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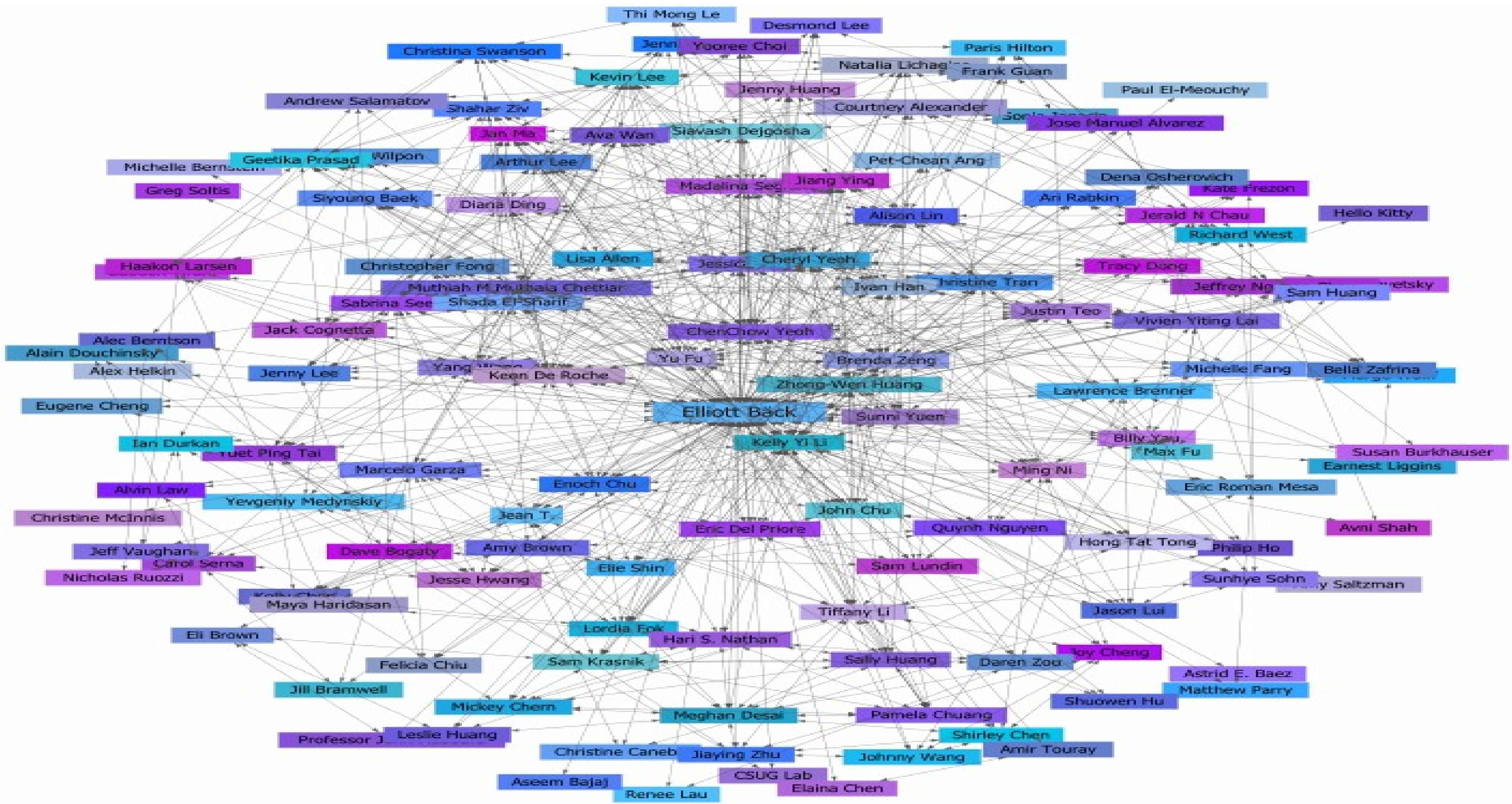

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/ $G = (V, E)$

$|V| = n$ ' $|E| = m$

$ 2 $w_e \geq 0, e \in E$

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0 $p = (v_1, \ldots, v_{|p|})$!

