Lemma 1. *If weight of edge (u, v) in G is decreased to obtain G', then for any $x \in V$, the set of shortest paths from x to u and from v to x is the same in G and G', and $d'(x, u) = d(x, u)$, $d'(v, x) = d(v, x)$; $\sigma'_{xu} = \sigma_{xu}$, $\sigma'_{vx} = \sigma_{vx}$.*

Lemma 2. *Let the weight of edge (u, v) be decreased to $\mathbf{w}'(u, v)$, and for any given pair of vertices s, t, let $D(s, t) = d(s, u) + \mathbf{w}'(u, v) + d(v, t)$. Then,*

1. *If $d(s, t) < D(s, t)$, then $d'(s, t) = d(s, t)$ and $\sigma'_{st} = \sigma_{st}$.*
   The shortest paths from s to t in G' are the same as in G.
2. *If $d(s, t) = D(s, t)$, then $d'(s, t) = d(s, t)$ and $\sigma'_{st} = \sigma_{st} + (\sigma_{su} \cdot \sigma_{vt})$.*
   The shortest paths from s to t in G' are a superset of the shortest paths G.
3. *If $d(s, t) > D(s, t)$, then $d'(s, t) = D(s, t)$ and $\sigma'_{st} = \sigma_{su} \cdot \sigma_{vt}$.*
   The shortest paths from s to t in G' are new (shorter distance).