Speed vs. Range: Exploring the Dynamics of the Tesla Roadster's Performance in the Context of Electric Vehicles

Reece Iriye

October 2, 2023

1: Introduction

Electric vehicles (EVs) represent a transformative shift in the automotive industry, promising a future of sustainable transportation. However, the adoption of EVs is often stalled by concerns about their range, known as "range anxiety." This concern stems from the fear that an electric vehicle will not have enough battery power to reach its destination, a problem exacerbated by factors like battery capacity, charging infrastructure, and efficiency under varying driving conditions [1]. As the automotive industry evolves, addressing these concerns has become paramount to increase the acceptance and usability of EVs. The Tesla Roadster, as a pioneering electric sports car, plays a crucial role in changing the narrative around EVs, demonstrating that they can be not only environmentally friendly but also high-performing and reliable over long distances.

The Tesla Roadster came out with its first model in 2008 [2]. It stands as a counterpoint to traditional concerns about electric vehicle range. Its advanced battery technology demonstrates that EVs can offer substantial driving ranges, challenging the preconceptions of limited mobility [2]. This paper aims to explore the Tesla Roadster's range variability in relation to speed, a crucial factor influencing an electric vehicle's effective range. By examining detailed data on the Roadster's performance at various speeds, as well as at other important mechanical variables, I intend to develop a comprehensive understanding of how speed and these other variables impact range. This analysis is critical in addressing the broader issue of range anxiety and in contributing to the development of more efficient and reliable electric vehicles.

2: A Basic Polynomial Fit

Before diving deep into the polynomial fits, it is essential to closely examine the raw data set that captures the Tesla Roadster's performance across different speeds. The data represents real-world figures released by Tesla themselves, showcasing the vehicle's range capabilities under varying velocities. This initial analysis is critical, as it lays the groundwork for subsequent analytical models and provides a baseline against which the accuracy and relevance of those models can be measured. The raw data plot will reveal the fundamental relationship between speed and range before any complex modeling is applied, offering insights into the Roadster's performance characteristics.

