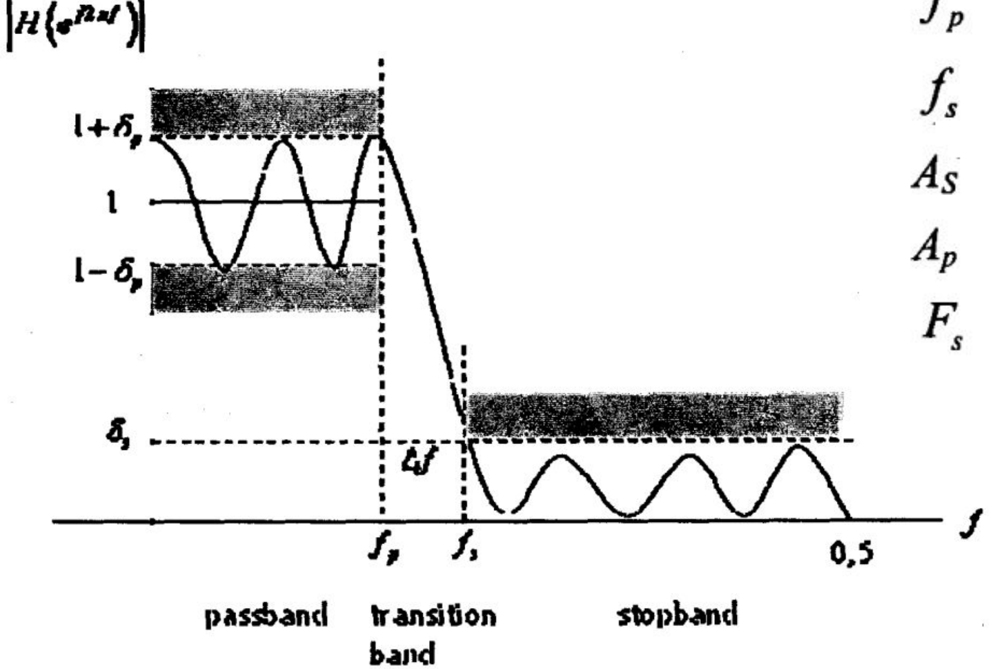
Implementasi Filter IIR

Pengolahan Sinyal

Course Outline

1. Filter Analog.
2. Desain filter IIR dari filter analog.
3. Implementasi Filter IIR.
4. Implementasi Filter FIR

Filter Non-IDeal



δ_p = deviasi *passband*

δ_S = deviasi *stopband*

f_p = frekuensi batas *passband*

f_s = frekuensi batas *stopband*

A_S = atenuasi *stopband* = $-20\log_{10}\delta_S$

A_p = *ripple passband* = $20\log_{10}(1+\delta_p)$ (dB)

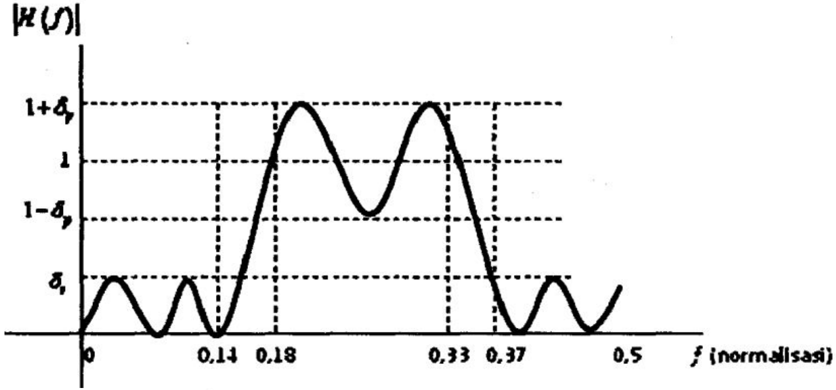
F_s = frekuensi sampling

Seperti yang telah disebutkan sebelumnya, dalam merancang filter selalu digunakan frekuensi normalisasi, yaitu f/F_s.

