Random Sequences Project

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The Dice Rolling Game

For this version of the game, we imagine rolling a 6 sided die until the first time the value decreases. That is, the following roll must be greater than or equal to the previous roll. How many rolls would we expect to get? Siehler derives the formula for a n-sided to be:

$$E[X] = \left(\frac{n}{n-1}\right)^n$$

Since, we are using a 6-sided die, we plug 6 into the formula, which gives us an expected value of:

$$\left(\frac{6}{5}\right)^6 = 2.985984$$

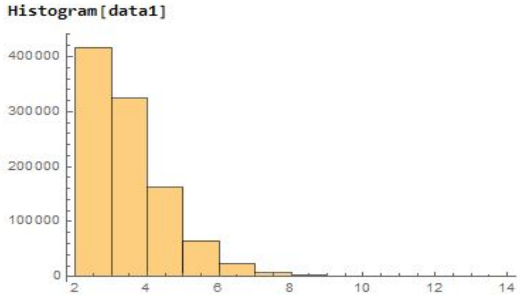
Now, that's nice and all, but let's simulate this stochastically, and see what we get! Using mathematica, we use the following code, and obtain the follow histogram:

```
In[321]:= Y := Module[{},
            count = 0;
            curr = 0;
            prev = -1;
            While[curr >= prev, ++count; prev = curr; curr = X;];
            Return[count];
           ];

In[346]:= data1 = Table[Y, 1000000];

In[347]:=
          Histogram[data1]
```

Out[347]=




```
In[328]:= Mean[data1] // N
Out[328]= 2.98686
```

Notice, the mean of the data is 2.98686, which is quite close to the expected value of 2.985984. Thus, our stochastic model, and Siehler's analytical model are in agreement!

A Variation: Strictly Increasing Values

Not much has changed from the standard dice rolling as described above, however, instead of considering values that are greater than or equal to the previous, we require the values must be *strictly* greater than the previous. How will this change our results? Our histogram? What will the expected value be? Siehler again derives the formula for this case with an n-sided die to be:

$$E[X] = \left(\frac{n+1}{n}\right)^n$$

Again, we are using a 6-sided die. Plugging in the value 6, we have:

$$\left(\frac{7}{6}\right)^6 = 2.52162$$

Now, let's simulate this situation stochastically with Mathematica, and see what we get!

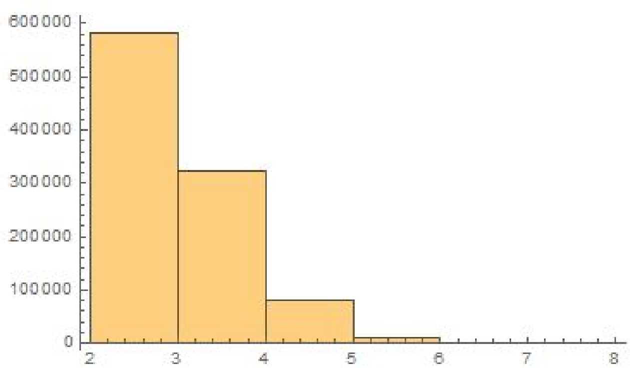
In[329]:=

```
Z := Module[{},
   count = 0;
   curr = 0;
   prev = -1;
   While[curr > prev, ++count; prev = curr; curr = X;];
   Return[count];
  ];
```

In[343]:= `data2 = Table[Z, 1000000];`

In[344]:= `Histogram[data2]`

Out[344]=



In[345]:= `Mean[data2] // N`

Out[345]= `2.52106`

Notice the mean is 2.52106! This is very close to the expected value of 2.52162! Our stochastic model is in agreement with Siehler's analytical one!

The Continuous Game

For our final variation, we imagine the situation where we keep sampling a *continuous* random variable, rather than one with discreet values, until it decreases. That is, the next value must be either greater than or equal to the previous. What would we expect the value to be? How will the histogram change? Is there something significant about the expected value? Sieheler derives the formula for the expected value of this situation to be:

$$\lim_{n\to\infty}\left(\frac{n}{n-1}\right)^n$$

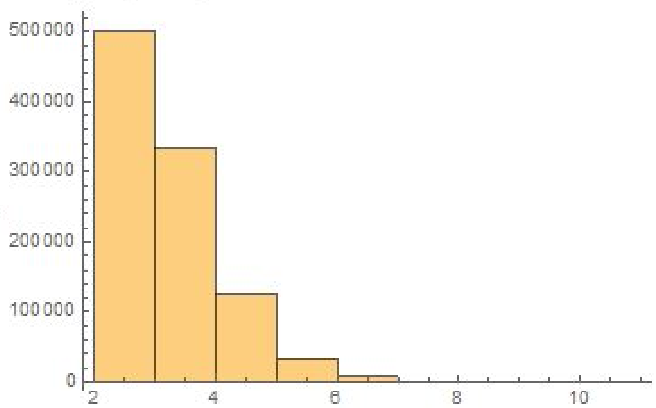
If you've seen this before, you already know what value this is, and you know it's significance. If you don't know what this is, then it's important you know! This is a well known limit expression for e! So, will we get this with our stochastic model? Let's find out! Using Mathematica once more, we generate a model for this situation.

```
In[332]:= A := Module[{},
            count = 0;
            curr = 0;
            prev = -1;
            While[curr >= prev, ++count; prev = curr; curr = U;];
            Return[count];
           ];

In[339]:= data3 = Table[A, 1000000];

In[341]:= Histogram[data3]
```

Out[341]=




```
In[342]:=
          Mean[data3] // N

Out[342]= 2.71773
```

The mean is 2.71773 which is close to the expected value of e, which is 2.71828! Thus, our stochastic model aligns with Sieheler's analytical one!