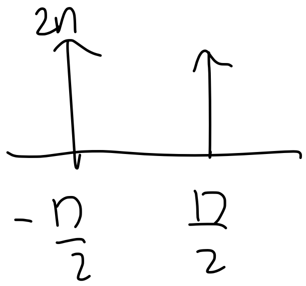
$$y_2[n] = e^{-j\frac{\pi}{2}n} \cdot \frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{2}n}}$$



$$y[n] = \cancel{e^{j\frac{\pi n}{2}}}(j)^n \cdot \frac{1}{1-\frac{1}{2}\cancel{e^{-j\frac{\pi n}{2}}}(-j)^n} + \cancel{e^{-j\frac{\pi}{2}n}}(-j)^n \cdot \frac{1}{1-\frac{1}{2}\cancel{e^{j\frac{\pi}{2}n}}(j)^n}$$

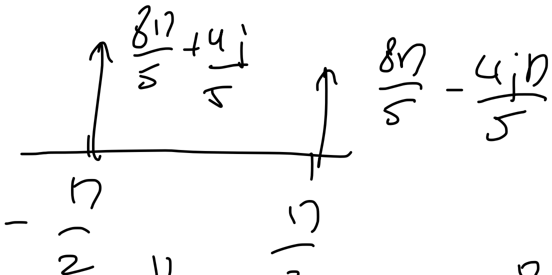
$$\left[\frac{j^n}{1+\frac{1}{2}(j)^n} + \frac{(-j)^n}{1-\frac{1}{2}(j)^n} = y[n]\right]$$

Otra manera



$$\cdot \frac{1}{1-\frac{1}{2}e^{-j\Omega}} = \begin{cases} \dfrac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{2}}}\cdot 2\pi = \dfrac{8\pi}{5} - \dfrac{4j\pi}{5} \\[2ex] \dfrac{1}{1-\frac{1}{2}e^{j\frac{\pi}{2}}}\cdot 2\pi = \dfrac{8\pi}{5} + \dfrac{4j\pi}{5} \end{cases}$$

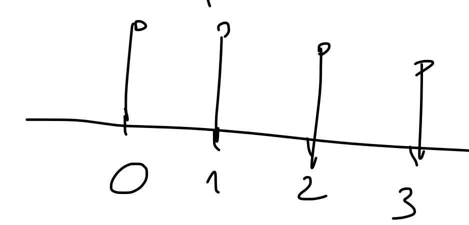
$$\hookrightarrow 2\pi\left(\delta\left(\Omega+\frac{\pi}{2}\right)+\delta\left(\Omega-\frac{\pi}{2}\right)\right)$$



$$\boxed{y[n] = \left(\frac{4}{5}+\frac{2}{5}j\right)e^{j\frac{\pi}{2}n} + \left(\frac{4}{5}-\frac{2}{5}j\right)e^{-j\frac{\pi}{2}n}}$$

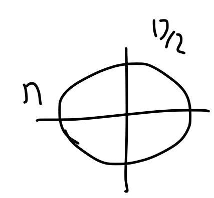
b) Salida SLI-B

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$



$$y[n] = x[n] * h[n]$$

$$x[n] = 2\cos\left(\frac{\pi n}{2}\right) = e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}$$



$$y[n] = \left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right) + \left(e^{j\frac{\pi}{2}(n-1)} + e^{-j\frac{\pi}{2}(n-1)}\right) + \left(e^{j\frac{\pi}{2}(n-2)} + e^{-j\frac{\pi}{2}(n-2)}\right) + \left(e^{j\frac{\pi}{2}(n-3)} + e^{-j\frac{\pi}{2}(n-3)}\right)$$

$$y[n] = 2\cos\left(\frac{\pi}{2}n\right) + 2\cos\left(\frac{\pi}{2}(n-1)\right) + 2\cos\left(\frac{\pi}{2}(n-2)\right) + 2\cos\left(\frac{\pi}{2}(n-3)\right)$$