Contoh Merancang Spesifikasi Suatu Filter

Suatu filter FIR mempunyai spesifikasi sebagai berikut:

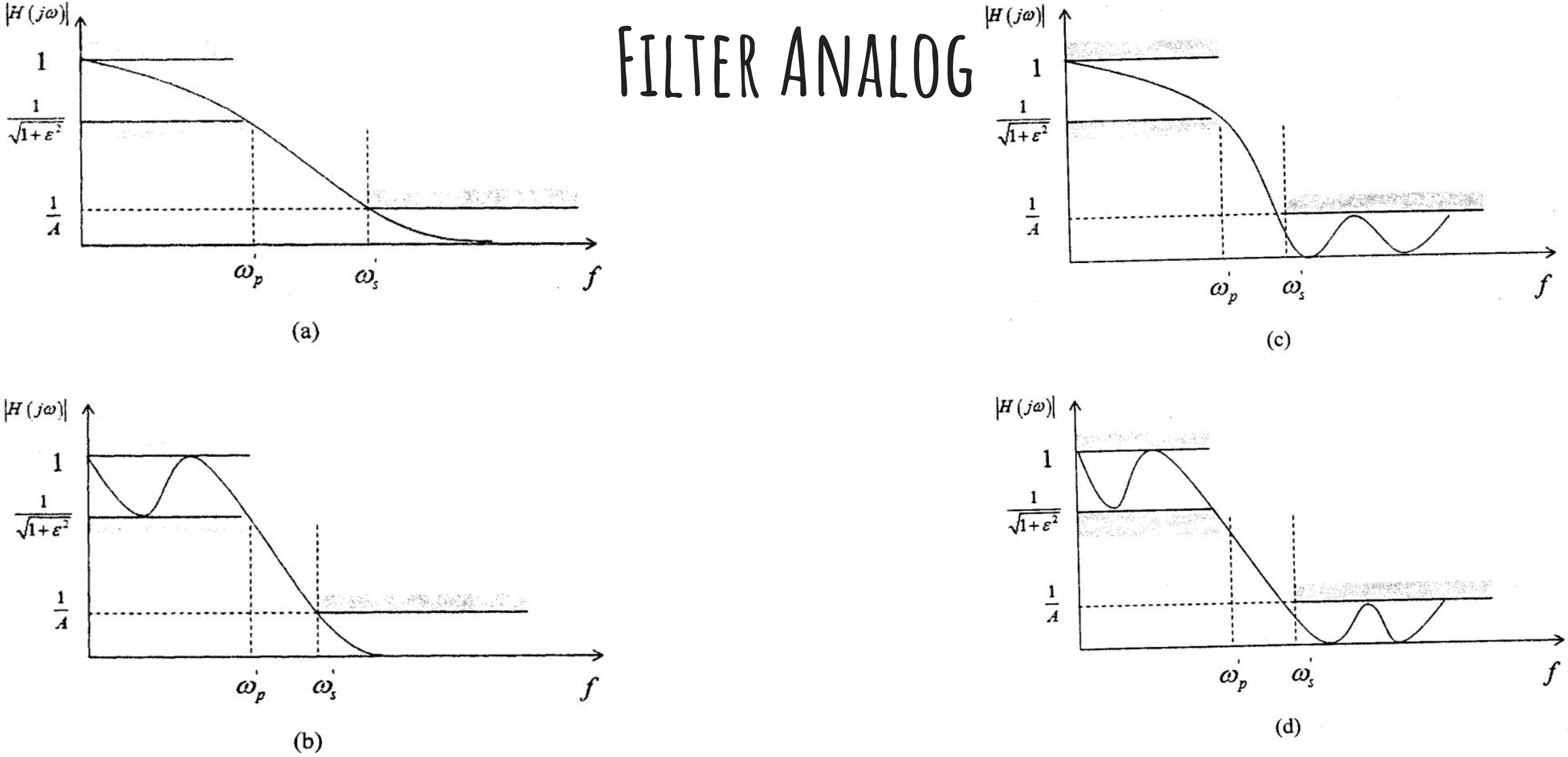
Passband	0,18 – 0,33 (normalisasi)
Lebar transisi	0,04 (normalisasi)
Deviasi *stopband*	0,001
Deviasi *passband*	0,05



Sketsa filter dengan karakteristik tersebut tampak pada Gambar 6.5. Jika frekuensi sampling diberikan adalah 10 kHz maka frekuensi sebenarnya dapat dihitung dengan mengalikan frekuensi normalisasi dengan frekuensi sampling, sebagai berikut:

Passband	1,8 – 3,3 kHz
Stopband	0 – 1,4 kHz dan 3,7 – 5 kHz
Atenuasi *stopband*	$-20\log_{10}(0{,}001) = 60$ dB
Ripple passband	$20\log_{10}(1+0{,}05) = 0{,}42$ dB

Filter Analog



Gambar 7.8 *Respons frekuensi filter analog (a) Butterworth, (b) Chebyshev tipe I,(c) Chebyshev tipe II, (d) Elliptic*

Ada dua parameter tambahan dalam filter analog, yaitu yang pertama adalah rasio transisi atau parameter selektivitas yang didefinisikan sebagai rasio dari frekuensi batas *passband* dan frekuensi batas *stopband*, atau

$$k = \frac{\omega_p'}{\omega_s'} \tag{7.40}$$

Parameter tersebut bernilai $k < 1$ untuk filter *lowpass*. Parameter yang ke dua adalah parameter diskriminasi, yaitu

$$k_1 = \frac{\varepsilon}{\sqrt{A^2 - 1}} \tag{7.41}$$

yang biasanya bernilai $k_1 << 1$.

Filter Butterworth

Respons frekuensi filter Butterworth *lowpass* diberikan sebagai

$$\left|H(\omega')\right|^2 = \frac{1}{1+\left(\dfrac{\omega'}{\omega_p^p}\right)^{2N}} \tag{7.42}$$

dengan N adalah orde dari filter tersebut dan ω_p^p is frekuensi batas 3 dB (*cutoff*) lowpass. Untuk frekuensi normalisasi maka $\omega_p^p = 1$. Orde filter dapat ditentukan dengan

$$N \geq \frac{\log\left(\dfrac{A^2-1}{\varepsilon^2}\right)}{2\log\left(\dfrac{\omega_s^p}{\omega_p^p}\right)} = \frac{\log\left(\dfrac{1}{k_1}\right)}{\log\left(\dfrac{1}{k}\right)} \tag{7.43}$$

dengan ω_s^p adalah frekuensi batas *stopband* (lihat Gambar (7.8) di atas). Fungsi alih analog filter Butterworth, $H(s)$ mempunyai *zero* di tak terhingga dan *N pole* pada posisi

$$p_l = e^{j\pi(2l+N-1)/2N} = \cos\left[\frac{(2l+N-1)\pi}{2N}\right] + j\sin\left[\frac{(2l+N-1)\pi}{2N}\right] \tag{7.44}$$

dengan $l = 1, 2, \ldots, N$.

Tabel 7.1 menunjukkan penyebut untuk filter analog prototipe Butterworth *lowpass* **normalisasi.** **Bagian** pembilangnya adalah selalu 1. Sebagai contoh filter Butterworth orde 2 mempunyai respons frekuensi $H(s)=\dfrac{1}{s^2+\sqrt{2}s+1}$.

Tabel 7.1 *Penyebut untuk filter analog Butterworth normalisasi*

Orde, N	**Penyebut**
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$\left(s^2+s+1\right)\left(s+1\right)$
4	$\left(s^2+0.76536s+1\right)\left(s^2+1.84776s+1\right)$
5	$\left(s+1\right)\left(s^2+0.6180s+1\right)\left(s^2+1.6180s+1\right)$
6	$\left(s^2+0.5176s+1\right)\left(s^2+\sqrt{2}s+1\right)\left(s^2+1.9318s+1\right)$
7	$\left(s+1\right)\left(s^2+0.4450s+1\right)\left(s^2+1.2456s+1\right)\left(s^2+1.8022s+1\right)$
8	$\left(s^2+0.3986s+1\right)\left(s^2+1.1110s+1\right)\left(s^2+1.6630s+1\right)\left(s^2+1.9622s+1\right)$

Filter Chebyshev

Respons frekuensi filter Chebyshev tipe 1 dapat ditulis sebagai

$$\left|H(\omega')\right|^2 = \frac{1}{1+\varepsilon^2 T_N^2\left(\omega'/\omega_p'\right)}$$

$$T_N(\omega') = \cos\left(N\cos^{-1}\omega'\right), \quad |\omega'| \le 1$$
$$= \cosh\left(N\cosh^{-1}\omega'\right), \quad |\omega'| > 1$$

$$T_r(\omega') = 2\omega' T_{r-1}(\omega') - T_{r-2}(\omega'), \quad r \ge 2$$

dengan $T_0(\omega') = 1$ dan $T_1(\omega') = \omega'$. Tabel 7.2 menunjukkan beberapa polinomial Chebyshev.

