A Concept of Time-varying FIR Notch Filter with Non-zero Initial Conditions Based on Linear Kalman Notch Filter Prototype

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***Abstract*—In this paper a digital FIR notch filter used for removing power-line interference from biomedical recordings is presented. A concept of finite impulse response narrow band-stop time-varying filter with non-zero initial conditions, based on linear Kalman notch prototype filter, is proposed. In order to reduce the duration of the transient response of the proposed FIR notch filter, a parameter of the Kalman notch prototype filter is temporarily varied in time. Moreover, optimal initial conditions for the proposed FIR filter have been determined. The presented filters are tested using sinusoidal and EEG signal. Computer simulations demonstrate that the proposed time-varying FIR filters outperform traditional time-invariant FIR filters with initial conditions set to zero.**

I. Introduction

Notch filters find many applications in signal processing. For a notch filter, the maximum attenuation occurs at a single frequency, leaving other frequency components unchanged [1]. The selectivity of the filter can be described as the ratio of the attenuation at the notch frequency to the bandwidth between the edges of the passband [2]. This kind of filters allows significant reduction of the power-line interference from measured signals. Typical applications of notch filters include measurements [3], communications [4], multimedia [5] and biomedical devices [6], [7]. A notch filter can be built as Infinite Impulse Response (IIR) or Finite Impulse Response (FIR) filter as well. IIR filters are recursive filters that allow to perform very narrow bandwidths at notch frequency [1]. On the other hand, IIR filters are unstable in some conditions. The FIR filters are non-recursive filters and cannot perform bandwidth as narrow as IIR filters, but unconditional stability and linear phase are among the advantages of FIR filters. However, IIR filters require lower orders to obtain narrower bandwidth at notch frequency [1]. Using IIR filter as prototype for FIR filter allows to obtain narrow rejection bandwidth [8].

It is difficult to obtain clean and high-quality signal from biological sources. Biomedical signals have low amplitudes and are easily corrupted by power-line interferences [9]. The electroencephalography (EEG) is one of the tool used for observing brain activity. In comparison to other medical examination cost of EEG is low [10]. The spectrum of EEG signals contains frequency components from 35 to 100 Hz. The power-line interference lies inside the spectrum of these signals [11]. One of the strategies employed to deal with distorted EEG signals is to use a notch filter with notch frequency set to the power-line interference frequency.

Design methods of FIR notch filters are well known and are widely described in literature [2], [8], [12], [13]. In the works devoted to the FIR filters authors mainly focus on analysing the frequency response. Properties of these filters in the time domain are rarely discussed in the literature. Using FIR notch filters to remove interferences from signals brings some problems related to the transient response. The filter output signal has two components: a transient response and a steady state response. The transient response of a filter is a secondary effect of the approximation of the frequency response [14]. It may significantly corrupt the initial portion of the processed signal. The duration of the transient in time-invariant FIR filters depends on the filter order and the pole radius. The commonly used filters are, in many cases, inappropriate due to the long duration of transients [15]. The improvement of the transient response is a problem which has been considered in many fields of engineering.

In this paper two methods for improving the transient response are presented. In first method, the transient response will be suppressed by means of the time-varying parameter of the Kalman filter prototype. In second method, non-zero initial condition for the FIR notch filter will be determined. To obtain FIR notch filter, linear Kalman filter prototype has been used. This filter allows to obtain narrow bandwidth and good selectivity. Moreover, a reduction of the transient response is provided. FIR notch filters proposed in this article can be used as the power-line interference eliminators for biomedical signals.

II. FIR notch filter based on linear Kalman prototype filter

The filter considered in this paper is based on the linear Kalman notch filter proposed in [16]. In article by Sameni the steady state analysis of Kalman filter is done. This analysis give the transfer function of the discrete-time linear Kalman notch filter. The transfer function of the filter presented by Sameni is given as follows:

$$H(z) = \beta \frac{1 - 2\cos(\omega_0) z^{-1} + z^{-2}}{1 - \frac{4\beta}{\beta+1}\cos(\omega_0) z^{-1} + \beta z^{-2}} \tag{1}$$

