Digital Q-Varying Notch IIR Filter With Transient Suppression

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***Abstract*—In many measurement applications, it is required to have notch filters that simultaneously possess a very selective magnitude response (high quality factor Q) and a transient response of short duration. However, increasing the quality factor also increases the duration of the transient process in the filter after the action of the excitation. This paper presents a new concept of digital IIR notch filters, whose quality factor changes with time. Owing to a temporary change in the value of the quality factor, the transient can considerably be reduced. Simulations verifying the effectiveness of the proposed Q-varying IIR notch filter are presented and compared with the performance of the traditional Q-constant filter using ECG signals with unwanted sinusoidal interference as a study case.**

***Index Terms*—Electrocardiogram (ECG) signals, infinite-impulse response (IIR) filters, notch filters, parameter-varying technique, time-varying systems, transient behavior.**

I. Introduction

NOTCH filters are useful in applications wherein a specific frequency must be eliminated. In other words, these filters are designed to have high attenuation for the signals in a given frequency range but transmit all others with no or minimal loss. The notch filter reduces the magnitude of the output not just at one frequency but over a band of frequencies, and in this sense, it is band elimination in nature. In many measurement and signal processing applications, it is desired to remove the narrowband or single-frequency sinusoidal interference while leaving the broadband signal unchanged. Typical application areas include biomedical engineering and communications. A specific example is that of removing the power line or other interference in the ECG recording system [1]–[6], filtering of humming Global System for Mobile communications mobile telephone noise [7], and estimation of the power system frequency [8].

The design of notch filters can be a difficult task, particularly when signals on a very narrow frequency band must be suppressed and those in the immediate neighborhood should be maintained. Moreover, this task may further be complicated by requiring linear phase response, short signal delay, or short transient. Digital notch filters can be constructed as either finite-impulse response (FIR) or IIR filters. In general, IIR filter structures can be designed with a much lower order than their FIR counterparts for meeting equivalent magnitude specifications. However, in general, IIR filters are potentially unstable and do not provide linear phase response. On the other hand, FIR filters are unconditionally stable and can be designed to give exact linear phase. However, to meet the amplitude specifications, typically, a high FIR filter length is required. The signal delay is proportional to the filter length, which is often intolerable for the applications [9]. For more details concerning IIR and FIR notch filter design, the interested reader may consult, for instance, [10]–[12].

Digital notch filters come in a variety of different forms that can be used for various purposes and that achieve different results. Fixed notch filters (IIR and FIR) [3], [9]–[12] are set to a given frequency and filter the signals at that frequency. Tunable notch filters [13] are similar to fixed notch filters as they are set to a given frequency. However, tunable notch filters have a range of frequencies that they can be set to and then fixed at that frequency. Adaptive notch filters (ANFs) [1]–[5], [8], [14]–[22] are used in situations wherein the characteristics of the processed signals are variable in frequency and depend on events over time. Accordingly, ANFs automatically adjust their frequency response depending upon the circumstances.

Adaptive notch filtering is a well-studied technique. Such filters have time-variant coefficients that are continuously updated by an optimization criterion. The least mean-square (LMS) ANF [3], [4] is one of the popular approaches in performing sinusoidal interference removal. Although this approach is rather simple and convenient to implement, it does have a drawback in the form of a tradeoff between the initial convergence and the notch bandwidth. If the LMS algorithm is replaced by the recursive least-square algorithm [5], then both rapid initial convergence and narrow bandwidth can be achieved. The cost for using this technique, however, is an increased computation burden that discourages real-time applications. In addition, there are many other adaptive filtering techniques, such as autoregressive-based algorithm [22]. Generally, ANFs remove interference using a reference signal. This filtering method leaves the source signal undistorted, but it cannot follow fast changes in the interference amplitude, producing an undesired ringing effect. A detailed characterization of the various adaptive notch filtering methods can be found in [23] and [24].