$$N \ge \frac{\cosh^{-1}\left(\sqrt{A^2-1}/\varepsilon\right)}{\cosh^{-1}\left(\omega_s'/\omega_p'\right)} = \frac{\cosh^{-1}(1/k_1)}{\cosh^{-1}(1/k)}$$

Tabel 7.2 *Polinomial Chebyshev* $C_N(x)$

N	$C_N(x)$
0	1
1	x
2	$2x^2-1$
3	$4x^3-3x$
4	$8x^4-8x^2+1$
5	$16x^5-20x^3+5x$

Respons magnitude filter Chebyshev tipe 2 disebut juga respons invers Chebyshev, diberikan oleh

$$\left|H(\omega')\right|^2 = \frac{1}{1+\varepsilon^2\left[\dfrac{T_N\left(\omega_s'/\omega_p'\right)}{T_N\left(\omega_s'/\omega'\right)}\right]^2}$$

Table 8.4 Chebyshev Lowpass Prototype Transfer Functions with 0.5 dB Ripple ($\varepsilon = 0.3493$)

n	$H_P(s)$
1	$\dfrac{2.8628}{s + 2.8628}$
2	$\dfrac{1.4314}{s^2 + 1.4256s + 1.5162}$
3	$\dfrac{0.7157}{s^3 + 1.2529s^2 + 1.5349s + 0.7157}$
4	$\dfrac{0.3579}{s^4 + 1.1974s^3 + 1.7169s^2 + 1.0255s + 0.3791}$
5	$\dfrac{0.1789}{s^5 + 1.1725s^4 + 1.9374s^3 + 1.3096s^2 + 0.7525s + 0.1789}$
6	$\dfrac{0.0895}{s^6 + 1.1592s^5 + 2.1718s^4 + 1.5898s^3 + 1.1719s^2 + 0.4324s + 0.0948}$

Table 8.5 Chebyshev Lowpass Prototype Transfer Functions with 1 dB Ripple ($\varepsilon = 0.5088$)

n	$H_P(s)$
1	$\dfrac{1.9652}{s+1.9652}$
2	$\dfrac{0.9826}{s^2+1.0977s+1.1025}$
3	$\dfrac{0.4913}{s^3+0.9883s^2+1.2384s+0.4913}$
4	$\dfrac{0.2456}{s^4+0.9528s^3+1.4539s^2+0.7426s+0.2756}$
5	$\dfrac{0.1228}{s^5+0.9368s^4+1.6888s^3+0.9744s^2+0.5805s+0.1228}$
6	$\dfrac{0.0614}{s^6+0.9283s^5+1.9308s^4+1.20121s^3+0.9393s^2+0.3071s+0.0689}$

Filter Elliptic

Filter Elliptic disebut juga filter Cauer, mempunyai respons magnitudo

$$\left|H\left(\omega'\right)\right|^2 = \frac{1}{1+\varepsilon^2 R_N^2\left(\omega'/\omega_p'\right)} \tag{7.62}$$

dengan $R_N\left(\omega'\right)$ adalah fungsi rasional orde N yang memenuhi $R_N\left(1/\omega'\right) = 1/R_N\left(\omega'\right)$. Orde filter dapat dicari menggunakan

$$N \geq \frac{2\log\left(4/k_1\right)}{\log\left(1/\rho\right)} \tag{7.63}$$

Contoh 7.7

Dengan pendekatan Butterworth, kita akan menentukan orde filter dengan fungsi alih filter mempunyai karakteristik *lowpass* dengan frekuensi cut-off 1-dB sebesar 1 kHz dan atenuasi minimum 40 dB pada 5 kHz.

