Regular Expressions

- Regular Expressions are used for representing certain sets of strings in an algebric fashion.

Rules

$\boxed{a, b, c \ldots \wedge, \phi}$

1. Any terminal symbol i.e symbols ε, Σ, including $\wedge$ and ϕ are regular expressions.
   $\{\varepsilon, \Sigma, \wedge, \phi\}$

2. The union of two regular expressions is also a regular expression R_1, R_2 : $R_1 \cup R_2 \Rightarrow (R_1 + R_2)$

3. Concatation is also RE. : $R_1, R_2 \Rightarrow (R_1 \cdot R_2)$

4. Closure of RE is RE : $R \rightarrow R^*$

$a^* = \{\varepsilon / \wedge, a, aa, aaa \ldots\}$

$a^+ = \{a, aa, aaa \ldots\}$

$a^+ \cdot a = a^*$

ARDEN'S THEOREM (FA → RE)

(rev E,

Examples:

describe the following sets as Regular Expressions:

1) $\{0, 1, 2\}$: 0 or 1 or 2

$$R = 0 + 1 + 2$$

2) $\{\Lambda, ab\}$

$$R = \Lambda \quad ab$$

this is present.

3) $\{abb, a, b, bba\}$: abb or a or b or bba

$$R = abb + a + b + bba.$$

4) $\{\Lambda, 0, 00, 000 \ldots\}$: $R = 0^*$ all strings that can be formed with 0 including empty symbol.

5) $\{1, 11, 111, 1111, \ldots\}$: $R = 1^+$

Identities of Regular Expression

1) $\phi + R = R$

ϕ = empty set $\therefore$ $\phi \cup R = R$

2) $\phi . R + R \phi = \phi$

3) $\varepsilon R = R \varepsilon = R$ $\qquad$ $(\Lambda = \varepsilon)$

4) $\varepsilon^* = \varepsilon$ and $\phi^* = \varepsilon$

5) $R + R = R$

6) $R^* . R^* = R^*$

7) $RR^* = R^* R$

8) $(R^*)^* = R^*$

9) $\varepsilon + RR^* = \varepsilon + R^* R = R^*$

10) $(PQ^*)P = P(QP)^*$

11) $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

12) $R(P+Q) = RP + RQ$ and $(P+Q)R = PR + QR$.

THEOREM (FA

Q Design RE for following language over Σ,

1) Language accepting string of length exactly 2.

$L_1 = \{aa, ab, ba, bb\}$

$R_1 = aa + ab + ba + bb$

$a(a+b) + b(a+b) \Rightarrow \boxed{(a+b)(a+b)}$

2) Language accepting of length atleast 2.

$L_2 = \{aa, ab, ba, bb, aaa, bbb, abab \ldots\}$

abba

$R_2 = \underbrace{(a+b)(a+b)}_{\text{atleast } 2} (a+b)^*$

anything more than length 2. $\epsilon^*)$

3) Language accepting string of length atmost 2.

$L_3 = \{\epsilon, a, b, aa, bb, ab, ba\}$

$R_3 = \epsilon + a + b + aa + ab + bb + ba$

$= (\epsilon + a + b)(\epsilon + a + b)$

)+RP]

①

$\frac{1}{2}$ $a, b, c \to 0, 1, 2$ ab ba

$(a+b+c)^* a (a+b+c)^* b (a+b+c)^* + (a+b+c)^* b\, c$

$(0+1)^* \; 1 \; 0|1 \; 0|1$ _ _ _ _ _ _ _

$(0+1)^* 1 (0+1)(0+1)^{a}$ _ _ _

$(0+1)^{9}$

101. Σ^1 Σ^2 Σ^3.

$(0^* 1^* 00)^* 0^* 1^*$

$1\, \cancel{10}\, 0\, 1$

$\varepsilon, 0, 000$ $\varepsilon, 0$.

$1\; 1\boxed{1\;0\;1}$ $\underline{0100}$

$0^* = \varepsilon$.

$\{a^n b^{\boxed{2m+1}} : n \geq 0 \;\&\; m \geq 0.\}$

$(0^* 100)^*$

$\varepsilon, 0100, 01000100, 0$.

$a^0 b^{2\times0+1}$

$a^0 b^1$

$a^1 b^3$

$a^2 b^5$

$a^3 b^7$

a^4

$(aa)^*$ $(bb)^* b$.

0, aa, aaaa

bb b

bbbb b.

bbbbbb b.

ARDENS ...essions

ab ba

$(a+b)^*$

ϵ, a, b, aa, bbb, ab

R^*R

$P+$