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where ω_0 is the notch frequency, $\hat{d}$ is the ratio of the covariances of the model and observation noises in the steady state, and $\beta = 1/(\hat{d}+1)$. Transfer function (1) can be rewritten in the following form [8]:

$$H_0(z) = K_0 H_{0\mathrm{N}}(z) H_{0\mathrm{D}}(z) \tag{2a}$$

where

$$H_{0\mathrm{N}}(z) = \beta - 2\beta\cos\omega_0 z^{-1} + \beta z^{-2} \tag{2b}$$

$$H_{0\mathrm{D}}(z) = \frac{1}{1 - \frac{4\beta}{\beta+1}\cos\omega_0 z^{-1} + \beta z^{-2}} \tag{2c}$$

Using long division, function (2c) can be written as follows:

$$\begin{aligned} H_{0\mathrm{D}}(z) &= 1 + az^{-1} + (a^2 - b)z^{-2} + (a^3 - 2ab)z^{-3} \\ &+ (a^4 - 3a^2 b + b^2)z^{-4} + (a^5 - 4a^3 b + 3ab^2)z^{-5} \\ &+ (a^6 - 5a^4 b + 6a^2 b^2 - b^3)z^{-6} + \dots \end{aligned} \tag{3}$$

where $a = \frac{4\beta}{\beta+1}\cos\omega_0$ and $b = \beta$. Equation (3) can be also put in the following form:

$$H_{0\mathrm{D}}(z) = \sum_{i=0}^{\infty} h_2(i) z^{-i} \tag{4a}$$

where

$$h(i) = \sum_{m=0}^{\lfloor i/2 \rfloor} (-1)^m \binom{i-m}{m} \left(\frac{4\beta}{\beta+1}\cos\omega_0\right)^{(i-2m)} \beta^m \tag{4b}$$

In equation (4b), notation $\lfloor x \rfloor$stands for the integral part of x. According to [8], by truncating the series of equation (4a) at $i = N$, the FIR filter can be described by the following transfer function:

$$N_0(z) = \sum_{i=0}^{N} D(i) z^{-i} \tag{5a}$$

where

$$D(i) = \beta h(i) - 2\beta\cos\omega_0 h(i-1) + \beta h(i-2),\ i = 2, 3, .., L \tag{5b}$$

and

$$D(0) = \beta h_2(0), \quad D(1) = [-2\cos\beta\omega_0 h(0) + \beta h(1)] \tag{5c}$$

It should be noted that the method proposed in [8] has some limitations. Let us assume that the sampling rate is equal to 250 Hz. In this case, if parameter $\hat{d}$ of the Kalman filter prototype is lower than 0.233, the ripples in the passband of the calculated FIR filter can be observed. The order of the filter cannot be greater than 170. Moreover, the attenuation factor of the obtained FIR filter is not always equal to 1 in the passband. The filters designed by the presented method have to be multiplied by scaling factor $K = \frac{1}{\max(G)}$, where G is the attenuation of the obtained filter in the passband. In Fig. 1 Bode diagrams of the Kalman filter prototype and the proposed FIR filter are presented.

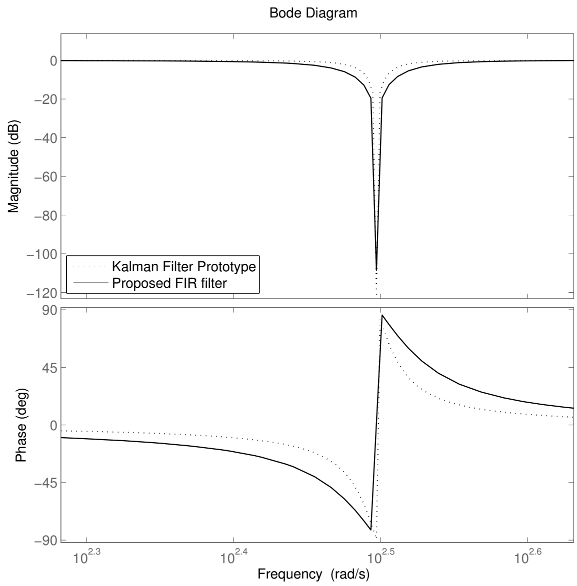

Figure 1: The Kalman filter prototype and the proposed FIR filter.