Pertama kita akan menentukan nilai ε dan A sebagai berikut. Berdasarkan persamaan (7.42) maka didapatkan

$$\left|H\left(\omega_p^{'}\right)\right|^2 = \frac{1}{1+\left(\omega_p^{'}/\omega_p^p\right)^{2N}} = \frac{1}{1+\varepsilon^2}$$

$$\left|H\left(\omega_s^{'}\right)\right|^2 = \frac{1}{1+\left(\omega_s^{'}/\omega_p^p\right)^{2N}} = \frac{1}{A^2}$$

Frekuensi *cut-off* pada 1 dB berarti 1 dB di bawah respons magnitudo maksimum, yaitu 1, atau 0 dB. Dengan kata lain, pada frekuensi *cut-off*, respons magnitudonya adalah -1 dB.

$$10\log\left(\frac{1}{1+\varepsilon^2}\right) = -1$$

sehingga didapatkan $\varepsilon^2 = 0,25895$

Ingat bahwa atenuasi minimum adalah $-20\log(1/A)$

$$10\log\left(\frac{1}{A^2}\right) = -40$$

sehingga didapatkan $A = 10000$.

Nilai k diperoleh dengan persamaan (7.40), yaitu

$$k = \frac{\omega_p'}{\omega_s'} = \frac{1000}{5000} = \frac{1}{5}$$

Nilai k_1 diperoleh dengan persamaan (7.41), yaitu

$$k_1 = \frac{\varepsilon}{\sqrt{A^2 - 1}} = \frac{\sqrt{0,25895}}{\sqrt{10000-1}} = 0.0051$$

sehingga orde filter

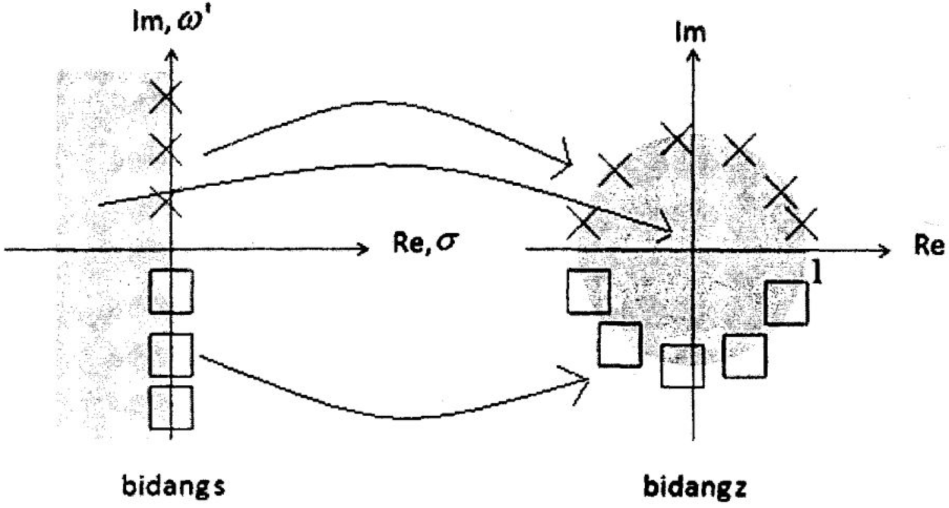
$$N \geq \frac{\log(1/k_1)}{\log(1/k)} = \frac{\log(196,0784)}{\log(5)} = 3,2797$$