Signal processing by traditional digital filtering techniques brings some problems, such as the transient response at the beginning of the signal. Its duration mainly depends on the filter order. This causes problems when particularly short signals are filtered or when the initial part of a processed signal is of great

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importance. In this case, the useful signal can considerably be distorted owing to the transient response, or it can entirely be lost in the transient. Due to this problem, filters of possibly small order are often used. As it has been mentioned before, IIR filter structures can be designed with a much lower order than their FIR counterparts for meeting equivalent magnitude specifications, so the IIR filter type will be considered in this paper.

Many measurement applications require notch filters that simultaneously possess a very selective magnitude response and a transient response of short duration. In the design of these systems, however, selective magnitude response (high value of the quality factor Q) and transient response of short duration are design specifications that are contradictory to each other and, therefore, are difficult to simultaneously tune.

In the literature, one can only find a few works that examine the tradeoff between the transient response and the width of the rejection band of the notch filter. In [6] and [25], the authors dynamically tune various filter parameters to create an optimum tradeoff between suppressing the transient response and having a narrow rejection band. They achieve better transient suppression by decomposing the signal into the useful signal part and the sinusoidal noise components. The useful component is then used to create initial values for the notch filter while simultaneously maintaining a real-time system. The results they obtained clearly show that this method is effective at filtering ECG signals. They minimize the rejection bandwidth while still reliably dealing with transient suppression. In [26], the authors present a method of improving the transient performance of IIR filters using strategic pole/zero placement. The proposed method is based on the addition of pole/zero pairs to the existing notch filter. Two methods of evaluating the compensation filter roots have been presented. The first method presents LMS-optimal solutions of the roots, and the second presents a closed-form suboptimal method of evaluating the roots. On the other hand, in [27], the authors present the optimization of the transient response by presetting the initial conditions of the inner state description.

The simultaneous improvement of the filter properties in the time and frequency domains is not possible, taking into account traditional time-invariant notch filters. However, it is possible to attain a significant reduction of the transient response duration of a notch filter to a given input signal by varying its quality factor with time. The Q-varying digital filters have been presented in the literature in, for instance, [28]–[30]. In [28], the authors present the controllable Q filter, which helps the system recover from faults. A Q-varying filter has been also used in [29] for the detection of a single sinusoid corrupted by Gaussian noise. In [30], the authors propose an adjusted-Q digital graphic equalizer employing the opposite filters. In the proposed adjusted-Q equalizer, the authors adjust the Q factor of the equalizer filter depending on the gain, yielding an improved equalizer performance.

In this paper, a new class of digital IIR Q-varying notch filters with a transient response of short duration will be presented. The improvement attained by this class of filters is based on a temporary change in the value of their quality factor Q. The strategy proposed for the variation of filter parameters was previously used in the past with some modifications in a number of applications, mainly in the analog technique. For instance, in [31], a parameter-varying low-pass filter was used to eliminate the oscillatory response exhibited by load cells used in weighting applications. Another parameter-varying filter was used in [32] to reduce the time employed in the acquisition of evoked potentials generated through auditive stimuli. Moreover, the parameter-varying technique was used in [33] and [34] to reduce the transient of low-pass filters with compensated group delay response.

The rest of this paper is organized as follows: In Section II, digital IIR notch filters are discussed in more detail. The idea of the second-order IIR notch filter with time-varying Q factor is presented in Section III. In Section IV, an example of using the proposed Q-varying digital filter to filtering the ECG signal with unwanted interference is given. Finally, some concluding remarks are given in Section V.

