Exploring Momentum and Energy: Investigating Non-Relativistic and Relativistic Behavior of Radioactive Isotopes Using Scintillator Detection

Zachary Morrison

PHY 353L Modern Laboratory
Department of Physics
The University of Texas at Austin
Austin, TX 78712, USA

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Abstract

We investigated the relationships between momentum and energy of radioactive isotopes, both in non-relativistic and relativistic regimes. By utilizing a sodium iodide (NaI) scintillator as a detection system, we measured the energy spectra emitted by radioactive sources. The observed spectra were then analyzed to extract momentum information, and we found our results **** to be consistent with accepted literature. Our findings provide valuable insights into the transition from non-relativistic to relativistic regimes and contribute to a deeper understanding of fundamental principles in nuclear physics.

1 Introduction

1.1 Physics Motivation

We explore the fundamental connection between momentum and energy in radioactive isotopes, encompassing both non-relativistic and relativistic regimes. It takes inspiration from the foundational contributions of Sir Isaac Newton and Albert Einstein in understanding this relationship. Newton's laws of motion provided a framework for comprehending the interdependence of momentum and energy within the realm of classical mechanics. However, as our understanding of the physical world expanded, Einstein's theory of relativity emerged, revolutionizing our comprehension of high-energy phenomena. By investigating momentum and energy spectra emitted by radioactive sources and analyzing their characteristics, this study aims to deepen our understanding of these fundamental concepts in nuclear physics, bridging the gap between classical mechanics and the principles put forth by Einstein's theory of relativity.

1.2 Theoretical background

In Newton's relationship between energy and momentum,

$$E = \frac{p^2}{2m},$$

where E is the kinetic energy of a particle, p is its momentum, and m is its mass. We also know that the relationship between a photon's energy and momentum is $P_\gamma = \frac{E_\gamma}{c}$, where P_γ is the momentum of the photon, E_γ is the energy of the photon, and c is the speed of light. In a head-on collision between a photon and an electron (just as we'll be seeing in our scintillator), momentum should be conserved. Because the direction of the reflected photon will be opposite to the direction of the incident photon, we can say that $P_\gamma = P_e - P_\gamma^{'}$, where P_e is the momentum of the electron and $P_\gamma^{'}$ is the scalar momentum of the reflected photon. Solving for the momentum of the electron, we get

$$P_e = P_\gamma + P_\gamma^{'} = \frac{E_\gamma + E_\gamma^{'}}{c}.$$

Combining these equations, we arrive at our classical relationship between the electron momentum and the energies.

$$P_e = \frac{2E_\gamma - E_e}{c}$$

Using the equation above, we can now calculate the momentum of the electron using the energies of the photon and the electron.

For Einstein's relativistic prediction ,

$$(E_e + mc^2)^2 = P_e^2 c^2 + m^2 c^4,$$

which becomes $E_e^2 + 2E_e mc^2 = P_e^2 c^2$. We can further reduce our equation to be in terms of the electron momentum and it becomes,

$$\frac{P_e^2}{2m} = \frac{E_e + E_e^2}{1.02}\mathrm{MeV}^{-1}.$$

Our goal will be to compare these two relationships for the momentum of the electron and see which relationship better describes our experimental data.

2 Experimental setup

2.1 Apparatus

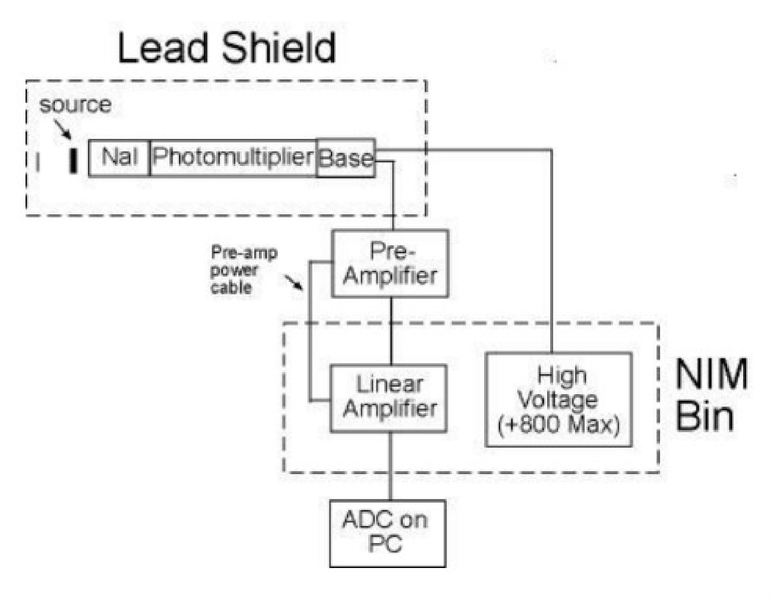

Figure 1: Apparatus setup for our experiment.

