# Using hydrogen and deuterium spectroscopy to determine the mass of a neutron.

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**Abstract**

We used spectroscopy to observe the structure of hydrogen atoms and determine the mass of a neutron. Knowing de Broglie's equation, $\lambda = \frac{h}{p}$, we experimentally measured a difference in wavelengths between the highest wavelength emissions of hydrogen and deuterium. We analyzed our results and calculated the mass of a neutron based on our data. Our determined mass, $1.674 \times 10^{-27} kg$, was consistent with accepted values of the neutron mass and reinforced our current understanding of atomic structure.

# 1 Introduction

## 1.1 Physics Motivation

The determination of the mass of a neutron holds significant importance in the field of nuclear physics and contributes to our fundamental understanding of atomic structure. The neutron, along with the proton, forms the nucleus of an atom and plays a crucial role in determining an atom's properties. The study of hydrogen and deuterium, provides a unique opportunity to investigate the properties of the neutron.

## 1.2 Theoretical background

Through hydrogen and deuterium spectroscopy, this experiment aims to utilize de Broglie's equation, $\lambda = \frac{h}{p}$ where $\lambda$ = wavelength, $h$ = Planck's constant, and $p$ = momentum , to experimentally measure the variation in wavelengths between the highest wavelength emissions of hydrogen and deuterium [1]. It can be shown that this relationship will allow us to find the mass of the neutron using the following derivation.

We're going to start with the equation for kinetic energy of an object,

$$E = \frac{1}{2}mv^2 .$$

We can then manipulate this equation to get it into terms of momentum,

$$2E = mv^2$$
$$2Em = m^2v^2$$
$$2Em = (mv)^2.$$

We know that momentum is equal to mass times velocity, so we can then make

$$2Em = p^2$$
$$p = \sqrt{2Em}.$$

With this understanding, we can use de Broglie's equation to relate our measured wavelength difference and energy to the mass of the atoms. We're going to call the mass of the deuterium $m_D$ and the mass of the hydrogen $m_H$.

$$\Delta\lambda = \lambda_H - \lambda_D = \frac{h}{\sqrt{2E_H m_H}} - \frac{h}{\sqrt{2E_D m_D}}$$
$$\frac{h}{\sqrt{2E_D m_D}} = \frac{h}{\sqrt{2E_H m_H}} - \Delta\lambda$$
$$\frac{\sqrt{2E_D m_D}}{h} = \frac{1}{\frac{h}{\sqrt{2E_H m_H}} - \Delta\lambda} = \frac{\sqrt{2E_H m_H}}{h - \frac{\Delta\lambda}{\sqrt{2E_H m_H}}}$$
$$\sqrt{2E_D m_D} = \frac{h\sqrt{2E_H m_H}}{h(1 - \frac{\Delta\lambda}{h\sqrt{2E_H m_H}})} = \frac{\sqrt{2E_H m_H}}{1 - \frac{\Delta\lambda}{h\sqrt{2E_H m_H}}}$$
$$2E_D m_D = \frac{2E_H m_H}{(1 - \frac{\Delta\lambda}{h\sqrt{2E_H m_H}})^2}$$
$$m_D = \frac{E_H m_H}{E_D(1 - \frac{\Delta\lambda}{h\sqrt{2E_H m_H}})^2}.$$

As we can see, our mass of deuterium can be calculated given we know the difference in wavelengths of our two emission lines. We also know that the energy for the measured emission line is , $E = h\nu = \frac{hc}{\lambda}$, where $\nu$ is the frequency of the light. Now let's isolate the masses of the subatomic particles making up these atoms in the equation.

