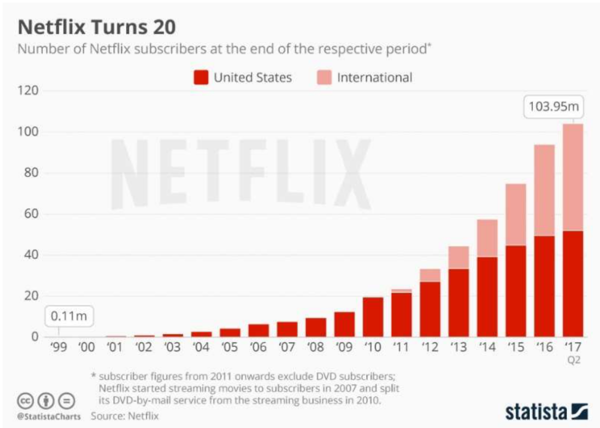
1 Netflix

In a review article, Mahajan, Muller, and Bass discuss *Bass's model* of the adoption over time of durable goods. Take a look at each of these papers. `Mahajan et al` present the basic differential equation of Bass's model as

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = p[m - N(t)] + \frac{q}{m}N(t)[m - N(t)]. \tag{1}$$

Here $N(t)$ is the number of consumers that have adopted the good at time t. The parameter m is the number who will ultimately adopt the good. Some consumers –Bass calls them *innovators*– adopt the product without being influence by other consumers who have already adopted it; they're not influence by word-of-mouth. The parameter p is the fraction of such consumers who adopt the product per unit time. Other consumers –Bass calls them imitators– adopt the product because of word-of-mouth. The fraction of word-of-mouth adopters who adopt the product per unit time is a function of the number of consumers who have already adopted the product. In Bass's model that fraction depends on the parameter q, it's the term $(qN)/m$ on Eq. (1). The parameter q can be interpreted as the *probability per unit time* that an imitator adopts the product at the point in time at which almost all of the adopters have adopted it.



1.1 First Question

Solve the basic differential equation, assuming time $t = 0$ is when the product is introduced. (So, at $t = 0$ nobody has adopted the product.) It can be solved in closed form –you almost certainly solved it in a calculus class or an ODE class– but you may solve it numerically if you'd prefer to. Discuss the influences of the parameters on the solution, and interpret those influences in terms of the product adoption model.

Solution. Let us go the numerical route, since I personally find it more interesting. We will have to make certain assumptions in our code, however, since there would be too many free parameters otherwise. To that end, we shall use the data provided by Statista as a guidance. Let us start by collecting all the parameters from Eq. (1) and discuss our approach:

$N(t)$: number of consumers that have adopted the good at time t. This is the solution that we are looking for, hence no a priori assumptions will be made (other than the initial value $N(0) = 0$, of course). [1]

m: number of consumers who will ultimately adopt the good. Let us focus on US numbers from the Statista graph, where it shows $m \approx 50$ million subscribers by the year 2017. Thus we shall set $m \equiv 5 \times 10^6$ in our code.

p & q: fraction of "innovators" (p) and "imitators" (q) who adopt the product per unit time. It makes sense (in my head at least!) that the innovators are far less than the imitators. Usually when a new product hits the market there is only a small group willing to pull the trigger without having first heard good word-of-mouth, read some favorable reviews, etc. [2] So, with that being said, lets us declare the values $p = 0.3$ and $q = 0.7$.

In our code we will use a straightforward Euler method, where we rewrite Eq. (1) as

$$\text{RHS} = p[m - N_i] + \frac{q}{m}N_i[m - N_i] \tag{2a}$$

$$N_{i+1} = \Delta t \cdot \text{RHS} + N_i. \tag{2b}$$

Here $N_i = N(t_i)$ is the usual FDM notation used to denote the value of the function N at the i^{th} time step $t_i = t_0 + i\Delta t$, while Eq. (2b) comes from discretizing the derivative

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \frac{N_{i+1} - N_i}{\Delta t}. \tag{3}$$

[1] Speaking of which, we will let the code run from $t = 0$ (1997) to $t = 20$ (2017), so that we cover the 21 years of data that the graph shows.

[2] You could make a case that in times of crisis, like the one we are currently in, people are more willing to try new things right off the bat because they are bored at home or what not (TikTok anyone??), but let's better not go there ...

Here is a snippet of my C++ code:

```cpp
#include <iostream>
#include <fstream>
#include <cmath>
#include <vector>

using namespace std;

int main(int argc, const char * argv[]) {


    //SET PARAMETERS
    const double m {5.0e6};
    const double p {0.3};
    const double q {0.7};
    const double dt {0.1};

    double N {0.0}; // (N(0) = 0)
    vector <double> Ndata {};  //data vector where we store our numerical results
    double RHS {}; //initialization of variable RHS

    double t {0.0};  //time from 0 to 20 years

    for (int i {0}; i <= 200; i++){
        t = i * dt;
        Ndata.push_back(N);
        cout << "N = " << N << " at time " << t << " years." << endl;
        RHS = p * (m - N) + (q/m) * N * (m - N);
        N = N + dt * RHS;
    }

   ofstream myfile ("num_data.csv");
    for (int i{0}; i <= 200; i++) {
        if (i != 200) {
            myfile << Ndata.at(i) << ",";
        } else {
            myfile << Ndata.at(i) << endl;
        }

    }

    return 0;
}
```

The numerical results obtained are (very) interesting; refer to Fig. 1. On the one hand, my assumption about the disparity between innovators and imitators was confirmed ... but, on the other hand, I (highly) underestimated it! The figure on the left, which uses parameter values $\{p = 0.3, q = 0.7\}$, shows a much faster adoption rate than what the Statista graph shows (Netflix would have certainly loved this!). On the other hand, with values $\{p = 0.000001, q = 0.999999\}$, the numbers on the right figure match quite nicely the numbers we see on the Statista graph for U.S. subscribers. Our conclusion is that we are surrounded by imitators! As I previously mentioned, I was expecting such disparity, but certainly not by this much!

