Homework #2b: Power Law Models

Math 4334: Mathematical Modeling
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This homework will have you look at six data sets from a variety of academic disciplines. For each data set, an appropriate transform involving logarithms (i.e., either a log/log plot or one of the semi-log plots) can make the data appear linear. This will allow you to construct an empirical model relating the two included variables. For each data set, construct and state a reasonable empirical model, including all fitted model parameters. Then answer the problem-specific follow-up prompt. Your answer to each question should consist of a plot, a paragraph describing your modeling process, and a discussion of the follow-up question.

Problem #1. Demographics. The file `us-population-1610-2020.xlsx` contains data on the *date*, and the population of the US between 1610 and 2020. Find a fit for the for the data points between 1660 to 1860.[1]

Problem #2. Aerospace Engineering. The file `rocket-delta-v.xlsx` contains data on the fuel ratio of a rocket (the ratio of total weight to payload weight), and the change in velocity obtained by burning all of that fuel.[2]

Problem #3. Astronomy. The file `planet-orbital-periods.xlsx` contains data on the orbital *radius* of the planets, and their orbital *period*.[3]

Problem #4. Biology. The file `fish-swimming-speeds.xlsx` contains data on the *mass* of many species of fish, and their *maximum swimming speed*.[4]

Problem #5. Physiology. The file `animal-mass-heartrates.xlsx` contains data on the *mass* of several different animals, and their *heartrate*.[5]

Problem #6. Anthropology. The file `us-city-populations-2017.xlsx` contains data on the *population ranking* of US cities, and the population of cities at those ranks.[6]

[1] Source: https://en.wikipedia.org/wiki/Demographic_history_of_the_United_States
[2] Source: I made this data up, but it is consistent with the physics of rocket propulsion using typical fuel sources.
[3] Source: https://nssdc.gsfc.nasa.gov/planetary/factsheet/
[4] Source: M.R. Hirt et al., Nature Ecology and Evolution 2017. DOI: 10.1038/s41559-017-0241-4.
[5] Source: A.J. Clark, Comparative Physiology of the Heart (New York: Macmillan, 1977), p. 99.
[6] Source: https://www.census.gov/data/tables/time-series/demo/popest/2010s-total-cities-and-towns.html