MATH 4334: Mathematical Modeling (HW #2b)

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Exercise 1

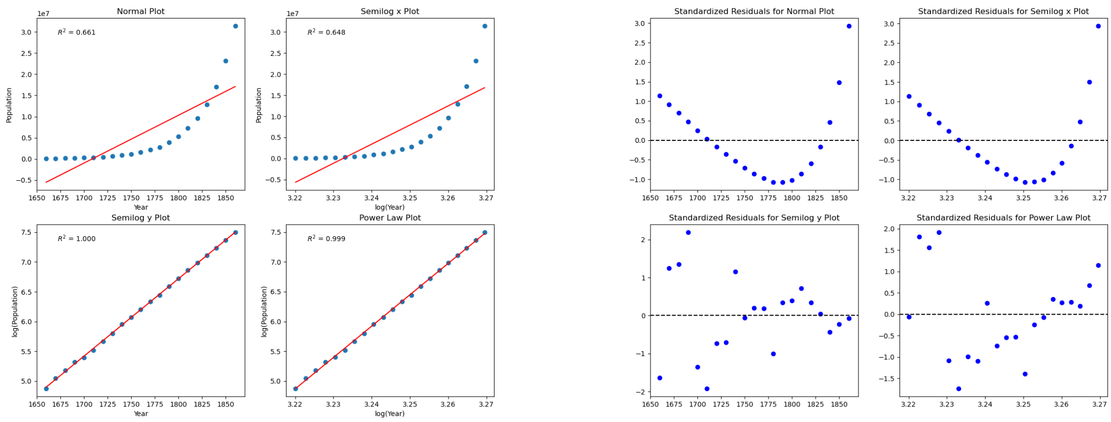

Figure 1: Log Graphs (left) and Residuals (right) for Problem 1

The semilog-y plot and the power law plot both have strong fits to the data. Any performance boost will be extremely minimal when making our plot a power law plot in comparison to the semilog-y plot, and the EVR value is also rounded to be slightly higher but probably insignificant. Additionally, there's identical residual plots.

Hence, I will prefer a semilog-y plot in this scenario, since it's less complex than the power law plot. The formula is as follows:

$$\begin{aligned}
\log_{10}(\widehat{Population}) &= 0.012992 \cdot (Years\ since\ 1610) + 4.250554 \\
\implies \widehat{Population} &= 17805.496 \cdot 10^{0.013 \cdot (Years\ since\ 1610)} \\
\implies \widehat{Population} &= 17800 \cdot 10^{\frac{1}{100} \cdot (Years\ since\ 1610)}
\end{aligned}$$

I've opted for slight modifications to better reflect the semilog-y plot, which seems more intuitive and likely mirrors the real-world scenario more closely. This preference comes from the notion that natural processes often adhere to simpler and more elegant rules.

Exercise 2

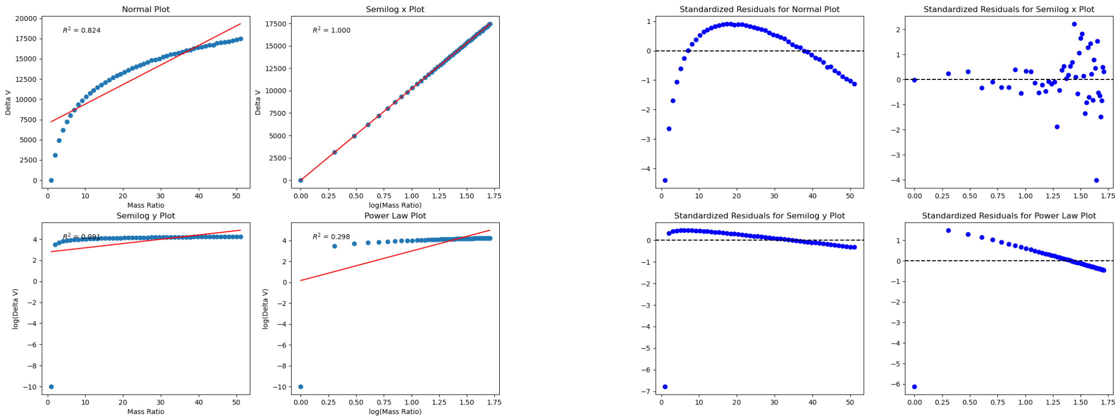

Figure 2: Log Graphs (left) and Residuals (right) for Problem 2

The EVR of the semilog-x plot is almost 1, in comparison to the other plots which all have strong patterns appearing in residuals. The spread in the residual makes sense, given we are looking at relative certainty as opposed to absolute uncertainty. All other plots have much smaller EVR's and patterns in residuals. Hence, I will choose the following function to represent ΔV as a function of the Mass Ratio.

