
Algorithm 4: VIRTUAL: BC with virtual vertices

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...
 $\ell \leftarrow 0$ 
▷Forward phase
while  $cont = \text{true}$  do
     $cont \leftarrow \text{false}$ 
    ▷Forward-step kernel
    for each virtual vertex  $u_{vir}$  in parallel do
         $u \leftarrow \text{vmap}[u_{vir}]$ 
        if  $d[u] = \ell$  then
1         for each  $v \in \Gamma_{vir}(u_{vir})$  do
2             if  $d[v] = -1$  then
3                  $d[v] \leftarrow \ell + 1, cont \leftarrow \text{true}$ 
                 if  $d[v] = \ell + 1$  then  $\sigma[v] \overset{atomic}{\leftarrow} \sigma[v] + \sigma[u]$ 
         $\ell \leftarrow \ell + 1$ 
...
▷Backward phase
while  $\ell > 1$  do
     $\ell \leftarrow \ell - 1$ 
    ▷Backward-step kernel
    for each virtual vertex  $u_{vir}$  in parallel do
         $u \leftarrow \text{vmap}[u_{vir}]$ 
        if  $d[u] = \ell$  then
             $sum \leftarrow 0$ 
4             for each  $v \in \Gamma(u)$  do
5                 if  $d[v] = \ell + 1$  then  $sum \leftarrow sum + \delta[v]$ 
6              $\delta[u] \overset{atomic}{\leftarrow} \delta[u] + sum$ 
    ▷Update bc values by using Equation (5)
...
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