III. Time-varying FIR notch filter

In order to improve the time-domain response of the FIR notch filter it was assumed that parameter $\hat{d}$ of the Kalman filter prototype is temporary varied in time. The proposed method gives an opportunity to suppress the transient response of the filter while also satisfying the frequency specification.

As the value of parameter $\hat{d}$ of the prototype filter is smaller, the rejection bandwidth of the filter is narrower, and the filter cuts off tighter spectrum of frequencies. However, if the rejection bandwidth is narrow, the duration of the transient response is longer. The main idea of the variation parameter of the notch filter is to change parameter $\hat{d}$ from maximal value to minimal in order to obtain the notch filter with suppressed transient response and narrow bandwidth. The largest value of $\hat{d}$ considered in this paper is one. If $\hat{d}$ is larger that one, the bandwidth does not change, but the attenuation factor of frequencies higher than notch frequency is major than one. The smallest value of $\hat{d}$ is 0.233. Below this value, ripples in the passband occur, and the attenuation factor at the notch frequency is smaller than 100 dB. In Fig. 2 FIR filters with different values of parameter $\hat{d}$ are presented. All signals used for simulations in this paper are sampled at 250 Hz and distorted by 50-Hz power-line interference. Therefore, the notch filter with notch frequency at 50 Hz are considered. Also, the double-notch filter, with notch frequencies at 50 and 100 Hz is proposed. Let us assume that m is the transient duration in samples of time-invariant filter. Parameter $\hat{d}$ of the proposed notch filter prototype vary in m samples.

In order to vary in time parameter $\hat{d}$ of the prototype Kalman filter the 3rd order Bezier curve was selected. The Bezier curve

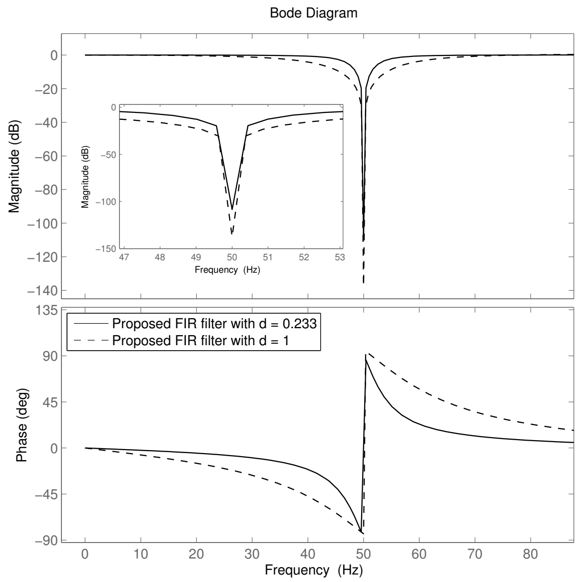

Figure 2: FIR filters with different value of the parameter $\hat{d}$.

has been assumed in the following form:

$$P(k) = P_1(1-k)^3 + 3P_2k(1-k)^2 + 3P_3k^2(1-t) + P_4k^3 \quad (6)$$

where $P_1 = (P_{1x}, P_{1y})$ are the Cartesian coordinates of the curve starting point, whereas, P_{1x} is the first sample of time-varying the parameter and P_{1y} is the value of parameter $\hat{d}$ at the start. The coefficients $P_2 = (P_{2x}, P_{2y})$ and $P_3 = (P_{3x}, P_{3y})$ are the coordinates of control points and $P_4 = (P_{4x}, P_{4y})$ are the coordinates of the final point of the curve. The values of k are ranged from 0 to 1 with $\frac{1}{m}$ step. Equation (6) can be rewritten in the following form:

$$P_x(k) = P_{1x}(1-k)^3 + 3P_{2x}k(1-k)^2 + 3P_{3x}k^2(1-t) + P_{4x}k^3 \quad \text{(7a)}$$

$$P_y(k) = P_{1y}(1-k)^3 + 3P_{2y}k(1-k)^2 + 3P_{3y}k^2(1-t) + P_{4y}k^3 \quad \text{(7b)}$$