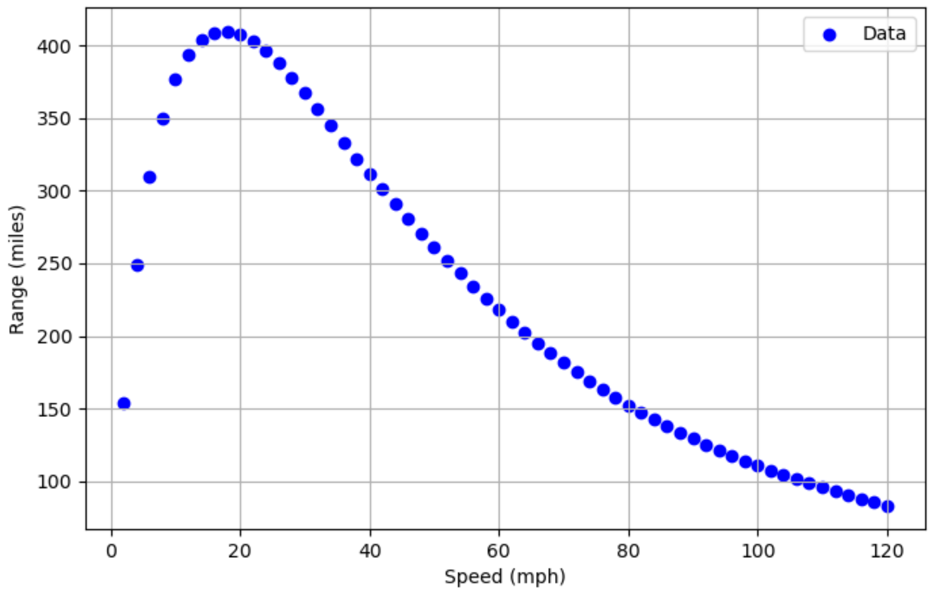


Figure 1: *Tesla Roadster Range (miles) vs. Speed (mph)*

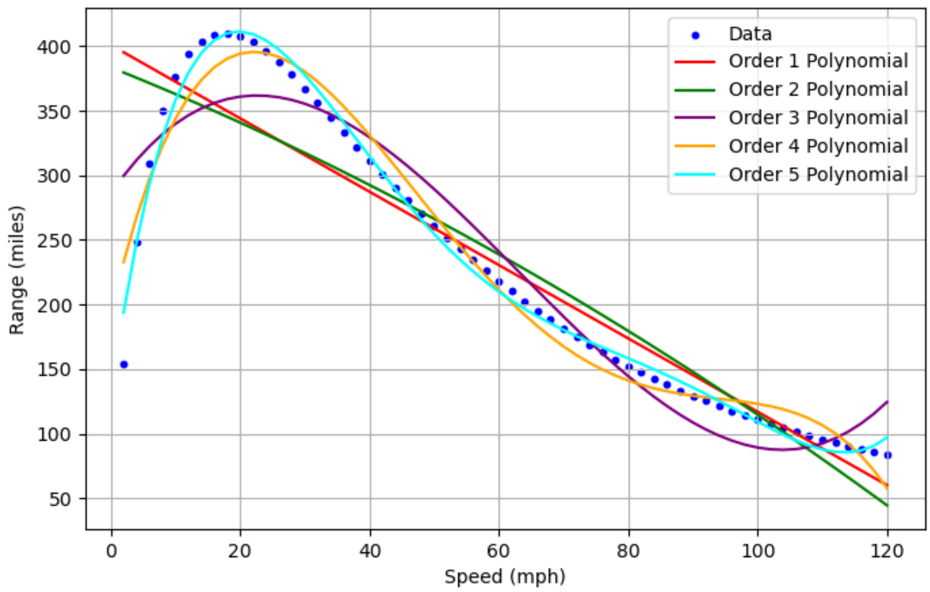
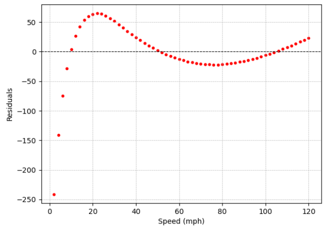
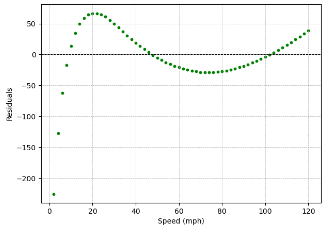


Figure 2: *Polynomial Fits to the Range vs Speed Data*

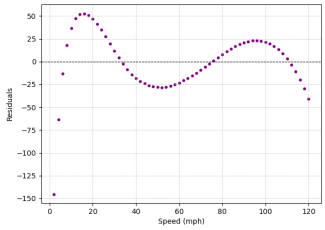
The distribution of these points in **Figure 1** showcases a non-linear trajectory, where the range increases from 150 miles to just over 400 miles when the speed increases from 2 mph to 18 mph, then once the roadster exceeds beyond the 20 mph speed, the range decreases and begins to converge. With this understanding of the underlying relationship between speed and range, I proceed to plot polynomial fits in **Figure 2**, which mathematically models and encapsulates this nuanced relationship. In creating a model to fit the Tesla Roadster data from **Figure 1**, I will develop the capability to interpolate likely Tesla Roadster travel ranges based on the speed at which they travel.



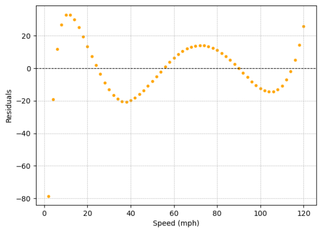
(a) *Degree 1 Residuals*



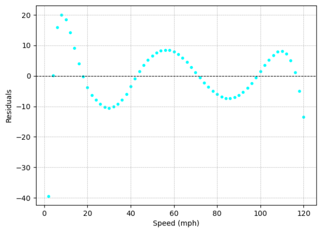
(b) *Degree 2 Residuals*



(c) *Degree 3 Residuals*



(d) *Degree 4 Residuals*



(e) *Degree 5 Residuals*

Figure 3: *Residual Plots for Polynomial Fits of Degree 1 to 5 Predicting Range from Speed*

In progressing from a first-degree to a fifth-degree polynomial model, there exists a visual improvement in the model's ability to mirror the Tesla Roadster's range data. This enhancement is evident in the progression in **Figure 2** from the first to fifth order polynomial, where higher-order polynomials fit more strongly to the initial rise and the noticeable plateaued convergence trend as speed increases seen in the data in **Figure 1**. Specifically, the fifth-degree polynomial—illustrated in **Figure 2(e)**—not only shows a reduction in the residual magnitudes but also maintains a consistent pattern, indicating a robust fit that transcends mere data overfitting. Additionally, while the structures in the residuals from **Figure 3(a)** to **Figure 3(e)**, the magnitudes in the residuals decrease significantly, showcasing a stronger hug to the curve itself in the higher degree polynomials.

$$
\begin{aligned}
R(v) \approx\ & 0.00000051 \cdot v^5 \\
& - 0.00018335 \cdot v^4 \\
& + 0.02460346 \cdot v^3 \\
& - 1.49905947 \cdot v^2 \\
& + 35.68312506 \cdot v \\
& + 128.208929
\end{aligned}
\tag{1}
$$

A sequence of Fisher tests evaluating the models seen in **Figure 2** and **Figure 3** statistically validates the fifth-degree polynomial's superior performance in comparison to all of the other models. This superiority is not a result of overfitting—where the model would be capturing random noise rather than the true underlying trend seen in the Tesla roadster data—but instead because it succeeds in capturing aspects of the vehicle's range dynamics that simpler models overlook such as the initial increase and steady plateau. It is important to note that this modeling approach is centered on interpolation within the observed data range. While this chosen model excels in this context, I would caution against using it for extrapolation beyond the scope of the recorded data, as its predictive power is likely to decrease outside the established range of the data at hand. It would be impractical to evaluate speeds outside of the 2-120 mph range anyways, as these ranges would not be practical to evaluate in the context of understanding the performance of the Tesla Roadster EV in the context of potential range based on speed.

