Bubble Nucleation

Vacuum Decay in the Early Universe

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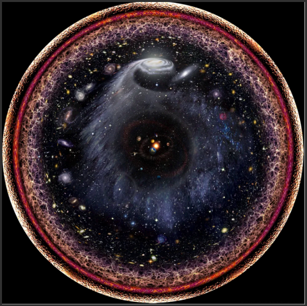
Overview

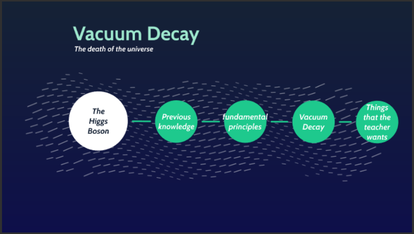


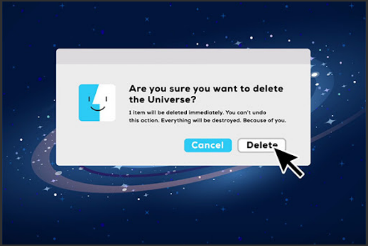
Introduction

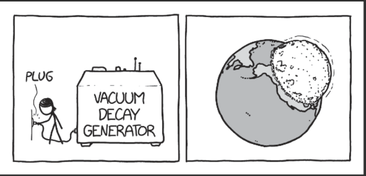
Equations...Equations ...Equations anyone?

$$
\begin{aligned}
\partial_t \bar{A}_{ij} = \chi[\alpha(R_{ij} + KK_{ij} - 2K_{ik}K^k{}_j) \\
- 8\pi\alpha(S_{ij} - \frac{1}{2}\underbrace{\gamma_{ij}}_{\text{no TF}}(S-\rho)) - D_iD_j\alpha \\
+ \mathcal{L}_{\vec{\beta}}K_{ij}]^{\mathrm{TF}} + \chi^{-1}\bar{A}_{ij}\,[\frac{2}{3}\chi(\alpha K - \partial_i\beta^i) \\
+ \beta^i\partial_i\chi] \\
= [\alpha\chi R_{ij} + \alpha\chi\chi^{-1}K\left(\bar{A}_{ij} + \frac{1}{3}\bar{\gamma}_{ij}K\right) \\
- 2\alpha\chi\chi^{-1}\left(\bar{A}_{ik} + \frac{1}{3}\bar{\gamma}_{ik}K\right)\left(\bar{A}^k{}_j + \delta^k{}_jK\right)\ldots \\
\cdots - 8\pi\alpha\chi S_{ij} - \chi D_iD_j\alpha]^{\mathrm{TF}} + \ldots
\end{aligned}
$$

Dramatic much?



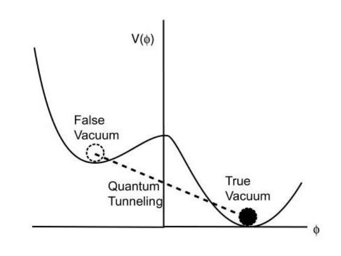




The Model

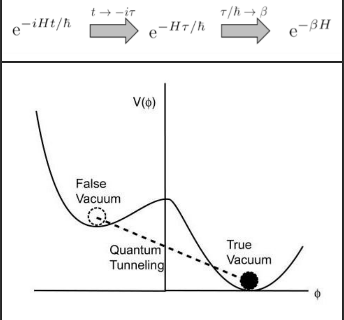
Semiclassical Vacuum Decay

$$e^{-iHt/\hbar} \xrightarrow{t \to -i\tau} e^{-H\tau/\hbar} \xrightarrow{\tau/\hbar \to \beta} e^{-\beta H}$$



$$\Gamma/V = Ae^{-B/\hbar}[1 + \mathcal{O}(\hbar)]$$

Semiclassical Vacuum Decay

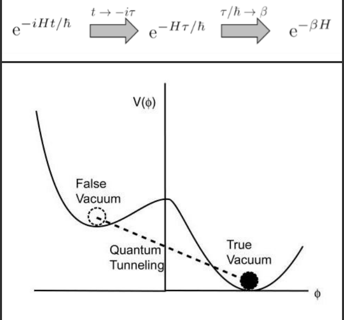


$$\Gamma/V = Ae^{-B/\hbar}[1 + \mathcal{O}(\hbar)]$$

$$B = S_E = \int \left[\frac{1}{2}\mathring{\nabla}^a\phi\mathring{\nabla}_a\phi + V(\phi)\right] \mathrm{d}\tau\, \mathrm{d}\vec{x}$$

$$V(\phi) = \frac{1}{2}\lambda^2 \sin^2\phi - \cos\phi - 1$$

Semiclassical Vacuum Decay



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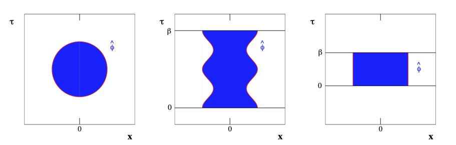
$$V(\phi) = \frac{1}{2}\lambda^2 \sin^2\phi - \cos\phi - 1$$

$$0=\mathring{\nabla}^2\phi - \partial_\phi V$$

Numerical Methods

Good Ole shooting methods?