$$\cos\left(\frac{\pi n}{2} - \frac{\pi}{2}\right) \qquad \cos\left(\frac{\pi n}{2} - \pi\right)$$

$$\operatorname{sen}\left(\frac{\pi}{2}n\right)$$

$$y[n] = 2\cos\left(\frac{\pi}{2}n\right) + 2\operatorname{sen}\left(\frac{\pi n}{2}\right) - 2\cos\left(\frac{\pi}{2}n\right) + 2\operatorname{sen}\left(\frac{-\pi}{2}n\right) = 0 //$$

c) $x[n] \to \boxed{H_1} \to \boxed{H_2} \to y[n]$

$x[n] \to \boxed{H_2} \to \boxed{H_1} \to y[n]$

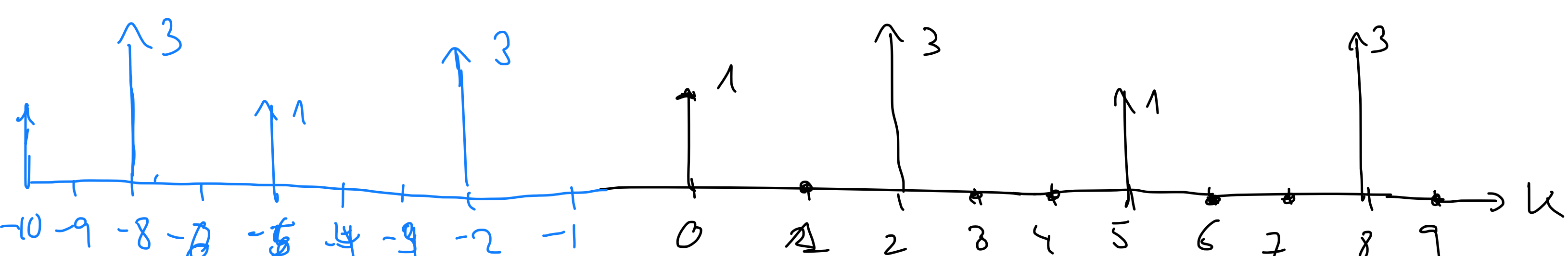
Entonces $x[n] \to \boxed{H_2[n]} \to 0$

$0 \to \boxed{H_1[n]} \to y[n]$

Entonces es 0

Ejercicio 2

$$a_k = \delta[k] + 3\delta[k-2] + 3\delta[k-8] + \delta[k-5]$$



a) ¿$x[n]$ es par?

$x[n]$ es real y par porque los coef son real y par

b) a_0 es el valor medio

$x[0]$ es la suma de todos los a_k $\qquad a_k e^{jk\cancel{\Omega n}}$ $\quad \Omega = 0$