3: A Transform from Efficiency to Consumption

Transitioning from the analysis of the Tesla Roadster's range, I now pivot our focus to modeling its energy consumption patterns in terms of kWh per mile across varying speeds. This shift offers a more direct measure of the vehicle's efficiency, providing a nuanced understanding of how speed affects energy usage. The underlying premise is that by understanding the consumption rate, we can indirectly but accurately gauge the vehicle's range under different driving conditions [3]. Moreover, while ranges are more interpretable to the average person, understanding how speed affects battery consumption provides a stronger comparison metric and objective for an engineer to minimize and evaluate.

To achieve this, I reinterpreted the range data to represent energy consumption. This was accomplished by inversely relating the range per 55 kWh battery pack to derive the consumption in kWh per mile. Polynomial regression, ranging from first to fifth degrees, was then applied to this transformed data. This method was chosen for its ability to capture the complex, non-linear relationship between speed and energy consumption. The polynomial coefficients, optimized through a least squares fitting method, offer a precise mathematical description of this relationship.

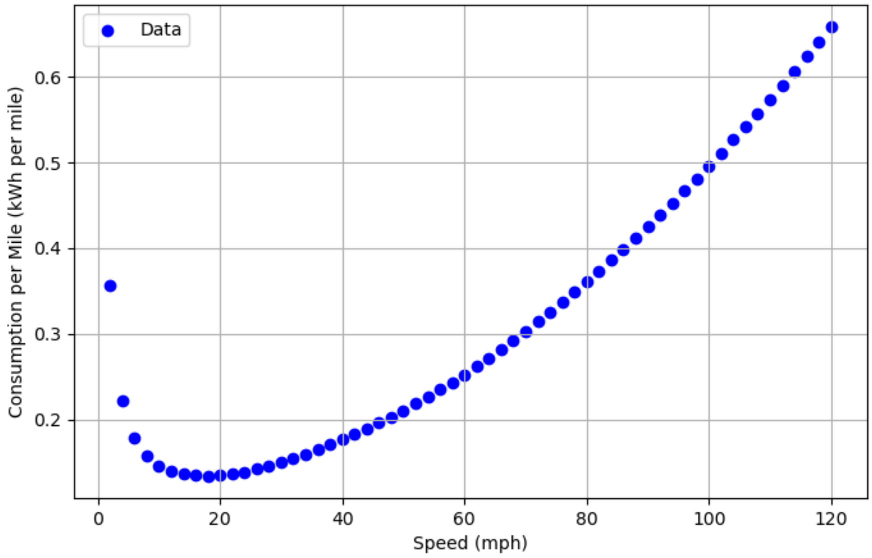


Figure 4: *Tesla Roadster Energy Consumption per Mile (kWh per mile) vs. Speed (mph)*

The representation of energy consumption against speed, as seen in **Figure 4**, showcases a key aspect of EV performance. It highlights that the efficiency of the Tesla Roadster is not constant but varies with speed, just as range varied. The relationship between energy consumption and speed is once again non-linear, but as speed initially increases, energy consumption decreases. Energy consumption then hits a global minimum of 0.134212 kWh per mile at 18 mph then increases gradually at higher magnitudes as speed continues to increase.

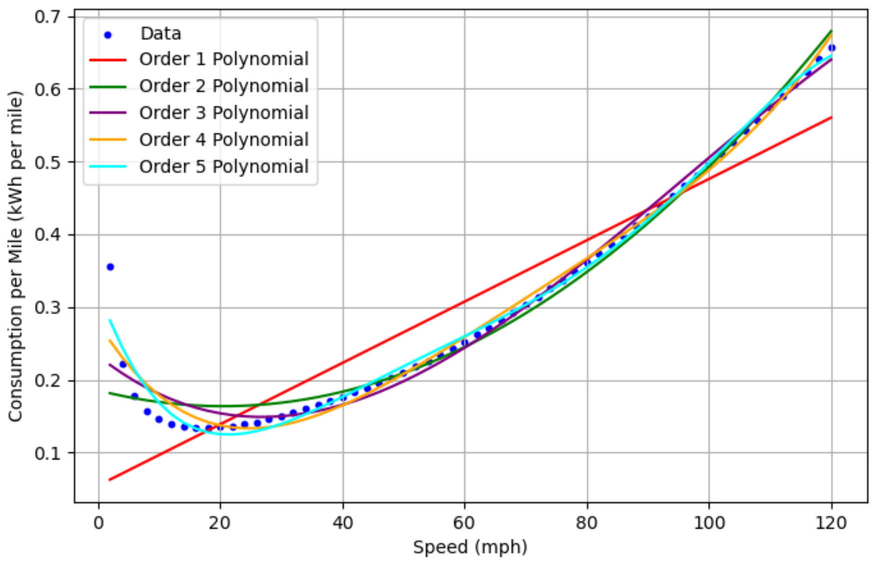
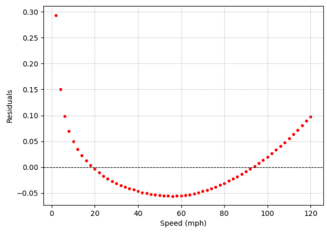
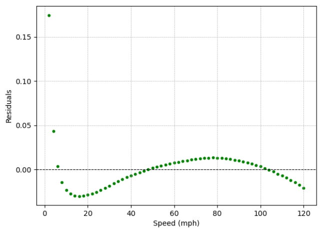


