

Let E^* be the set of edges in G that lie on shortest paths, let $m^* = |E^*|$, and let ν^* be the maximum number of edges that lie on shortest paths through any single vertex. Here is our main result:

Theorem 1. *After an incremental update on an edge or a vertex in a directed or undirected graph with positive edge weights, the betweenness centrality of all vertices can be recomputed in:*

1. $O(\nu^* \cdot n)$ time using $O(\nu^* \cdot n)$ space;
2. $O(m^* \cdot n)$ time using $O(n^2)$ space.

$$O(mn + n^2)$$