II. Digital IIR Notch Filters

The frequency response specification for ideal digital notch filter is given by

$$\left|H(e^{j\Omega})\right| = \begin{cases} 0, & \text{for } \Omega = \Omega_0 \\ 1, & \text{for } \Omega \neq \Omega_0 \end{cases} \tag{1}$$

where $\Omega = 2\pi f/f_s$ is the normalized digital frequency with respect to the sampling frequency f_s, and Ω_0 is the digital center notch frequency. Therefore, the digital notch frequency and the digital bandwidth can be written in the following forms:

$$\Omega_0 = \frac{2\pi f_0}{f_s} \quad \Delta\Omega = \frac{2\pi\Delta f}{f_s} \tag{2}$$

where f_0 is the analog notch frequency, and Δf is the analog notch bandwidth of the filter. This bandwidth is usually defined as the ratio of the center frequency f_0 and the quality factor Q, i.e.,

$$\Delta f = \frac{f_0}{Q}. \tag{3}$$

Using bilinear transformation, the transfer function of the second-order IIR notch filter can be written in the following form:

$$H_N(z) = \frac{1}{1+C} \cdot \frac{z^2 - 2\beta z + 1}{z^2 - \frac{2\beta}{1+C} z + 1 - \frac{2}{1+C}} \tag{4}$$

where

$$\beta = \cos(\Omega_0) \quad C = \tan(0.5\,\Delta\Omega). \tag{5}$$

Taking into account (2) and (3), the constant C can also be written in the following form:

$$C = \tan(0.5\,\Omega_0 Q^{-1}). \tag{6}$$

The higher the quality factor Q of the notch filter, the narrower the bandwidth and the greater the selectivity. However, as the quality factor is increased, the transient duration of the filter is also increased. This phenomenon is particularly noticeable at low frequencies, where the transient process duration can take as long as several seconds.

In some situations, a small-Q notch filter can be used, which, as a result, gives the transient at an acceptable level. However, for small-Q notch filters, the amplitude of frequency components close to the unwanted notch frequency will be affected by the filter to some extent. Therefore, if the notch frequency is close or inside the useful signal frequency spectrum, the high-Q filter should be applied.

III. Idea of Q-Varying Digital IIR Notch Filter

To improve the time-domain response of the notch filter, it was assumed that its quality factor Q is varied in time. The second-order Q-varying digital IIR notch filter may mathematically be represented by the following time-varying difference equation:

$$[1+C(n)]\,y(n)=2\beta y(n-1)-[1-C(n)]\,y(n-2)+x(n)-2\beta x(n-1)+x(n-2) \quad (7)$$

where

$$C(n)=\tan\left(0.5\,\Omega_0 Q^{-1}(n)\right). \quad (8)$$

In (7), $x(n)$ and $y(n)$ are the input and output of the filter, respectively. The function $Q(n)$ in (8) defines the variation of the quality factor Q. It should be noticed that the digital notch frequency Ω_0 is not time varying, since it is important to preserve the damping of the same notch frequency during all the time of filtering.

It is well known that for smaller values of the quality factor, the duration of the transient behavior of the notch filter is diminished. If this rule is taken as a departure point, then it may be concluded that to improve the dynamic behavior of the notch filter, a temporary decrease of the quality factor has to take place when the filter is expected to display transient behavior at its output. Therefore, the function responsible for the variation of the quality factor has been formulated as follows:

$$Q(n)=\overline{Q}\cdot\left[1+(d_Q-1)\cdot\exp\left(-\frac{nt_s}{r}\right)\right], \qquad n\geq 0 \quad (9)$$

where $\overline{Q}=\lim_{n\to\infty}Q(n)$ is the quality factor that comes from the notch filter approximation. The coefficient d_Q defines the variation range of the function $Q(n)$. This parameter is given by

$$d_Q=\frac{Q(0)}{\overline{Q}} \quad (10)$$

and it is always smaller that unity since $Q(0)$ is smaller than $\overline{Q}$. The constant r can be denoted as the exponential variation rate of the function $Q(n)$.

In this paper, the second-order digital IIR notch filter with center notch frequency $f_0 = 15$ Hz and quality factor $Q = 20$ will be considered as a case study. The sampling frequency has been chosen to equal 1 kHz. Fig. 1 presents the comparison of responses to the notch frequency for Q-constant and Q-varying IIR filter. It is easy to notice that the Q-varying filter is able to suppress the notch frequency considerably faster than the traditional Q-constant filter.