- Shooting method very effective under $O(D)$ assumption

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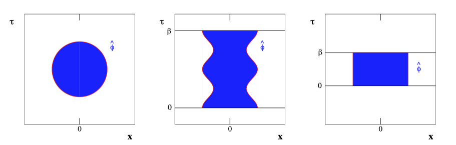

- Shooting method very effective under $O(D)$ assumption
- ...but utterly useless otherwise ...

Relaxation Techniques

Introduce auxiliary scalar $\Phi(s, \tau, x)$ (so that $\lim_{s\to\infty} \Phi = \phi_b$) and set

$$\mathscr{F} \equiv \mathring{\nabla}^2\Phi - \partial_\Phi V$$

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First attempt:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}s} = \mathcal{O}\mathscr{F}$$

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First attempt:

$$\begin{aligned}
\frac{\mathrm{d}\Phi}{\mathrm{d}s} &= \mathscr{F} \\
\Phi^{n+1}_{i,j} &= \varsigma \left[\Phi^n_{i+1,j} + \Phi^n_{i-1,j} + \Phi^n_{i,j+1} + \Phi^n_{i,j-1}\right] + \Phi^n_{i,j}\left[1 - 4\varsigma\right] \\
&\quad - \Delta s \left[\frac{\lambda^2}{2}\sin\left(2\Phi^n_{i,j}\right) + \sin\Phi^n_{i,j}\right] \\
\varsigma &\equiv \frac{\Delta s}{\mathrm{h}^2}
\end{aligned}$$

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&\quad -\Delta s\left[\frac{\lambda^2}{2}\sin\left(2\Phi^n_{i,j}\right)+\sin\Phi^n_{i,j}\right] \\
\varsigma &\equiv \frac{\Delta s}{\mathrm{h}^2}
\end{aligned}$$

Unstable algorithm...try gain ...☹

Relaxation Techniques

Second attempt: Examine the response of the field near the solution Φ_b under the effect of a slight perturbation $\delta\Phi$:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}s} = \frac{\mathrm{d}}{\mathrm{d}s}\left(\Phi_b + \delta\Phi\right) = \underbrace{\frac{\mathrm{d}\Phi_b}{\mathrm{d}s}}_{=0} + \frac{\mathrm{d}(\delta\Phi)}{\mathrm{d}s} = \frac{\mathrm{d}(\delta\Phi)}{\mathrm{d}s}.$$

Hence the behavior of the system close to the bubble solution Φ_b is governed by a second-order operator.

Relaxation Techniques

Second attempt: Make $\mathcal{O}$ a 2nd-order operator $\Delta^\dagger$ so that

$$\Delta^\dagger \mathscr{F} \equiv (\mathscr{F}')^\dagger \mathscr{F} = -\mathring{\nabla}^2 \mathscr{F} + \partial_\Phi^2 V \mathscr{F}.$$

Thus, we have

$$\frac{\mathrm{d}\Phi}{\mathrm{d}s} = \Delta^\dagger \mathscr{F}.$$

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Thus, we have

$$\frac{\mathrm{d}\Phi}{\mathrm{d}s} = \Delta^\dagger \mathscr{F}.$$

Much better this time! ...But not good enough ...☹
(To have stability, vonNeumann shows $\Delta s \sim O(\mathrm{h}^4)$)

Relaxation Techniques

Third time's the charm!: Put

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}s^2} + k\frac{\mathrm{d}\Phi}{\mathrm{d}s} = \Delta^\dagger \mathscr{F}$$

Relaxation Techniques

Third time's the charm!: Put

$$\frac{d^2\Phi}{ds^2} + k\frac{d\Phi}{ds} = \Delta^\dagger \mathscr{F}$$

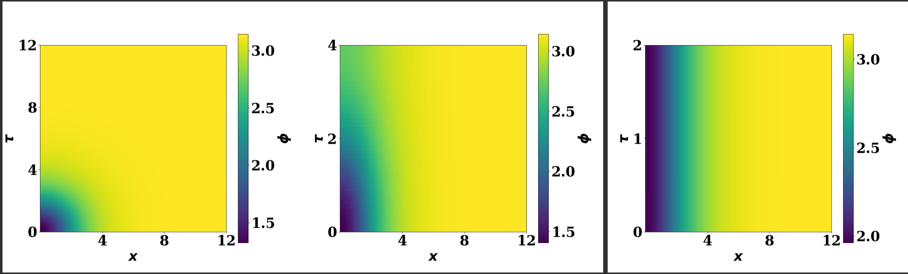
Then

$$\boxed{\Phi_{i,j}^{n+1} = \frac{1}{1+\frac{k}{2}\Delta s}\left\{2\Phi_{i,j}^{n} - \Phi_{i,j}^{n-1}\left[1-\frac{k}{2}\Delta s\right] + \Delta^\dagger \mathscr{F}(\Phi_{i,j}^{n})\Delta s^2\right\}}$$

