Solution manual of 'Introduction to Commutative algebra, Atiyah-Macdonald'

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1. Let A be a ring, M an $A-$module. The support of M is defined to be the set Supp (M) of prime ideals p of A such that $M_p \neq 0$. Prove the following results:

   (a) $M \neq 0 \Leftrightarrow$ Supp $(M) \neq \emptyset$.

   (b) $V(a) =$ Supp (A/a).

   (c) If $0 \to M' \to M \to M'' \to 0$ is an exact sequence, then Supp $(M) =$ Supp $(M') \cup$ Supp (M'').

   (d) If $M = \sum M_i$, then Supp $(M) = \cup$ Supp (M_i).

   (e) If M is finitely generated, then Supp $(M) = V(\mathrm{Ann}_A(M))$(and is therefore a closed subset of spec A).

   (f) If M and N are finitely generated then Supp $(M \otimes_A N) =$ Supp $(M) \cap$ Supp (N). [Use Chapter 2, Exercise 3.]

   (g) If M is finitely generated and a is an ideal of M then Supp $(M/aM) = V(a + \mathrm{Ann}_A(M))$.

   (h) If $f : A \to B$ is a ring homomorphism and M is a finitely generated $A-$module, then Supp $(B \otimes_A M) = f^{*-1}$(Supp (M)).

   Answer.

   (a) dfd

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