LAPORAN PRAKTIKUM
ANALISIS ALGORITMA

Disusun Oleh :

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Studi Kasus 5: Mencari Pasangan Titik Terdekat (Closest Pair of Points)

1. Buatlah program untuk menyelesaikan problem closest pair of points menggunakan algoritma divide & conquer yang diberikan. Gunakan bahasa C++

   Program:

```
#include <bits/stdc++.h>
using namespace std;

// A structure to represent a Point in 2D plane
class Point
{
public:
	int x, y;
};

/* Following two functions are needed for library function qsort().
Refer: http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */

// Needed to sort array of points
// according to X coordinate
int compareX(const void* a, const void* b)
{
	Point* p1 = (Point*)a, * p2 = (Point*)b;
	return (p1->x - p2->x);
}

// Needed to sort array of points according to Y coordinate
int compareY(const void* a, const void* b)
{
	Point* p1 = (Point*)a, * p2 = (Point*)b;
	return (p1->y - p2->y);
}

// A utility function to find the
// distance between two points
float dist(Point p1, Point p2)
{
	return sqrt((p1.x - p2.x) * (p1.x - p2.x) +
		(p1.y - p2.y) * (p1.y - p2.y)
	);
}

// A Brute Force method to return the
// smallest distance between two points
// in P[] of size n
float bruteForce(Point P[], int n)
{
	float min = FLT_MAX;
	for (int i = 0; i < n; ++i)
		for (int j = i + 1; j < n; ++j)
			if (dist(P[i], P[j]) < min)
				min = dist(P[i], P[j]);
	return min;
}
```

```
// A utility function to find
// minimum of two float values
float min(float x, float y)
{
	return (x < y) ? x : y;
}


// A utility function to find the
// distance beween the closest points of
// strip of given size. All points in
// strip[] are sorted accordint to
// y coordinate. They all have an upper
// bound on minimum distance as d.
// Note that this method seems to be
// a O(n^2) method, but it's a O(n)
// method as the inner loop runs at most 6 times
float stripClosest(Point strip[], int size, float d)
{
	float min = d; // Initialize the minimum distance as d

	qsort(strip, size, sizeof(Point), compareY);

	// Pick all points one by one and try the next points till the difference
	// between y coordinates is smaller than d.
	// This is a proven fact that this loop runs at most 6 times
	for (int i = 0; i < size; ++i)
		for (int j = i + 1; j < size && (strip[j].y - strip[i].y) < min; ++j)
			if (dist(strip[i], strip[j]) < min)
				min = dist(strip[i], strip[j]);

	return min;
}

// A recursive function to find the
// smallest distance. The array P contains
// all points sorted according to x coordinate
float closestUtil(Point P[], int n)
{
	// If there are 2 or 3 points, then use brute force
	if (n <= 3)
		return bruteForce(P, n);

	// Find the middle point
	int mid = n / 2;
	Point midPoint = P[mid];

	// Consider the vertical line passing
	// through the middle point calculate
	// the smallest distance dl on left
	// of middle point and dr on right side
	float dl = closestUtil(P, mid);
	float dr = closestUtil(P + mid, n - mid);

	// Find the smaller of two distances
	float d = min(dl, dr);

	// Build an array strip[] that contains
```

```
        // points close (closer than d)
        // to the line passing through the middle point
        Point strip[n];
        int j = 0;
        for (int i = 0; i < n; i++)
                if (abs(P[i].x - midPoint.x) < d)
                        strip[j] = P[i], j++;

        // Find the closest points in strip.
        // Return the minimum of d and closest
        // distance is strip[]
        return min(d, stripClosest(strip, j, d));
}

// The main functin that finds the smallest distance
// This method mainly uses closestUtil()
float closest(Point P[], int n)
{
        qsort(P, n, sizeof(Point), compareX);

        // Use recursive function closestUtil()
        // to find the smallest distance
        return closestUtil(P, n);
}

// Driver code
int main()
{
        Point P[] = { {4, 2}, {12, 30}, {40, 50}, {5, 5}, {3, 4} };
        int n = sizeof(P) / sizeof(P[0]);
        cout << "The smallest distance is " << closest(P, n);
        return 0;
}
```

Screenshot:

```
The smallest distance is 2.23607
Process returned 0 (0x0)   execution time : 0.599 s
Press any key to continue.
```

2. Tentukan rekurensi dari algoritma tersebut, dan selesaikan rekurensinya menggunakan metode recursion tree untuk membuktikan bahwa algoritma tersebut memiliki Big-O (n lg n)

   Jawaban:

   Kompleksitas Waktu
   Kompleksitas waktu dari algoritma tersebut adalah T (n). Asumsikan algoritma pengurutan O (nLogn). Algoritma tersebut membagi semua titik dalam dua set dan secara rekursif memanggil dua set. Setelah membelah, strip ditemukan dalam waktu O (n), mengurutkan strip dalam waktu O (nLogn) dan akhirnya menemukan titik terdekat dalam strip dalam waktu O (n). Jadi T (n) dapat dinyatakan sebagai berikut

$$T(n) = 2T(n / 2) + O(n) + O(nLogn) + O(n)$$
$$T(n) = 2T(n / 2) + O(nLogn)$$

T (n) = T (n x Logn x Logn)

Studi Kasus 6: Algoritma Karatsuba untuk Perkalian Cepat

1. Buatlah program untuk menyelesaikan problem fast multiplication menggunakan algoritma divide & conquer yang diberikan (Algoritma Karatsuba). Gunakan bahasa C+

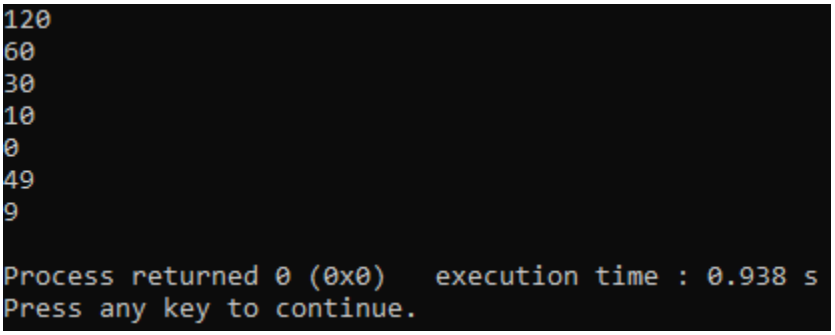
   Program:

```
#include<iostream>
#include<stdio.h>

using namespace std;

// FOLLOWING TWO FUNCTIONS ARE COPIED FROM http://goo.gl/q0OhZ
// Helper method: given two unequal sized bit strings, converts them to
// same length by adding leading 0s in the smaller string. Returns the
// the new length
int makeEqualLength(string& str1, string& str2)
{
	int len1 = str1.size();
	int len2 = str2.size();
	if (len1 < len2)
	{
		for (int i = 0; i < len2 - len1; i++)
			str1 = '0' + str1;
		return len2;
	}
	else if (len1 > len2)
	{
		for (int i = 0; i < len1 - len2; i++)
			str2 = '0' + str2;
	}
	return len1; // If len1 >= len2
}

// The main function that adds two bit sequences and returns the addition
string addBitStrings(string first, string second)
{
	string result; // To store the sum bits

	// make the lengths same before adding
	int length = makeEqualLength(first, second);
	int carry = 0; // Initialize carry

	// Add all bits one by one
	for (int i = length - 1; i >= 0; i--)
	{
		int firstBit = first.at(i) - '0';
		int secondBit = second.at(i) - '0';

		// boolean expression for sum of 3 bits
		int sum = (firstBit ^ secondBit ^ carry) + '0';

		result = (char)sum + result;
```

```
                // boolean expression for 3-bit addition
                carry = (firstBit & secondBit) | (secondBit & carry) | (firstBit & carry);
        }

        // if overflow, then add a leading 1
        if (carry) result = '1' + result;

        return result;
}

// A utility function to multiply single bits of strings a and b
int multiplyiSingleBit(string a, string b)
{
        return (a[0] - '0') * (b[0] - '0');
}

// The main function that multiplies two bit strings X and Y and returns
// result as long integer
long int multiply(string X, string Y)
{
        // Find the maximum of lengths of x and Y and make length
        // of smaller string same as that of larger string
        int n = makeEqualLength(X, Y);

        // Base cases
        if (n == 0) return 0;
        if (n == 1) return multiplyiSingleBit(X, Y);

        int fh = n / 2; // First half of string, floor(n/2)
        int sh = (n - fh); // Second half of string, ceil(n/2)

        // Find the first half and second half of first string.
        // Refer http://goo.gl/lLmgn for substr method
        string Xl = X.substr(0, fh);
        string Xr = X.substr(fh, sh);

        // Find the first half and second half of second string
        string Yl = Y.substr(0, fh);
        string Yr = Y.substr(fh, sh);

        // Recursively calculate the three products of inputs of size n/2
        long int P1 = multiply(Xl, Yl);
        long int P2 = multiply(Xr, Yr);
        long int P3 = multiply(addBitStrings(Xl, Xr), addBitStrings(Yl, Yr));

        // Combine the three products to get the final result.
        return P1 * (1 << (2 * sh)) + (P3 - P1 - P2) * (1 << sh) + P2;
}

// Driver program to test aboev functions
int main()
{
        printf("%ld\n", multiply("1100", "1010"));
        printf("%ld\n", multiply("110", "1010"));
        printf("%ld\n", multiply("11", "1010"));
        printf("%ld\n", multiply("1", "1010"));
        printf("%ld\n", multiply("0", "1010"));
```

```
    printf("%ld\n", multiply("111", "111"));
    printf("%ld\n", multiply("11", "11"));
}
```

