# Math 260 HW # 7


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### Section 3.3

(2) For the following homogenous system of linear equations, find the dimension of and a basis for the solution set :

d) $2x_1 + x_2 - x_3 = 0$
$x_1 - x_2 + x_3 = 0$
$x_1 + 2x_2 - 2x_3 = 0$

Solution:
First let's find the solution set...

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & -2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & -3 \\ 0 & 3 & -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus we have
$x_2 - x_3 = 0 \Longrightarrow x_2 = x_3$
$x_1 - x_2 + x_3 = 0 \Longrightarrow x_1 = 0$

Hence, since $x_3$ is a free variable we let $x_3 = t \in \mathbb{R}$. Then we have

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Thus a basis for the solution set is the singleton $\{(0,\ 1,\ 1)\}$ and that implies that the dimension of the solution set is 1.

(3) Find all solutions to the following system :

d) $2x_1 + x_2 - x_3 = 5$
$x_1 - x_2 + x_3 = 1$
$x_1 + 2x_2 - 2x_3 = 4$

Solution:

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 5 \\ 1 & -1 & 1 & 1 \\ 1 & 2 & -2 & 4 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -3 & 3 \\ 0 & 3 & -3 & 3 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_2 - x_3 = 1 \Longrightarrow x_2 = 1 + x_3$

$x_1 - (1 + x_3) + x_3 = 1 \Longrightarrow x_1 = 2$

Letting $x_3 = t \in \mathbb{R}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \operatorname{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

## Section 3.4

(2) Use Gaussian elimination to solve the following system of linear equations :

c) $x_1 + 2\,x_2 + 2\,x_4 = 6$

$3\,x_1 + 5\,x_2 - x_3 + 6\,x_4 = 17$

$2\,x_1 + 4\,x_2 + x_3 + 2\,x_4 = 12$

$2\,x_1 - 7\,x_3 + 11\,x_4 = 7$

Solution:

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 3 & 5 & -1 & 6 & 17 \\ 2 & 4 & 1 & 2 & 12 \\ 2 & 0 & -7 & 11 & 7 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -4 & -7 & 7 & -5 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 7 & -1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$$

Using back substitution we have

$x_4 = -1$

$x_3 - 2(-1) = 0 \Longrightarrow x_3 = -2$

$x_2 + (-2) = 1 \Longrightarrow x_2 = 3$

$x_1 - 2(-2) + 2(-1) = 4 \Longrightarrow x_1 = 2$

Hence our solution is $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -2 \\ -1 \end{pmatrix}$ ✡

## Section 4.4

(4) Evaluate the determinant of the following matrix by using any legitimate method :

g) $\begin{pmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix}$

Solution:

$$\begin{pmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -5 & 11 \\ 0 & 4 & -1 & 1 \\ 0 & 3 & 4 & -5 \end{pmatrix}$$

Hence $\det\begin{pmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix} = \det\begin{pmatrix} 1 & -5 & 11 \\ 4 & -1 & 1 \\ 3 & 4 & -5 \end{pmatrix}$

Now we simplify this further

$$\begin{pmatrix} 1 & -5 & 11 \\ 4 & -1 & 1 \\ 3 & 4 & -5 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -5 & 11 \\ 0 & 19 & -43 \\ 0 & 19 & -38 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -5 & 11 \\ 0 & 19 & -43 \\ 0 & 0 & 5 \end{pmatrix}$$

Hence finally we have

$$\det\begin{pmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix} = \det\begin{pmatrix} 1 & -5 & 11 \\ 0 & 19 & -43 \\ 0 & 0 & 5 \end{pmatrix} = 1 \cdot 19 \cdot 5 = 95$$