In the next step, in order to find the values of function $P_x = F(P_y(k))$ at query points for every sample of the transient response, an interpolation process is used. If the value of P_{1x} is equal to 1, the parameter variation starts at the beginning of filtration process. The value of P_{4x} is equal to m. The values of P_{1y} and P_{4y} are chosen in such a way that P_{1y} is the largest value of parameter $\hat{d}$, that ensures wide bandwidth (in the cases considered in this paper it is unity), while P_{4y} is the smallest value that provides attenuation at the notch frequency equal to 100 dB. In Fig. 3 the Beizer curve and the control points are presented. To find the values of P_{2x}, P_{2y}, P_{3x}, P_{3y} the genetics algorithm is used. The goal of genetic algorithm is to find mentioned before values that ensure minimization of the transient response duration of time-varying notch filter. Let us assume that the duration of the transient response m_v is the number of first sample of the FIR filter output signal, for with the attenuation at the notch frequency is equal to 100 dB, while the filter input signal is the sinusoidal one with frequency equal to the notch frequency of the filter. In Table I the proposed values of the parameters for the proposed time-invariant and time-varying FIR notch filters based on the linear Kalman notch filter prototype with notch frequencies at 50 and 100 Hz are presented.

Table I: Parameters of time-invariant FIR filter prototype and proposed time-varying FIR filter

	notch at 50 Hz	notch at 100 Hz
N	125	60
Time-invariant FIR filter		
$\hat{d}$	0.233	0.55
m	86	38
Proposed time-varying FIR filter		
(P_{1x}, P_{1y})	(0,1)	(0,1)
(P_{2x}, P_{2y})	(75,0.7354)	(32,0.9828)
(P_{3x}, P_{3y})	(40,0.4115)	(37,0.9810)
(P_{4x}, P_{4y})	(86,0.2330)	(38,0.5500)
m_v	28	26

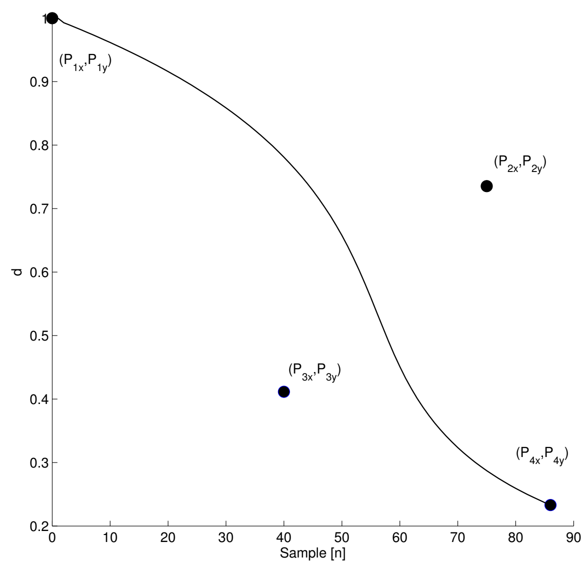

Figure 3: The Beizer curve and the control points.

IV. Non-zero initial conditions for the FIR filter

A discrete signal distorted by a single frequency sinusoidal noise (e.g. power-line interference) may be considered as a sum of the useful signal $u(n)$ and the sinusoidal one $s(n)$ with known frequency:

$$x(n) = u(n) + s(n) \quad (8)$$

The interference $s(n)$ with amplitude A_0, frequency ω, and phase ϕ is given by:

$$s(n) = A_0 sin(\omega n + \phi) \tag{9}$$

In the method proposed by Pei and Tseng [17] the first l samples of input signal X are decomposed into useful signal component U and sinusoidal disturb component S as follows:

$$X = U + S \tag{10}$$

where notations X, U and S stand for:

$$X = [x(0)\, x(1)\, \ldots\, x(l-1)]^T \tag{11a}$$

$$U = [u(0)\, u(1)\, \ldots\, u(l-1)]^T \tag{11b}$$

$$S = [s(0)\, s(1)\, \ldots\, s(l-1)]^T \tag{11c}$$

In equations (11a-11c) notation T stands for the transposition. Equation (9) can be rewritten in the following form:

$$s(n) = A_0[\cos(\phi)]\sin(\omega_0 n) + \sin(\phi)\cos(\omega_0 n)] \tag{12}$$

