EMPIRICAL ANALYSIS OF A MANY-GOODS-TO-ONE-AGENT ALLOCATION PROBLEM

Data Analysis Lab
Richard Kempert

Introduction

- Non-price based matching markets
  e.g., organ donations, student houses, university courses…
- Basic Setting
  - Finite set of agents
  - Finite set of goods
  - Agents have preferences over goods
- Project aim
  analysing empirically a
  MANY-GOODS-TO-ONE-AGENT ALLOCATION PROBLEM

Relevance

Simple case
Each agent can allocate only one good
= Serial Dictatorship (SD)
→ proven efficient allocation mechanism

Complex case
Each agent can allocate several goods
SD is not efficient anymore (Manea, 2007)

Empirical analysis of such mechanisms are rare

Operationalization

Sample Data

Agents = trainers

Goods = course seats

Trainers have preferences over course seats (Yes/No)

NO utility of each course seat for each trainer

Judging efficiency of allocation by:

Ratio of assigned course seats

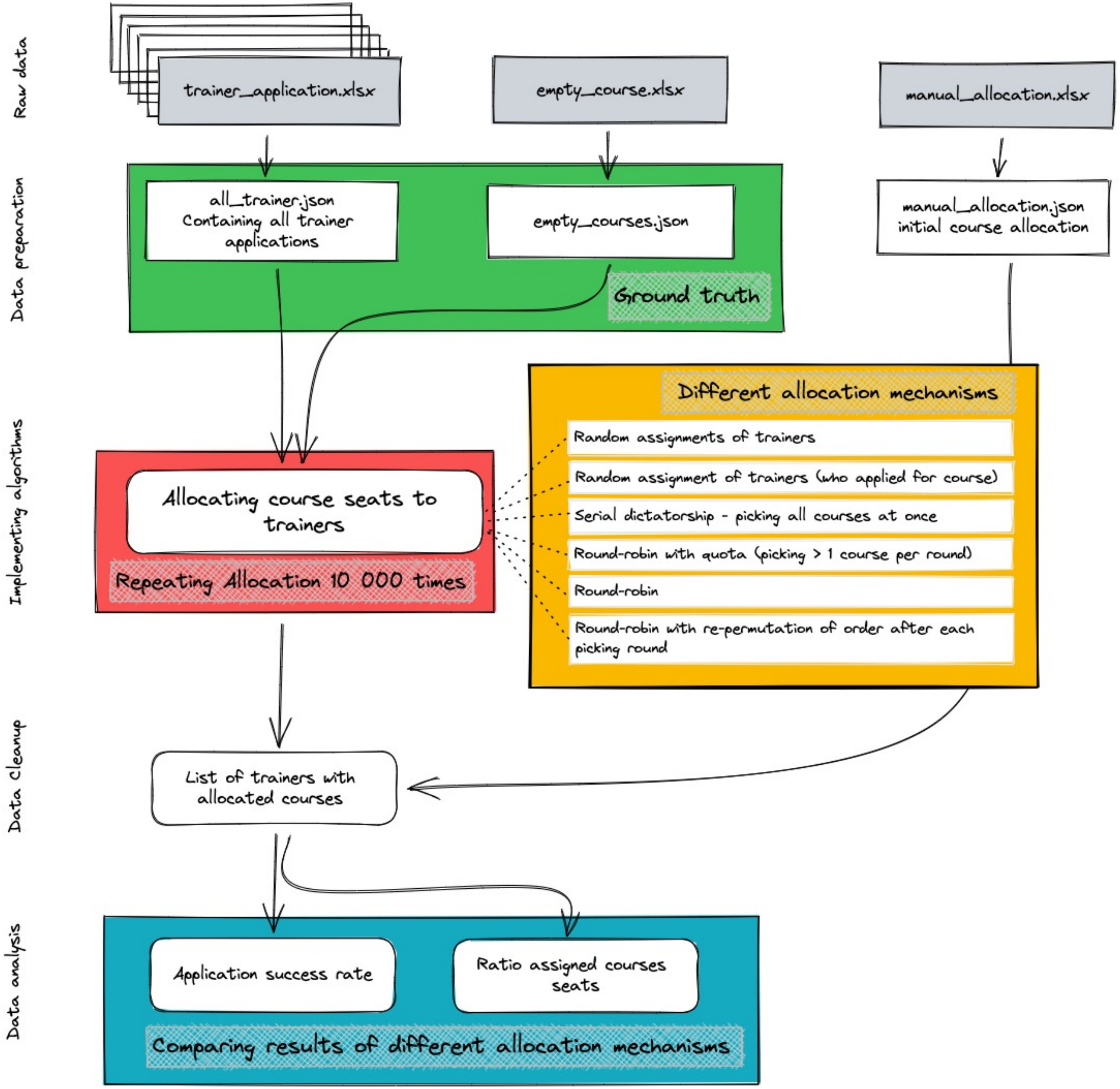
Coefficient of variation of application success rate – combining:

- Spread of values (fairness between trainers)
- Mean (performance of allocation)
- → Low value = high mean and/or low spread

Hypothesis

H_0 1:	The ratio of assigned course seats does not differ between different allocation mechanisms.
H_1 1:	The ratio of assigned course seats does differ between different allocation mechanisms.
H_0 2:	The coefficient of variation of application success rate does not differ between different allocation mechanisms.
H_1 2:	The coefficient of variation of application success rate does differ between different allocation mechanisms.

Workflow


Raw data
Data preparation
Implementing algorithms
Data cleanup
Data analysis
trainer_application.xlsx
empty_course.xlsx
manual_allocation.xlsx
all_trainer.json
Containing all trainer applications
empty_courses.json
Ground truth
manual_allocation.json
initial course allocation
Different allocation mechanisms
Random assignments of trainers
Random assignment of trainers (who applied for course)
Serial dictatorship - picking all courses at once
Round-robin with quota (picking > 1 course per round)
Round-robin
Round-robin with re-permutation of order after each picking round
Allocating course seats to trainers
Repeating Allocation 10 000 times
List of trainers with allocated courses
Application success rate
Ratio assigned courses seats
Comparing results of different allocation mechanisms

Implemented Allocation Mechanisms

Serial Dictatorship	Round-robin	Random from course side
• permute order of trainers • 1_{st} trainer allocates **all** preferred courses that are available • 2_{nd} trainer … • n_{th} trainer …	• permute order of trainers • 1_{st} trainer allocates **one** preferred courses that are available • 2_{nd} trainer … • n_{th} trainer … • restart with 1st trainer • stop if no more available courses are preferred	• for each course assign as much trainers as required
	• Version 1: quota 1 good • Version 2: quota x goods per picking round • Version 3: re-permute order of trainers	• Version 1: assigning from set of trainers • Version 2: assigning from set of trainers who applied for the course

Descriptive Analysis Ground Truth

Course Type	Pay Rate (in €) per Day	Number of Courses	Mean Duration (in Days)
School courses	288.8	34	3.2
One day courses	243.2	5	1.0
Individual courses	319.2	5	4.2
Trainee courses	395.2	4	3.8

Clustering Trainers

	Few and short courses trainer	Long courses trainer	Workhorses
Number of Trainers	9	9	2
One day courses	0.44	0.22	2.50
Individual courses	0.11	1.56	1.50
School courses	4.56	3.67	25.00
Trainee courses	0.33	1.22	1.00
Duration	2.94	3.69	3.04
Total Number of Course Applications	5.44	6.67	30.00

Data analysis

Each allocation mechanism was performed 10 000 times

Calculating for each allocation:

- (1) the ratio of assigned course seats
- (2) the coefficient of variation
  - standard deviation as a measurement of **fairness** between trainers
  - mean as a measurement of **performance**