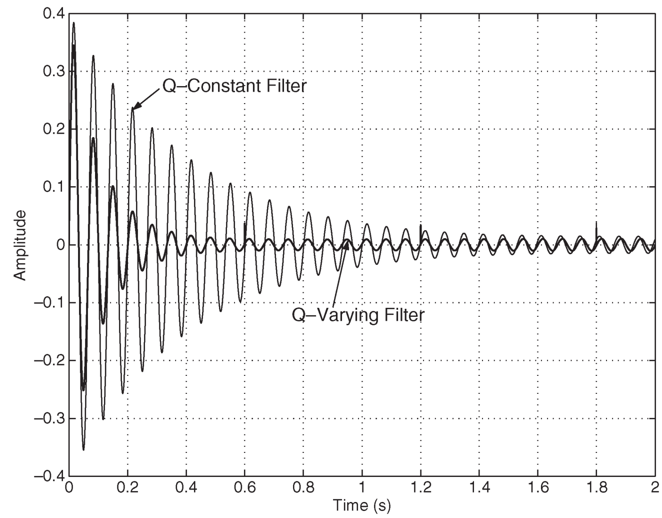

Fig. 1. Responses to the notch frequency for Q-constant and Q-varying filter.

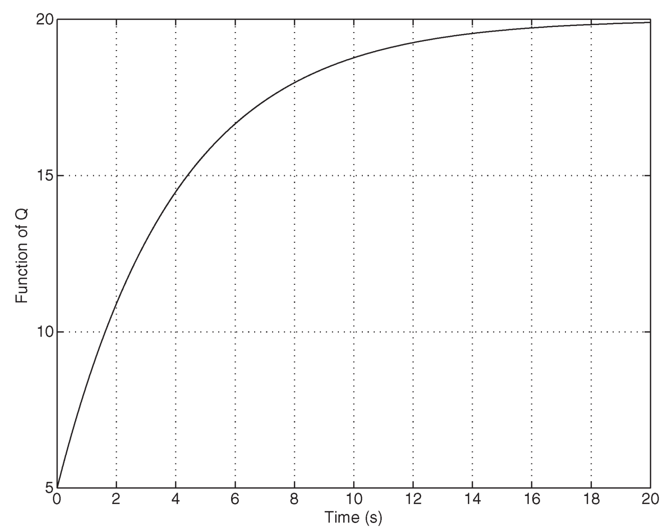

Fig. 2. Assumed function of Q for the considered digital IIR notch filter.

One of the important problems of the synthesis of the Q-varying filter is to assort the parameters of the function that varies the quality factor Q. As it has been mentioned before, the function of Q is characterized by variation range d_Q and variation rate r. The variation range d_Q has been chosen to be equal to 0.25, which means that at the beginning of the variation process the quality factor is equal to 5. The variation rate describes how long the quality factor is being varied. It has been examined, via a set of simulations, that the variation time should be ten times greater than the transient duration of the Q-constant filter. For example, if the transient of the Q-constant filter takes 2 s, then the variation process should take 20 s. A shorter variation period may result in the introduction of distortion as a consequence of unstable filter behavior. The assumed function of Q is presented in Fig. 2.

The proposed Q-varying filter has a time-varying nature. Such systems cannot be described by means of a transfer

