Class Notes

Rippy & Joey

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Hyperbolic & Trig Functions

Useful Definitions

$$\sinh z = \frac{e^z - e^{-z}}{2} \tag{1}$$

$$\cosh z = \frac{e^z + e^{-z}}{2} \tag{2}$$

$$\sin(z) = \frac{e^{-z} - e^{-iz}}{2i} \tag{3}$$

$$\cos(z) = \frac{e^{iz} + e^{iz}}{2} \tag{4}$$

Useful Derivatives

$$\frac{d}{dz}\sin(z) = \cos(z)$$
$$\frac{d}{dz}\cos(z) = -\sin(z)$$
$$\frac{d}{dz}\tan(z) = \sec^2(z)$$
$$\frac{d}{dz}\cot(z) = -\csc^2(z)$$
$$\frac{d}{dz}\sec(z) = \sec(z)\tan(z)$$
$$\frac{d}{dz}\csc(z) = -\csc(z)\cot(z)$$
$$\frac{d}{dz}\sinh z = \cosh z$$
$$\frac{d}{dz}\cosh z = \sinh z$$
$$\frac{d}{dz}\tanh z = \mathrm{sech}^2 z$$
$$\frac{d}{dz}\mathrm{sech} z = -\mathrm{sech} z \tanh z$$
$$\frac{d}{dz}\mathrm{coth} z = -\mathrm{csch}^2 z$$
$$\frac{d}{dz}\mathrm{csch} z = -\mathrm{csch} z \coth z$$

Useful Identities

Conversions

$$\sin(iy) = i\sinh(y)$$
$$\cos(iy) = \cosh(y)$$
$$-i\sinh iz = \sin z$$
$$-i\sin iz = \sinh z$$
$$\cosh iz = \cos z$$
$$\cos iz = \cosh z$$

Useful Identities

$$
\begin{aligned}
e^{iz} &= \cos(z) + \sin(z) \\
\sin(-z) &= -\sin(z) \\
\cos(-z) &= \cos(z) \\
\sin(z_1 + z_2) &= \sin(z_1)\cos(z_2) + \cos(z_1)\sin(z_2) \\
\cos(z_1 + z_2) &= \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2) \\
\sin(2z) &= 2\sin(z)\cos(z) \\
\cos(2z) &= \cos^2(z) - \sin^2(z) \\
\sin^2(z) + \cos^2(z) &= 1 \\
\sin(z + 2\pi) &= \sin(z) \\
\cos(z + 2\pi) &= \cos(z) \\
\sin(z) &= \sin(x)\cosh(y) + i\cos(x)\sinh(y) \\
\cos(z) &= \cos(x)\cosh(y) - i\sin(x)\sinh(y) \\
|\sin(z)|^2 &= \sin^2(x) + \sinh^2(y) \\
|\cos(z)|^2 &= \cos^2(x) + \sinh^2(y) \\
\sinh -z &= -\sinh z \\
\cosh -z &= \cosh z \\
\cosh^2 z - \sinh^2 z &= 1 \\
\sinh z_1 + z_2 &= \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2 \\
\cosh z_1 + z_2 &= \cosh z_1 \cosh z_2 + \sinh z_1 \sinh z_2 \\
\sinh z &= \sin x \cos y + i \cosh x \sin y \\
\cosh z &= \cosh x \cos y + i \sinh x \sin y \\
|\sinh z|^2 &= \sinh^2 x + \sin^2 y \\
|\cosh z|^2 &= \sinh^2 x + \cos^2 y
\end{aligned}
$$

Useful Inverses

$$
\begin{aligned}
\cosh^{-1} z &= \log[z + (z^2 - 1)^{1/2}] \\
\sinh^{-1} z &= \log[z + (z^2 + 1)^{1/2}] \\
\sin^{-1}(z) &= -i\log[iz + (1 - z^2)^{1/2}] \\
\cos^{-1}(z) &= -i\log[z + i(1 - z^2)^{1/2}] \\
\tan^{-1}(z) &= \frac{i}{z}\log\left(\frac{i+z}{i-z}\right)
\end{aligned}
$$