

LAPORAN

PRAKTIKUM KOMPUTASI BIOMEDIS

Chapter 8 : Numerical Integration: Trapezoid & Simpson 1/3 Rule

Pelaksanaan Praktikum:

Hari: Selasa

Tanggal: 31 Oktober 2019

Jam ke: 9-10



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A. TUJUAN

Mahasiswa dapat menentukan nilai integrasi numerik menggunakan metode trapezium dan metode simpson.

B. DASAR TEORI

Sepertihalnya dengan differensial, dalam keperluan geometri, perubahan nilai per-satuan waktu atau jarak serta kecepatan dalam kalkulus yang digunakan adalah Integral(Munir,2010). Dalam kemajuan teknologi saat ini, kebutuhan penghitungan integral menjadi lebih rumit jika harus dihitung secara manual. Dari kondisi tersebut, maka digagas sebuah metode integrasi numerik dengan pendekatan kontinu yang diterapkan pada computer yang kemudian dapat membantu perhitungan turunan secara lebih cepat dan efisien meski menghasilkan nilai galat/ eror hasil perhitungan.

Terdapat tiga metode yang umum diterapkan dalam menghitung integrasi numerik yaitu

1. Metod Persegi panjang

$$\int_a^b f(x)dx \approx \frac{h}{2} (f_0 + 2f_1 + 2f_2 + \dots + 2f_{n-1} + f_n) = \frac{h}{2} (f_0 + 2 \sum_{i=1}^{n-1} f_i + f_n)$$

2. Metode Trapezoid

$$\approx \frac{h}{2} (f_0 + 2 \sum_{i=1}^{n-1} f_i + f_n)$$

3. Metode Simpson 1/3

$$\approx \frac{h}{3} (f_0 + 4 \sum_{i=1,3,5}^{n-1} f_i + 2 \sum_{i=2,4,6}^{n-2} f_i + f_n)$$

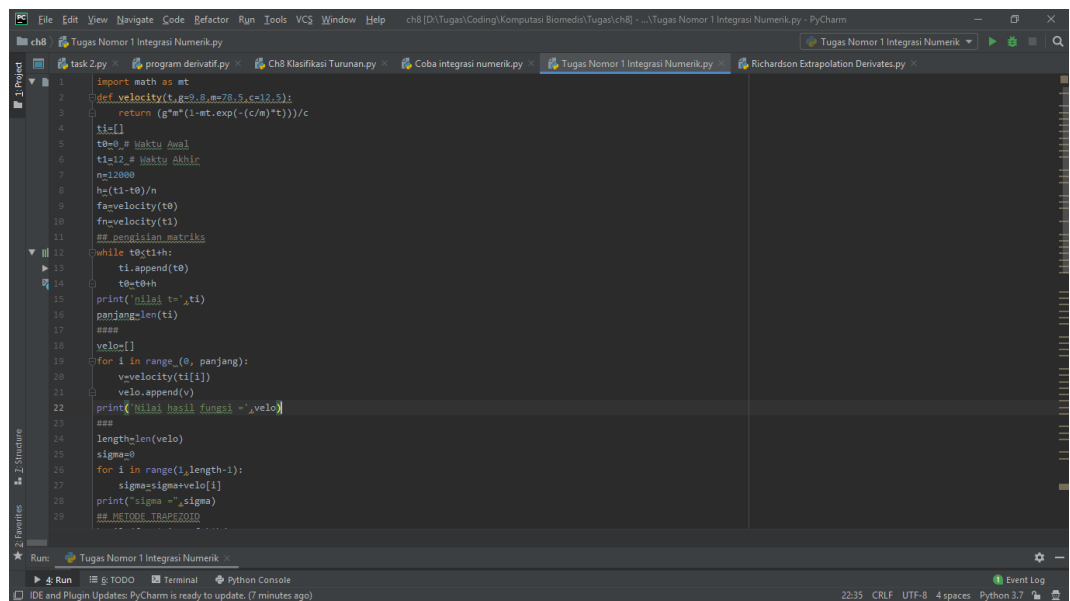
C. TUGAS

1. From the problem, which method is the most accurate? Explain!
2. From the problem, calculate the time that the IV Drip needed to be changed! (When the z=5cm), Use all integration methods for this problem!