$$\hat{\Delta V} = 10225.290305 \cdot \log(Mass\ Ratio) + 0.636494$$

Exercise 3

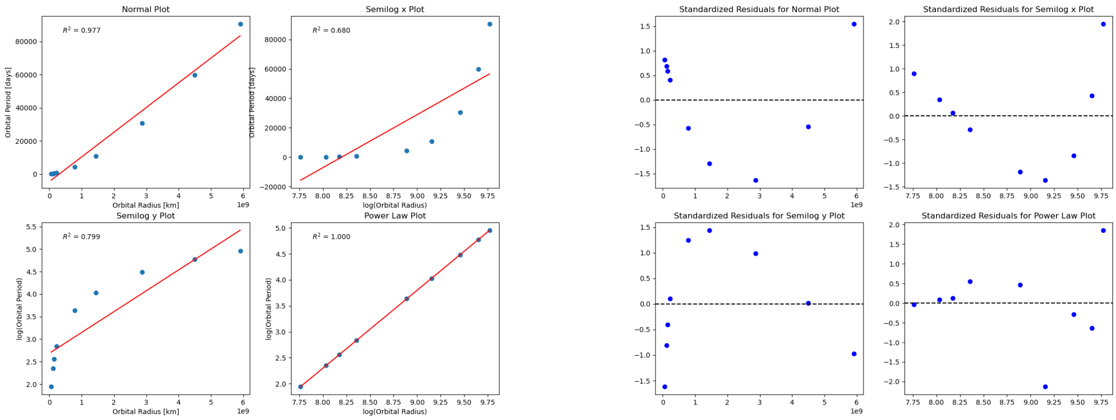

Figure 3: Log Graphs (left) and Residuals (right) for Problem 3

The power law plot vastly outperforms the rest of the plots, as it fits the data clearly and creates randomness in the residual unlike the rest of the plots. Hence, I will choose the following function to represent the orbital period of a planet based on the radius in km.

$$\begin{aligned} \log_{10}(\widehat{Orbital\ Period}) &= 1.498799523351 \cdot \log_{10}(Radius) + -9.690180361864 \\ \implies \widehat{Orbital\ Period} &= 0.000000000204 \cdot Radius^{1.498799523351} \\ \implies \widehat{Orbital\ Period} &= 2 \cdot 10^{-10} \cdot Radius^{\frac{3}{2}} \end{aligned}$$

I've opted for slight modifications to better reflect the power law plot, which seems more intuitive and likely mirrors the real-world scenario more closely. This preference comes from the notion that natural processes often adhere to simpler and more elegant rules.

Exercise 4

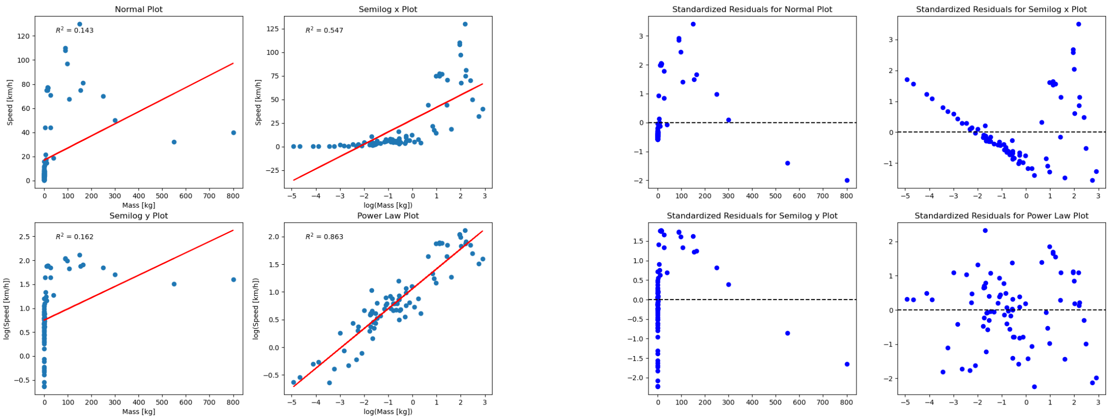

Figure 4: Log Graphs (left) and Residuals (right) for Problem 4

The power law plot vastly outperforms the rest of the plots. All other models have stronger patterns in residuals, whereas the power law plot has a random residual with a high EVR for capturing maximum fish speeds based on their mass. Hence, I will choose the following function to represent the orbital period of a planet based on the radius in km. The equation can represented as:

$$\log(Max\ Speed) = 0.359453289273 \cdot \log(Mass) + 1.057278563421$$
$$\implies Max\ \hat{S}peed = 11.409813966057 \cdot Mass^{0.359453289273}$$
$$\implies Max\ \hat{S}peed = 11.41 \cdot Mass^{1/3}$$

I've opted for slight modifications in the power coefficient to better reflect the power law plot, which seems more intuitive and likely mirrors the real-world scenario more closely. This preference comes from the notion that natural processes often adhere to simpler and more elegant rules.