Using equation (12), matrix A can be defined as follows:

$$A = \begin{bmatrix} 1 & 0 \\ \cos(\omega_0) & \sin(\omega_0) \\ \cos(2\omega_0) & \sin(2\omega_0) \\ \vdots & \vdots \\ \cos[(l-1)\omega_0] & \sin[(l-1)\omega_0] \end{bmatrix} \tag{13}$$

In this matrix vector S is in the row space for any initial phase ϕ of $s(n)$. Next, the projection matrix B is defined as:

$$B = A(A^T A)A^{-1}A^T \tag{14}$$

The initial conditions vector $\hat{U}$ for the FIR filter is calculated using following formula:

$$\hat{U} = (I - B)X \tag{15}$$

where I is the identity matrix.

The N-th order FIR filter output $y(n)$ is given by:

$$y(n) = [b_0(n) + b_1(n-1) + \ldots + b_N(n-N)]x(n) \tag{16}$$

where $b_0(n), b_1(n-1), \ldots, b_N(n-N)$ are the filter coefficients, $y(n)$ and $x(n)$ are the filter output and input, respectively. The first m output samples of the proposed filter are calculated by:

$$y(n) = [y(0)y(1)\ldots y(l-1)] = \hat{U}^T = [(I-B)X]^T \tag{17}$$

In the case of improper choice of the value of l, unintentional ripples in the initial conditions occur. To avoid the problems related to the computational complexity and delays of the output signals, the minimum value of m that ensures the smallest value of the MSE should be selected. Therefore, a properly chosen value of m guarantees good quality of filtering and suppression of the transient response. The local minima $^*l$ of the MSE are correlated with the sampling rate f_s and distortion frequency f_0 as follows:

$$^*m = \frac{f_s}{f_o}k, \qquad k = 1, 2, 3, \ldots \tag{18}$$

For the signals used for simulation the sampling rate f_s is 250 Hz and distortion frequencies are 50 and 100 Hz respectively. The length of the initial conditions vector l has to fulfil a condition stated in the form of equation (18) and has to be larger than the duration of transient response m_v of the time-varying FIR filter. For the notch filter with notch at 50 Hz, $^*l = 5k$ and $m_v = 28$ (taken from Table I), the value of l should be equal to 30. For the FIR filter with notch at 100 Hz the value of l should be equal to 27.

V. DOUBLE-NOTCH FIR NOTCH FILTER WITH NON-ZERO INITIAL CONDITIONS

To remove the fundamental frequency of power-line interference and the second harmonic from the useful signal spectrum, the double-notch is proposed. The distorted signals considered in this paper are sampled at 250 Hz. Therefore, the proposed double-notch FIR is sufficient in presented situation. To obtain double-notch filter, two FIR filters have to be calculated. The first one with notch frequency at $f_1 = 50$ Hz, and the second with notch frequency at $f_2 = 100$ Hz. The design parameters for these filters are presented in Table I. Let us assume that, m is the transient duration of time-invariant FIR filter prototype, N is the double-notch FIR filter order and h_1 and h_2 are the FIR filters coefficients vectors with notch frequency at 50 and 100 Hz, respectively. The double-notch time-varying FIR filter coefficients vector h_m can be obtain from the following equations:

$$h_m(i,m) = \sum_{j=1}^{i} h_1(j,m)h_2(i-j,m),\ i = 0, 1, \ldots, \frac{N}{2} \tag{20a}$$

$$h_m\left(\frac{N}{2}+1, m\right) = \sum_{j=i}^{\frac{N}{2}} h_1\left(\frac{N}{2}+1-j, m\right) h_2(j),\ i = 1, 2, \ldots, \frac{N}{2} \tag{20b}$$

As can be seen the order of the double-notch FIR filter has to be even. In the case presented in this paper the order of double-notch FIR filter is $N = 250$.