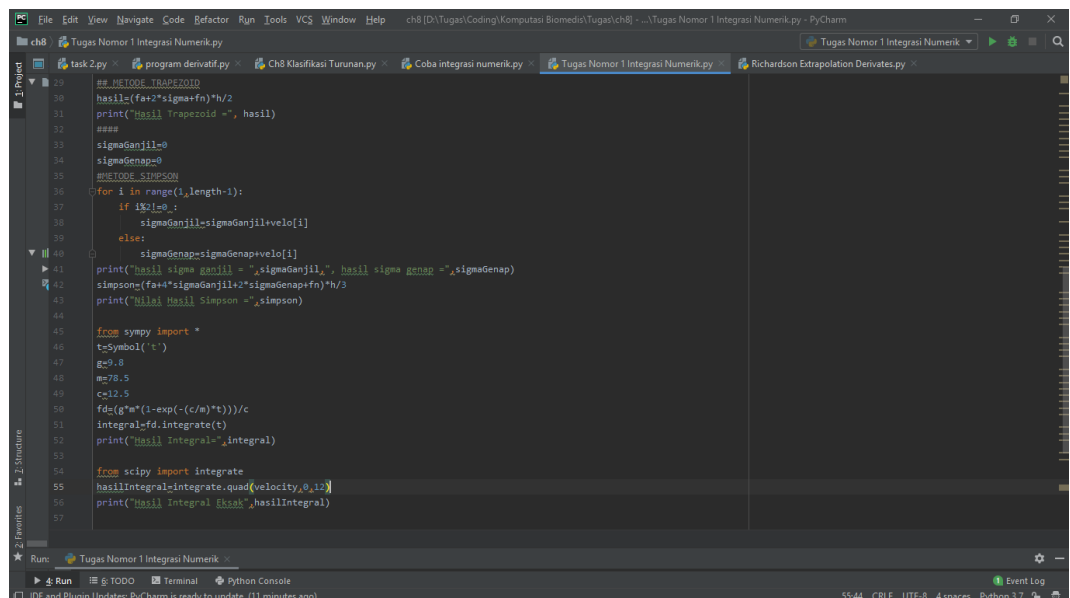
- From the proble, use extrapolate to an even more accurate solution by eliminating $O(h^2)$ error term, where h is the panel width.