Karena N harus bilangan bulat dipilih $N = 4$

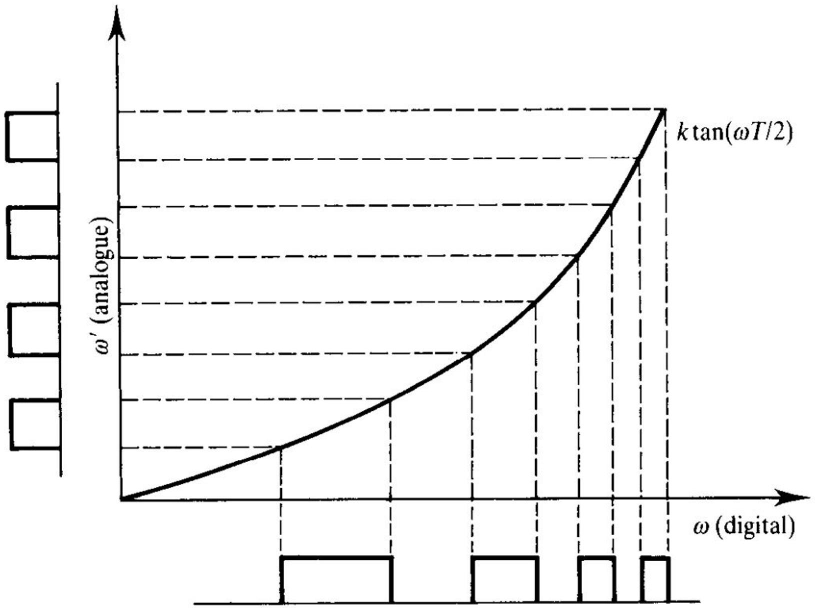
Bilinear Z-Transform

$$s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$\omega' = k\tan\left(\frac{\omega T}{2}\right), \quad k = 1 \text{ or } \frac{2}{T}$$



Gambar 7.6 *Pemetaan bidang s ke bidang z*



Secara umum langkah-langkah metode BZT adalah:

1. Gunakan spesifikasi filter digital untuk menentukan filter prototipe analog *lowpass* yang dinormalisasi. Filter yang dijadikan referensi selalu filter *lowpass*. Jenis-jenis filter lainnya dapat dicari dengan menggunakan transformasi pada filter *lowpass*.
2. Tentukan frekuensi *prewarp* dari frekuensi-frekuensi batas. Untuk filter *bandpass* dan *bandstop* terdapat dua frekuensi batas, yaitu ω_{p1} dan ω_{p2}.
3. Denormalisasi filter prototipe analog dengan menggunakan transformasi

$$s = \frac{s}{\omega_p^{'}} \qquad \textit{lowpass} \text{ ke } \textit{lowpass} \tag{7.35}$$

$$s = \frac{\omega_p^{'}}{s} \qquad \textit{lowpass} \text{ ke } \textit{highpass} \tag{7.36}$$

$$s = \frac{s^2 + \omega_0^2}{Ws} \quad \textit{lowpass} \text{ ke } \textit{bandpass} \tag{7.37}$$

$$s = \frac{Ws}{s^2 + \omega_0^2} \quad \textit{lowpass} \text{ ke } \textit{bandstop} \tag{7.38}$$

dengan $\omega_0^2 = \omega_{p1}^{'}\omega_{p2}^{'}$ dan $W = \omega_{p2}^{'} - \omega_{p1}^{'}$

4. Gunakan transformasi $s = k\left(\frac{z-1}{z+1}\right)$ untuk mendapatkan fungsi alih filter digital.

5. Nilai k sebenarnya tidak berpengaruh, karena pada dasarnya akan habis akibat pembagian.

Sebagai contoh jika diberikan $H(s)=\frac{1}{s+1}$ dengan frekuensi *cutoff* ω_p maka frekuensi *prewarp*-nya adalah $\omega_p' = k\tan\left(\frac{\omega_p}{2}\right)$ sehingga fungsi alihnya berubah menjadi

$H'(s)=H(s)\Big|_{s=s/\omega_p'}=\frac{1}{s/k\tan(\omega_p/2)+1}$. Langkah selanjutnya mengubah s menjadi $s=k\left(\frac{z-1}{z+1}\right)$ sehingga menghasilkan $H(z)=\frac{1}{[\cancel{k}(z-1)/(z+1)]/\cancel{k}\tan(\omega_p/2)+1}$.

Dengan demikian nilai k tidak berpengaruh dan untuk menyederhanakan kita ambil $k=1$.

6. Dilihat dari langkah-langkah sebelumnya maka kita harus melakukan dua kali substitusi, untuk menyederhanakan dapat dilakukan satu kali substitusi yaitu dengan $s=\cot\left(\frac{\omega_p}{2}\right)\frac{z-1}{z+1}$. Perlu diperhatikan bahwa substitusi ini berlaku hanya untuk *lowpass* ke *lowpass*.

7. Untuk filter *lowpass* dan *highpass*, orde $H(z)$ akan sama dengan orde $H(s)$, namun untuk filter *bandpass* dan *bandstop*, orde $H(z)$ akan dua kali orde $H(s)$.