2 p $\{v_1, v_{|p|}\}$

: ! 2 $\mathsf{Int}(p) = p \setminus \{v_1, v_{|p|}\}$

$\mathcal{S}_{uv}$ 2 u v

$\sigma_{uv} = |\mathcal{S}_{uv}|$ u v

$$\mathbb{S}_G = \bigcup_{(u,v) \in V \times V, u \neq v} \mathcal{S}_{uv}$$ 2 G

3 $! $v \in V$

4 6 !

v

3 $! $v \in V$

4 6 !

v

C $\mathcal{T}_v = \{p \in \mathbb{S}_G \ : \ v \in \mathsf{Int}(p)\}$

! v

$$\boxed{\mathsf{b}(v) = \frac{1}{n(n-1)} \sum_{p_{uw} \in \mathbb{S}_G} \frac{1_{\mathcal{T}_v}(p_{uw})}{\sigma_{uw}} = \frac{1}{n(n-1)} \sum_{p_{uw} \in \mathcal{T}_v} \frac{1}{\sigma_{uw}}}$$

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2 $\Theta(n^3)$

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                2 $O(nm)$ $O(nm + n^2 \log n)$

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2 $\Theta(n^3)$

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2 $O(nm)$ $O(nm + n^2 \log n)$

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(ε, δ)1 $

! $\tilde{\mathsf{b}}(v)$ $v \in V$

$$\boxed{\Pr\left(\exists v \in V \ : \ |\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \varepsilon\right) < \delta}$$

ε ! !! !

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0 ! $\tilde{\mathsf{b}}(w), w \in V$

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0 ! $\tilde{\mathsf{b}}(w), w \in V$

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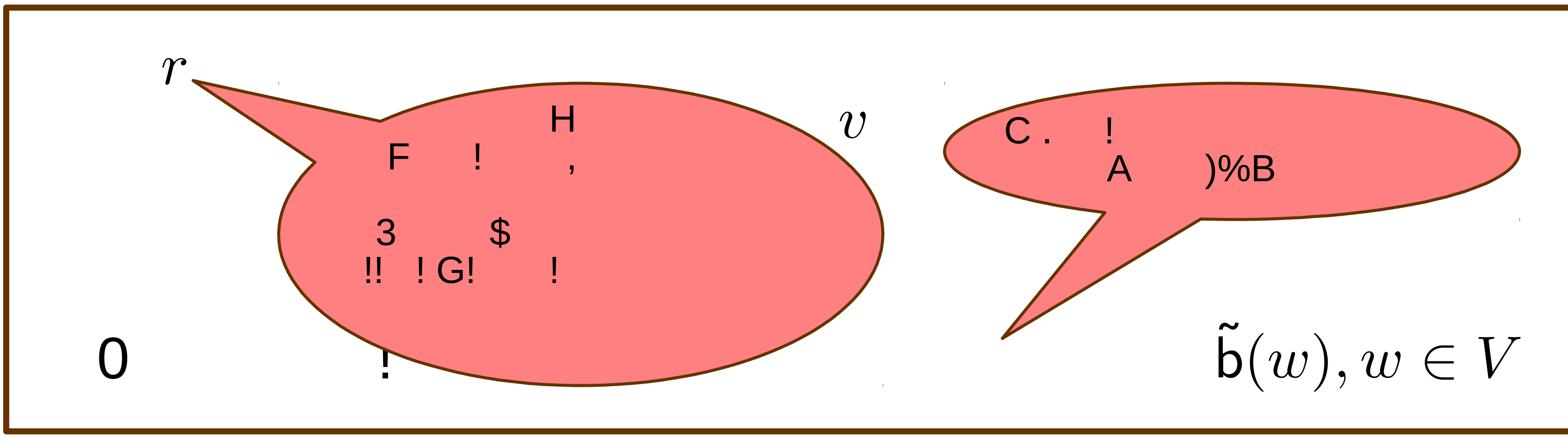

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$$\Pr(|\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \varepsilon) < 2e^{-2r\varepsilon^2}$$

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$$\Pr(|\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \varepsilon) < 2e^{-2r\varepsilon^2}$$