D. PEMBAHASAN

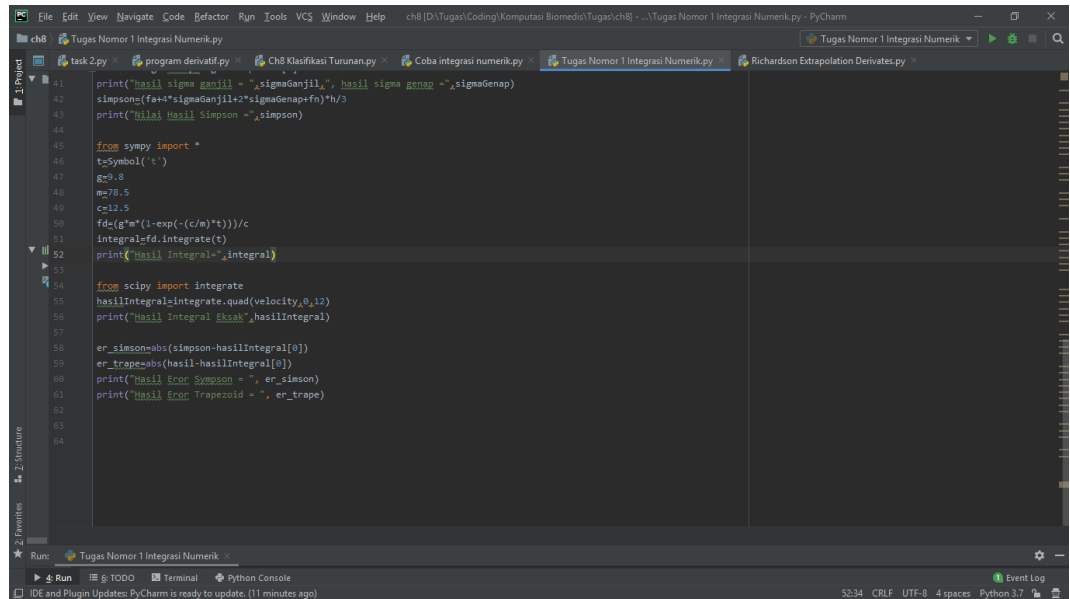
- Dalam penentuan nilai integrasi numerik pada persoalan pertama, maka definisikan terlebih dahulu persamaan $v(t)$ pada sebuah fungsi. Jika yang ditanyakan adalah jarak, maka jarak(s) adalah integral pertama dari fungsi kecepatan. Berikut adalah kode program dalam Python 3.7 dengan jumlah pias(n) sebanyak 12000 pias:



```
1 import math as mt
2 def velocity(t,g=9.8,m=78.5,c=12.5):
3     return (g*m*(1-mt.exp(-(c/m)*t)))/c
4 ti=[]
5 t0=0 # Waktu Awal
6 t1=12 # Waktu Akhir
7 n=12000
8 ht=(t1-t0)/n
9 fa=velocity(t0)
10 fn=velocity(t1)
11 ## pengisian matriks
12 while t0<t1+ht:
13     ti.append(t0)
14     t0=t0+ht
15     print('nilai t=',ti)
16     panjang=len(ti)
17     $$$
18     velo=[]
19     for i in range(0, panjang):
20         velocity(ti[i])
21         velo.append(v)
22     print('Nilai hasil fungsi =',velo)
23     $$$
24     length=len(velo)
25     sigma=0
26     for i in range(1,length-1):
27         sigma=sigma+velo[i]
28     print('sigma =',sigma)
29     ## METODE TRAPEZOID
```

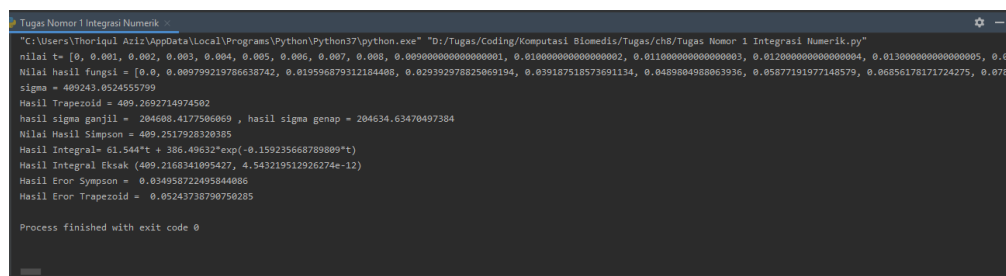


```
30 hasil=(fa+2*sigma+fn)*ht/2
31 print('Hasil Trapezoid =', hasil)
32 $$$
33 sigmaGanjil=0
34 sigmaGenap=0
35 ##METODE SIMPSON
36 for i in range(1,length-1):
37     if i%2!=0:
38         sigmaGanjil=sigmaGanjil+velo[i]
39     else:
40         sigmaGenap=sigmaGenap+velo[i]
41 print('hasil sigma ganjil =',sigmaGanjil,', hasil sigma genap =',sigmaGenap)
42 simpson=(fa+4*sigmaGanjil+2*sigmaGenap+fn)*ht/3
43 print('Nilai Hasil Simpson =',simpson)
44
45 from sympy import *
46 t=Symbol('t')
47 g=9.8
48 m=78.5
49 c=12.5
50 fd=(g*m*(1-exp(-(c/m)*t)))/c
51 integral=fd.integrate(t)
52 print('Hasil Integral=',integral)
53
54 from scipy import integrate
55 hasilIntegral=integrate.quad(velocity,0,12)
56 print('Hasil Integral Exact',hasilIntegral)
57
```



```
41 print("hasil sigma ganjil = ", sigmaGanjil, ", hasil sigma genap = ", sigmaGenap)
42 simpsonc(fa+sigmaGanjil+2*sigmaGenapfa)*h/3
43 print("Nilai Hasil Simpson = ", simpson)
44
45 from sympy import *
46 t=Symbol('t')
47 g=9.8
48 m=78.5
49 c=12.5
50 fd=(g*m*(1-exp(-(c/m)*t)))/c
51 integral=fd.integrate(t)
52 print("Hasil Integral=", integral)
53
54 from scipy import integrate
55 hasilIntegral=integrate.quad(velocity,0,12)
56 print("Hasil Integral Eksak", hasilIntegral)
57
58 er_simpson=abs(simpson-hasilIntegral[0])
59 er_trapezoid=abs(hasil-hasilIntegral[0])
60 print("Hasil Error Simpson = ", er_simpson)
61 print("Hasil Error Trapezoid = ", er_trapezoid)
62
63
64
```