Figure 5: *Polynomial Fits to the Energy Consumption Rate Data*

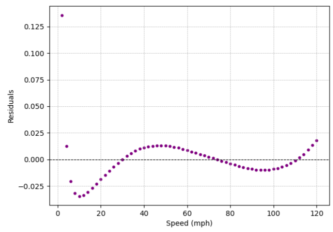
In **Figure 5**, the fitted polynomial models reveal the relationship between speed and energy consumption rate. The models, spanning from a 1-degree polynomial to a 5-degree polynomial, underscore the non-linear and dynamic nature of the Roadster's energy efficiency. Especially noteworthy are the polynomials with larger coefficients' ability to reflect the nuanced changes in energy consumption patterns across the spectrum of operational speeds.



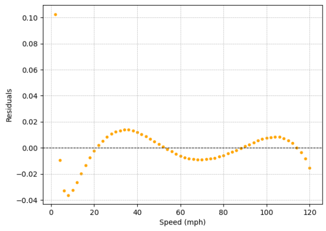
(a) *Degree 1 Residuals*



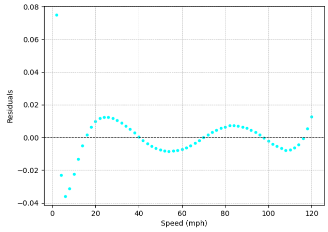
(b) *Degree 2 Residuals*



(c) *Degree 3 Residuals*



(d) *Degree 4 Residuals*



(e) *Degree 5 Residuals*

Figure 6: *Residual Plots for Polynomial Fits of Degree 1 to 5 Predicting Energy Consumption Rate*

Upon examining the residual plots for polynomial fits of degree 1 to 5, as presented from **Figure 6(a)** to **Figure 6(b)**, some difficulty emerged in identifying exactly which model is performing the best out of the five. Notably, structure can be seen in all residual plots, insinuating an imperfect fit with a polynomial. The higher degree polynomials, however, are superior in fitting as close to the curved pattern in the data as they possibly can in comparison to lower degree polynomials. The 4th and 5th degree polynomials residuals, as observed in **Figure 6(d)** and **Figure 6(e)**, hug the line more closely across the speed domain, showcasing minimal deviation from the expected trend. The consistency and reduced magnitude of these residuals indicate a strong fit that aligns well with the actual data, which genuinely captures the underlying physical phenomena governing the Roadster's energy consumption.

In comparing the 4th and 5th degree polynomials, the 5th degree polynomial reigns victorious because of its ability to capture trends that the 4th degree polynomial fails to capture cannot be understated. The improvement to a more complex model, in other words, is not statistically negligable within this domain. This preference for the 5th degree polynomial is not rooted in the complexity for complexity's sake. Instead, it takes into account the 5th degree polynomial's capacity to predict energy consumption rate from the speed of the Tesla Roadster—a relationship that is inherently non-linear and multifaceted.

$$
\begin{aligned}
C(v) \approx (&-0.26116546 \cdot 0.01694815^5) \cdot v^5 \\
&+ 0.00000012 \cdot v^4 \\
&- 0.00001587 \cdot v^3 \\
&+ 0.00096136 \cdot v^2 \\
&- 0.02382886 \cdot v \\
&+ 0.32534113
\end{aligned} \tag{2}
$$

In evaluating the polynomial models predicting the energy consumption rate of the Tesla Roadster, choosing the optimal polynomial was a slightly more difficult challenge compared to the range prediction scenario. While the Fisher test statistic for predicting range, when comparing the 4th-degree to the more complex 5th-degree polynomial, was a substantial 149.8049, the same statistic for predicting energy consumption rate was 30.4733. Despite being lower, this value is still considerably high and statistically significant, suggesting that the patterns captured were not due to noise, but actual inherent patterns in the data. The 5th-degree polynomial, therefore, was chosen as it successfully captures these intricacies in the relationship between speed and energy consumption within our measured speed domain. The shape of the consumption per mile data is nuanced due to the swooshed shape it presents, but the shape is not as complex as the original range data where range initially increases, then continues to decreases but at a smaller rate as speed increases. It is crucial to note, however, that the polynomial depicted by **Equation (2)**, should be approached with caution and not be used for extrapolation beyond the observed domain of data. The predictive power of this model, while strong within the tested parameters, is likely to dissipate if applied outside the established domain of speed and energy consumption data.

