Electromagnetic Radiation

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Abstract

This paper will endeavor to give a explanation of what electromagnetic radiation is, how it is created, the properties it has, and a few of the applications it can be used for. Specifically, this will focus on dipole radiation, how it is created, and its application through the dipole antenna.

I. INTRODUCTION: ELECTROMAGNETIC RADIATION, WHAT IS IT?

Electromagnetic radiation, referred to as EM radiation or EMR, denotes the waves of electromagnetic fields propagating through space. Familiar types of electromagnetic radiation are radio waves, microwaves, infrared light, visible light, ultraviolet light, X-rays, and gamma rays.
Through a classical lens, electromagnetic radiation consists of electromagnetic waves, which are synchronized oscillations of electric and magnetic fields. In a vacuum, electromagnetic waves travel at the speed of light. In homogeneous, isotropic media, the oscillations of the two fields (electric and magnetic) are perpendicular to each other and perpendicular to the direction of energy and wave propagation, forming a transverse wave. (See Fig. 1)

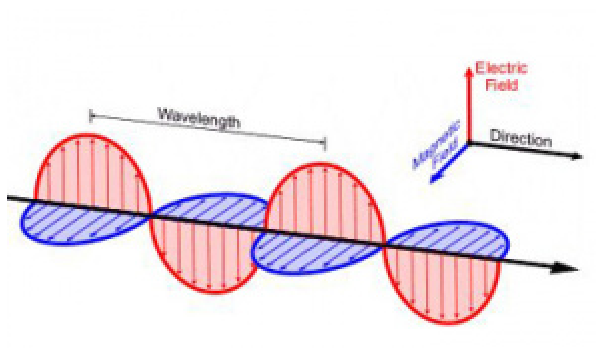

FIG. 1. A visualization of a electromagnetic wave [1]

The position of a wave within the electromagnetic spectrum is characterized by either its frequency or its wavelength. Radio waves have some of the longest wavelengths, followed by microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and finally some of the shortest wavelengths, gamma rays. (See Fig. 2)

Electromagnetic waves are created from accelerating charged particles. Electromagnetic waves carry energy, momentum, and angular momentum away from their source particle and can affect those same quantities in any system it interacts with. Electromagnetic radiation is often associated with electromagnetic waves that radiate without the continuing influence of the moving charges that produced them, as they have moved far enough away from their source. On this same note, electromagnetic radiation is also referred to as the far field, since they are far from the source that produced them, and conversely, the near field refers to

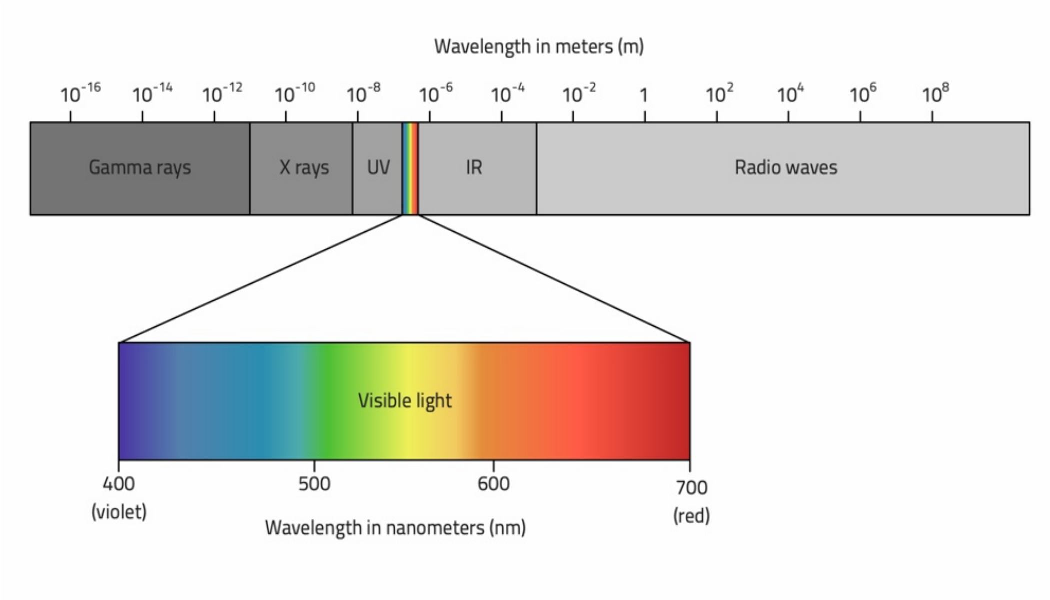

FIG. 2. The Electromagnetic Spectrum of EM radiation [1]