Dari kode program tersebut, hasil tampilan pada *command window* :



```
"C:\Users\Thorlup Azis\AppData\Local\Programs\Python\Python37\python.exe" "D:/Tugas/Coding/Komputasi Biomedis/Tugas/ch8/Tugas Nomor 1 Integrasi Numerik.py"
nilai t= [0, 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009000000000000001, 0.010000000000000002, 0.011000000000000003, 0.012000000000000004, 0.013000000000000005, 0.014000000000000006, 0.015000000000000007, 0.016000000000000008, 0.017000000000000009, 0.01800000000000001, 0.019000000000000011, 0.020000000000000012, 0.021000000000000013, 0.022000000000000014, 0.023000000000000015, 0.024000000000000016, 0.025000000000000017, 0.026000000000000018, 0.027000000000000019, 0.02800000000000002, 0.029000000000000021, 0.030000000000000022, 0.031000000000000023, 0.032000000000000024, 0.033000000000000025, 0.034000000000000026, 0.035000000000000027, 0.036000000000000028, 0.037000000000000029, 0.03800000000000003, 0.039000000000000031, 0.040000000000000032, 0.041000000000000033, 0.042000000000000034, 0.043000000000000035, 0.044000000000000036, 0.045000000000000037, 0.046000000000000038, 0.047000000000000039, 0.04800000000000004, 0.049000000000000041, 0.050000000000000042, 0.051000000000000043, 0.052000000000000044, 0.053000000000000045, 0.054000000000000046, 0.055000000000000047, 0.056000000000000048, 0.057000000000000049, 0.05800000000000005, 0.059000000000000051, 0.060000000000000052, 0.061000000000000053, 0.062000000000000054, 0.063000000000000055, 0.064000000000000056, 0.065000000000000057, 0.066000000000000058, 0.067000000000000059, 0.06800000000000006, 0.069000000000000061, 0.070000000000000062, 0.071000000000000063, 0.072000000000000064, 0.073000000000000065, 0.074000000000000066, 0.075000000000000067, 0.076000000000000068, 0.077000000000000069, 0.07800000000000007, 0.079000000000000071, 0.080000000000000072, 0.081000000000000073, 0.082000000000000074, 0.083000000000000075, 0.084000000000000076, 0.085000000000000077, 0.086000000000000078, 0.087000000000000079, 0.08800000000000008, 0.089000000000000081, 0.090000000000000082, 0.091000000000000083, 0.092000000000000084, 0.093000000000000085, 0.094000000000000086, 0.095000000000000087, 0.096000000000000088, 0.097000000000000089, 0.09800000000000009, 0.099000000000000091, 0.1, 0.10100000000000009, 0.10200000000000009, 0.10300000000000009, 0.10400000000000009, 0.10500000000000009, 0.10600000000000009, 0.10700000000000009, 0.10800000000000009, 0.10900000000000009, 0.11, 0.11100000000000009, 0.11200000000000009, 0.11300000000000009, 0.11400000000000009, 0.11500000000000009, 0.11600000000000009, 0.11700000000000009, 0.11800000000000009, 0.11900000000000009, 0.12]
sigma = 409243.0524555799
Hasil Trapezoid = 409.2692714974502
hasil sigma ganjil = 204608.4177506009 , hasil sigma genap = 204634.63470497384
Nilai Hasil Simpson = 409.2517928320385
Hasil Integral= 61.544*t + 386.49632*exp(-0.159235668789809*t)
Hasil Integral Eksak (409.2168341095427, 4.543219512926274e-12)
Hasil Error Simpson = 0.034958722495844886
Hasil Error Trapezoid = 0.05243738790758285

Process finished with exit code 0
```