4: A Transform from Energy to Power

Diving deeper into the dynamics of the Tesla Roadster's performance, it becomes increasingly evident that analyzing power usage, measured in kilowatts (kW), against speed might offer a more direct insight into the vehicle's efficiency. Power, as a measure of energy consumption rate, can provide a real-time snapshot of the vehicle's performance, particularly how it translates electrical energy into motion. This metric is pivotal in understanding the instantaneous demands placed on the Roadster's battery and motor system at various speeds, offering a more dynamic perspective compared to the cumulative energy consumption or range metrics. In essence, analyzing power usage enables us to capture the intensity of energy utilization at any given moment, which is crucial for optimizing the Roadster's performance and enhancing its efficiency in real-world driving conditions. The data in **Figure 7** showcases a more simple pattern in comparison to all the previous data observed in this paper. The shape resembles a

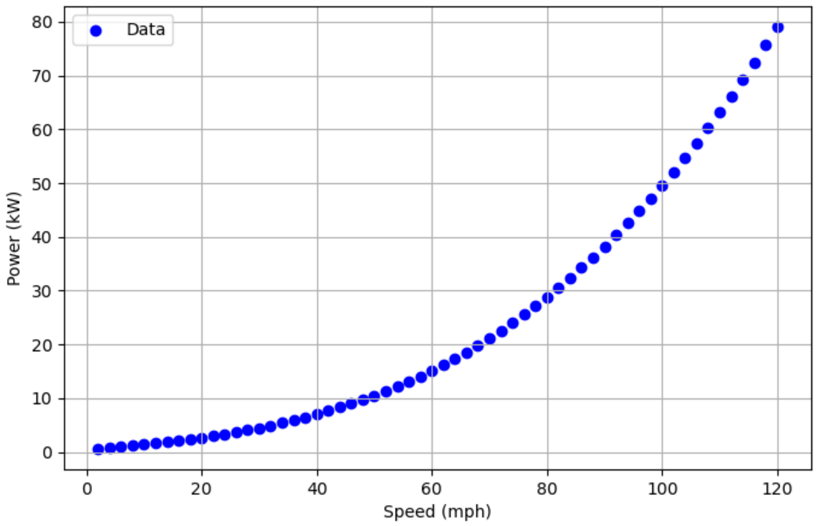


Figure 7: *Tesla Roadster Power Usage (kW) vs. Speed (mph)*

clear polynomial trend unlike the previous data for range versus speed in **Figure 1** and energy consumption per mile versus speed in **Figure 4**. In light of this, fitting polynomial models to power data will be a crucial step in my analysis. Similar to my previous approaches, we will employ polynomial regression, ranging from 1st to 5th degrees, to model this monotonically increasing relationship between power usage and speed. The objective is to uncover the underlying patterns that govern the Roadster's power dynamics across different speeds, providing a comprehensive understanding of its performance characteristics.

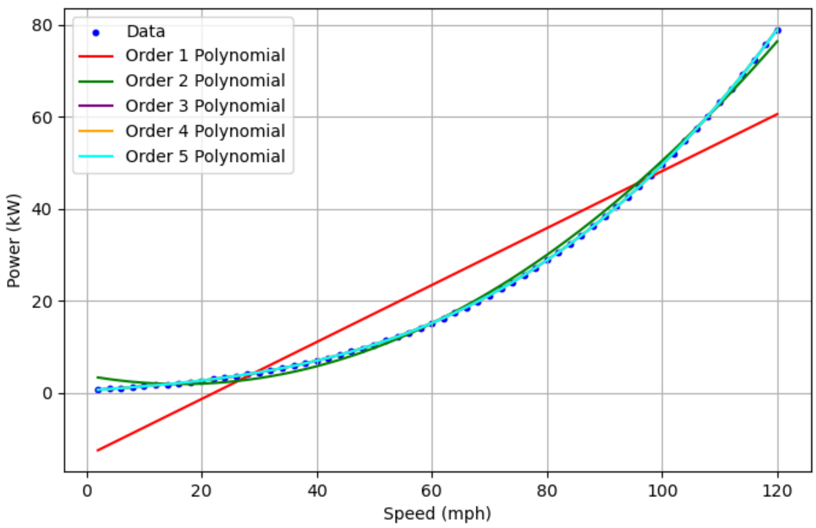
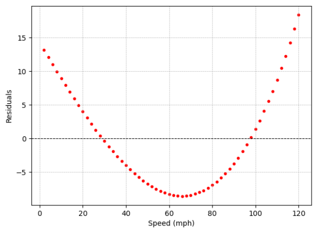