For our apparatus, we used a Spectrum Technique Model RSS 8 Gamma Source Kit, a 2" NaI detector, a Power Design High Voltage Supply to enable our photomultiplier to convert the emitted photons from the NaI to an electrical signal, an ORTEC 113 Photomultiplier Preamplifier to amplify to electric signal while minimizing noise, an ORTEC 672 Linear Amplifier to increase our signal while also preserving its linearity, a NIM bin which provided power to our shaping amplifier system, an ORTEC 926 Multichannel analyzer for our data collection, 50 Ohm cables with BNC connectors to connect preamplifier, amplifier, and photomultiplier, a high voltage cable for our high voltage power supply, a Tektronix Digital Oscilloscope to setup our electronics, our lab desktop computer with an EG&G ADC card to convert our analog signals into digital format, and Maestro software for multichannel analysis.

2.2 Data Collection

We first connected our NaI detector to our ORTEC 113, and then we connected it to our ORTEC 672. Next, we connected the 672 to our ORTEC 926, and plugged in our High Voltage power supply using the high voltage cable. Then, to check our amplifier's output, we used our Tektronix Oscilloscope to check for anomalies in our signal. When none were found, we used the Maestro software to collect data for each of radioactive isotopes.

We began with the Na-22 sample to calibrate our machine. We did this because the photopeaks for the Na-22 have well-known values. This allowed us to calibrate our spectra peaks to the correct values using the tuning knob on our setup. We then reset our data collection, and we recorded a data for the Na-22 until we got a smooth curve for its energy vs counts graph.

3 Data Analysis and Results

3.1 Data Processing and Results

Using the data that we gathered for each of our radioactive isotopes, we were able to construct graphs of the energy vs the counts on our scintillator. An example of the graph with it's appropriate fits is shown below:

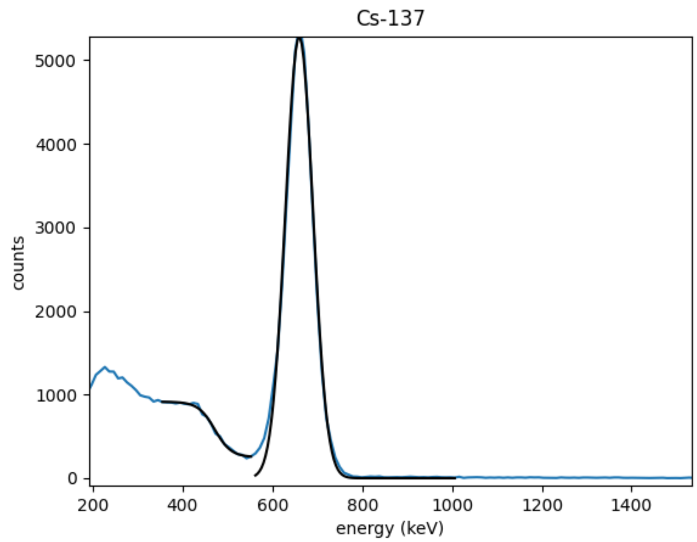

Figure 2: Cs-137 counts vs keV graph

In this graph, you can see the photopeak being fit to a Gaussian and the Compton edge on the left being fit to a negative logistic function. Using this method, we were able to acquire the values for the peak energy and the Compton edge energy for all of the elements in our kit. A table of these values and their uncertainties can be seen below:

Radioactive Isotope	Photo peak (E_γ in keV)	Compton Edge (K in keV)
Cd-109	665.20 $\pm$0.82	495.90 $\pm$30.93
Co-57	629.53 $\pm$2.18	379.22 $\pm$15.23
Co-60	1143.46 $\pm$2.48	932.62 $\pm$1.72
Cs-137	665.86 $\pm$0.37	477.70 $\pm$1.58
K-40	654.93 $\pm$1.25	454.58 $\pm$4.32
Mn-54	815.04 $\pm$1.49	456.58 $\pm$22.37
Na-22	511.18 $\pm$1.06	341.87 $\pm$1.72
Zn-65	664.12 $\pm$0.77	451.40 $\pm$7.86

Figure 3: Table of values for photopeaks and Compton edges for each isotope in our dataset.