$x[10] = x[0]$ $\qquad \sum a_k$

$$x[n] = \sum a_k \cdot \cancel{e^{jk\frac{2\pi}{10}n}} \to 1 \qquad n = 0$$

$= 1 + 3 + 1 + 3 = 8$

$x[10] = 8$

c) $P = \sum |a_k|^2 = 1^2 + 3^2 + 1^2 + 3^2 = 20\ W$

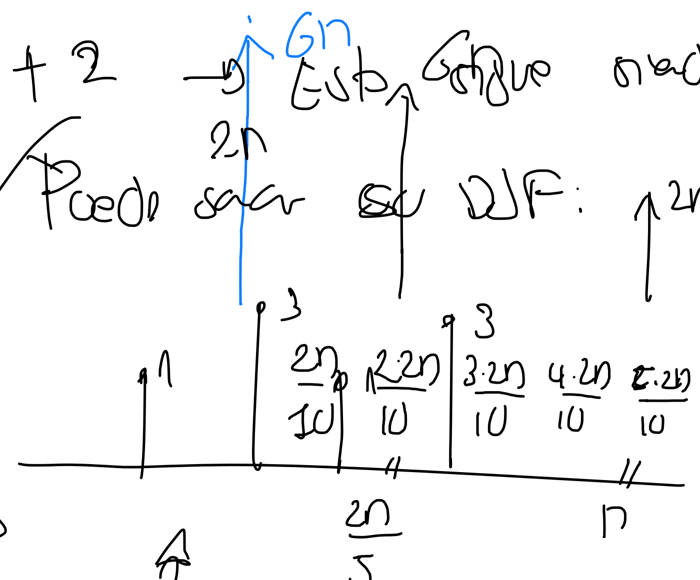
Es la DTF, viene de una señal periódica

$\boxed{E = \infty}$

d) $y[n] = x[n] + 2 \rightarrow$ Esto sigue siendo periódica

$\boxed{E = \infty}$

Puedo sacar su DTF:



su DSF

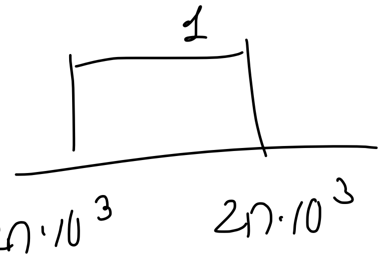
TF: $2\pi \cdot 2\delta[n]$

DSF $= \frac{4\pi\delta[n]}{2\pi}$ $\quad 2\delta[n]$

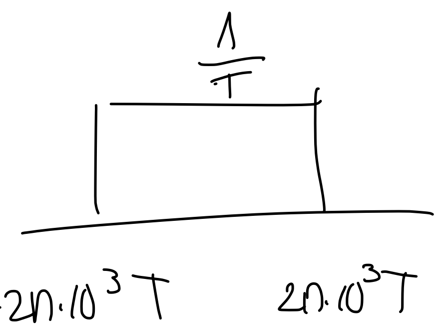
Un a_k en 0

$\sum |a_k|^2 = (1+2)^2 + 3^2 + 1^2 + 3^2 = 28\ W$

Ejercicio 3

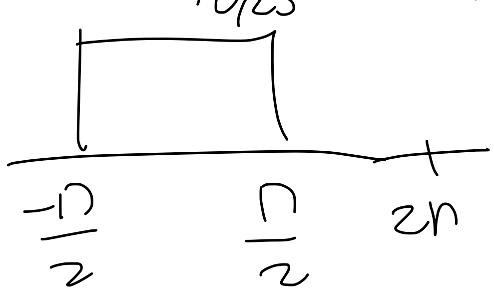


a)



$$2\pi\cdot\cancel{10^3}\cdot0{,}25\cdot\cancel{10^{-3}} = \frac{2\pi}{4} = \frac{\pi}{2}$$

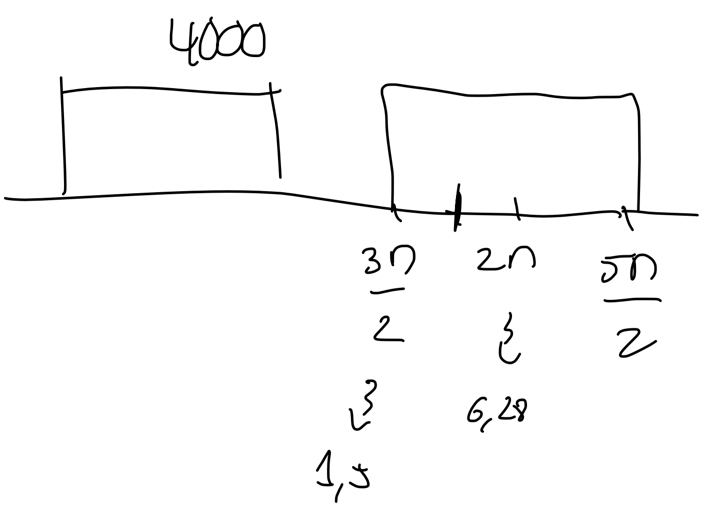
$1/0{,}25 \cdot 1000 = \underline{4000}$