Alternatives to the BZT for the bandpass and bandstop filters are the following biquadratic transformations (Gold and Rader, 1969, Gray and Markel, 1976):

$$s = \cot\left[\frac{(\omega_2 - \omega_1)T}{2}\right]\left[\frac{z^2 - 2z\cos\gamma + 1}{z^2 - 1}\right] \quad \text{lowpass to bandpass} \tag{7.23a}$$

$$s = \tan\left[\frac{(\omega_2 - \omega_1)T}{2}\right]\left[\frac{z^2 - 1}{z^2\ 2z\cos\gamma + 1}\right] \quad \text{lowpass to bandstop} \tag{7.23b}$$

$$\cos\gamma = \cos\left[\frac{(\omega_2 + \omega_1)T}{2}\right] \Big/ \cos\left[\frac{\omega_2 - \omega_1)T}{2}\right]$$

An illustration of the BZT method Determine, using the BZT method, the transfer function and difference equation for the digital equivalent of the resistance–capacitance (RC) filter. Assume a sampling frequency of 150 Hz and a cutoff frequency of 30 Hz.

Solution

The normalized transfer function for the RC filter is

$$H(s) = \frac{1}{s + 1}$$

The critical frequency for the digital filter is $\omega_p = 2\pi \times 30$ rad. The analogue frequency, after prewarping, is $\omega_p' = \tan(\omega_p T/2)$. With $T = 1/150$ Hz, $\omega_p' = \tan(\pi/5) = 0.7265$. The denormalized analogue filter transfer function is obtained from $H(s)$ as

$$H'(s) = H(s)|_{s=0.7265} = \frac{1}{s/0.7265 + 1} = \frac{0.7265}{s + 0.7265}$$

$$H(z) = H'(s)|_{s=(z-1)/(z+1)} = \frac{0.7265(1 + z)}{(1 + 0.7265)z + 0.7265 - 1}$$

$$= \frac{0.4208(1 + z^{-1})}{1 - 0.1584z^{-1}}$$

The difference equation is

$$y(n) = 0.1584y(n - 1) + 0.4208[x(n) + x(n - 1)]$$

Further illustration of the BZT method It is required to design a digital filter to approximate the following analogue transfer function:

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Using the BZT method obtain the transfer function, $H(z)$, of the digital filter, assuming a 3 dB cutoff frequency of 150 Hz and a sampling frequency of 1.28 kHz.

Solution

The critical frequency is $\omega_\mathrm{p} = 2\pi \times 150$, giving the prewarped analogue frequency of

$$\omega'_\mathrm{p} = \tan(\omega_\mathrm{p} T/2) = 0.3857$$

The prewarped analogue filter is given by:

$$H'(s) = H(s)|_{s=s/\omega'_\mathrm{p}} = \frac{1}{(s/\omega'_\mathrm{p})^2 + \sqrt{2}s/\omega'_\mathrm{p} + 1}$$

$$= \frac{\omega_\mathrm{p}'^{2}}{s^2 + \sqrt{2}\omega'_\mathrm{p}s + \omega_\mathrm{p}'^{2}} = \frac{0.1488}{s^2 + 0.5455s + 0.1488}$$

Applying the BZT gives

$$H(z) = \frac{0.0878z^2 + 0.1756z + 0.0878}{z^2 - 1.0048z + 0.3561}$$

$$= \frac{0.0878(1 - 2z^{-1} + z^{-2})}{1 - 1.0048z^{-1} + 0.3561z^{-2}}$$

Obtain the transfer function of a lowpass digital filter meeting the following specifications:

passband	0–60 Hz
stopband	>85 Hz
stopband attenuation	>15 dB

Assume a sampling frequency of 256 Hz and a Butterworth characteristic.

(1) The critical frequencies for the digital filter are

$$\omega_1 T = \frac{2\pi f_1}{F_s} = \frac{2\pi 60}{256} = 2\pi \times 0.2344$$

$$\omega_2 T = \frac{2\pi f_2}{F_s} = \frac{2\pi 85}{256} = 2\pi \times 0.3320$$

(2) The prewarped equivalent analogue frequencies are:

$$\omega_1' = \tan\left(\frac{\omega_1 T}{2}\right) = 0.906\,347;\ \omega_2' = \tan\left(\frac{\omega_1 T}{2}\right) = 1.715\,80$$

