Problem Set 1

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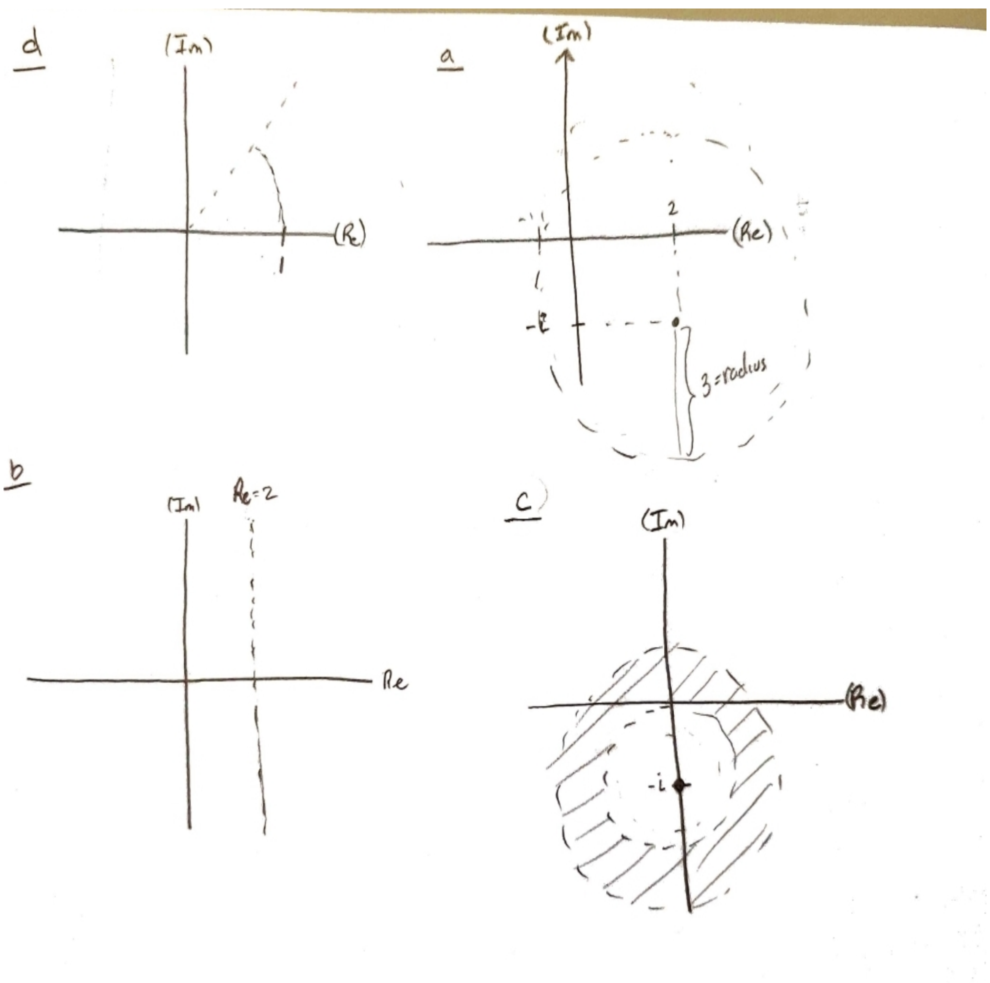
Problem 1



Problem 2

First, we start with De Moivre's Formula:

$$(\cos(\theta) + i\sin(\theta))^n = (\cos(n\theta) + i\sin(n\theta))$$

Subbing in 3 for n:

$$(\cos(\theta) + i\sin(\theta))^3 = (\cos(3\theta) + i\sin(3\theta))$$

Expanding out, we get:

$$\cos^3(\theta) - 3\sin^2(\theta)\cos(\theta) + i(3\cos^2(\theta)\sin(\theta) - \sin^3(\theta)) = (\cos(3\theta) + i\sin(3\theta))$$

Splitting these into the real and imaginary parts, we get:

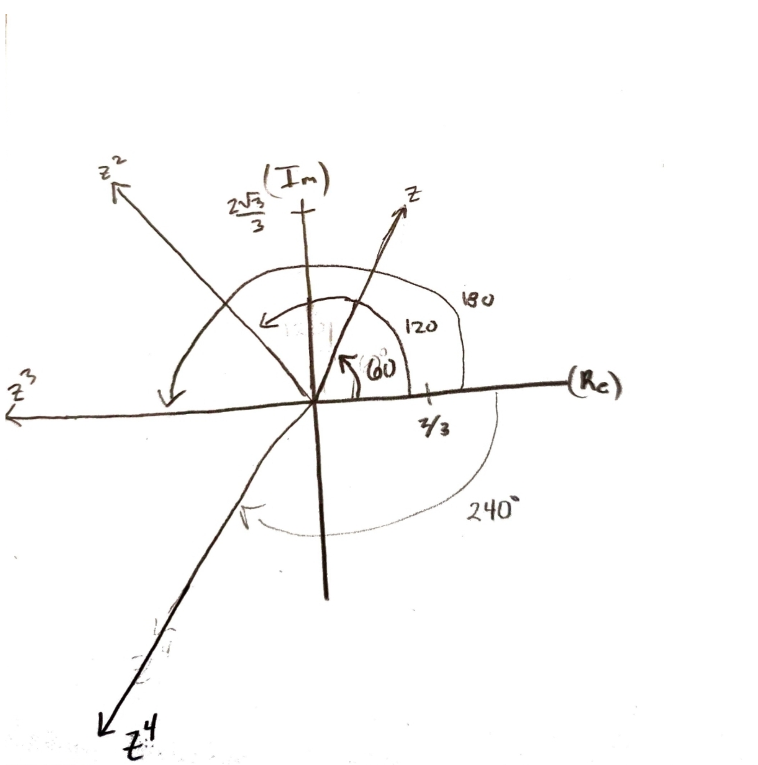
$$\cos(3\theta) = \cos^3(\theta) - 3\sin^2(\theta)\cos(\theta)$$
$$\sin(3\theta) = 3\cos^2(\theta)\sin(\theta) - \sin^3(\theta)$$

Which, shows the identity we'd like.