Exercise 5

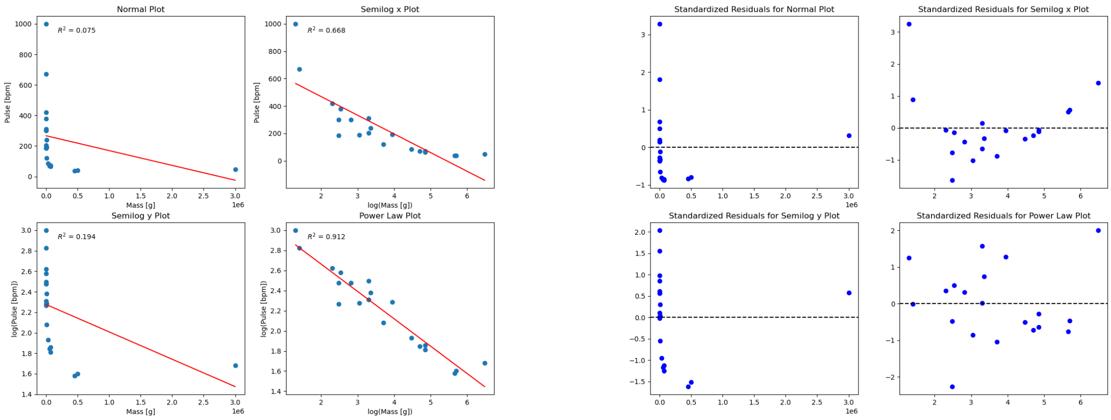

Figure 5: Log Graphs (left) and Residuals (right) for Problem 5

The power law plot vastly outperforms the rest of the plots. All other models have stronger patterns in residuals, whereas the power law plot has a random residual with a high EVR for capturing pulse based on their mass. Hence, I will choose the following function to represent the pulse of an animal based on its mass. The equation can represented as:

$$\log(\hat{Pulse}) = -0.271993914151 \cdot \log(Mass) + 3.207344979290$$
$$\implies \hat{Pulse} = 1611.925550429522 \cdot Mass^{-0.271993914151}$$
$$\implies \hat{Pulse} = 1611.926 \cdot Mass^{-1/4}$$

I've opted for slight modifications in the power coefficient to better reflect the power law plot, which seems more intuitive and likely mirrors the real-world scenario more closely. This preference comes from the notion that natural processes often adhere to simpler and more elegant rules.

Exercise 6

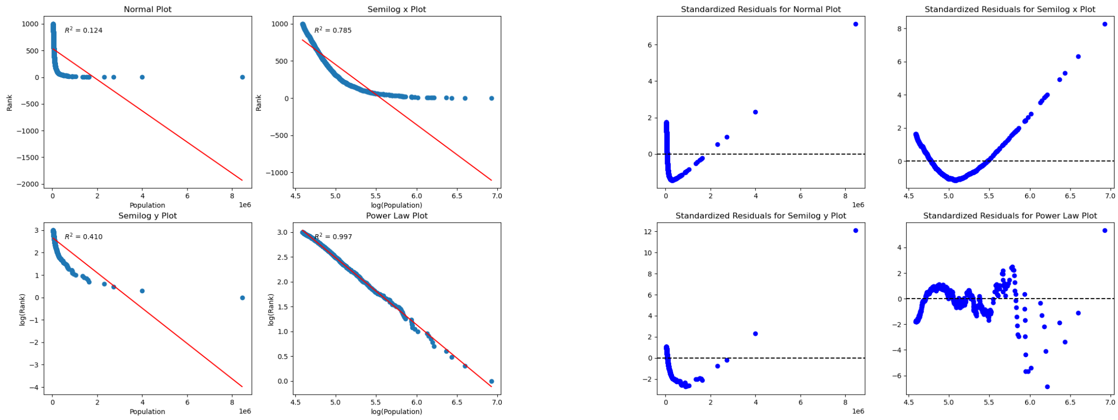

Figure 6: Log Graphs (left) and Residuals (right) for Problem 6

The power law plot vastly outperforms the rest of the plots. There unfortunately still exists a pattern in the residual for the power law plot, but in the actual plot of the data, it appears to be very strong in comparison to the rest of the models following the pattern in the data with an EVR of 0.997. Hence, I will choose the following function to represent the rank of a city's population size based on the actual size of the city. The model equation is:

$$\log(Rank) = -1.348047636819 \cdot log(Population) + 9.222291773199$$

$$\implies Rank = 1668367699.958117 \cdot Population^{-1.348047636819}$$

$$\implies Rank = 1668367700 \cdot Population^{-4/3}$$

I've opted for slight modifications in the power coefficient to better reflect the power law plot, which seems more intuitive and likely mirrors the real-world scenario more closely. This preference comes from the notion that natural processes often adhere to simpler and more elegant rules.