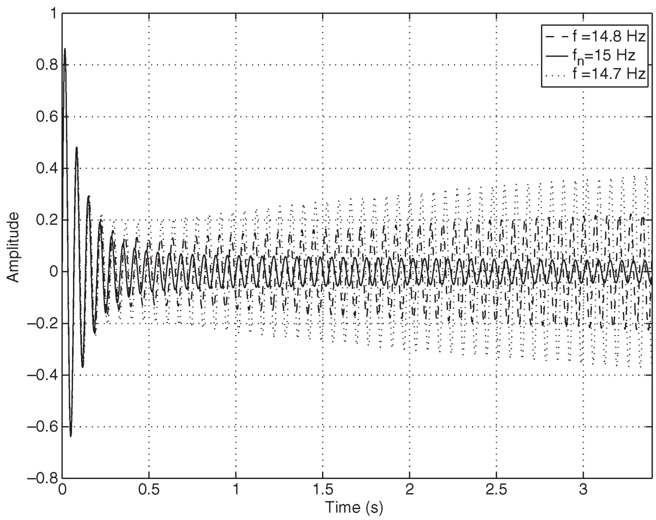

Fig. 3. Time–frequency analysis for the proposed second-order IIR notch filter.

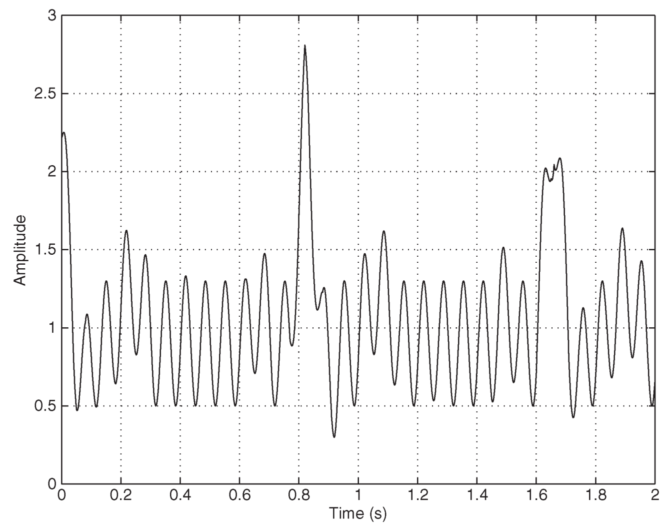

Fig. 4. Simulated ECG signal with 15-Hz interference.

function, so they cannot also be described by the frequency response. However, to present a time–frequency analysis for the proposed Q-varying notch filter, the responses to sinusoidal signals with unit amplitude and frequencies close to the notch frequency are presented in Fig. 3.

IV. Filtering of ECG Signal With Unwanted Interference

The ECG is a time-varying signal reflecting the ionic current flow, which causes the cardiac fibers to contract and subsequently relax. The surface of the ECG is obtained by recording the potential difference between two electrodes placed on the surface of the skin. A single normal cycle of the ECG represents the successive atrial depolarization/repolarization and ventricular depolarization/repolarization, which occur with every heart beat.

A major problem in the recording of ECGs is that the measurement signal is degraded by various additive interferences. A well-known method capable of reducing these interferences is the use of a notch filter characterized by a unit gain at all frequencies except at notch frequency, where the gain is zero. The problem of the elimination of interferences from the ECG signals was considered using various methods. In [6], the authors present a technique that uses vector projection to find better initial values for notch filters. Other methods, such as adaptive notch filtering [3]–[5], nonlinear adaptation [2], nonlinear wavelets [35], genetic algorithms [36], morphological filtering [37], and iterative learning filtering [38], have also been applied for ECG denoising. For more details concerning the comparison of ECG signal filtering, the interested reader may consult, for instance, [3], where the author compares the complexity and performance of two notch filter implementations, an adaptive filter with internally generated reference, and a nonadaptive second-order notch filter.

The notch filter is designed to remove specific unwanted frequencies from a signal but allow other frequencies to pass undisturbed. However, both the amplitude and the phase of frequency components close to the unwanted frequency will be affected by the filter to some extent. When a corrupted signal passes through a notch filter for the removal of sinusoidal interference, the transient response distorts the filter output on startup. Typically, a notch filter with narrower rejection bandwidth has a longer transient response at the filter output. However, we usually prefer a notch filter with narrower rejection bandwidth to faithfully separate the sinusoidal and broadband components. Hence, a tradeoff between the transient duration and the distortion of separateness has to be considered, taking into account traditional time-invariant notch filters.