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F $ 4 $
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$$\Pr(|\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \varepsilon) < 2e^{-2r\varepsilon^2}$$

$! n

H 1 r (ε, δ)

$$r \geq \frac{1}{2\varepsilon^2}\left(\ln n + \ln 2 + \ln\frac{1}{\delta}\right)$$

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! (u, v)

' $\mathcal{S}_{uv}$ u v

! p $\mathcal{S}_{uv}$

! $w \in \mathsf{Int}(p)$

$\tilde{\mathsf{b}}(w) \leftarrow \tilde{\mathsf{b}}(w) + 1/r$

| =

3 VD(G) G
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! 2 G

$$\boxed{\mathsf{VD}(G) = \max\{|p| \;:\; p \in \mathbb{S}_G\}}$$

: 2 $\mathsf{VD}(G) = \Delta_G + 1$

- G

3 2

/ $\varepsilon, \delta \in (0,1)$

$$r \geq \frac{1}{\varepsilon^2}\left(\lfloor \log_2(\mathsf{VD}(G)-2)\rfloor + 1 + \ln\frac{1}{\delta}\right)$$

$$\Pr\left(\exists v \in V \ : \ |\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \varepsilon\right) < \delta$$

+ 1 (ε, δ)

3 2

/ $\varepsilon, \delta \in (0,1)$

$$r \geq \frac{1}{\varepsilon^2}\left(\lfloor \log_2(\mathsf{VD}(G) - 2) \rfloor + 1 + \ln \frac{1}{\delta}\right)$$

$$\Pr\left(\exists v \in V \ : \ |\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \right) < \delta$$

+ 1 (ε, δ)

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/ $\varepsilon, \delta \in (0,1)$

$$r \geq \frac{1}{\varepsilon^2}\left(\lfloor \log_2(\mathsf{VD}(G) - 2) \rfloor + 1 + \ln \frac{1}{\delta}\right)$$

$$\Pr\left(\exists v \in V \ : \ |\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \right) < \delta$$

+ 1 (ε, δ)

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$\mathcal{R}$ B

' $C \subseteq B$ $P_C = \{C \cap F \ : \ F \in \mathcal{R}\}$

C $P_C = 2^C$

3 I 1= $\mathcal{R}$ H

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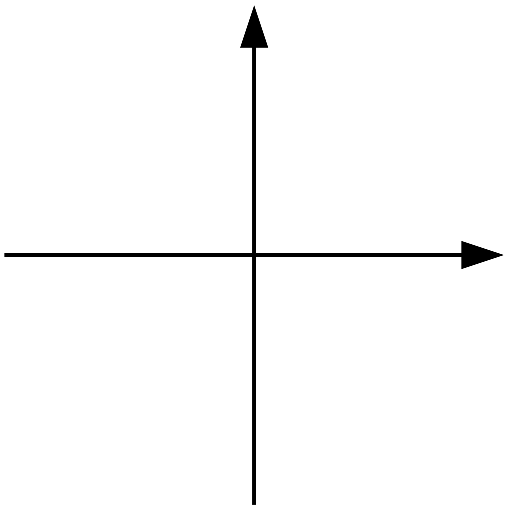
I 1=

$$B = \mathbb{R}^2$$



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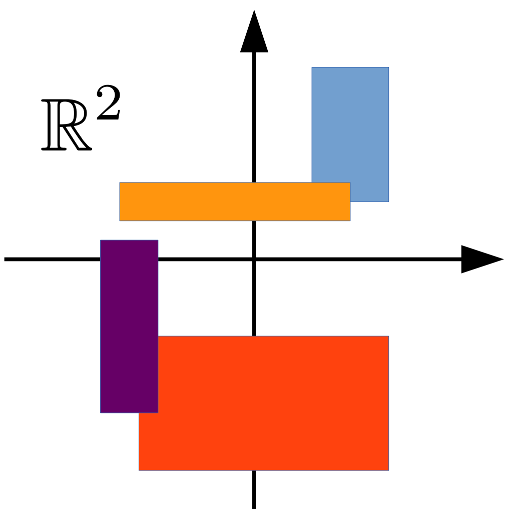
$B = \mathbb{R}^2$

$\mathcal{R}$J 1 !



