

Games
Minimax
α-β Pruning
Imperfect Choice
Stochastic Games
Partially Observable Games

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• A natural application!
• Two different kinds:
• Single agent "solitaire" games
• Adversarial multi-agent games
• The most common – turn-based, two-player, zerosum games with perfect environment information.
• Example: Chess
• Chance, imperfect information, multi-agent,
cooperative-agent, non-deterministic aspects can be
added.
• Frequently: hard to solve!

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Adversarial Games

- Typically, we will still consider a tree for the state space, start with an initial configuration of our game and then the successors is each possible move from that configuration.
- Big issue: size of search tree:
  - Chess branching factor of ~35, games of 50+ moves per player common. 35^100 or 10^154 possible search space (10^40 possible configurations)
- What do we do? Good enough solutions. Pruning. Better evaluation/heuristic functions.

Defining Games

S<sub>0</sub> – initial state

PLAYER(s) – player that has the move at s

ACTIONS(s) – set of legal moves at s

RESULT(s, a) – resulting state per the transition function

TERMINAL-TEST(s) – function that determines whether or not the game is over.

UTILITY(s, p) – utility/objective/payoff function for player p at terminal state s. Examples:

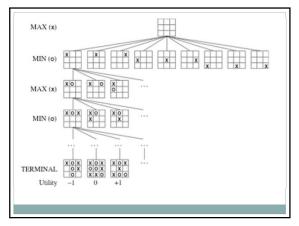
Chess – 0, 1, 1/2
Backgammon – 0 to 192

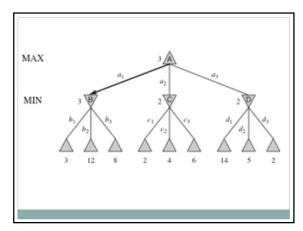
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## Minimax

- Label our players MAX and MIN. This represents the target utility value in reference to our first player.
- MAX the first player wants to maximize his or her utility, the higher the better (traditionally).
- MIN our second player wants to minimize the first player's utility with their move.
- Traditionally expand all of our nodes then work backwards.
- We assume that our opponent will make optimal moves – minimax value represents best possible payoff against optimal opponent.

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Minimax

return the a in Actions(state) maximizing Min-Value(Result(a, state))

function MINIMAX-DECISION(state) returns an action inputs: state, current state in game

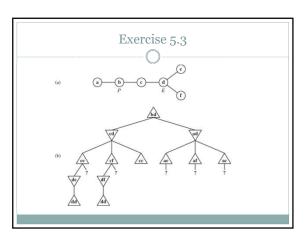
 $\begin{array}{l} \text{function } \underline{\text{Max-Value}}(state) \text{ returns } a \text{ } utility \text{ } value \\ \text{if } \underline{\text{TERMINAL-TEST}}(state) \text{ then } \text{return } \underline{\text{UTILITY}}(state) \end{array}$ 

function Min-Value(state) returns a utility value if Terminal-Test(state) then return Utility(state)

for a, s in Successors(state) do  $v \leftarrow \text{Max}(v, \text{Min-Value}(s))$ 

 $v \leftarrow \infty \\ \text{for } a, \ s \ \text{in Successors}(state) \ \text{do} \ v \leftarrow \text{Min}(v, \ \text{Max-Value}(s))$ 

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Properties of Minimax

Complete only if tree is finite.
Optimal against an optimal opponent.
Time complexity – exponential!
Space complexity – linear!

## α-β Pruning

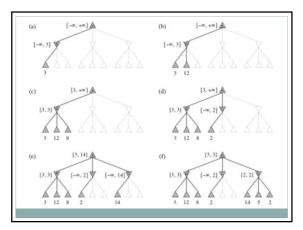
- Trouble with Minimax time! Exponential in the depth of the tree.
- How do we trim this? Pruning!
- Effectively cuts the time in half (still exponential).
- Pruning elimination of subtrees/possible states without examining them due to some factor.
- Eliminate branches that cannot affect our final solution – still returns the same solution as minimax.

α-β Pruning

- General principal consider a node n such that the player has a choice to moving to that node. If player has a better choice at that branch (m) or at any choice further up, n will never actually be reached!
- · Basically the same properties as minimax.

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Dealing with Complexity

- Size is an issue (isn't it always)? How do we deal with it?
- Option 1 cutoff test use a heuristic to estimate the utility of a given move at the set maximum depth. If that heuristic meets a threshold (dependent on if that level is a min or a max) then keep it, otherwise, discard
- Option 2 forward pruning consider only a selection of n best moves, prune all others.
- Neither option is guaranteed to be optimal!

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## Games of Chance

- Frequently, our games will include some element of chance (commonly, dice).
- We can still use minimax/ $\alpha$ - $\beta$  pruning in this case, but a small adjustment is required.
- Between each max and min we will add a chance branch – this represents the roll that the player at that level could make, including the probabilities (for instance, with 2 die, 7 is the most common roll at ~17%).
- We can only calculate expected utility here!

## Partially Observable Games

- In other games, only part of my environment is known for instance, card games where the opponent's cards are hidden.
- Typically just figure out all possible configurations and probabilities, and go from there.
- Choose the action that has the highest expected utility regardless of the deal for your opponent.
- Called averaging over clairvoyance assumes that the environment becomes fully observable to both players immediately or soon after the first action.

Problems with AOC

- Averaging over clairvoyance can lead you astray –
- Day 1 Road A leads to a heap of gold, Road B leads to a fork. Take the left fork and it leads to a bigger heap of gold. Take the right fork and you'll be run over by a bus.
- Day 2 Road A leads to a heap of gold, Road B leads to a fork. Take the right fork and it leads to a bigger heap of gold. Take the left fork and you'll be run over by a bus.
- Day 3 Road A leads to a heap of gold, Road B leads to a fork. One of the fork leads to a bigger heap of gold, but the other has that darned bus. Which fork do you take?

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