Newton's second law tells us that the force exerted by a body, $p_1$, on another body, $p_2$, (particle 1 on particle 2) is $F_{21} = m_2 a_2$ and the force of $p_2$ on $p_1$ is $F_{12} = m_1 a_1$. From Newton's third law, we know that these forces are equal and opposite to each other, $F_{12} = -F_{21}$. If we look at the relative acceleration between these two bodies, we find,

$$a = a_1 - a_2 = (1 + \frac{m_1}{m_2})a_1 = \frac{m_2+m_1}{m_1 m_2} m_1 a_1 = \frac{F_{12}}{\mu},$$

where $\mu$ is defined as the reduced mass of the system. Now that we have a basic model for representing the nuclei of our atoms, let's relate it to deuterium and hydrogen. Below, we use $m_n$, $m_p$, and $m_e$ to represent the masses of the neutron, proton, and electron respectively.

$$m_H = \frac{m_p m_e}{m_p+m_e},\ m_D = \frac{(m_p+m_n)m_e}{(m_p+m_n+m_e)}.$$

Next, we'll plug these identities into our mass of deuterium

$$m_D = \frac{(m_p+m_n)m_e}{(m_p+m_n+m_e)} = \frac{E_H \frac{m_p m_e}{m_p+m_e}}{E_D(1-\frac{\Delta\lambda}{h\sqrt{2E_H \frac{m_p m_e}{m_p+m_e}}})^2}$$

$$m_n + m_p = (m_p + m_n + m_e)\frac{E_H \frac{m_p m_e}{m_p+m_e}}{E_D(1-\frac{\Delta\lambda}{h\sqrt{2E_H \frac{m_p m_e}{m_p+m_e}}})^2}.$$

Let's simplify our equation a little bit by defining a new variable,

$$\zeta = \frac{E_H \frac{m_p m_e}{m_p+m_e}}{E_D(1-\frac{\Delta\lambda}{h\sqrt{2E_H \frac{m_p m_e}{m_p+m_e}}})^2},$$

$$m_n = (m_p + m_n + m_e)\zeta - m_p,$$

$$m_n - \zeta m_n = \zeta(m_p + m_e) - m_p,$$

$$m_n = \frac{\zeta(m_p+m_e)-m_p}{1-\zeta}.$$

This means that our mass of the neutron, when accounting for every subatomic particle's mass, is

$$m_n = \frac{\frac{E_H \frac{m_p m_e}{m_p+m_e}}{E_D(1-\frac{\Delta\lambda}{h\sqrt{2E_H \frac{m_p m_e}{m_p+m_e}}})^2}(m_p+m_e)-m_p}{1-\frac{E_H \frac{m_p m_e}{m_p+m_e}}{E_D(1-\frac{\Delta\lambda}{h\sqrt{2E_H \frac{m_p m_e}{m_p+m_e}}})^2}}.$$

And if we decide to use the reduced mass assumption on our mass of hydrogen, we can simplify to

$$m_n = \frac{\frac{E_H m_e}{E_D(1-\frac{\Delta\lambda}{h\sqrt{2E_H m_e}})^2}(m_p+m_e)-m_p}{1-\frac{E_H m_e}{E_D(1-\frac{\Delta\lambda}{h\sqrt{2E_H m_e}})^2}}.$$

We can now accurately calculate the mass of a neutron and compare it to accepted values after measuring the difference in wavelengths of our spectra, and we can also compare the difference of our mass calculated with the reduced mass assumption and without it.

# 2 Experimental setup

## 2.1 Apparatus

We used a system designed to ionize low-pressure gas tubes. This system consisted of a high voltage transformer that takes in current from the wall and increases the voltage potential from 120 V to the order of 10 kV. The transformer runs this potential across our gas tube holder where the energy from the current knocks the electrons in the atoms off of their orbits causing them to emit photons. This emission can be seen by the glowing of the gas tube. A simplified drawing of our apparatus is shown below:

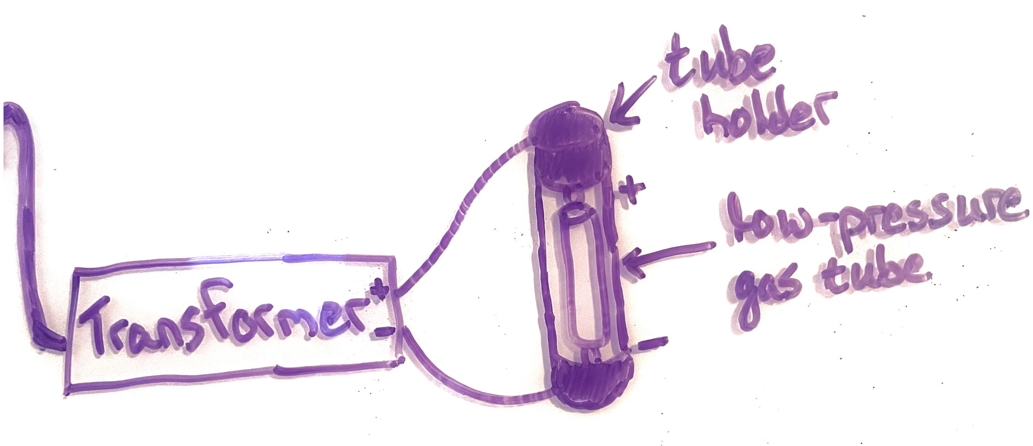



Figure 1: Experimental apparatus. Transformer on the left running current through low-pressure gas tube on the right.

## 2.2 Data Collection

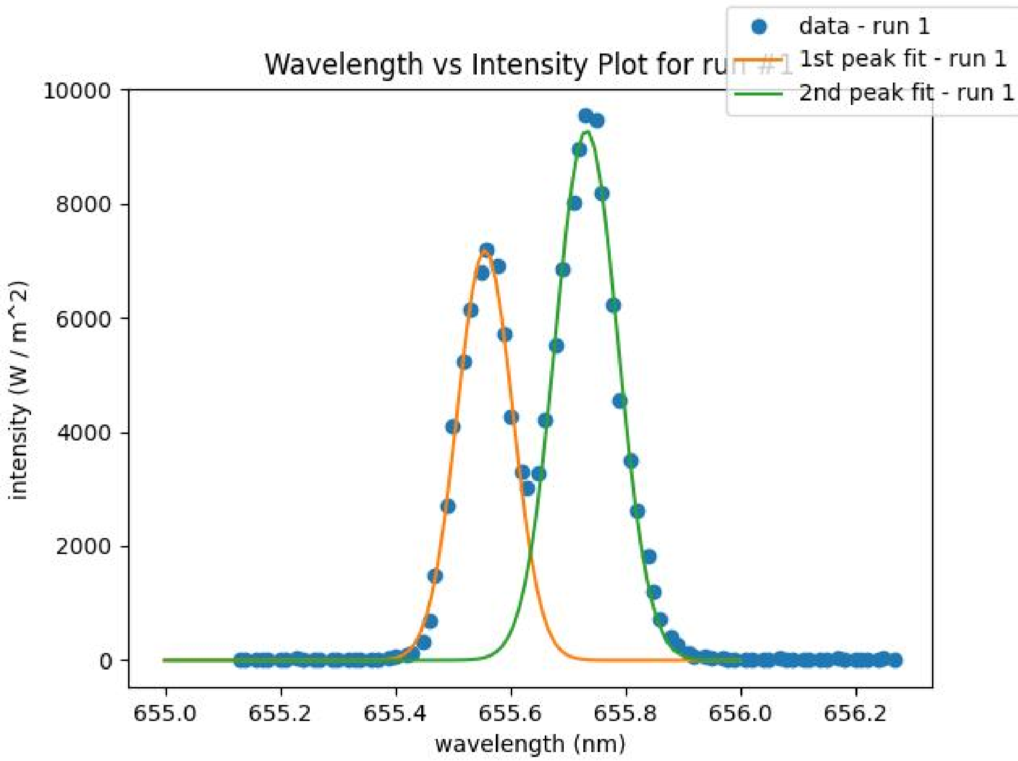
We began the experiment by installing a low-pressure gas tube of $H_2$ and $D_2$ into our tube holder that's connected to the Luminous Tube Transformer. We then hooked up our Ocean Optics Spectrometer model HR4000 to the computer that we used to collect our spectroscopy data. This device is particularly useful for us because it has a bandwidth of 630 nm to 683 nm, which happens to be the range where the emission lines that we'll be looking at lie. We calibrated our sensor by comparing the wavelength of our Spectrometer readings to a spectroscopy line generator that's calibrated to certain wavelengths for different elements. We decided that the calibration of our instrument was acceptable for collecting the data that we'd be using.