electromagnetic fields near the charges and current that produced them. From a quantum mechanics perspective, we view EMR as consisting of photons, the uncharged elementary particles that have zero rest mass, which are the quanta of electromagnetic force, responsible for all electromagnetic interactions. Quantum electrodynamics combines the two, and is the theory of how EMR interacts with matter on an atomic level. Quantum effects provide additional sources of EMR not taken into account in classical, like the transition of electrons to lower energy levels in an atom and black-body radiation. The energy of an individual photon is quantized and increases along with the frequency of photons per Planck's equation, $E = hf$, where E is the energy per photon, f is the frequency of the photon, and h is Planck's constant. However, for the context of this paper, we will be looking at electromagnetic radiation through a classical lens.

II. KEY EQUATIONS, IDEAS, AND PROPERTIES OF ELECTROMAGNETIC RADIATION

Power Radiated

As I mentioned earlier, when charges accelerate, the field they produce can transport energy *irreversibly* out to infinity, this is the process we define as radiation. In order to calculate the energy that is radiating at some t_0, we construct a spherical surface with radius r around the accelerating charge and take the surface integral of the charge's Poynting vector. This coupled with the fact that electromagnetic events travel at the speed of light results in a $t_0 = t - r/c$, (we call this retarded time), which gives a resulting radiation power of

$$P_{\text{rad}}(t_0) = \lim_{r \to \infty} P(r, t_0 + \frac{r}{c}). \tag{1}$$

This resulting energy per unit time is carried away from the source particle and *never comes back* [5].

Dipole Radiation

A dipole radiator is essential a thin wire, with two oppositely charged spheres that are driven back and forth. The resulting radiation looks like:

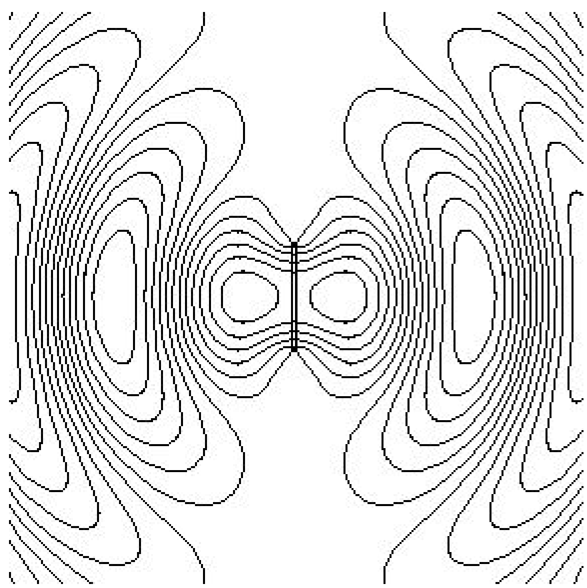

FIG. 3. Dipole Radiation Intensity Wave-fronts [2]

Using the oscillating charge, we can derive an equation for the oscillating dipole to be

$$\boldsymbol{p}(t) = p_0 \cos(\omega t)\hat{\boldsymbol{z}}. \tag{2}$$

Using this dipole with the electric potential of a dipole

$$V_{\text{dip}} = \frac{1}{4\pi\epsilon_0} \frac{\boldsymbol{p} \cdot \hat{r}}{r^2}. \tag{3}$$

Further, using these equations with the oscillating dipole, we can derive the potential to be:

$$V(r, \theta, t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\big(\cos(\theta)\big)\sin(\omega(t - r/c)). \tag{4}$$

EM waves in vacuum travel at the speed of light. The retarded time, $t - r/c$, is the time when the field began to propagate from the point where it was emitted to an observer.

The Vector potential is determined using the current flowing through the wire, then integrated, to finally yield

$$\boldsymbol{A}(r, \theta, t) = -\frac{\mu_0 p_0, \omega}{4\pi r}\sin(\omega(t - r/c))\hat{\boldsymbol{z}}. \tag{5}$$

We then use this vector potential and the scalar potential to find $\boldsymbol{E}$ and $\boldsymbol{B}$.

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\big(\frac{\sin(\theta)}{r}\big)\cos(\omega(t - r/c))\hat{\boldsymbol{\theta}} \tag{6}$$

and

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\big(\frac{\sin(\theta)}{r}\big)\cos(\omega(t - r/c))\hat{\boldsymbol{\phi}} \tag{7}$$

As expected, these fields are perpendicular, as $\hat{\boldsymbol{k}} = \hat{\boldsymbol{r}}$ so $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{k}} = \hat{\boldsymbol{r}}$, which creates the expected electromagnetic wave.