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$B = \mathbb{R}^2$

$\mathcal{R}$J 1 !

K 2

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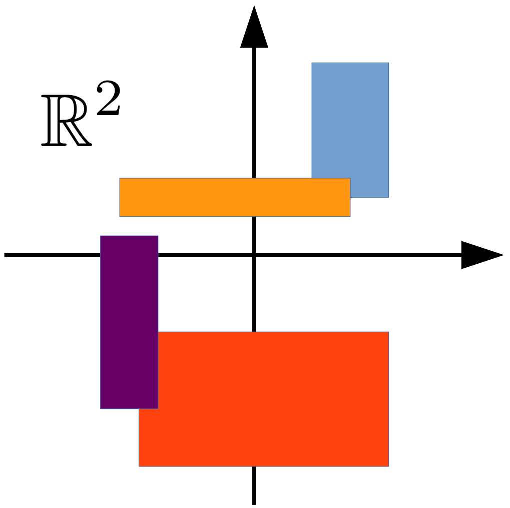

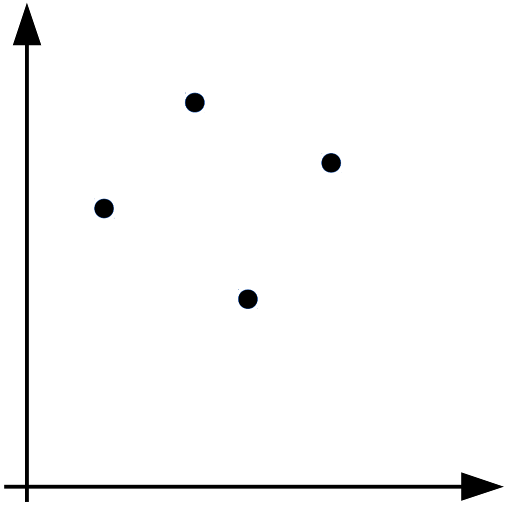

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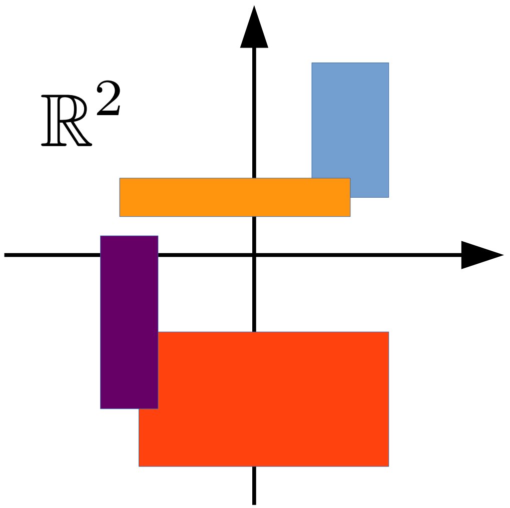
$B = \mathbb{R}^2$

$\mathcal{R}$J 1 !



K 2

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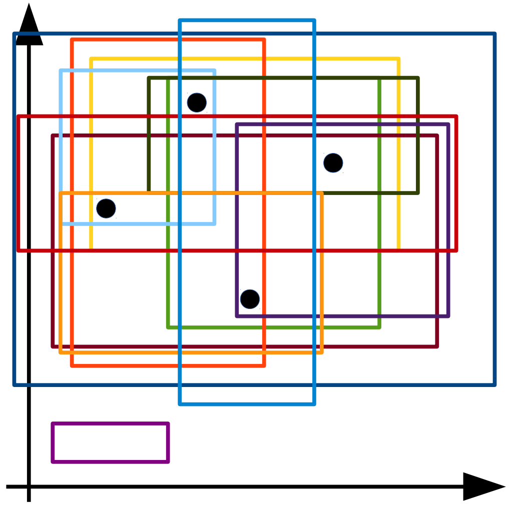