Screenshot:



2. Rekurensi dari algoritma tersebut adalah T (n) = 3T (n / 2) + O (n), dan selesaikan rekurensinya menggunakan metode substitusi untuk membuktikan bahwa algoritma tersebut memiliki Big-O (n lg n)

Jawaban:

- Let's try divide and conquer
  - Divide each number into two halves
    - $x = x_H\, r^{n/2} + x_L$
    - $y = y_H\, r^{n/2} + y_L$
  - then:

    $xy = (x_H\, r^{n/2} + x_L)\, y_H\, r^{n/2} + y_L$

    $= x_H y_H r^n + (x_H y_L)\, r^{n/2} + x_L\, y_L$
  - runtime
    - $T(n) = 4\, T(n/2) + O(n)$
    - $T(n) = O(n^2)$
- Instead of 4 subproblems, we only need 3 (with the help of clever insight)
- Three subproblems:
  - $a = x_H\, y_H$
  - $d = x_L\, y_L$
  - $e = (x_H + x_L)\, (y_H + y_L) - a - d$
- Then $xy = a\, r^n + e\, r^{n/2} + d$
- $T(n) = 3\, T(n/2) + O(n)$
- $T(n) = O(n^{\log 3}) = O(n^{1.584\ldots})$

Studi Kasus 7: Permasalahan Tata Letak Keramik Lantai (Tilling Problem)

1. Buatlah program untuk menyelesaikan problem tilling menggunakan algoritma divide & conquer yang diberikan. Gunakan bahasa C++

Program:

```cpp
#include
<bits/stdc++.h>

                    using namespace std;

                    // function to count the total number of ways
                    int countWays(int n, int m)
                    {

                        // table to store values
                        // of subproblems
                        int count[n + 1];
                        count[0] = 0;

                        // Fill the table upto value n
                        for (int i = 1; i <= n; i++) {
                            // recurrence relation
                            if (i > m)
                                count[i] = count[i - 1] + count[i - m];

                            // base cases
                            else if (i < m)
                                count[i] = 1;

                            // i = = m
                            else
                                count[i] = 2;
                        }

                        // required number of ways
                        return count[n];
                    }

                    // Driver program to test above
                    int main()
                    {
                        int n = 7, m = 3;
                        cout << "Number of ways = "
                             << countWays(n, m);
                        return 0;
                    }
```

Screenshot:

```
Number of ways = 9
Process returned 0 (0x0)   execution time : 0.330 s
Press any key to continue.
```

2. Relasi rekurensi untuk algoritma rekursif di atas dapat ditulis seperti di bawah ini. C adalah konstanta. T (n) = 4T (n / 2) + C. Selesaikan rekurensi tersebut dengan Metode Master

   Jawaban:
   Kompleksitas Waktu:

   Relasi perulangan untuk algoritma rekursif di atas dapat ditulis seperti di bawah ini. C adalah konstanta.
   T (n) = 4T (n / 2) + C
   Rekursi di atas dapat diselesaikan dengan menggunakan Metode Master dan kompleksitas waktu adalah O (n2)

   Pengerjaan algoritma Divide and Conquer dapat dibuktikan menggunakan Mathematical Induction. Biarkan kuadrat input berukuran 2k x 2k di mana k> = 1.
   Kasus Dasar: Kita tahu bahwa masalahnya dapat diselesaikan untuk k = 1. Kami memiliki 2 x 2 persegi dengan satu sel hilang.
   Hipotesis Induksi: Biarkan masalah dapat diselesaikan untuk k-1.
   Sekarang perlu dibuktikan untuk membuktikan bahwa masalah dapat diselesaikan untuk k jika dapat diselesaikan untuk k-1. Untuk k, ditempatkan ubin berbentuk L di tengah dan memiliki empat subsqure dengan dimensi 2k-1 x 2k-1 seperti yang ditunjukkan pada gambar 2 di atas. Jadi jika dapat menyelesaikan 4 subskuares, dapat menyelesaikan kuadrat lengkap.