Appendix B
Laplace Transforms and Properties of the Fourier Transform

B.1 LAPLACE TRANSFORMS

Some useful Laplace transforms are given here. A more extensive list can be found in several tables available online.

1.	1	$\frac{1}{s}$
2.	t^n	$\frac{n!}{s^{n+1}}$
3.	$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
4.	$(1-e^{-\alpha t})$	$\frac{\alpha}{s(s+\alpha)}$
5.	$\cos \beta t$	$\frac{s}{s^2+\beta^2}$
6.	$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$
7.	$\frac{1}{\beta^2}[1-\cos(\beta t)]$	$\frac{1}{s\left(s^2+\beta^2\right)}$
8.	$t-\frac{1}{\beta}\left(1-e^{-\beta t}\right)$	$\frac{\beta}{s^2(s+\beta)}$
9.	$e^{-\alpha t}-e^{-\gamma t}$	$\frac{\gamma-\alpha}{(s+\alpha)(s+\gamma)}$
10.	$t-\frac{1}{\alpha}(1-e^{-\alpha t})$	$\frac{\alpha}{s^2(s+\alpha)}$
11.	$\left(\frac{b\beta-b\alpha+c}{2\beta}\right)e^{-(\alpha-\beta)t}+\left(\frac{b\beta+b\alpha-c}{2\beta}\right)e^{-(\alpha+\beta)t}$	$\frac{bs+c}{s^2+2\alpha s+\alpha^2-\beta^2}$
12.	$e^{-\alpha t}t[b+(c-b\alpha)t]$	$\frac{bs+c}{(s+\alpha)^2}$
13.	$*e^{-\alpha t}\left(\frac{c-b\alpha}{\beta}\right)\sin \beta t+b\cos \beta t$	$\frac{bs+c}{s^2+2\alpha s+\alpha^2+\beta^2}$
14.	$*1-e^{-\alpha t}\left(\frac{\alpha-b}{\beta}\right)\sin \beta t+\cos \beta t$	$\frac{bs+\alpha^2+\beta^2}{s(s^2+2\alpha s+\alpha^2+\beta)}$
15.	$*\frac{\omega_n}{\sqrt{1-\delta^2}}\left[e^{-\delta\omega_n t}\sin\left(\omega_n\sqrt{1-\delta^2}\,t\right)\right]$	$\frac{\omega_n^2}{s^2+2\delta\omega_n+\omega_n^2}$
16.	$*1-\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\sin\left(\omega_n\sqrt{\left(1-\delta^2\right)}\,t+\theta\right)$ where $\theta=\tan^{-1}\left(\frac{\sqrt{1-\delta^2}}{\delta}\right)$	$\frac{\omega_n^2}{s\left(s^2+2\delta\omega_n+\omega_n^2\right)}$

*Roots are complex.

B.2 PROPERTIES OF THE FOURIER TRANSFORM

The Fourier transform has a number of useful properties. A few of the properties discussed in the book are summarized here.

Linearity:

$$z(t) = ax(t) + by(t) \Rightarrow Z(\omega) = aX(\omega) + bY(\omega)$$

Differentiation:

$$\frac{dx(t)}{dt} \Rightarrow j\omega X(\omega)$$

Integration:

$$\int_{-\infty}^{t} x(\tau)d\tau \Rightarrow \frac{X(\omega)}{j\omega}$$

Time shift:

$$x(t-\tau) \Rightarrow X(\omega)e^{-j\omega\tau}$$

Time scaling:

$$x(at) \Rightarrow \frac{1}{a}X\left(\frac{\omega}{a}\right)$$

Convolution:

$$\int x(\tau)y(t-\tau)d\tau \Rightarrow X(\omega)Y(\omega)$$

Multiplication:

$$x(t)y(t) \Rightarrow \frac{1}{2\pi}\int X(v)Y(\omega - v)dv$$

where ω and v are frequencies.