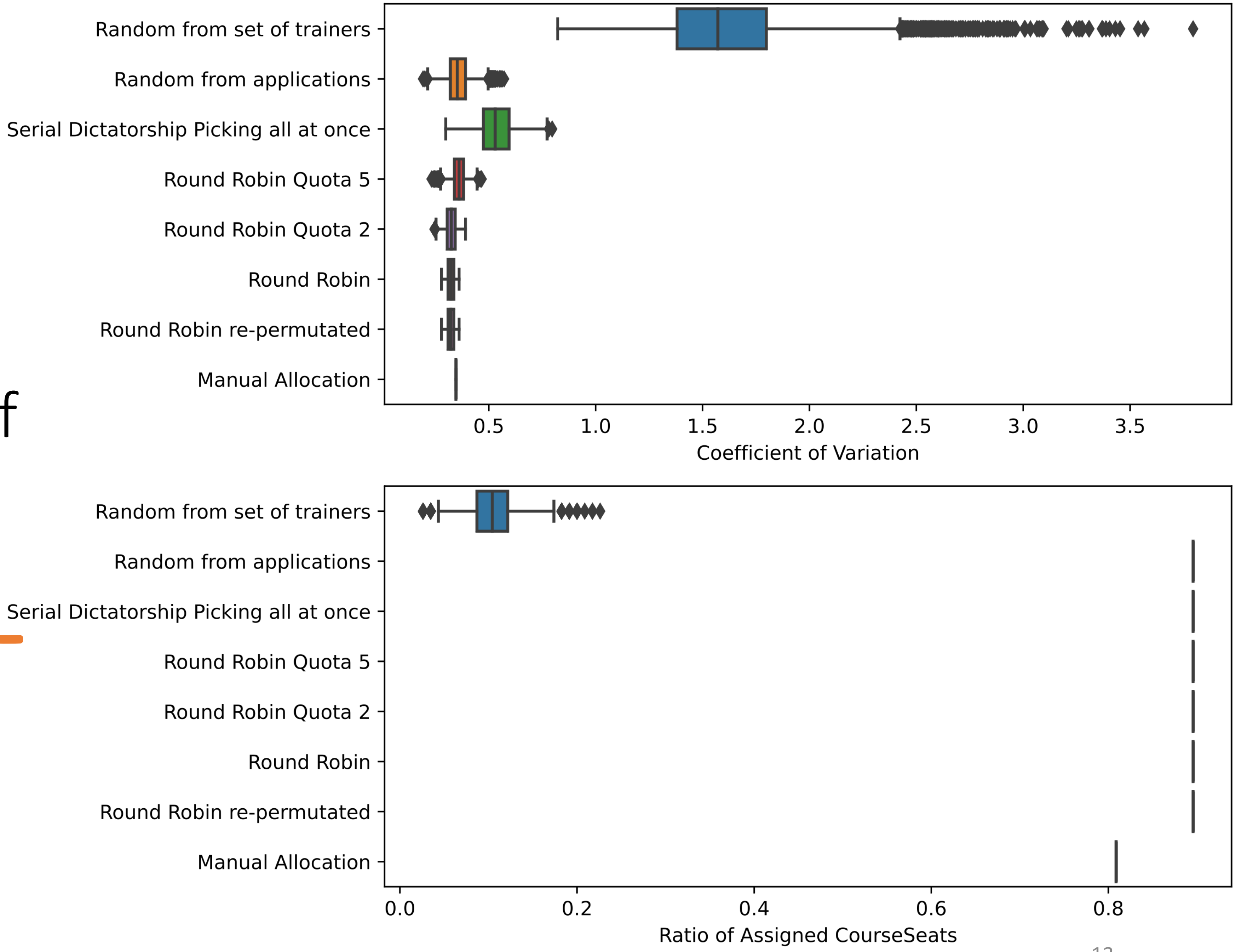
Comparing both metrics for each allocation mechanisms