Now, we calculate the momentum for each of these isotopes using the classical expression and find: 0.00276168 +- 1.03e-7 N*s for Cd-109, 0.0029128 +- 5.28e-8 N*s for Co-57, 0.00282677 +- 5.84e-9 N*s for Cs-137, 0.00449435 +-1.75e-8 N*s for Co-60, 0.00283097 +-1.66e-8 N*s for K-40, 0.00389168 +- 7.52e-8 N*s for Mn-54, 0.0022483 +-9.11e-9 N*s for Na-22, and 2.89947013e-06 +-2.67e-8 N*s for Zn-65. We want to see how this varies from our relativity description of momentum, so we graph the relativity values and the classical values vs energy and we find:

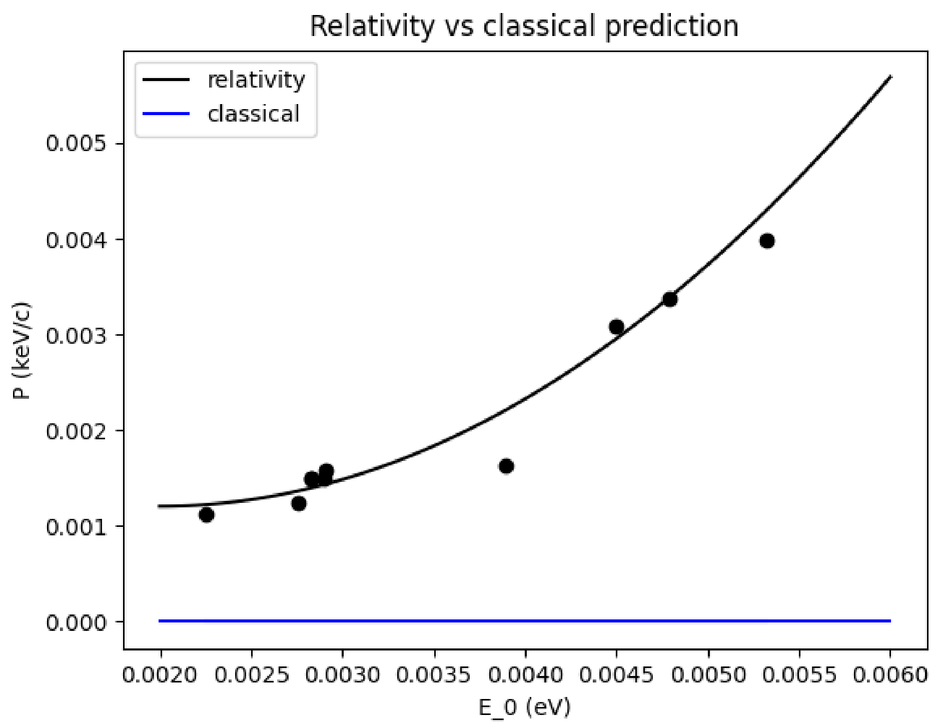

Figure 4: Relativity prediction vs classical prediction for the momentum values as a function of energy.

As can be seen in the graph above, relativity predicts for the momentum to scale quadratically as a function of the energy while classical mechanics predicts it to scale linearly at a much slower rate. Our data agrees more with the relativity model, so we believe that Einstein's predictions were correct.

4 Summary and conclusions

In conclusion, we used a NaI scintillator to gather data on the momentum of Compton scattered electrons from different radioactive isotopes. We found the Compton edges and photopeaks of each of these isotopes from the data. With these values, we found a stronger correlation between the relativity model and our data than what we found with our classical model. This leads us to believe that Einstein's model for momentum of subatomic particles is more accurate than the one derived from classical Newtonian physics.

I'd like to end this paper with a special thanks to my lab partner Surendra Anne, my TAs Dominick Stec and Jeffrey Vit, and to my professor Greg Sitz.

References

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