Dari hasil tersebut dapat diketahui bahwa, hasil eror integrasi numerik dengan metode sympon dan metode trapezium terhadap nilai eksaknya menunjukan bahwa eror dengan metode sympon lebih kecil yaitu 0.034 dibanding dengan metode trapezium yaitu 0.052. Sehingga, dari nilai keakuratan, metode sympon lebih akurat akibat nilai error yang diberikan lebih rendah. Hal ini diakibatkan karena metode simpson dalam menentukan nilai integrasi numerik menggunakan dua trapezoid dalam pembatasan pias, sehingga nilai hilang penghitungan menjadi lebih kecil.

2. Dari soal yang diberikan, maka dapat dihitung nilai dari waktu yang dibutuhkan menggunakan metode intgrasi simpson dan metode trapezoid. Berikut adalah kode program dalam Python 3.7 dengan jumlah pias 5000:

```
File Edit View Navigate Code Refactor Run Tools VCS Window Help ch8 [D:\Tugas\Coding\Komputasi Biomedis\Tugas\ch8] - ...Tugas Nomor 2 Integrasi Numerik.py - PyCharm
Tugas Nomor 2 Integrasi Numerik.py
1 import math as mt
2 def poisel(z, L=0.1, d4=0.01, d=0.1, p=1, g=0.01, R=10):
3     a=64*L*u/(p*d**2)
4     return (8*R**2)/(d**2)*(-a+mt.sqrt(a**2+8*g*(z+L)))
5
6 t0=5 # Waktu Awal
7 t1=50 # Waktu Akhir
8 n=5000
9 h=(t1-t0)/n
10 fa=poisel(t0)
11 fn=poisel(t1)
12 ## pengisian matriks
13 while t0<t1+h:
14     t1.append(t0)
15     t0=t0+h
16     print('Nilai t=', t1)
17     panjang=len(t1)
18     pois=[]
19     for i in range(0, panjang):
20         vp=poisel(t1[i])
21         pois.append(v)
22     print('Nilai hasil fungsi =', pois)
23     length=len(pois)
24     sigma=0
25     for i in range(1, length-1):
26         sigma=sigma+pois[i]
27     print('sigma =', sigma)
28
29 Run: Tugas Nomor 2 Integrasi Numerik
Run TODO Terminal Python Console
PEP 8: blank line at end of file 14:1 CRLF UTF-8 4 spaces Python 3.7
```

```
File Edit View Navigate Code Refactor Run Tools VCS Window Help ch8 [D:\Tugas\Coding\Komputasi Biomedis\Tugas\ch8] - ...Tugas Nomor 2 Integrasi Numerik.py - PyCharm
Tugas Nomor 2 Integrasi Numerik.py
24
25 length=len(pois)
26 sigma=0
27 for i in range(1, length-1):
28     sigma=sigma+pois[i]
29     print('sigma =', sigma)
30 ## METODE TRAPEZOID
31 hasil=(fa+2*sigma+fn)*h/2
32 print('Hasil Trapezoid =', hasil)
33
34 sigmaGanjil=0
35 sigmaGenap=0
36 ##METODE SIMPSON
37 for i in range(1, length-1):
38     if i%2!=0:
39         sigmaGanjil=sigmaGanjil+pois[i]
40     else:
41         sigmaGenap=sigmaGenap+pois[i]
42     print('hasil sigma ganjil =', sigmaGanjil, 'hasil sigma genap =', sigmaGenap)
43 simpson=(fa+4*sigmaGanjil+2*sigmaGenap+fn)*h/3
44 print('Nilai Hasil Simpson =', simpson)
45
46 from sympy import *
47 z=Symbol('z')
48 L=0.1, d4=0.01, d=0.1, p=1, g=0.01
49 for i in range(0, panjang)
50
51 Run: Tugas Nomor 2 Integrasi Numerik
Run TODO Terminal Python Console
IDE and Plugin Updates: PyCharm is ready to update. (today 10:11 PM) 21:20 CRLF UTF-8 4 spaces Python 3.7
```

```

40: else:
41:     sigmaGenap=sigmaGenapppols[i]
42:     print("hasil sigma ganjil = ",sigmaGanjil," , hasil sigma genap = ",sigmaGenap)
43:     simpson=(fa+sigmaGanjil+2*sigmaGenap+fn)*h/3
44:     print("Nilai Hasil Simpson = ",simpson)
45:
46:     from sympy import *
47:     z=Symbol('z')
48:     L=91.44
49:     u=0.01
50:     d=0.1
51:     p=1
52:     g=0.01
53:     R=10
54:     a=64*L*u/(p*d**2)
55:     fd=(8*R**2)/(d**2)*(-a+sqrt(a**2+8*g*(z+L)))
56:     integral=fd.integrate(z)
57:     print("Hasil Integral=",integral)
58:
59:     from scipy import integrate
60:     hasilIntegral=integrate.quad(poisel,5,50)
61:     print("Hasil Integral Eksak",hasilIntegral)
62:
63:     er_simpson=abs(simpson-hasilIntegral[0])
64:     er_trapezoid=abs(hasil-hasilIntegral[0])
65:     print("Hasil Error Simpson = ", er_simpson)
66:     print("Hasil Error Trapezoid = ", er_trapezoid)
67:
68:

```

Berikut hasil dari *command window* program :

```

Run: "C:\Users\Thorliq\Ari\AppData\Local\Programs\Python\Python37\python.exe" "D:\Tugas\Coding\Komputasi Biomedis\tugas\ch8\Tugas Nomor 2 Integrasi Numerik.py"
nilai t= [5, 5.009, 5.018000000000001, 5.027000000000001, 5.036000000000001, 5.045000000000002, 5.054000000000002, 5.063000000000003, 5.072000000000003, 5.081000000000003, 5.090000000000003, 5.099000000000003, 5.108000000000003, 5.117000000000003, 5.126000000000003, 5.135000000000003, 5.144000000000003, 5.153000000000003, 5.162000000000003, 5.171000000000003, 5.180000000000003, 5.189000000000003, 5.198000000000003, 5.207000000000003, 5.216000000000003, 5.225000000000003, 5.234000000000003, 5.243000000000003, 5.252000000000003, 5.261000000000003, 5.270000000000003, 5.279000000000003, 5.288000000000003, 5.297000000000003, 5.306000000000003, 5.315000000000003, 5.324000000000003, 5.333000000000003, 5.342000000000003, 5.351000000000003, 5.360000000000003, 5.369000000000003, 5.378000000000003, 5.387000000000003, 5.396000000000003, 5.405000000000003, 5.414000000000003, 5.423000000000003, 5.432000000000003, 5.441000000000003, 5.450000000000003, 5.459000000000003, 5.468000000000003, 5.477000000000003, 5.486000000000003, 5.495000000000003, 5.504000000000003, 5.513000000000003, 5.522000000000003, 5.531000000000003, 5.540000000000003, 5.549000000000003, 5.558000000000003, 5.567000000000003, 5.576000000000003, 5.585000000000003, 5.594000000000003, 5.603000000000003, 5.612000000000003, 5.621000000000003, 5.630000000000003, 5.639000000000003, 5.648000000000003, 5.657000000000003, 5.666000000000003, 5.675000000000003, 5.684000000000003, 5.693000000000003, 5.702000000000003, 5.711000000000003, 5.720000000000003, 5.729000000000003, 5.738000000000003, 5.747000000000003, 5.756000000000003, 5.765000000000003, 5.774000000000003, 5.783000000000003, 5.792000000000003, 5.801000000000003, 5.810000000000003, 5.819000000000003, 5.828000000000003, 5.837000000000003, 5.846000000000003, 5.855000000000003, 5.864000000000003, 5.873000000000003, 5.882000000000003, 5.891000000000003, 5.900000000000003, 5.909000000000003, 5.918000000000003, 5.927000000000003, 5.936000000000003, 5.945000000000003, 5.954000000000003, 5.963000000000003, 5.972000000000003, 5.981000000000003, 5.990000000000003, 6.000000000000000]
sigma = 318921591.91097873
Hasil Trapezoid = 2870868.499965744
hasil sigma ganjil = 159492694.44257247 , hasil sigma genap = 159428897.4684047
Nilai Hasil Simpson = 2870868.4999659215
Hasil Integral = -468172800.0*z + 679.578661230037*(78.48*z + 34254952.8768)**(3/2)
Hasil Integral Eksak (2870868.4999663965, 3.187304309333779e-08)
Hasil Error Simpson = 4.7497451305389404e-07
Hasil Error Trapezoid = 6.523914635181427e-07

Process finished with exit code 0