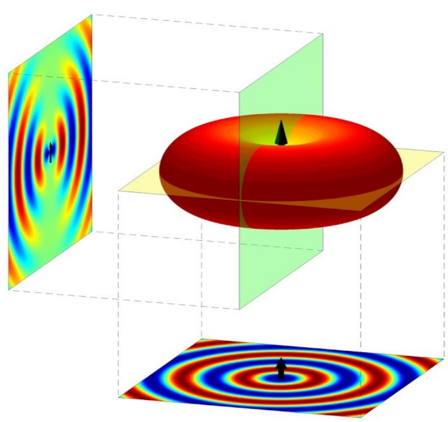

FIG. 4. A 3D Render of Dipole Radiation Intensity Wavefronts, with 2D cross sections [3]

Recall that $\boldsymbol{S}$ is the Poynting vector $\boldsymbol{S} = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B})$. We can find the equation for $\boldsymbol{S}(\boldsymbol{r}, t)$ by plugging in $\boldsymbol{B}, \boldsymbol{E}$, which gives us the energy radiated by an oscillating electric dipole:

$$\boldsymbol{S}(\boldsymbol{r}, t) = \frac{\mu_0}{c}\Big(\frac{p_0 \omega^2}{4\pi}\big(\frac{\sin(\theta)}{r}\big)\cos(\omega(t - r/c))\Big)^2\hat{\boldsymbol{r}} \tag{8}$$

The intensity $\langle \boldsymbol{S} \rangle$ is then obtained by averaging over time over a complete cycle, which gives us:

$$\langle \boldsymbol{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2(\theta)}{r^2} \hat{\boldsymbol{r}}. \tag{9}$$

The resulting wave is spherical, and looks like a sphere being pinched into a torus, and then back into a sphere as the charges oscillate back and forth. There is **no** radiation on the axis of the dipole. [5] It is Eq. (9) that can be seen graphed 2D (Fig. 3) and 3D (Fig. 4). Finally, in order to find the total power, we just integrate $\langle \boldsymbol{S} \rangle$ over a sphere of radius r, giving us

$$P_{\text{tot}} = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}. \tag{10}$$

Point Source Radiation

Here, we are going to use a point charge to see how radiation is created from a moving charge. The fields derived for a point charge in arbitrary motion are:

$$\boldsymbol{E}(\boldsymbol{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{r}{(\boldsymbol{r} \cdot \boldsymbol{u})^3} \Big((c^2 - v^2)\boldsymbol{u} + \boldsymbol{r} \times (\boldsymbol{u} \times \boldsymbol{a}) \Big) \tag{11}$$

where $\boldsymbol{u} = c\hat{\boldsymbol{r}} - v$ and

$$\boldsymbol{B}(\boldsymbol{r}, t) = \frac{1}{c} \hat{\boldsymbol{r}} \times \boldsymbol{E}(\boldsymbol{r}, t) \tag{12}$$

The first term in Eq. (11) is called the velocity field, and the second one is called the acceleration field. We calculate the Poynting vector to be:

$$\boldsymbol{S} = \frac{1}{\mu_0 c} (E^2 \hat{\boldsymbol{r}} - (\hat{\boldsymbol{r}} \cdot \boldsymbol{E})\boldsymbol{E}). \tag{13}$$

It is the acceleration field, or radiation field, that we are interested in, since the velocity field is the near field, and this is not radiation. Upon further examination we have the radiation field equal to:

$$\boldsymbol{E}_{\text{rad}} = \frac{q}{4\pi\epsilon_0} \frac{r}{(\boldsymbol{r} \cdot \boldsymbol{u})^3} (\boldsymbol{r} \times (\boldsymbol{u} \times \boldsymbol{a})). \tag{14}$$

From this, we find the radiation Poynting vector to be:

$$\boldsymbol{S}_{\text{rad}} = \frac{1}{\mu_0 c} E_{\text{rad}}^2 \hat{\boldsymbol{r}} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \left(\frac{\sin^2 \theta}{r^2} \right) \hat{\boldsymbol{r}}. \tag{15}$$

By then taking the spherical surface integral of $\boldsymbol{S}_{\text{rad}}$. we can derive an expression for the power radiated to be:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}, \tag{16}$$

where a is the acceleration of the particle. This equation is called the **Larmor formula**. It is important to note that the rate at which energy passes through a sphere surrounding the source is not the same as the rate at which energy left the particle, since the rate is affected by movement of the source. (like the Doppler effect)[5].

