Geometrized Units

In *Planck* (or *geometrized*) units, the *Newtonian gravitational constant* G, the *speed of light*, c, and the *reduced Planck constant* $\hbar$, are all set to unity; i.e., $G = c = \hbar = 1$. A straightforward dimensional analysis shows

$$[G] = \frac{L^3}{MT^2} \tag{1a}$$

$$[c] = \frac{L}{T} \tag{1b}$$

$$[\hbar] = \frac{ML^2}{T}, \tag{1c}$$

where M, L, and T refer to mass, length, and time, respectively.

The whole point of using these units is that we can just focus on our calculations without worrying about all these cumbersome units of mass, length, and whatever else floating around, and then afterwards we may perform a straightforward dimensional analysis to recover physical units (and to make sure that what we calculated isn't complete nonsense!). We illustrate this concept through an example, by calculation the *Planck length* ℓ_p:

We write ℓ_p in terms only of combinations of powers of the three fundamental quantities discussed above,

$$\ell_p = G^\alpha c^\beta \hbar^\gamma. \tag{2}$$

We now rewrite this equation in terms of the units of measurement of each quantity,

$$\begin{aligned} L &= (L^3 M^{-1} T^{-2})^\alpha \, (LT^{-1})^\beta \, (ML^2 T^{-1})^\gamma \\ &= L^{3\alpha+\beta+2\gamma} \, M^{-\alpha+\gamma} \, T^{-2\alpha-\beta-\gamma}. \end{aligned}$$

So we have a linear system of equations

$$\begin{aligned} 3\alpha + \beta + 2\gamma &= 1 \\ -\alpha + \gamma &= 0 \\ -2\alpha - \beta - \gamma &= 0, \end{aligned}$$

which has the solution

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}.$$

Thus equation (2) is

$$\ell_p = G^{1/2} c^{-3/2} \hbar^{1/2} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35} \text{ meters}. \tag{3}$$

Similar derivations yield other fundamental geometric quantities, such as the *Planck mass* m_p and *Planck time* t_p. Planck units are in some sense the most natural choice of units because they are not based on any arbitrarily chosen physical system.