We started up our transformer and our gas ionized. We then attached a black tube, made of construction paper, around the tip of our detector to block external light from leaking into our detector. With this attachment, we held the front of our detector in front of the ionized gas tube and began to analyze our results. Because of the lower bandwidth of our detector we had a much greater sensitivity than expected, and our data came out to look like a square wave with intensity on the y-axis and wavelength on the x-axis. We came to the conclusion that the light was over saturating our detector. We decided to use a neutral density filter (rated ND = 1.0) to reduce the intensity of the light that we were feeding into our detector. We attached it to the detecting aperture on our detector and remeasured the data. Two distinct Gaussian peaks were visible in the data, and we knew that one of each of those peaks represented hydrogen and deuterium respectively.

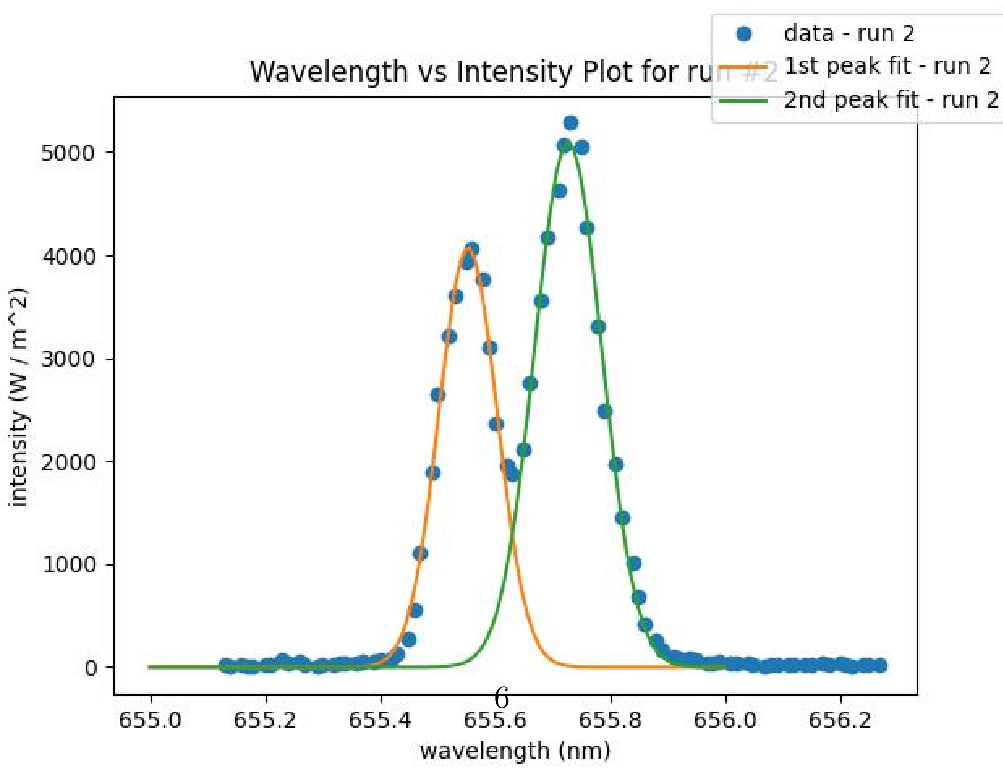
# 3 Data Analysis and Results

## 3.1 Data Processing and Hypothesis Testing

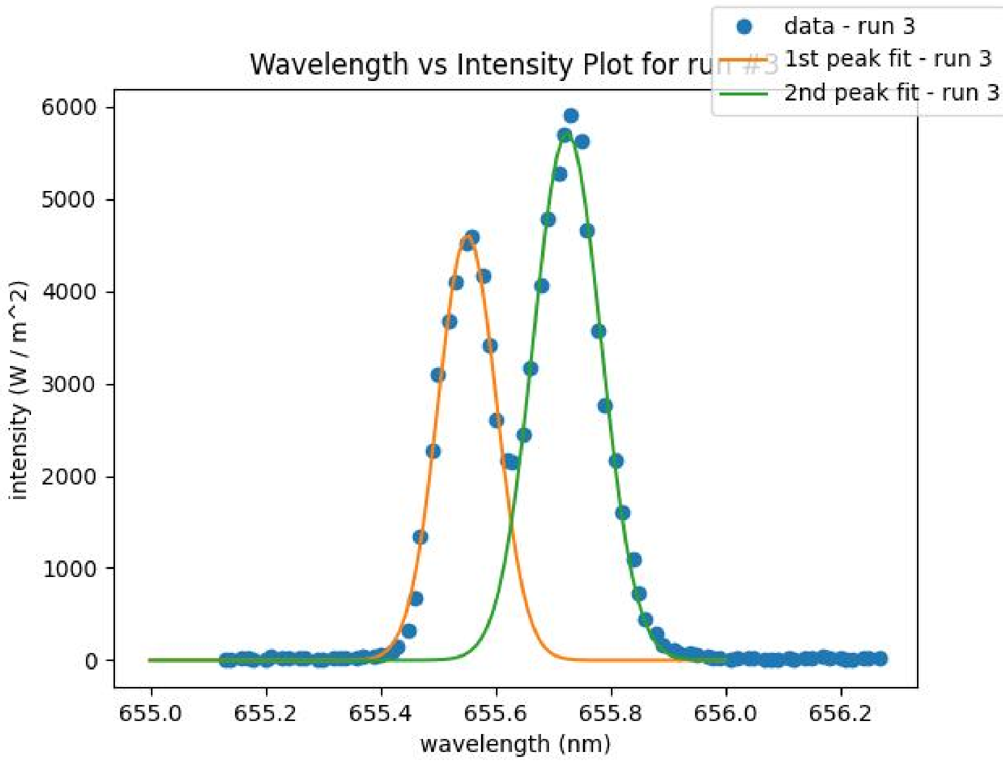
Our Gaussian peaks can be seen below:




(a) Wavelength vs. intensity plot for our first data collection run.




(b) Wavelength vs. intensity plot for our second data collection run.




(c) Wavelength vs. intensity plot for our third data collection run.

We extracted our maximum intensity points from each plot in terms of their wavelength, and calculated the error for each of these plots based on the instrument's reported error of $\pm 0.025$nm and the standard deviation of our fit data from the recorded data values.