Nyquist ✓

$\frac{\pi}{2} < \pi$ ✓

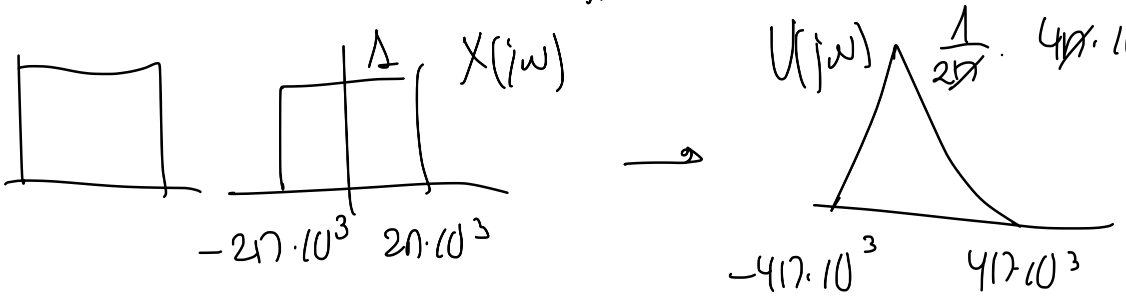
¿$1{,}75\pi$?
↳ $\frac{7\pi}{4}$



<u>Vale 4000</u>

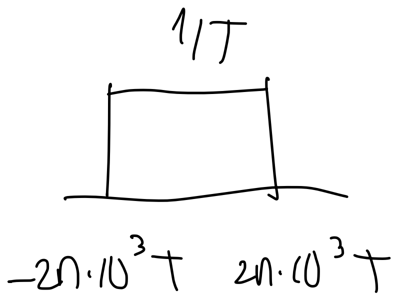
b)

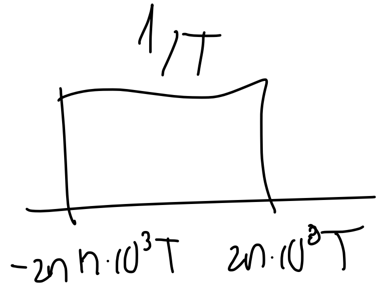
$x(t)\cdot x(t) \quad \frac{1}{2\pi}\{X(j\omega) * X(j\omega)\}$



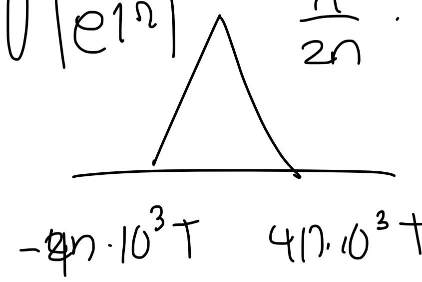
$\frac{1}{2\cancel{\pi}}\cdot4\cancel{\pi}\cdot10^3\cdot1 = 2\cdot10^3 = 2000$

$x[n]\cdot x[n] \quad \frac{1}{2\pi}\{X(e^{j\Omega}) * X(e^{j\Omega})\}$





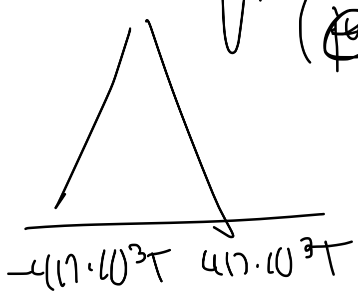
$U(e^{j\Omega})$



$\frac{1}{2\pi}\cdot 4\pi\cdot 10^3 \cancel{T}\cdot\frac{1}{\cancel{T}}$

Entonces si nuestra $U(jw)$

$U'(e^{j\Omega})$



que sea que al → Te vale cualquier T

Ejercicio 4

$256 - 137 = 119$

Si $k > \frac{N}{2}$ (Mitad de arriba) tienes que usar

$$k = \frac{-f_c N + N f_s}{f_s}$$

a) $k = \frac{f_c N}{f_s} \Rightarrow f_c = \frac{k\cdot f_s}{N}$

$$f_c = \frac{119\cdot 1000}{256} = \underline{464{,}84\ Hz}$$

La otra se calcula como:

→ El mayor se resta.

$$f_c = \frac{137\cdot 1000}{256} = \underline{535{,}156\ Hz}$$

$1000 - 535{,}156 \rightarrow$

$= \underline{464{,}84\ Hz}$

En realidad basta con hacerlo con 1.

b) Tiene buena resolución espectral.

c) Si tiene pocas muestras se pierde resolución espectral. Faltan muestras

Ejercicio 5

$$H(z) = \frac{2\left(z - e^{+j\frac{\pi}{4}}\right)\left(z - e^{-j\frac{\pi}{4}}\right)}{z\left(z - Ae^{j\frac{\pi}{4}}\right)\left(z - Ae^{-j\frac{\pi}{4}}\right)}$$

$$e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

$$e^{-j\frac{\pi}{4}} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$$

a) Si $A > 0$

Ceros:

$$z - e^{-j\frac{\pi}{4}} = 0 \quad \left[z = e^{-j\frac{\pi}{4}} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} \right]$$

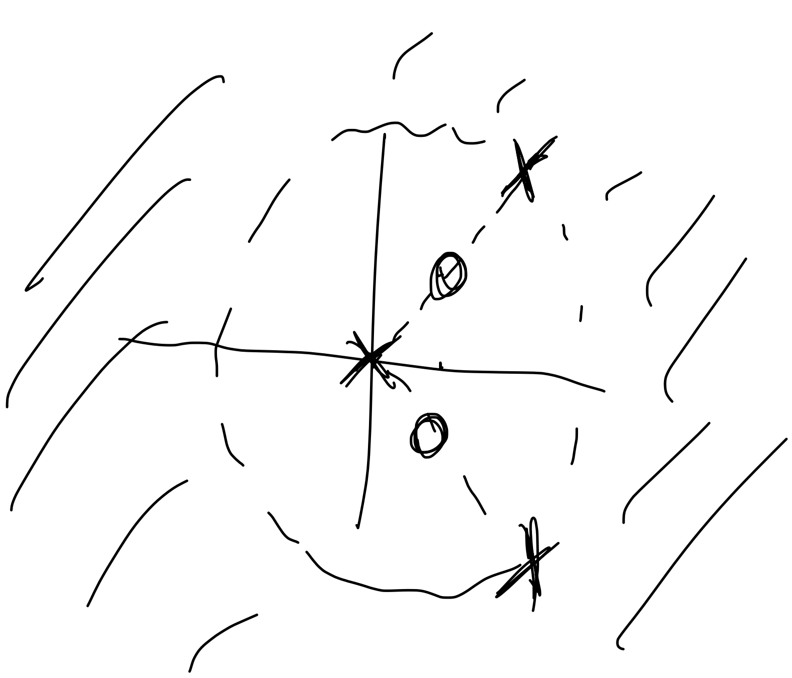
$$z - e^{j\frac{\pi}{4}} = 0 \quad z = e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

Polos

$\underline{z = 0}$

$$z = A \cdot e^{j\frac{\pi}{4}} = A \cdot \frac{\sqrt{2}}{2} + Aj\frac{\sqrt{2}}{2}$$

$$A \cdot \frac{\sqrt{2}}{2} - Aj\frac{\sqrt{2}}{2}$$



b) $$\frac{2\left(z-e^{i\frac{\pi}{4}}\right)\left(z-e^{-i\frac{\pi}{4}}\right)}{z\left(z-A\cdot e^{i\frac{\pi}{4}}\right)\left(z-A\cdot e^{-i\frac{\pi}{4}}\right)}\cdot\frac{z^{-3}}{z^{3}} =$$

$$= \frac{2z^{-1}\left(1-e^{i\frac{\pi}{4}}z^{-1}\right)\left(1-e^{-i\frac{\pi}{4}}z^{-1}\right)}{1\left(1-A\cdot e^{i\frac{\pi}{4}}z^{-1}\right)\left(1-A\cdot e^{-i\frac{\pi}{4}}z^{-1}\right)}$$

c) ¿Valores para los que el sistema es estable?