```

Dari hasil tersebut dapat diketahui bahwa nilai waktu yang dibutuhkan adalah 2870868,499 detik untuk metode trapezoid dan metode simpson. Keduanya memiliki nilai eror terhadap nilai eksaknya hingga ordo 10^{-8} .

3. Dari persoalan tersebut, sehingga dapat dibuat sebuah kode pemrograman pada Python 3.7 sebagai berikut:

```

File Edit View Navigate Code Refactor Run Tools VCS Window Help ch8 [D:\Tugas\Coding\Komputasi Biomedis\Tugas\ch8] - ...Tugas Nomor 3 Integrasi Numerik.py - PyCharm
Tugas Nomor 3 Integrasi Numerik.py
1 import math as mt
2 def poly(r,N=20,b=1):
3     return (mt.exp(-3*r**2/(2*N*b**2))*4*mt.pi*r**2)*((3/(2*mt.pi*N*b**2))**(3/2))
4
5 ti=[]
6 t0=0 # Waktu Awal
7 t1=1 # Waktu Akhir
8 n=10
9 hc=(t1-t0)/n
10 fa=poly(t0)
11 fn=poly(t1)
12 ## pengisian matriks
13 while t0<t1+h:
14     ti.append(t0)
15     t0=t0+h
16 print('Nilai t=',ti)
17 panjang=len(ti)
18 ##
19 velo=[]
20 for i in range(0, panjang):
21     vp=poly(ti[i])
22     velo.append(v)
23 print('Nilai hasil fungsi =',velo)
24 ##
25 length=len(velo)
26 sigma=0
27 for i in range(1,length-1):
28     sigma=sigma+velo[i]
29 print('sigma =',sigma)
30 ## METODE TRAPEZOID
31
Run: Tugas Nomor 3 Integrasi Numerik
Run TODO Terminal Python Console
IDE and Plugin Updates: PyCharm is ready to update. (today 10:11 PM) 17:5 CRLF UTF-8 4 spaces Python 3.7