| | Run 1 | Run 2 | Run 3 |
|---|---|---|---|
| First peak | 655.557nm $\pm$ 0.025nm | 655.553nm $\pm$ 0.025nm | 655.552 $\pm$ 0.025nm |
| Second peak | 655.733nm $\pm$ 0.025nm | 655.728nm $\pm$ 0.025nm | 655.725nm $\pm$ 0.025nm |

Because the standard deviation of our fit data was significantly lower than the instrument's measurement error it ended up rounding off. We then averaged our peak measurements and got two wavelengths. This gives us

$$\lambda_D = 655.554\text{nm} \pm 0.014\text{nm},$$
$$\text{and } \lambda_H = 655.729\text{nm} \pm 0.014\text{nm}.$$

Using these values, we can calculate the mass of a neutron

$$m_n = 1.67353 \times 10^{-27}\text{kg} \pm 0.00001 \times 10^{-27}\text{kg}.$$

This value is extremely close to the mass of a proton, as is the regularly accepted value for the mass of a neutron.

# 4 Summary and conclusions

Our value of $m_n = 1.67353 \times 10^{-27}\text{kg} \pm 0.00001 \times 10^{-27}\text{kg}$ is within $0.001 \times 10^{-27}\text{kg}$ of the accepted value for the mass of a neutron. I believe that if we'd taken a little more time to correct the calibration of our Spectrometer we could have measured our mass to an even more precise degree.

I'd like to end this paper with a special thanks to my lab partner Surendra Anne, my TAs Dominick Stec and Jeffrey Vit, and to my professor Greg Sitz.

# References

[1] L. de Broglie, Recherches sur la théorie des quanta (Researches on the quantum theory), Thesis (Paris), 1924; L. de Broglie, Ann. Phys. (Paris) 3, 22 (1925).