In this section, the proposed second-order Q-varying digital IIR notch filter will be used to remove the frequency-constant 15-Hz interference from the ECG signal. For this purpose, two kinds of ECG signals will be used: 1) simulated and 2) real.

A. Simulated ECG Signal

In this section, the simulated ECG signal will be considered. The frequency range of this signal is stated to be from 0.05 Hz to about 15 Hz. The interference has been assumed to be 15 Hz. Therefore, the sinusoidal component, which has to be removed from the useful signal, lies on the border of the spectrum of the ECG signal. The ECG signal, simulated with the help of Matlab software, with the unwanted interference of 15 Hz is presented in Fig. 4.

Figs. 5 and 6 present the results of filtering using traditional Q-constant digital IIR notch filters. The first filter has the quality factor equal to 20. The selectivity of this filter is appropriate; however, the transient duration is too long. The second filter has the quality factor equal to 5. Here, the transient duration is acceptable; however, as one can notice in Fig. 6, the selectivity of the filter is at an unacceptable level since the filter also affects the frequency components of the useful signal. This too small selectivity manifests in cutting the peaks of the ECG signal.

The results of the Q-varying digital filtering are presented in Fig. 7. The quality factor of the filter is varied from $Q = 5$ to $Q = 20$. It is easy to notice that the use of this filter causes that both the transient duration and the selectivity of the filter

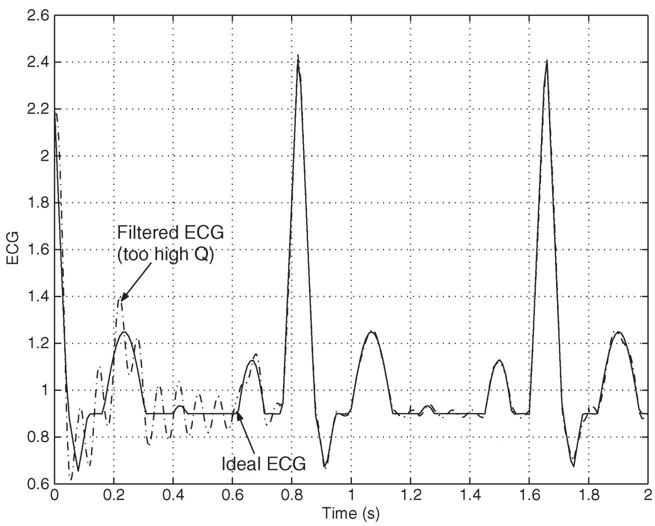

Fig. 5. Results of filtering using traditional IIR filter with too high Q.

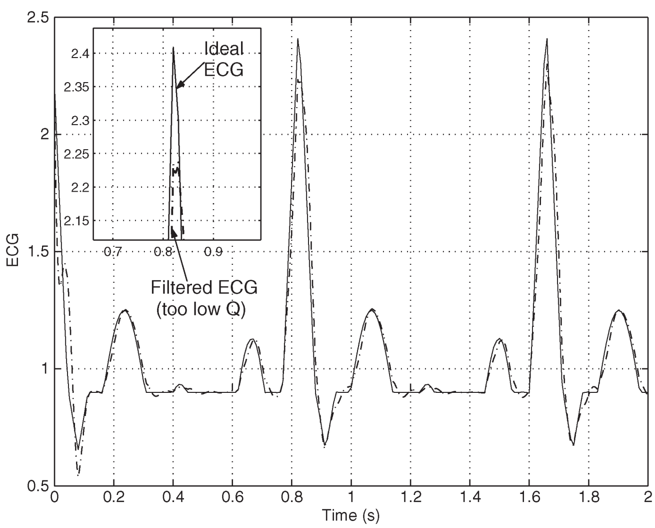

Fig. 6. Results of filtering using traditional IIR filter with too low Q.