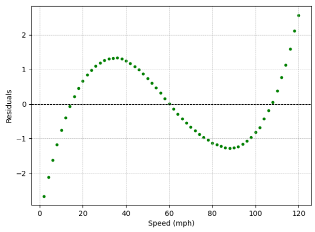
Figure 8: *Polynomial Fits to the Range vs Power Data*

Immediately starting at the 2nd degree polynomial and up to the 5th degree polynomial in

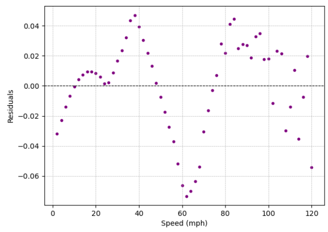
Figure 8, there seems to be a much closer fit to the data in comparison to previous approaches, where 2nd and 3rd degree polynomials failed to capture obvious nuanced patterns in the Tesla Roadster power versus speed data. This trend alone showcases how this data was more receptive to a polynomial fit, as opposed to the previous data I analyzed and observed regarding energy consumption and travel range.



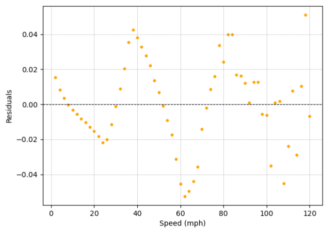
(a) *Degree 1 Residuals*



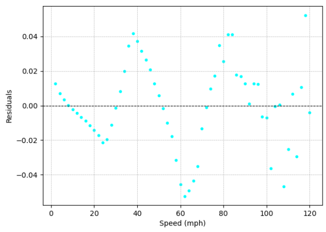
(b) *Degree 2 Residuals*



(c) *Degree 3 Residuals*



(d) *Degree 4 Residuals*



(e) *Degree 5 Residuals*

Figure 9: *Residual Plots for Polynomial Fits of Degree 1 to 5 Predicting Power from Speed*

In **Figure 9**, there appears to be a strong structure in each residual plot—which has been true for the previous models for other range energy variables for the Tesla Roadster as well. Upon reaching **Figure 9(c)**, the residual structure attains a jagged shape and maintains it as the degree polynomial increases. This structure is profound but may signify a significant increase in the prediction accuracy of our model once polynomials of the 3rd degree and above are applied to predicting the Tesla Roadster power based on speed alone.

A series of Fisher Tests concludes that the 4th degree polynomial performs best in capturing the relevant patterns in the Tesla Roadster power data using speed without overfitting to noise.

$$\begin{aligned} P(v) \approx (0.22992121 \cdot 0.01694915^4) \cdot v^4 \\ + 0.00003369 \cdot v^3 \\ + 0.00040258 \cdot v^2 \\ + 0.08635882 \cdot v \\ + 0.52238372 \end{aligned} \tag{3}$$

The curve representing power usage in relation to speed, as illustrated in **Figure 7** and further analyzed through polynomial fitting in **Figure 8**, presented a more straightforward pattern for polynomial fitting compared to the earlier cases of range and energy consumption. The data's inherently simpler structure, characterized by a monotonically increasing trend, allowed for a more efficient and accurate polynomial fitting process. This simplicity is crucial because it directly translates to an enhanced ability to predict the Tesla Roadster's power requirements at different speeds with greater precision and reliability.

This improved model fit can be attributed to the power usage data's inherent characteristics. Unlike range or energy consumption, which exhibited complex behaviors such as initial increase followed by a plateau or a non-linear swoosh pattern, power usage displayed a more direct and consistent relationship with speed. This linear progression made it easier to model and predict using polynomial regression. The fourth-degree polynomial, as described in **Equation (3)**, emerged as the most suitable model. It captures the essence of the Roadster's power dynamics without falling into the trap of overfitting, which is often a risk with higher-degree polynomials.

5: Comparing Range Models

In evaluating the Tesla Roadster's range capabilities, three distinct models can be developed based on the equations for range, energy consumption, and power as seen in **Equation (1)**, **(2)**, and **(3)** respectively, as well as the Tesla Roadster's 55 kWH battery pack.

$$R_1(v) = R(v) \tag{4}$$

$$R_2(v) = \frac{55}{C(v)} \tag{5}$$

$$R_3(v) = \frac{55v}{P(v)} \tag{6}$$

Figure 10 illustrates each model's representation of the Tesla Roadster's EV range as a function of speed. By quickly glancing at each of the plots, **Equation (6)**, which represents the calculation of EV range based on power derived from speed, most closely fits to the data in comparison to the rest of the other models. **Equation (5)** oscillates around the data and fails to capture the curved pattern that appears in the EV range data, and **Equation (4)** fails to capture the true pattern which can be seen in the peak of the data—especially when compared to the power-to-range model.