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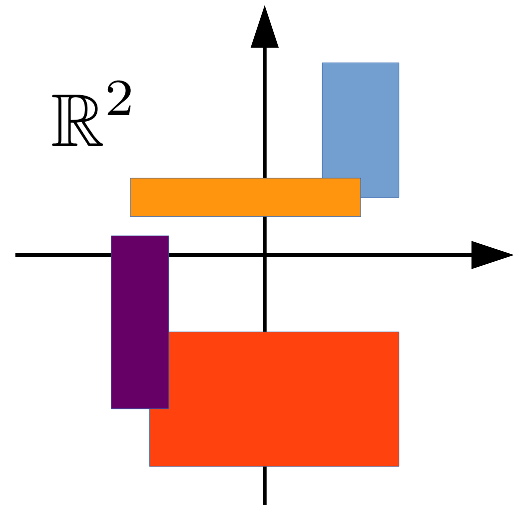

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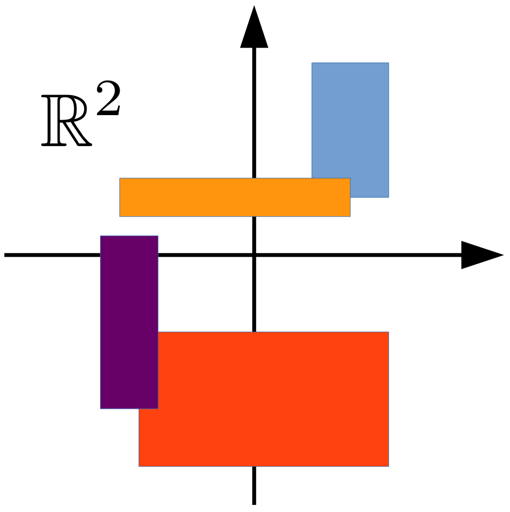

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$B = \mathbb{R}^2$

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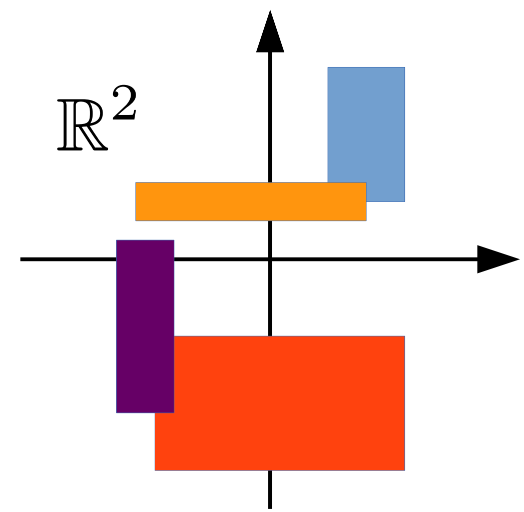

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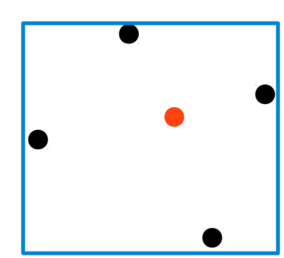
$\text{VC}(\mathcal{R}) = 4$



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C $\mathcal{R}$ $\text{VC}(\mathcal{R}) \leq d$

C π $ $ $ B$

C $\pi(A)$ $ $ $A \subseteq B$

/ $\varepsilon, \delta \in [0, 1]$! S

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$$\boxed{|S| \geq \frac{1}{\varepsilon^2}\left(d + \ln\frac{1}{\delta}\right)}$$

'

$$\boxed{\Pr\left(\exists A \in \mathcal{R} \;:\; \left|\pi(A) - \frac{1}{|S|}\sum_{s \in S} 1_A(s)\right| > \varepsilon\right) < \delta}$$

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C $\mathcal{R}$ $\text{VC}(\mathcal{R}) \leq d$

C π $ $ $ B$

C $\pi(A)$ $ $ $A \subseteq B$

/ $\varepsilon, \delta \in [0,1]$! S

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$$|S| \geq \frac{1}{\varepsilon^2}\left(d + \ln\frac{1}{\delta}\right)$$

'

$$\Pr\left(\exists A \in \mathcal{R} \ : \ \left|\pi(A) - \frac{1}{|S|}\sum_{s \in S} 1_A(s)\right| > \varepsilon\right) < \delta$$