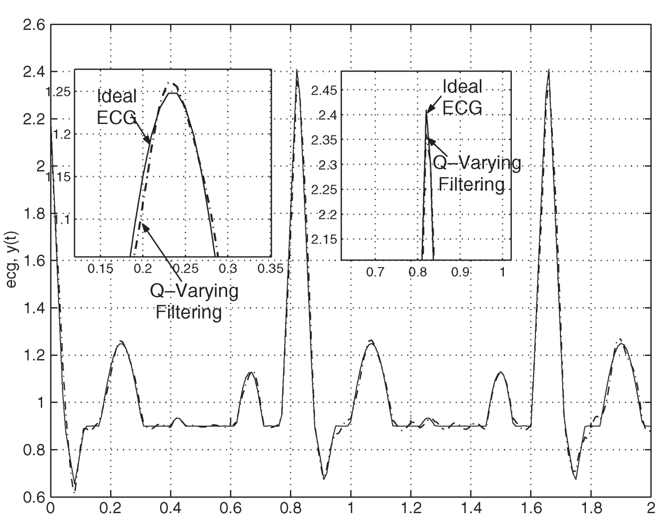

Fig. 7. Results of digital filtering using Q-varying IIR filter.

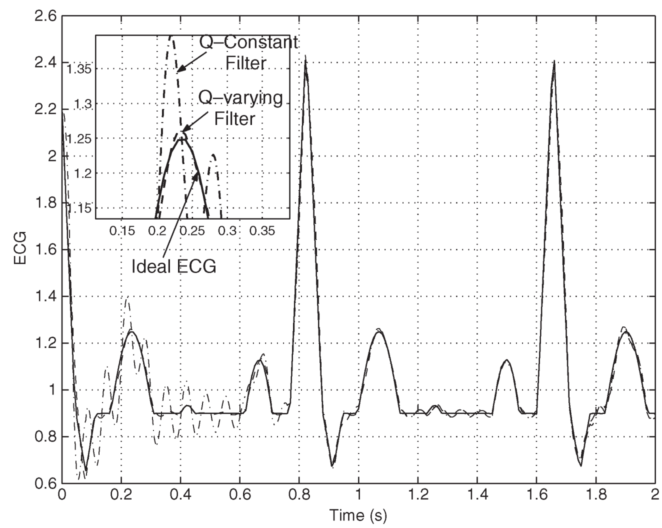

Fig. 8. Comparison of filtering results using Q-constant and Q-varying filters.

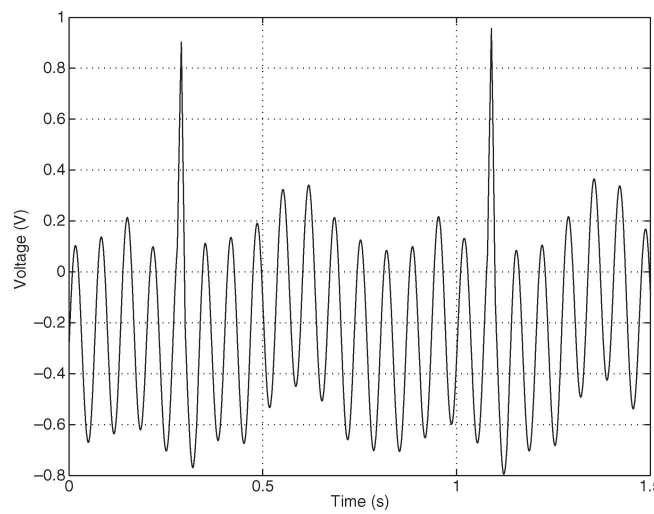

Fig. 9. Real ECG signal with 15-Hz interference.

are at an acceptable level. The comparison of the traditional Q-constant and the proposed Q-varying filter is presented in Fig. 8. Looking at the magnified peak of the ECG signal, it can be seen that the time-varying filter is significantly more effective compared with the traditional time-invariant filter.