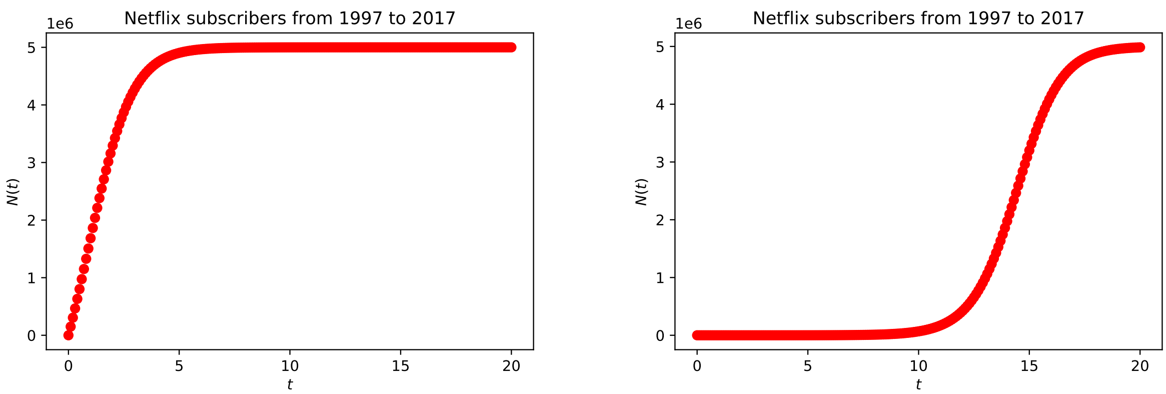


Figure 1: Numerical results show U.S. Netflix subscribers (in millions) from the year 1997 ($t = 0$) to 2017 ($t = 20$). The figure on the left has parameter values $\{p = 0.3, q = 0.7\}$, while the figure on the right has $\{p = 0.000001, q = 0.999999\}$. $\square$

1.2 Second Question

If $p = q$, how long does it take for half of the adoptions to take place?

Solution. Letting $p = q = 0.5$, we see that it takes only just over a year to reach half of the adoptions (≈ 2.5 million subscribers in our case); refer to Fig. 2. Comparing this with our previous results shown in Fig. 1 we see the same trend: more early adopters means less time the Netflix overlords have to wait to notice their pockets fattening.

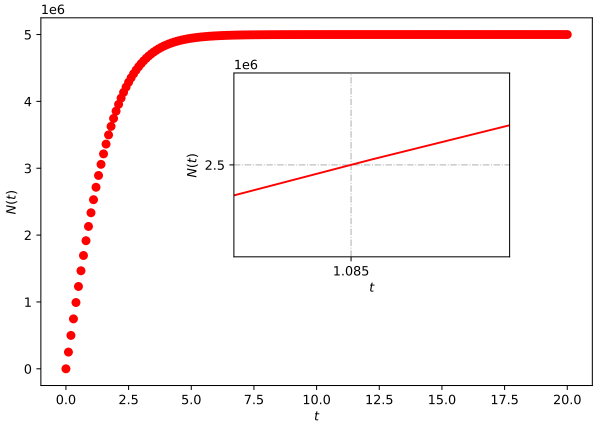


Figure 2: When letting $p = q = 0.5$, we see an even faster rate of adoption than we saw earlier in Fig. 1. In-set in this figure is a zoom-in of the graph at $t \approx 1$, which shows that it would take Netflix just over a year to reach 2.5 million subscribers if we had the same number of innovators and imitators in the model. □

1.3 Third Question

Show that your solution approaches an asymptotic value for large times. What is that asymptotic value? Interpret it in terms of the model.

Solution. This has already been shown above; the asymptotic value is, of course, m. This makes sense in the model; once the number of subsbribers reaches the value m, the rate of change of subscribers vanishes (i.e., N remains constant ($= m$)):

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = p[m - m] + \frac{q}{m}m[m - m] = 0.$$ □

1.4 Fourth Question

Take a look at the graph of Netflix subscribers included above (U.S. subscribers and international subscribers) each year from 1999 through 2017, a graph taken from the Statista website. On the assumption that a Netflix subscription is a durable good to which Bass's model applies, estimate m, p, and q for the U.S. or for international subscribers, or for both. Identify the asymptotic value of $N(t)$ and say when it will be reached. (You don't need to do highly-precise estimates of these parameters, and you don't need to do any fancy stats, or anything like that. Just come up with reasonable numbers.)

Solution. Answered in Question 1. □