To improve the quality of the filtering process non-zero initial conditions for the double-notch FIR filter were determined. The algorithm used to calculate the initial conditions is similar to the algorithm presented in Section IV. However, one change has been made. The matrix from equation (13) for the double-notch FIR filter has the form presented in equation (19), were $\omega_1 = 2\pi\frac{f_1}{f_s}$ and $\omega_2 = 2\pi\frac{f_2}{f_s}$.

VI. SIMULATIONS RESULTS

Simulations of the proposed time-varying FIR filter with non-zero initial conditions have been made using Mathworks Matlab 2012a. For examination, sinusoidal and EEG signals have been used. Firstly, the filter with time-varying parameter has been considered, next the non-zero initial conditions for

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ \cos(\omega_1) & \sin(\omega_1) & \cos(\omega_2) & \sin(\omega_2) \\ \cos(2\omega_1) & \sin(2\omega_1) & \cos(2\omega_2) & \sin(2\omega_2) \\ \vdots & \vdots & \vdots & \vdots \\ \cos[(l-1)\omega_1] & \sin[(l-1)\omega_1] & \cos[(l-1)\omega_2] & \sin[(l-1)\omega_2] \end{bmatrix} \tag{19}$$

presented FIR filter have been used to improve the quality of filtration. To evaluate the performance of the proposed FIR filters the mean square error (MSE) is calculated. The mean square error used in this paper is defined as:

$$ERR = \frac{1}{N}\sum_{n=1}^{N} |y(n) - u(n)|^2 \tag{21}$$

where N is the notch filter order, $y(n)$ the filter output signal, and $u(n)$ a clean, original biomedical signal. Usually the smaller the MSE is, the closer output signal is to original, and the shorter transient response occur.

In the following subsections single-notch FIR filter and double-notch FIR filter are considered. In simulations time-invariant, time-varying and time varying with non-zero initial condition FIR notch filters are used.

The notch frequency of the proposed single-notch FIR filter is equal to 50 Hz(the power-line interference frequency). The double-notch FIR filter has notch at 50 and 100 Hz. This allow to remove the first and the second harmonic of the power-line interference form the useful signal spectrum. For filters with time-invariant coefficients there are only small possibilities of shortening the transient state. In this simulations a technique for improving the transient response of the time-invariant FIR filters using varying filter parameter in time and non-zero initial conditions for the filter is presented. The MSEs of notch FIR filters are presented in Table II and Table III.

Table II: Values of MSEs of proposed notch FIR filters with notch frequency at 50 Hz.

	Sinusoidal	EEG
Time-invariant FIR notch	2.41376	0.00158
Time-varying FIR notch	0.81589	0.00065
Time-varying FIR single notch with non-zero initial conditions	1.09144e-07	0.00012

A. Sinusoidal signal

In Fig. 4 the transient responses of proposed FIR filters are presented. The input signals was 50-Hz sinusoid in the first simulation, and the sum of the 50- and 100-Hz sinusoids in the second simulation. As presented in Fig. 9, the filter with parameter varied in time and non-zero initial conditions outperforms traditional FIR filter. The transient responses of proposed filters are shorter than the transient response of time-invariant FIR notch filter. In Table III results of simulation using double-notch filter are presented.

Table III: Values of MSEs of proposed double-notch FIR filters with notch frequencies at 50 and 100 Hz .

	Sinusoidal	EEG
Time-invariant FIR double-notch	1.87335	0.00183
Time-varying FIR double-notch	1.23484	0.00143
Time-varying FIR double-notch with non-zero initial conditions	1.49874e-07	0.00044