Radiation Reaction

In general, the electromagnetic force of one part, A, of a charge distribution, on another part, B, is not equal and opposite to the force of B on A. The summation of all these imbalances gives us a net force on the charge on itself. This self-force consequently results in the breakdown of Newton's 3rd law, which accounts for the radiation reaction. Basically, the radiation is due to the force of the charge on itself, and due to conservation of momentum, the particle recoils after emitting radiation. [5].

III. APPLICATIONS

Dipole Antennae

A dipole antenna, is arguably the simplest and most widely used type antenna. A dipole antenna consists of two conductors of equal length oriented end to end, with the feedline connected between them. (Fig. 5) Driving current is applied for the transmitter between the two parts of the antenna, or in the case of a receiving antenna, the receiving signal is received between the two parts of the antenna. The most common antenna used is the center-fed half-wave dipole, which is just under a half wavelength long because of a number of effects associated with the fact that the radio frequency waveform is carried within a wire and also most likely not in a vacuum. The resulting radiation pattern of the half-wave dipole is maximum directly perpendicular to the conductor (as seen the the dipole radiation diagrams), and zero in the axial direction[6]. In the case of a linear dipole antenna, we have that intensity is approximately,

$$\langle \boldsymbol{S} \rangle = \frac{\mu_0 \omega^2 I_0^2 d^2}{32\pi^2 c_0} \Big(\frac{\sin^2\theta}{r^2} \Big) \hat{\boldsymbol{r}}. \tag{17}$$

However, this changes slightly from model to model.

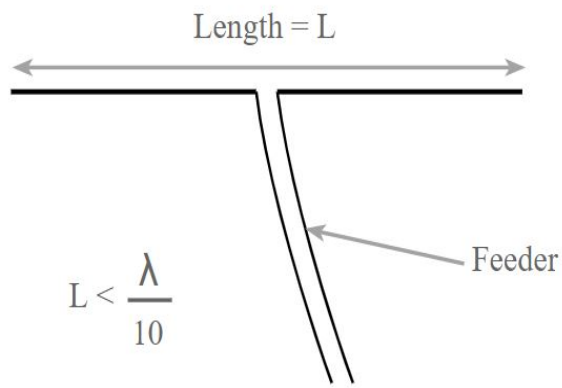

FIG. 5. A basic short dipole antenna [4]

There are many variants of dipole antennae. For example, multiple half wave dipole antennae, which have a length that is an odd multiple of the wavelength. This creates a much different radiation pattern. Folded dipole antennae are a simple dipole antenna folded back in on itself (Fig. 6). This design allows for the antenna to have a wider bandwidth than a standard antenna. [6] Short dipole antenna have a length that is much less than that

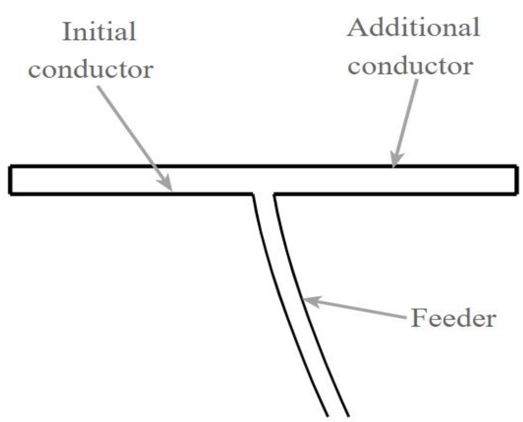

FIG. 6. A basic folded dipole antenna [4]

of the radio wavelength. The small length has many benefits for practical applications. The antenna's feed impedance is also less responsive to frequency changes [6].

Applications

These (and many other types) of dipole antennae are used in a variety of applications. Old TV signal receivers, FM radio receivers and transmitters, they are widely used by radio amateurs and short wave listeners in fixed locations due to their incredibly simple and inexpensive construction. Bluetooth transmitters and receivers are made from dipole antenna, due to, again their simplicity, cost-efficiency, and reliability [6].

IV. CONCLUSION

Electromagnetic radiation is all around us. Our very technological existence, so heavily depends on electromagnetic radiation, and our understanding of its uses and properties. Dipole antennas are in nearly everything, an understanding them can give you not only a useful bragging right, but an important understanding of some of the fundamental processes our technological society is based upon.

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[1] Youtube, *Electromagnetic Radiation spectrum*, 2015. https://www.youtube.com/watch?v=cEXgnLXnMy8

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