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C $\mathcal{R}$ $\mathrm{VC}(\mathcal{R}) \leq d$

C π $ $ $ $ $ $ $ B

C $\pi(A)$ $ $ $ $ $A \subseteq B$

/ $\varepsilon, \delta \in [0, 1]$! S :

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$$\boxed{|S| \geq \frac{1}{\varepsilon^2}\left(d + \ln \frac{1}{\delta}\right)}$$

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$$\boxed{\Pr\left(\exists A \in \mathcal{R} \ : \ \left|\pi(A) - \frac{1}{|S|}\sum_{s \in S} 1_A(s)\right| > \varepsilon\right) < \delta}$$

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$$\mathcal{R}_G = \{\mathcal{T}_v, v \in V\}$$

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$\mathcal{R}_G = \{\mathcal{T}_v, v \in V\}$

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$\mathcal{R}_G = \{\mathcal{T}_v, v \in V\}$

$ $ $ $\mathbb{S}_G$

$$\boxed{\pi_G(p_{uv}) = \frac{1}{n(n-1)}\frac{1}{\sigma_{uv}}}$$

3 !! π_G

l 1=

3 2 $\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2 \mathsf{VD}(G) - 2 \rfloor + 1$

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3 2 $\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2 \mathsf{VD}(G) - 2 \rfloor + 1$

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3 2 $\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2 \mathsf{VD}(G) - 2 \rfloor + 1$

0 2 3 ' ' A $|A| = d$

2^d $\mathcal{T}_v \in \mathcal{R}_G$

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3 2 $\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2 \mathsf{VD}(G) - 2 \rfloor + 1$

0 2 3 ' ' A $|A| = d$

2^d $\mathcal{T}_v \in \mathcal{R}_G$

$p \in A$ 2^{d-1}

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3 2 $\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2 \mathsf{VD}(G) - 2 \rfloor + 1$

0 2 3 ' ' A $|A| = d$

2^d $\mathcal{T}_v \in \mathcal{R}_G$

$p \in A$ 2^{d-1}

p ; $\mathcal{T}_v$ $v \in \mathsf{Int}(p)$

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3 2 $\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2 \mathsf{VD}(G) - 2 \rfloor + 1$

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2^d $\mathcal{T}_v \in \mathcal{R}_G$

$p \in A$ 2^{d-1}

p ; $\mathcal{T}_v$ $v \in \mathsf{Int}(p)$

p $|\mathsf{Int}(p)| \leq \mathsf{VD}(G) - 2$ $\mathcal{T}_v$

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3 2 $\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2 \mathsf{VD}(G) - 2 \rfloor + 1$

0 2 3 ' ' A $|A| = d$

2^d $\mathcal{T}_v \in \mathcal{R}_G$

$p \in A$ 2^{d-1}

p ; $\mathcal{T}_v$ $v \in \mathsf{Int}(p)$

p $|\mathsf{Int}(p)| \leq \mathsf{VD}(G) - 2$ $\mathcal{T}_v$

$\dot{A}$ ' \$ $2^{d-1} \leq \mathsf{VD}(G) - 2$

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! (u, v)

' $\mathcal{S}_{uv}$ u v

! p $\mathcal{S}_{uv}$

! $w \in \mathsf{Int}(p)$

$\tilde{\mathsf{b}}(w) \leftarrow \tilde{\mathsf{b}}(w) + 1/r$

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! $w \in \mathsf{Int}(p)$

$\tilde{\mathsf{b}}(w) \leftarrow \tilde{\mathsf{b}}(w) + 1/r$

$\tilde{\mathsf{b}}(w) \leftarrow \tilde{\mathsf{b}}(w) + 1/r$

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! (u, v)

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! p $\mathcal{S}_{uv}$

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! $w \in \mathsf{Int}(p)$

$\tilde{\mathsf{b}}(w) \leftarrow \tilde{\mathsf{b}}(w) + 1/r$

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$$r \geq \frac{1}{\varepsilon^2}\left(\lfloor \log_2(\mathsf{VD}(G) - 2)\rfloor + 1 + \ln\frac{1}{\delta}\right)$$