B. Real ECG Signal

In this section, a real ECG signal (MIT-BIH Database) will be considered as a case study. The basic real ECG signal has the frequency range from 0.05 to 100 Hz. Therefore, the 15-Hz interference, which has to be removed from the useful signal, lies inside the spectrum of the ECG signal. The real ECG signal with the 15-Hz interference is presented in Fig. 9.

Fig. 10 presents the results of filtering using traditional Q-constant IIR notch filters with various orders. As mentioned in Section I, the higher the order of the filter, the longer the transient, which can be observed in Fig. 10. This observation suggests that the second-order notch filter is the best solution if the transient duration is of great importance.

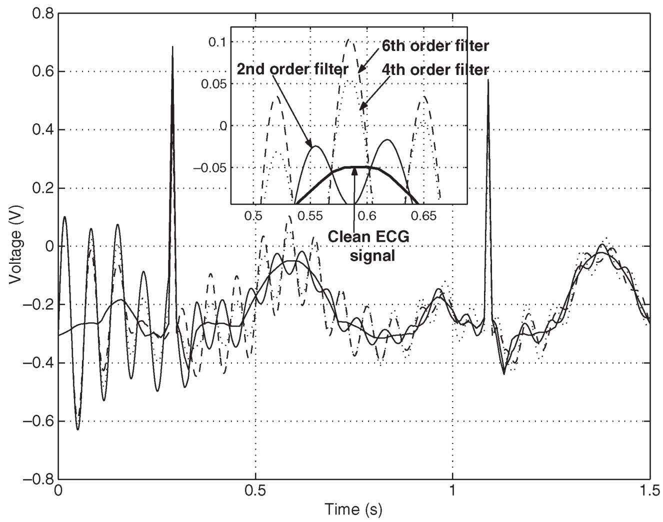

Fig. 10. Comparison of real ECG signal filtering using Q-constant notch filters.

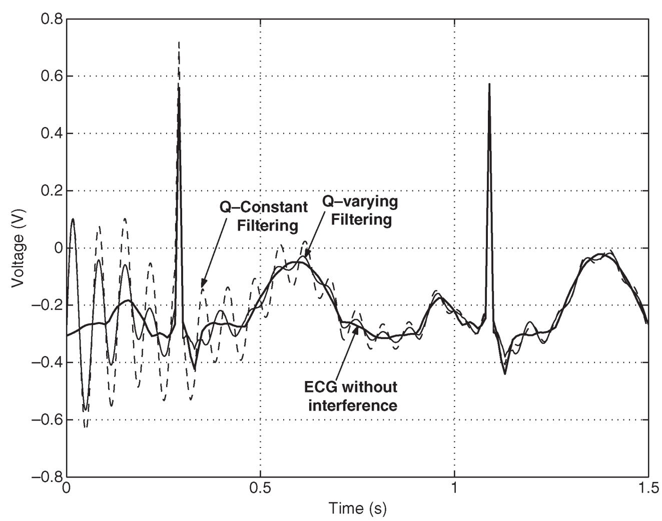

Fig. 11. Comparison of real ECG signal filtering using Q-constant and Q-varying filters.

The comparison of the traditional Q-constant and the proposed Q-varying filter is presented in Fig. 11. It can be seen that the Q-varying filter is significantly more effective compared with the traditional Q-constant filter.

V. Conclusion

In this paper, the parameter-varying technique has been used to generate a new class of digital Q-varying IIR notch filters with transient suppression. The proposed Q-varying digital notch filters possess selective magnitude response and transient response of short duration. It was demonstrated that this new class of filters achieved a considerable reduction of the duration of the transient response compared with the traditional time-invariant filter, which was used as a prototype. As an example, the proposed Q-varying digital IIR notch filter was used to remove the 15-Hz interference from the simulated and real ECG signals. The results of simulations confirmed that, using the proposed Q-varying notch IIR filter, both the transient duration and the selectivity of the filter are at an acceptable level.

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