```

```

File Edit View Navigate Code Refactor Run Tools VCS Window Help ch8 [D:\Tugas\Coding\Komputasi Biomedis\Tugas\ch8] - ...Tugas Nomor 3 Integrasi Numerik.py - PyCharm
Tugas Nomor 3 Integrasi Numerik.py
39 ## METODE TRAPEZOID
40 hasil=(fa+2*sigma+fn)*h/2
41 print('Hasil Trapezoid =', hasil)
42 ##
43 sigmaGanjil=0
44 sigmaGenap=0
45 ##METODE SIMPSON
46 for i in range(1,length-1):
47     if i%2!=0:
48         sigmaGanjil=sigmaGanjil+velo[i]
49     else:
50         sigmaGenap=sigmaGenap+velo[i]
51 print('hasil sigma ganjil =',sigmaGanjil, 'hasil sigma genap =',sigmaGenap)
52 simpson=(fa+4*sigmaGanjil+2*sigmaGenap+fn)*h/3
53 print('Nilai Hasil Simpson =',simpson)
54
55 from sympy import *
56 r=Symbol('r')
57 N=20
58 b=1
59 fd=(exp(-3*r**2/(2*N*b**2))*4*pi*r**2)*((3/(2*pi*N*b**2))**(3/2))
60 integral=fd.integrate(r)
61 print('Hasil Integral=',integral)
62
63 from scipy import integrate
64 hasilIntegral=integrate.quad(poly,0,1)
65 print('Hasil Integral Skala',hasilIntegral)
66
67 er_simpsonabs(simpson-hasilIntegral[0])
68
Run: Tugas Nomor 3 Integrasi Numerik
Run TODO Terminal Python Console
IDE and Plugin Updates: PyCharm is ready to update. (today 10:11 PM) 17:5 CRLF UTF-8 4 spaces Python 3.7

```

```

40: sigmaGenap=sigmaGenap+velo[i]
41: print("hasil sigma ganjil = ",sigmaGanjil," , hasil sigma genap = ",sigmaGenap)
42: simpson=(fa+sigmaGanjil+2*sigmaGenapfn)*h/3
43: print("Nilai Hasil Simpson = ",simpson)
44:
45: from sympy import *
46: r=Symbol('r')
47: N=20
48: b=1
49: fd=(exp(-3*r**2/(2*N*b**2)))**4*pi*r**2*((3/(2*pi*N*b**2)))**(3/2))
50: integral=fd.integrate(r)
51: print("Hasil Integral=",integral)
52:
53: from scipy import integrate
54: hasilIntegral=integrate.quad(poly,0,1)
55: print("Hasil Integral Eksak",hasilIntegral)
56:
57: er_simson=abs(simpson-hasilIntegral[0])
58: er_trape=abs(hasil-hasilIntegral[0])
59: print("Hasil Error Simpson = ", er_simson)
60: print("Hasil Error Trapezoid = ", er_trape)
61:
62:
63:

```

Berikut adalah hasil *command window* :

```

Run: Tugas Nomor 3 Integrasi Numerik
↑
↓
"C:\Users\Thoriqu\Ari\AppData\Local\Programs\Python\Python37\python.exe" "D:/Tugas/Coding/Komputasi Biomedis/Tugas/ch8/Tugas Nomor 3 Integrasi Numerik.py"
nilai te [0, 0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999, 0.9999999999999999, 1.0999999999999999]
Nilai hasil fungsi = [0.0, 0.00046318152598096425, 0.0018485621563876713, 0.004143696816986937, 0.007327998958606626, 0.01137297114316892, 0.01624252335192494, 0.021893374594753656,
sigma = 0.16990428843196437
Hasil Trapezoid = 0.01914060909237908
hasil sigma ganjil = 0.07320606671709504 , hasil sigma genap = 0.09669822171486933
Nilai Hasil Simpson = 0.017640810509392386

```

E. KESIMPULAN

Dari persoalan diatas dapat diketahui bahwa terdapat beberapa metode dalam melakukan integrase numerik diantaranya adalah metode persegi, metode trapezoid dan metode simpson 1/3. Dari ketiga metode tersebut, metode simpson 1/3 merupakan metode dengan tingkat akurasi tertinggi dari pada yang lain yang dibuktikan dengan nilai eror terhadap nilai eksak integrasi kecil.

F. DAFTAR PUSTAKA

- Capra, Steven C and Canale.1991. “**Numerical Methods for Engineers with Personal Computers Applications**”. MacGraw-Hill Book Company.
- King M.R and Mody N.A .2010. “**Numerical and Statical Methods for Bioengineering**”.Cambridge University Press. New York.

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