Problem 3

Doing the calculations, we get the following:

$$z = (1.\overline{33}e^{\frac{\pi}{3}i})$$
$$z^2 = (1.\overline{77}e^{\frac{2\pi}{3}i})$$
$$z^3 = (2.\overline{370}e^{\pi i})$$
$$z^4 = (3.16e^{\frac{4\pi}{3}i})$$



Problem 4

To compute the nth roots, we will use the polar form, and convert to normal form. Thus, we have:

$$r = (r_0)^{\frac{1}{n}}$$
$$\theta = \frac{\theta_0}{n} + \frac{2\pi k}{n}$$

Using this form, we find the nth roots for each. For (a) we have:

$$r = 1$$
$$\theta = \frac{\pi}{4} + \frac{\pi}{2}k$$

Which gives us the distinct values (in polar form):

$$n_0 = e^{\frac{1}{8}\pi i}$$
$$n_1 = e^{\frac{5}{8}\pi i}$$
$$n_2 = e^{\frac{9}{8}\pi i}$$
$$n_3 = e^{\frac{13}{8}\pi i}$$

Which are equivalent to these values (in standard form):

$$n_0 = 0.924 + 0.383i$$
$$n_1 = -0.383 + 0.924i$$
$$n_2 = -0.924 - 0.383i$$
$$n_3 = 0.383 - 0.924i$$

For b, since the angles weren't nice even fractions, I will express them in decimal form. For (b) we repeat the process, and have:

$$n_0 = 3.28e^{1.72i}$$
$$n_1 = 3.28e^{3.81i}$$
$$n_2 = 3.28e^{5.91i}$$

Which are equivalent to these values (in standard form):

$$n_0 = -0.49 + 3.24i$$
$$n_1 = -2.57 - 2.03i$$
$$n_2 = 3.05 - 1.20i$$

Problem 5

The geometric series can be rewritten as follows:

$$1 + c + c^2 ... c^{n-1} = \frac{1 - c^n}{1 - c}$$

Because c is an nth root of unity, $c^n = 1$. Thus, the expression becomes 0, since $1 - c^n = 0$.

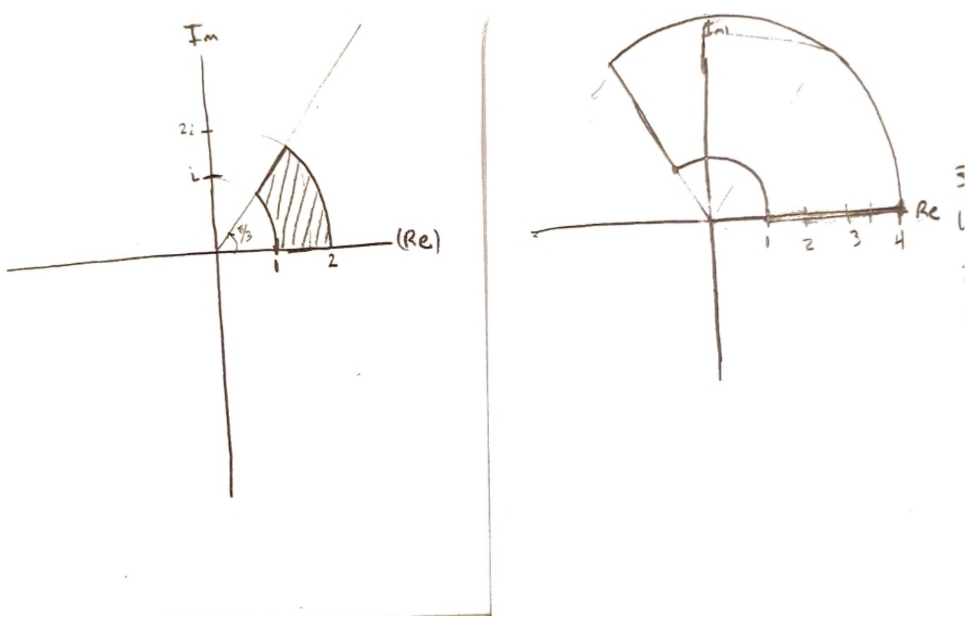
Problem 6

open and bounded: $\mid z \mid < 1$
neither open nor closed: $1 \leq \mid z \mid < 2$
closed and connected: $\mid z \mid \leq 1$
closed, bounded, and not connected $\mid z \mid \leq 1, \; 2 \leq \mid z \mid \leq 3$
unbounded and not connected $\mid z \mid < 1, \; \mid z \mid > 2$

Problem 7

Given S is a domain, then S is both open and connected. Thus, because S is connected, then any two points in S can be connected by a sequence of line segments that still remain in S. Let z_1, z_2 be arbitrary points in S. We use the fact S is connected to connect these two points with a continuous line. Because the line is continuous and remains in S, no matter what ϵ we choose, every deleted ϵ neighborhood around either point will always contain a point from the line, which is in S. Thus, every deleted ϵ neighborhood of $z \in S$ contains at least one point from S, and thus every point in S is an accumulation point.

Problem 8



Problem 9

The domain becomes the set defined by:

$$1 \leq x^2 - y^2 \leq 2 \cap 1 \leq 2xy \leq 2$$