Homework #3b: Scaling and Balancing

Math 4334: Mathematical Modeling
Dr. Scott Norris

Problem #1. Terminal Velocity, revisited. Consider the attached data on the relationship between the mass of a metal sphere and its terminal falling velocity. Using a force balance argument, build a model attempting to explain this data set. Assume that the spheres have the same density as pure iron. Construct a plot containing both the data and the predictions of your model. [You should be able to achieve very good agreement].

Problem #2. Braking Distance, revisited. Consider again the data on braking distance from HW#2a. Starting with an energy balance argument, obtain a theoretical prediction for the relationship between braking distance and velocity. Specifically, consider the following:

1. Use an energy balance argument (initial kinetic energy = work done during braking) to obtain a *theoretical* model of braking distance.
2. At this point your physical model should have one term (a quadratic), whereas your empirical model from unit #2a has two terms (a linear and a quadratic). Which do you think is more accurate? In other words,
   (a) should you eliminate the linear term from your empirical model? If so, why? Is this supported by an F-test?
   (b) or should you add a linear term to your physical model? If so, what additional effect would it describe?
3. Recall that in Homework #2a, you eliminated the constant term in your polynomial fit, based on the results of an F-test. From a physical perspective, why does that make perfect sense?
4. Based on the above considerations, decide upon a final model for the data. List ***and interpret*** all fit parameters in your model in terms of their physical meaning. Do the values of these parameters make seem reasonable? Do the differences in parameter values between the first and second experiment also seem reasonable? Discuss.

Problem #3. Planetary Orbital Periods, revisited. Consider again the data on planetary orbits from homework #2b. Using ideas of geometric similarity and force balance, obtain a theoretical prediction for the relationship between distance and orbital period. You should recall a few things from physics and earlier math classes:

- assume orbits are geometrically similar, perfect circles of radius R, centered on the sun
- a circle of radius R can be parameterized by the equations $\vec{\mathbf{r}}(t) = \langle R\cos(\omega t), R\sin(\omega t)\rangle$[1]
- the acceleration $\vec{\mathbf{a}}$ of a particle on a path $\vec{\mathbf{r}}(t)$ is given by the expression $\vec{\mathbf{a}}(t) = \vec{\mathbf{r}}''(t)$
- the acceleration provided by the sun is given by Newton's Law of Gravitation: $|\vec{\mathbf{a}}| = \frac{GM}{R^2}$

By equating the observed acceleration with the known acceleration, derive the relationship between the period $T = \frac{2\pi}{\omega}$ of a planet and its orbital radius R, in terms of its star's mass M. Look up the values of G and M to obtain a theoretical prediction for orbital periods of the Sun's planets (you may have to perform a unit conversion). Compare this prediction to the model you obtained empirically in HW#2b. Discuss.

[1] Here, ω is the *frequency*, given in radians of orbit per day. The *period* – days per orbit – satisfies $T = \frac{2\pi}{\omega}$.