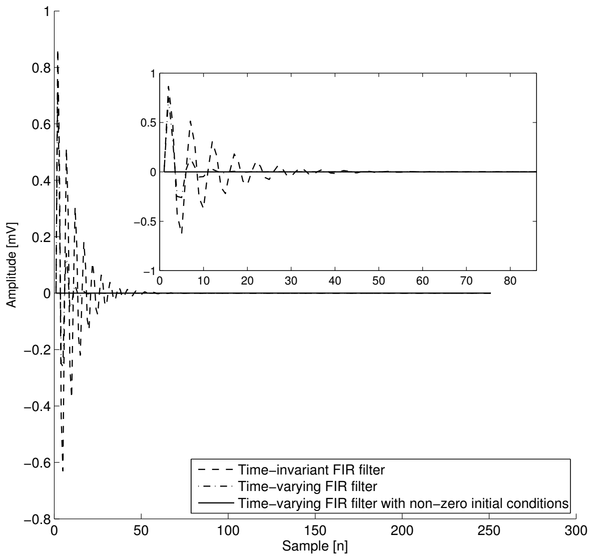

Figure 4: Results of simulation with sinusoidal input signal

B. EEG signal

In Fig. 5 the results of simulation with the single-notch FIR filter is presented. The time-invariant, time-varying and time-varying with non zero initial conditions FIR filters are used in the simulations, respectively. Fig. 6 presents the FFT of noised and filtered EEG signal. The duration of transient responses of proposed filters is shorter than the duration of transient response of time-invariant filter. Using the non-zero initial conditions gives an opportunity to eliminate the greater part of the transient response of the FIR notch filter based on linear Kalman notch filter prototype.

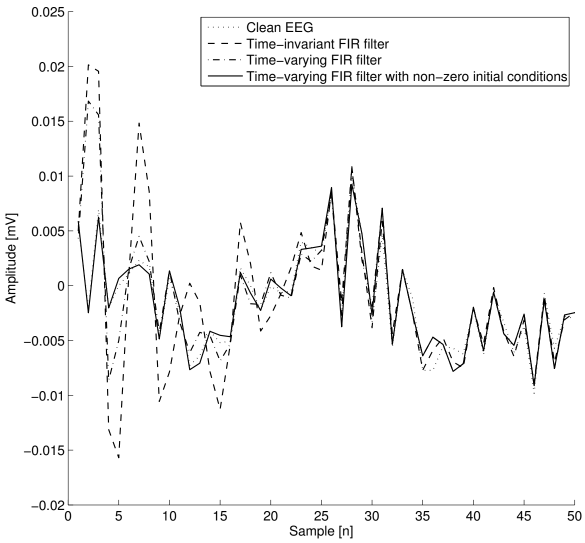

Figure 5: Results of simulation with the EEG input signal.

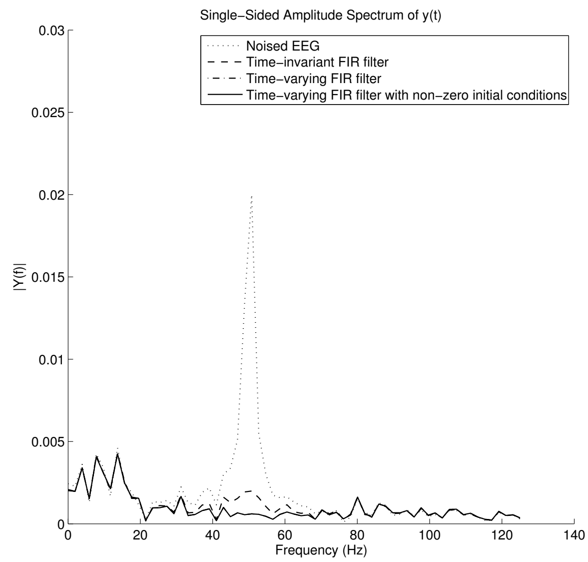

Figure 6: FFT of noised and filtered EEG signals.

VII. Conclusions

In order to reduce the duration of the transient response of the FIR notch filter based on linear Kalman notch filter prototype, the parameter-varying finite impulse response notch filter with the non-zero initial conditions has been proposed. To vary the prototype filter parameter in time the Beizer curves has been used. To compute initial conditions, the vector projection method has been used. The proposed FIR notch filter has been successfully used to reduce the transient response of signals distorted by 50-Hz and 100-Hz sinusoidal interferences. Simulations results prove, for EEG signals, that this technique can improve the performance of FIR notch filters based on linear Kalman prototype filter. Furthermore, the values of filter parameters and the values of Beizer curves parameters, have been determined. The proposed FIR notch filter has been successfully used to bound the transient response of signals disturbed by power line interferences. Furthermore, it may be employed as a filter used for interference cancellation in the biomedical signals. The proposed time-varying FIR filter concept based on linear Kalman notch filter will be extended to the class of multi-notch filters as harmonic eliminator.

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