MATH 4334: Mathematical Modeling (HW #3b)

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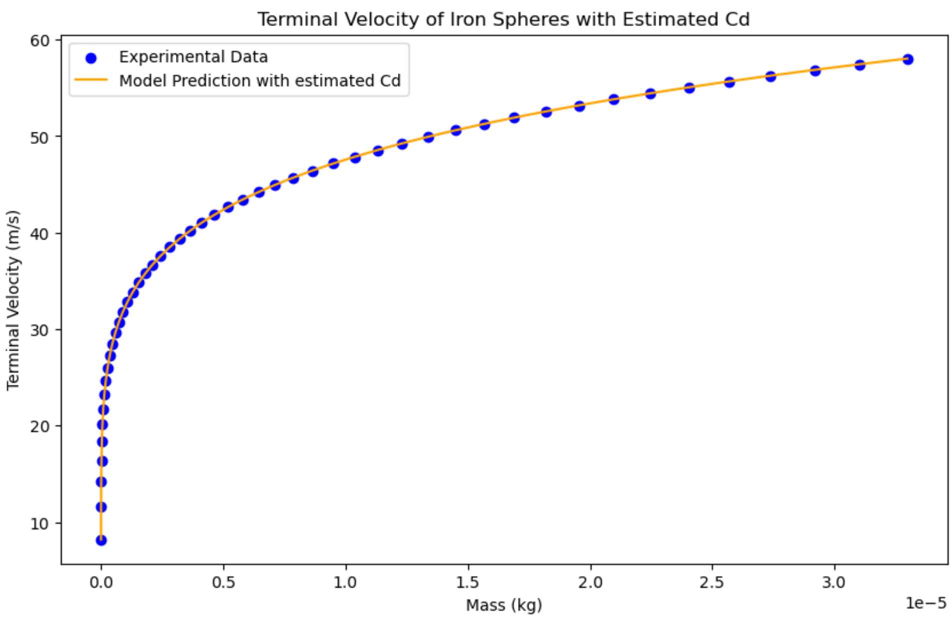
Exercise 1

Beginning with a force-balance argument equating gravitational force to drag force, we achieve $mg = \frac{1}{2}\rho v^2 AC_d$. Our model predicting velocity can then be rearranged as $v = \sqrt{\frac{2mg}{\rho AC_d}}$.

The mass is written in grams, so I converted it to kilograms. Then, I used the kilogram mass quantity to determine the radius of the sphere using the equation $r = (\frac{3m}{4\pi\rho_{iron}})^{1/3}$, then using that radius, I determined the quantity of A for each data point with the equation $A = \pi r^2$.

Now that I have these quantities, I am now able to determine the drag coefficient C_d. I reconstructed the velocity equation above to arrive at the equation $C_d = \frac{2mg}{\rho Av^2}$. For each sphere, I found the drag coefficient to be equal to 0.049907. Plugging this equation now into predicting terminal velocity for each metal sphere, I achieved the following graph.



Exercise 2

(1)

Equating kinetic energy of a car with the work needed for the vehicle to stop, we achieve the following set-up: $\frac{1}{2}mv_o^2 = F \cdot d$. Force is equivalent to the product of mass and acceleration, so we can rewrite the equation above as $\frac{1}{2}mv_0^2 = m \cdot a \cdot d$. Rearranging the equation above, we can arrive at the equation $d = \frac{v_o^2}{2a}$.

This shows that the braking distance is directly proportional to the square of the velocity and inversely proportional to the deceleration. It's important to note that this equation assumes a constant deceleration and no other forces acting on the vehicle (like air resistance or rolling resistance), and that the road surface conditions remain constant during braking.

(2)

The physical model is well-grounded in physics and represents how these physical properties work in the car breaking scenario. Because the physics model particularly is related to how the car operates as a body over time in nature while breaking, we should remove the linear term. We should remove the linear term, because its existence is not grounded in physics, despite the fact that an F-test supports it staying in our model.

(3)

Our equation for d only contains a single squared term and does not have a constant term from a physical standpoint that would make sense to exist in our model. Distance is governed solely by starting velocity and acceleration in the equation and should not include any other additional term in the theoretical scenario.

(4)

Included in our model is initial velocity as a squared term in the numerator along with acceleration multiplied by a factor of 2 in the denominator. that leads to distance being measured in terms of meters based on how velocity and acceleration are measured, which makes perfect sense. The absence of mass is due to friction not currently being considered in our model. I believe this also is a superior showing of how our model performs in comparison to our previous experiment that had both a linear and squared term for velocity. The prior model may have captured friction, but it might have captured more noise as well or overfit which is why I believe it makes more intuitive sense to use this model grounded in physics.

Exercise 3

$$
\begin{aligned}
&\vec{\boldsymbol{r}}'' = \langle -\omega^2 Rcos(\omega t), -\omega^2 Rsin(\omega t)\rangle \\
&\implies \vec{\boldsymbol{a}} = \vec{\boldsymbol{r}}'' \\
&\implies |\vec{\boldsymbol{a}}| = |\vec{\boldsymbol{r}}''| \\
&\implies \frac{GM}{R^2} = \sqrt{(-\omega^2 Rcos(\omega t))^2 + (-\omega^2 Rsin(\omega t))^2} \\
&\implies \frac{GM}{R^2} = \sqrt{(-\omega^4 R^2 (cos(\omega t))^2 + sin(\omega t))^2} \\
&\implies \frac{GM}{R^2} = R\omega^2 \\
&\implies GM = R^3 (\frac{2\pi}{T})^2 \\
&\implies GM = R^3 \frac{4\pi^2}{T^2} \\
&\implies T = \sqrt{\frac{4\pi^2 R^3}{GM}}
\end{aligned}
$$

From the Internet, I have found that $G = 6.674 \times 10^{-11}$, and $M = 1.989 \times 10^{30}$. Now, considering that I need to do conversions for km to m and seconds to days, these are the predictions for the orbital period of each of the Sun's planets that I have seen in Earth days:

'Mercury': 87.93539486428945,

'Venus': 224.63983095889623,

'Earth': 365.21050633813775,

'Mars': 686.6925888107595,

'Jupiter': 4336.281538534151,

'Saturn': 10832.842237573854,

'Uranus': 30728.115638267784,

'Neptune': 60152.6699804304,

'Pluto': 90600.40845354972

A clear indicator of correctness is that the Earth has an orbital period of about 365.21. This makes sense, given years are 365 days and we celebrate leap years every 4 years. The predictions are also super similar to my predictions in HW 2b, and my model is similar, because the radius is to the 3/2 power, and with the power law conversion, everything else is written as a coefficient that serves well as a predictor.