Sabemos que el sistema es causal

Polos dentro del círculo unidad.

$$A\cdot e^{i\frac{\pi}{4}} < 1 \Rightarrow A < \frac{1}{e^{i\frac{\pi}{4}}} = \frac{2}{\sqrt{2}} + j\frac{2}{\sqrt{2}}$$

Sí tiene TF

d) $$H(z) = \frac{Y(z)}{X(z)} = \frac{2\left(1-e^{i\frac{\pi}{4}}z^{-1}\right)\left(1-e^{-i\frac{\pi}{4}}z^{-1}\right)}{1\left(1-A\cdot e^{i\frac{\pi}{4}}z^{-1}\right)\left(1-Ae^{-i\frac{\pi}{4}}z^{-1}\right)}$$

$$\left(2-2e^{i\frac{\pi}{4}}z^{-1}\right)\left(1-e^{-i\frac{\pi}{4}}z^{-1}\right) = 2-2e^{i\frac{\pi}{4}}z^{-1} - 2e^{-i\frac{\pi}{4}}z^{-1} + \underbrace{2\cancel{e^{i\frac{\pi}{4}}}z^{-2}\cancel{e^{-i\frac{\pi}{4}}}}_{2z^{-2}}$$

$$\left(1-Ae^{i\frac{\pi}{4}}z^{-1}\right)\left(1-Ae^{-i\frac{\pi}{4}}z^{-1}\right) = 1 - Ae^{-i\frac{\pi}{4}}z^{-1} + Ae^{i\frac{\pi}{4}}z^{-1} + A^{2}z^{-2}$$

$$y[n] - A e^{-j\frac{\pi}{4}} y[n-1] + A e^{j\frac{\pi}{4}} y[n-1] + A^2 y[n-2] =$$

$$= 2x[n] - 2e^{j\frac{\pi}{4}} x[n-1] - 2 e^{-j\frac{\pi}{4}} x[n-1] + 2x[n-2]$$

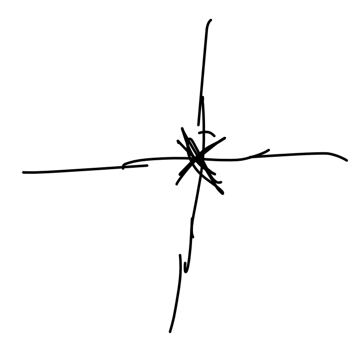
Ejercicio 6

$H(z) = 1 - z^{-1} = \frac{1-z^{-1}}{1} \cdot \frac{z}{z}$ Cero: $1 - \frac{1}{z} = 0$

$z = 1 \rightarrow \boxed{z = 1}$

$\frac{1 - z}{z}$ $\boxed{z = 1}$ $\boxed{z = 0}$

FIR → polos en cero

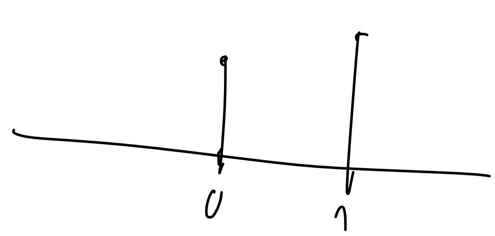


b) Causal incluye ∞

Entonces es causal

c) Estable, polos dentro del circulo unidad

d) $1 - z^{-1} \rightarrow \delta[n] - \delta[n-1]$



No es simétrico