(3) Next we need to obtain $H(s)$ with Butterworth characteristics, a 3 dB cutoff frequency of 0.906 347, and a response at 85 Hz that is down by 15 dB. For an attenuation of 15 dB, $\delta_s = 0.1778$ and so from Equation 7.16b $N = 2.468$. We use $N = 3$, since it must be an integer. A normalized third-order filter is given by

$$H(s) = \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s+1}\,\frac{1}{s^2+s+1}$$

$$= H_1(s)\ H_2(s)$$

$$\cot\left(\frac{\omega_1 T}{2}\right) = \cot\left(\frac{2\pi \times 0.2344}{2}\right) = 1.103\,155$$

Performing the transform in two stages, one for each of the factors of $H(s)$ above, we obtain

$$H_2(z) = H_2(s)\big|_{s = \cot(\omega_1 T/2)[z-1)/(z+1)]}$$
$$= 0.3012 \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.1307z^{-1} + 0.3355z^{-2}}$$

which we have arrived at after considerable manipulation. Similarly, we obtain $H_1(z)$ as

$$H_1(z) = 0.4754 \frac{1 + z^{-1}}{1 - 0.0490z^{-1}}$$

$H_1(z)$ and $H_2(z)$ may then be combined to give the desired transfer function, $H(z)$:

$$H(z) = H_1(z)H_2(z) = 0.1432 \frac{1 + 3z^{-1} + 3z^{-2} + z^{-3}}{1 - 0.1801z^{-1} + 0.3419z^{-2} - 0.0165z^{-3}}$$

Highpass filter design Convert the simple lowpass filter in Example 7.5 into an equivalent highpass discrete filter. The s-plane transfer function is given by

$$H(s) = \frac{1}{s+1}$$

Solution

$$\omega_{\mathrm{p}}' = \tan(\omega_{\mathrm{p}}T/2) = 0.7265$$

Using the LPF-to-HPF transformation of Equation 7.22b, the denormalized analogue transfer function is obtained as

$$H'(s) = H(s)\big|_{s=\omega_{\mathrm{p}}'/s} = \frac{1}{\omega_{\mathrm{p}}'/s + 1} = \frac{s}{s + 0.7265}$$

The z-plane transfer function is obtained by applying the BZT:

$$H(z) = H'(s)\big|_{s=(z-1)/(z+1)} = \frac{(z-1)/(z+1)}{(z-1)/(z+1) + 0.7265}$$

Simplifying, we have

$$H(z) = 0.5792 \frac{1 - z^{-1}}{1 + 0.1584z^{-1}}$$

The coefficients of the digital filter are

$$a_0 = 0.5792 \qquad b_1 = 0.1584$$

$$a_1 = -0.5792$$

Bandpass filter design A discrete bandpass filter with Butterworth characteristics meeting the following specifications is required. Obtain the coefficients of its transfer function, $H(z)$.

passband	200–300 Hz
sampling frequency	2000 Hz
filter order	2

Solution

The prewarped passband edge frequencies are given by

$$\omega_1' = \tan\left(\frac{\omega_1 T}{2}\right) = \tan(200\pi/2000) = 0.3249$$

$$\omega_2' = \tan\left(\frac{\omega_2 T}{2}\right) = \tan(300\pi/2000) = 0.5095$$

Thus $\omega_0^2 = 0.1655$ and $W = \omega_2' - \omega_1' = 0.1846$. A first-order normalized analogue lowpass filter is required (half the order of the bandpass filter). Thus we have

$$H(s) = \frac{1}{s + 1}$$

Using the lowpass-to-bandpass transformation (Equation 7.22c) we have

$$H'(s) = H(s)\big|_{s=(s^2+\omega_0^2)/Ws} = \frac{1}{(s^2 + \omega_0^2)/Ws + 1}$$

$$= \frac{Ws}{s^2 + Ws + \omega_0^2}$$

Applying the BZT to the analogue bandpass filter we have

$$H(z) = H'(s)|_{s=(z-1)/(z+1)} = \frac{W\,(z-1)/(z+1)}{[(z-1)/(z+1)]^2 + W(z-1)/(z+1) + \omega_0^2}$$

$$= \frac{W(z^2-1)/(1+W+\omega_0^2)}{z^2 + [2(\omega_0^2-1)/(1+W+\omega_0^2)]z + (1-W+\omega_0^2)/(1+W+\omega_0^2)}$$

Substituting the values of ω_0^2 and W and simplifying we have

$$H(z) = 0.1367\,\frac{1-z^2}{1-1.2362z^{-1}+0.7265z^{-2}}$$