TRRT
Workshop 5 Hand-In

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Problem 1 (Solution to Exercise 5)**.** *We are assuming that*

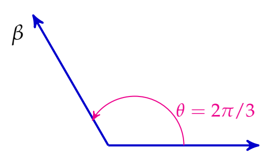
$$\frac{\pi}{2} \leq \theta := \measuredangle(\alpha, \beta) < \pi,$$

where θ is as large as possible, and from the table

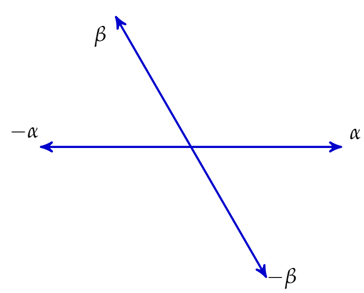
$\langle \beta, \alpha \rangle$	$\langle \alpha, \beta \rangle$	θ	$\frac{(\beta,\beta)}{(\alpha,\alpha)}$
0	0	$\frac{\pi}{2}$	*undefined*
1	1	$\frac{\pi}{3}$	1
-1	-1	$\frac{2\pi}{3}$	1
2	1	$\frac{\pi}{4}$	2
-2	-1	$\frac{3\pi}{4}$	2
3	1	$\frac{\pi}{6}$	3
-3	-1	$\frac{5\pi}{6}$	3

we have only four viable options for θ: $\pi/2$, $2\pi/3$, $3\pi/4$, $5\pi/6$.

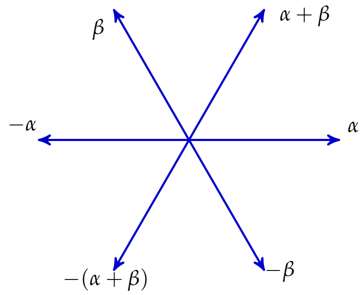
We now consider the case $\theta = 2\pi/3$, in which $|\beta| = |\alpha|$:



But the scalar multiples $-\alpha$ and $-\beta$ are also in R, so we extend the diagram to



Proceeding even further, by the result from Q4 we know that if $\pi/2 < \measuredangle(\alpha, \beta) < \pi$, then $\alpha + \beta \in R$ (and *so is the scalar multiple* $-(\alpha+\beta)$*). Thus we extend the diagram further:*



Now, by repeatedly applying the reflections s_α *(with* α *being a root on this diagram) and using the table above (or, equivalently, by calling on the result of Q4), we conclude that these roots exhaust all the elements of* R *for* $\theta = 2\pi/3$*.* □