MATH 722: Mathematical Modeling II
Problem Set 3

Due Date: Thursday, March 11th at 9:30 AM EST

Goal: In this problem set, you will create and solve a linear programming problem to find a selection of foods meeting stated nutritional requirements.

Directions: Please write up your solutions to the questions below, and then upload a PDF to the folder titled Problem Set 3 under the Assignment tab on myCourses.

In this assignment, you will track the foods you eat over the course of one day. From those food eaten, you will pick between 10 to 15 items. You will then build a diet plan for a day from these 10 to 15 items that meets certain nutritional requirements and minimizes costs of purchasing the foods. Note that Dr. Rooney helped design this assignment and said that he would be more than happy to discuss this assignment or the topics he discussed in class (linear programming/combinatorial optimization) with you. Dr. Rooney's email is `brooney@mail.rit.edu`.

1. The first set of nutritional requirements focus on the total amount of calories eaten over the course of the day and the sources of those calories.
   (a) Determine the number of calories you would like to eat over the course of a day, C.
   (b) Determine a range, (p_{min}, p_{max}), of your total energy input, C, to be derived from proteins. For example, in the pediatric diet paper, $p_{min} = 0.12$ and $p_{max} = 0.15$.
   (c) Determine a range, (c_{min}, c_{max}), of your total energy input, C, to be derived from carbohydrates. For example, in the pediatric diet paper, $c_{min} = 0.45$ and $c_{max} = 0.50$.
   (d) Determine a range, (l_{min}, l_{max}), of your total energy input, C, to be derived from lipids/fats. For example, in the pediatric diet paper, $l_{min} = 0.38$ and $l_{max} = 0.42$.
2. The second nutritional requirement focuses on limiting sodium intake.
   (a) Determine how you will measure sodium per serving size.
   (b) Determine a range, (s_{min}, s_{max}), of acceptable sodium intake over the course of the day.
3. Pick a third nutritional requirement you would like to enforce, determine how you will measure the requirement, and then determine a range of an acceptable values over the course of the day.
4. Pick a fourth nutritional requirement you would like to enforce, determine how you will measure the requirement, and then determine a range of an acceptable values over the course of the day.
5. Now, for each food item, x_i, determine the following quantities (make sure to include references):
   (a) A reasonable serving size;
   (b) A reasonable maximal number of helpings (measured in serving size);
   (c) Find the amount of energy from proteins per serving size;

(d) Find the amount of energy from carbohydrates per serving size;

(e) Find the amount of energy from lipids/fats per serving size;

(f) Find the amount of sodium per serving size;

(g) Find the amount of the first additional nutritional requirement per serving size;

(h) Find the amount of the second additional nutritional requirement per serving size.

(i) Determine the cost per serving size;

6. Write down the objective function computing the cost of the diet plan for your linear programming problem.

7. Write down the constraints of your linear programming problem:

   (a) Maximum number of helpings for each item;

   (b) Minimum/maximum amount of total energy derived from proteins;

   (c) Minimum/maximum amount of total energy derived from carbohydrates;

   (d) Minimum/maximum amount of total energy derived from lipids/fats;

   (e) Minimum/maximum amount of sodium intake;

   (f) Minimum/maximum amount of your first additional nutritional constraint;

   (g) Minimum/maximum amount of your second additional nutritional constraint;

8. Use a software package of your choice to solve your linear programming problem. Here is the link to the online program Dr. Rooney showed in class, `https://online-optimizer.appspot.com/?model=builtin:default.mod`, or you could modify the Juypter notebook created by Dr. Rooney. Describe any modifications you had to make to establish a non-empty feasible region and comment on the solution you found.

9. Now that you have your "optimal" diet what could you change to try to find a diet that uses as many of the available foods as possible?