Performing Kruskal Wallis & One-way ANOVA independence tests

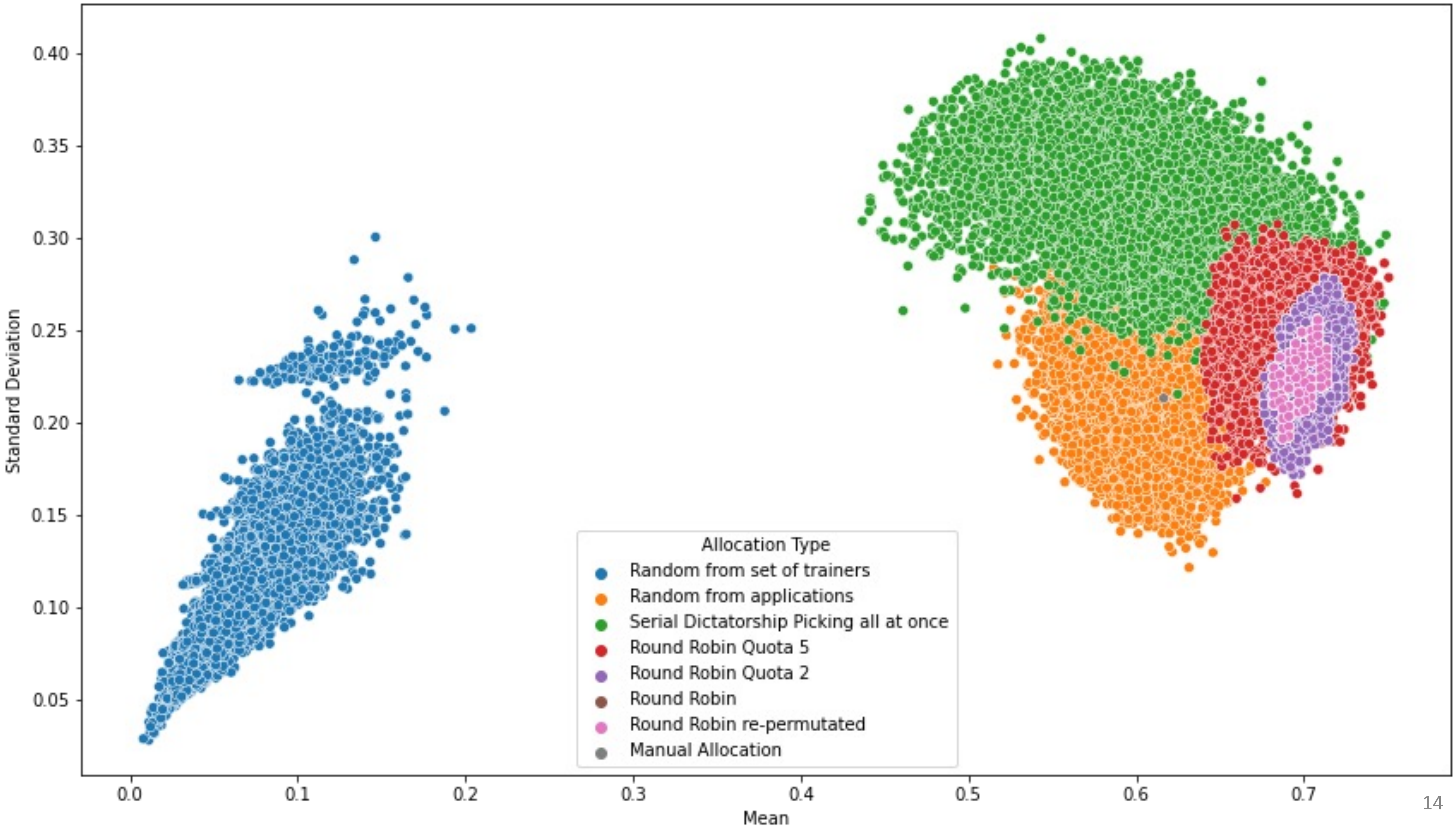
Mean, minimum and standard deviation of coefficient of variation and ratio assigned course seats per mechanism

Allocation Type	Coefficient of Variation			Ratio of Assigned Course Seats
	min	M	SD	M
Manual Allocation	0.35	0.35		0.81
Random from applications	0.19	0.36	0.05	0.90
Random from set of trainers	0.82	1.62	0.34	0.11
Round Robin	0.28	0.32	0.02	0.90
Round Robin Quota 2	0.25	0.32	0.03	0.90
Round Robin Quota 5	0.23	0.36	0.03	0.90
Round Robin re-permutated	0.28	0.32	0.02	0.90
Serial Dictatorship Picking all at once	0.30	0.54	0.08	0.90

Boxplots coefficient of variation, ratio of assigned course seats



Scatterplot mean and standard deviation of application success rate



Results - Independency tests

Ratio of assigned course seats

Number of allocations	Kruskal Wallis	One-way ANOVA
10 000 & 100	All mechanisms except manual allocation and “Random from set of trainers” do not differ significantly (p value >0.001) → partly rejecting H_0 1	

Coefficient of Variation – application success rate

Number of allocations	Kruskal Wallis	One-way ANOVA
10 000	All mechanisms differ significantly (p value >0.001) → rejecting H_0 2	
100	Round-robin mechanisms do not differ significantly (p value >0.001) → partly rejecting H_0 2	

Limitations

Only one vector of preferences per trainer → Interesting to compare results with different preferences (e.g. via random simulation)

The coefficient of variation as useful measurement of the efficiency of the allocation → More meaningful when having an independent utility value per course per trainer → BUT → increases complexity of algorithm – harder to implement

Practical Implications

Condition: similar data sample (e.g., with preferences but without utility)

Including trainers preferences into mechanism → yields max ratio of assigned course seats

Performance and spread of round-robin with quota of max 2 yield very similar results

Benefit of performing allocation programmatically → allowing for more efficient allocations

Implementing utility function allows for better assessment of the efficiency of allocations