Another true test of these models' effectiveness comes from their ability to extrapolate beyond the given data. Specifically, predicting the Roadster's range at a high speed of 150 mph was critical. The extrapolations yielded strikingly different results: 964.23 mi for $R_1(v)$, 285.32 mi for $R_2(v)$, and a more realistic 58.00 mi for $R_3(v)$. These varying predictions showcase the challenges that exist in modeling complex real-world phenomena like EV range. While $R_1(v)$ and $R_2(v)$ provided optimistic extrapolations, it was $R_3(v)$ that aligned closest with practical

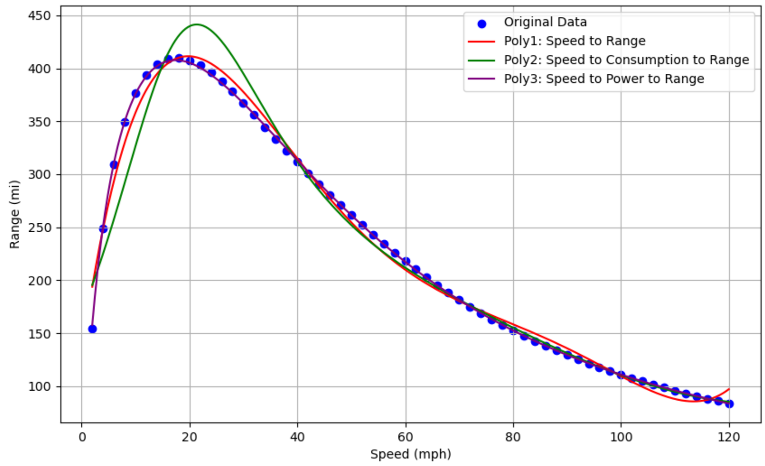


Figure 10: *Polynomial Fits to the Energy Consumption Rate Data*

expectations. The 58.00 mi EV range prediction falls in line with the monotonically decreasing and converging trend that can be seen with much higher speeds as speeds increase past typical interstate highway speeds. Meanwhile, both the 964.23 mi and 285.32 mi extrapolations for $R_1(v)$ and $R_2(v)$ are unreasonable, because they suggest that EV range increases after speed reaches 120 mph when this is likely not the case.

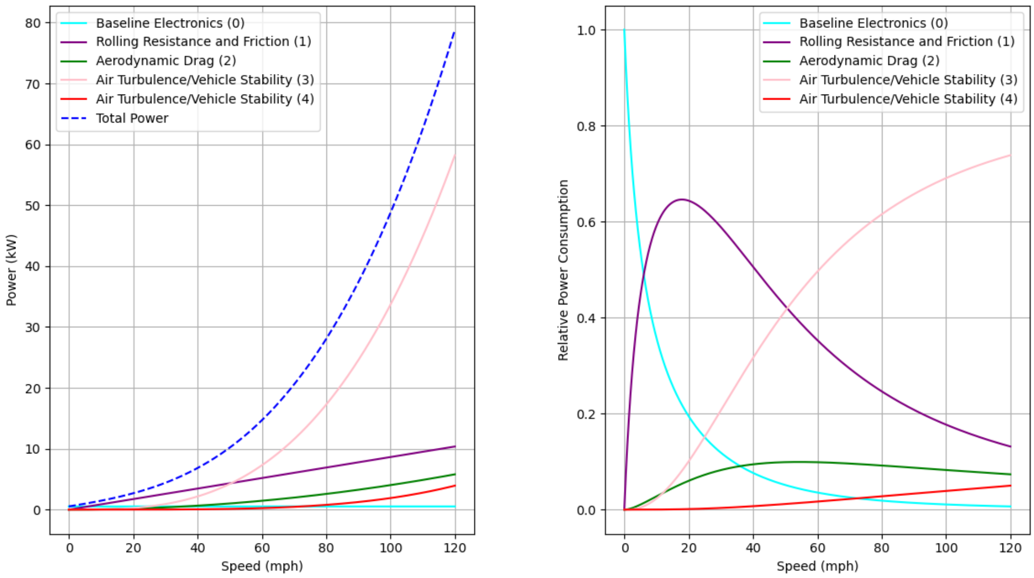
$R_3(v)$ most closely fits to the data and provides realistic extrapolations for EV range of the Tesla Roadster. Thus, this model is a strong predictor of total Tesla Roadster EV ranges based on speed, showcasing how this model excels between the speeds from 2-120 mph and also beyond in its extrapolation task.

6: Providing Context for Power

In my preferred for power usage of the Tesla Roadster from **Equation (6)**, denoted in the format $P(v) = a_4v^4 + a_3v^3 + a_2v^2 + a_1v + a_0$, each of the coefficients multiplied with a k^{th} order terms has a meaning.