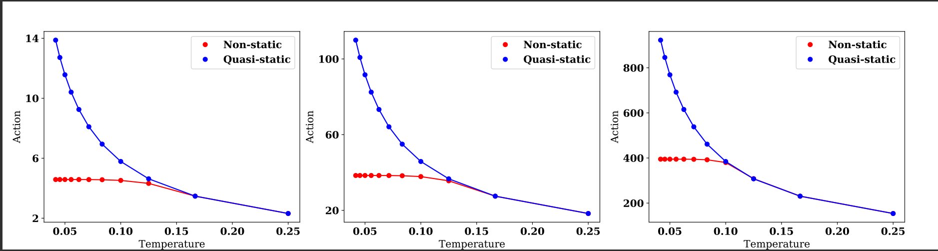
This is it! ☺
(To have stability, vonNeumann shows $\Delta s \sim O(\mathrm{h}^2)$)

Results

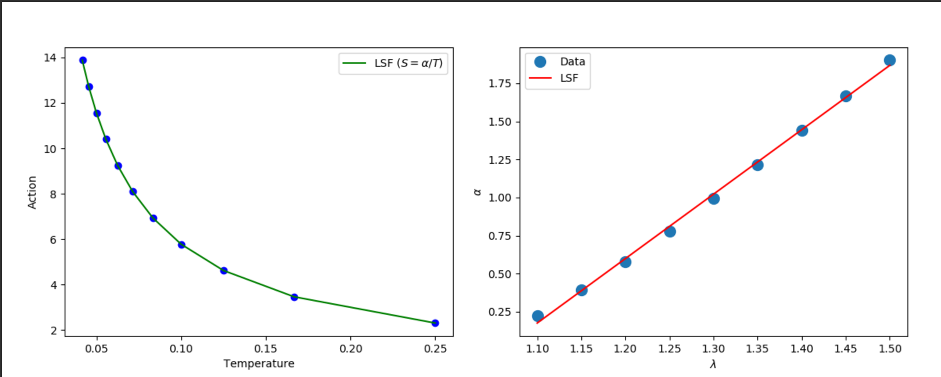
Transition from vacuum to thermal state



Action v Temperature (all three D cases)



The most intriguing result!



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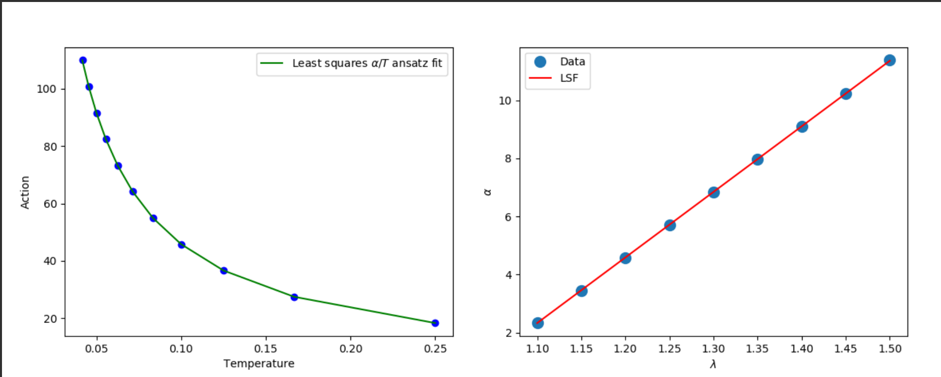

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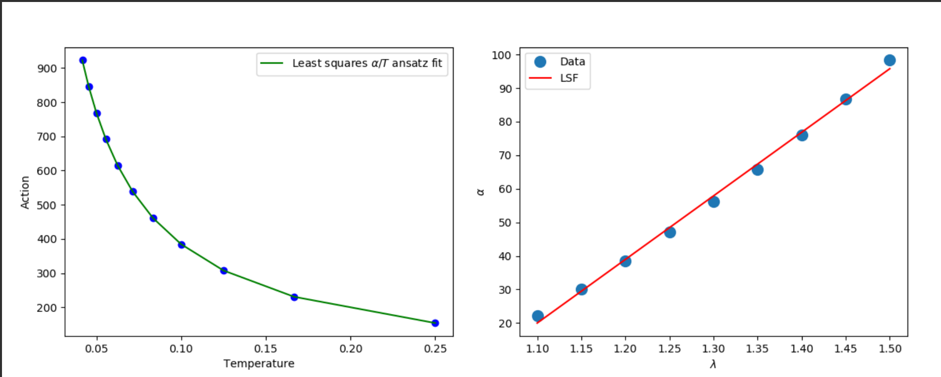

THANK YOU!