Testing the Fibonacci formula

The following code demonstrates that the algorithm used by *Mathematica* to find Fibonacci numbers and the formula $\frac{\left(1+\sqrt{5}\right)^n-\left(1-\sqrt{5}\right)^n}{2^n\sqrt{5}}$ yield very close though not exact results :

```
fibonaccilist = {};
formulalist = {};
For[n = 1, n ≤ 912, n++,
  actualfib = Fibonacci[n];
  formulafib = ((1 + √5)^n - (1 - √5)^n)/(2^n √5) // N;
  AppendTo[fibonaccilist, actualfib];
  AppendTo[formulalist, formulafib];
  WorkingPrecision → MachinePrecision];
If[fibonaccilist == formulalist,
 Print["The two lists are equal"],
 Print["The two lists are not equal"]
]
```

```
The two lists are not equal
```

I chose to test this up to the 912^{th} Fibonacci number because this is where Mathematica recognizes that the methods yield different results. For instance, if we test it up to the 911^{th} term, the numbers already differ quite noticeably, but *Mathematica* is incapable of discerning any difference (maybe this is due to the machine's precision?) . Let me show what I mean in the following ...

```
fibonaccilist⟦911⟧
formulalist⟦911⟧ // IntegerPart
```

```
10 920 820 416 459 328 443 221 811 072 207 854 881 435 307 575 238 350 852 564 743 215 922 283 ⋱
 679 178 069 381 290 262 679 629 826 259 806 842 091 635 423 101 972 412 825 783 917 076 523 888 ⋱
 382 721 659 090 269 202 596 043 912 832 242 330 879 807 510 689

10 920 820 416 459 476 630 607 439 164 474 618 260 512 823 666 698 852 594 164 536 094 413 391 ⋱
 039 391 457 475 919 099 657 836 814 646 612 952 841 536 916 240 359 258 297 583 874 657 732 792 ⋱
 326 536 402 225 375 405 989 830 497 729 268 815 447 218 716 672
```

```
fibonaccilist⟦911⟧ == formulalist⟦911⟧
```

```
True
```

For some reason *Mathematica* "sees" these two numbers as being equal when clearly they're not. I'm assuming this has something to do with the machine precision. Now we test the 912^{th} term and we see that *Mathematica* no longer "sees" the two outputs as the same :

```
fibonaccilist[[912]]
formulalist[[912]] // IntegerPart
fibonaccilist[[912]] == formulalist[[912]]
```

17 670 258 618 864 975 009 258 339 416 537 971 760 012 675 183 748 670 183 843 230 539 845 347 ⋱
074 301 607 362 326 211 014 092 320 301 155 064 788 141 495 664 328 603 298 786 390 904 066 437 ⋱
630 628 079 431 525 530 236 022 705 233 071 673 280 212 746 944

17 670 258 618 865 239 442 480 033 584 089 084 688 775 306 251 095 272 183 619 831 313 863 506 ⋱
888 290 756 822 740 523 196 581 858 363 459 669 482 138 218 685 136 202 496 864 266 276 387 703 ⋱
338 069 483 588 144 412 619 397 243 115 740 208 626 046 861 312

```
False
```

Now we test the formulas relating the golden ratio and Fibonacci numbers:

$$1+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\text{Fibonacci}[n]\ \text{Fibonacci}[n+1]}$$

```
GoldenRatio
```

$$\text{Limit}\left[\frac{\text{Fibonacci}[n]}{\text{Fibonacci}[n-1]},\ n\to\infty\right] == \text{GoldenRatio}$$

```
True
```