! ! $\tilde{\mathsf{b}}(v)$

(ε, δ)1 2

$$\Pr\left(\exists v \in V \ : \ |\tilde{\mathsf{b}}(v) - \mathsf{b}(v)| > \varepsilon\right) < \delta$$

!

$$r \geq \frac{1}{\varepsilon^2}\left(\lfloor \log_2(\mathsf{VD}(G) - 2) \rfloor + 1 + \ln\frac{1}{\delta}\right)$$

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$$\Pr\left(\exists v \in V \;:\; |\tilde{\mathsf{b}}(v)\right.$$

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! $\mathsf{VC}(\mathcal{R}_G) \leq 3$

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! $\text{VC}(\mathcal{R}_G) \leq 3$

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! VC$(\mathcal{R}_G) \leq 3$

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$$\text{VC}(\mathcal{R}_G) \leq \lfloor \log_2 k - 1 \rfloor + 1$$

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$$T(K,G) = \{v \in V \ : \ \mathsf{b}(v) \geq \mathsf{b}^{(K)}\}$$

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2 $T(K,G)$

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$$T(K,G) = \{v \in V \ : \ \mathsf{b}(v) \geq \mathsf{b}^{(K)}\}$$

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! $T(K,G)$

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$$\boxed{(1-\varepsilon)\mathsf{b}(v) \le \tilde{\mathsf{b}}(v) \le (1+\varepsilon)\mathsf{b}(v)}$$

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(2 $T(K, G)$
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! v

$$\boxed{(1-\varepsilon)\mathsf{b}(v) \leq \tilde{\mathsf{b}}(v) \leq (1+\varepsilon)\mathsf{b}(v)}$$

$ $ $1-\delta$

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$$\mathsf{VC}(\mathcal{R}_G) \leq \lfloor \log_2(\mathsf{VD}(G) - 2) \rfloor + 1$$

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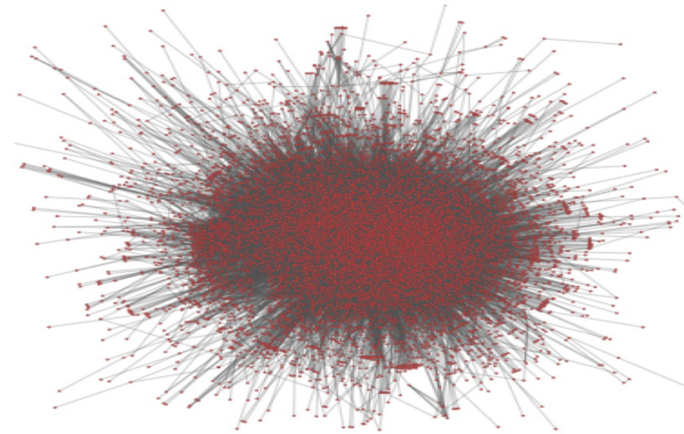

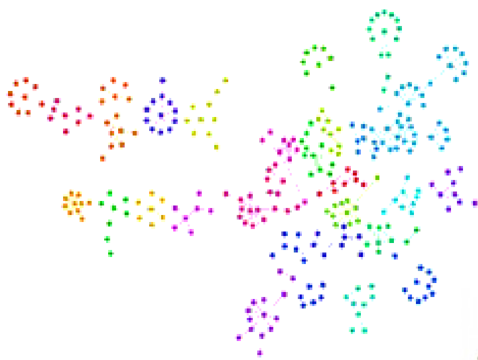

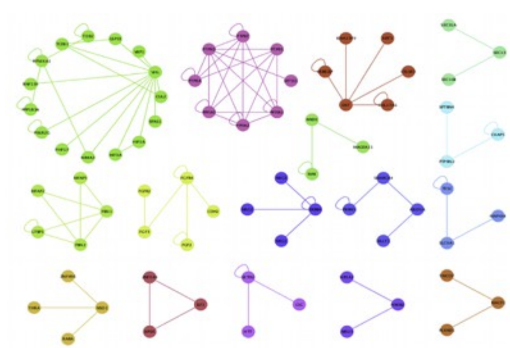

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