The constant term a_0, independent of speed, can be interpreted as the baseline power consumption. Often coined as the "bias" term in statistics, this term encompasses the energy requirements of the vehicle's electronics, such as lighting and air conditioning within an EV, which are essential for the vehicle's operation regardless of its movement [4]. The linear term a_1 is associated with rolling resistance and powertrain friction. These losses, which scale directly with speed, stem from the powertrain friction in the vehicle's moving parts, including the tires' interaction with the road surface. As the Tesla Roadster accelerates, these factors increasingly influence its power requirements [4,5]. An increase in weight in a car directly causes an increase in friction as well, as this increase would cause the normal force to increase which directly impacts the frictional force acting against the car.

The quadratic term a_2 relates to aerodynamic drag which is an extremely influential force at higher speeds. This resistance grows as a square of the speed, reflecting how air resistance exponentially increases with velocity, requiring even more power to maintain or increase speed [4,5]. The higher-order terms, a_3 and a_4, may represent more complex or compounded effects of these factors at higher speeds or specific aerodynamic characteristics of the Roadster. These terms could indicate how the vehicle's design and its interaction with the surrounding air evolve as it accelerates, possibly incorporating factors such as air turbulence and vehicle stability dynamics. Thus, wind resistance plays an impact in influencing a_3, particularly because wind resistance is a cubic term [6], but as of now, I will label a_3 and a_4 as influential quantities that may relate to turbulence and vehicle stability dynamics.



(a) *Total Power Consumption* (b) *Relative Power Consumption*

Figure 11: *Comparison Between Total Parameter Contributions and Absolute Contributions*

Figure 11(a) showcases the total power consumption by each individual coefficient in $P(v)$. Meanwhile, **Figure 11(b)** showcases the individual contribution of each mechanical factor in total power consumption and relatively how much they contributed in comparison to the rest of the factors. According to **Figure 11(b)**, at "city" speeds around 30 mph, rolling resistance and friction predominantly will affect power loss. Air turbulence and vehicle stability also play a key role in adding to the total power usage, followed by the baseline electronics output when the EV is travelling at 30 mph. At "highway" speeds around 70 mph, air turbulence and vehicle stability factors have much larger of an effect on power usage for EVs than the rest of the variables, but rolling resistance and friction are still undoubtedly influential nonetheless.

To enhance the Tesla Roadster's range, a focused approach on optimizing vehicle stability and aerodynamics is essential. This involves not only refining the physical design for reduced drag but also advancing the stability control systems to ensure efficient power usage. Such optimizations can lead to significant improvements in range, particularly at highway speeds, thus directly addressing the primary concern of long-distance capability in EVs. By prioritizing these aspects, manufacturers can significantly extend the practical travel distances of EVs, making them more appealing and reliable for longer journeys, which would decrease the prevalent "range anxiety" among potential EV consumers.

When looking for methods to optimize EV performance, particularly at higher speeds, it's imperative to compare these advancements with the most efficient form of mechanized land-based transportation: trains. Despite the technological advancements of cars, trucks, and EVs, trains hold a distinct edge in efficiency, especially for long-distance travel. This disparity can be highlighted when considering the performance of EVs, like the Tesla Roadster, at highway speeds.

Trains excel in efficiency due to several key factors. First, they have a much lower rolling resistance per ton than rubber-tired vehicles on roads [7]. This efficiency is partly due to the steel wheels on steel rails, which reduce the frictional forces compared to vehicles on roads. As a result of steel wheels being on steel rails, the rolling resistance can be reduced by around 85-99%, which is essential, especially considering how influential rolling resistance appears to be in **Figure 11(a)** and **Figure 11(b)**. Additionally, trains benefit significantly from the aerodynamic advantage provided by their streamlined design and the fact that they travel in a relatively straight line, facing less air turbulence and resistance than road vehicles, which often encounter varied terrain and traffic that disrupts airflow [8]. Even though paths tend to be relatively straightfoward for cars and trucks on county highways traveling large distances, they are still performing movements on asphalt roads that trains are not, such as swaying and moving within their lane or changing lanes. Especially at highway speeds when EVs are traveling 70 mph or higher, this turbulence that they are experiencing contributes heavily to their "range anxiety," while trains have the luxury of minimizing factors of power consumption due to the fact that they travel on steel rails with steel wheels.

7: Conclusion

In this paper, I evaluated the dynamics of the Tesla Roadster's performance, particularly focusing on the issue of range anxiety among EVs as a whole. Through a detailed analysis involving polynomial regression models, I investigated the relationship between speed and various aspects of EV performance, including range, energy consumption, and power usage.

This exploration revealed that the polynomial model $P(v)$, which predicts the Roadster's power usage based on speed, stands out in its ability to provide an accurate representation of the vehicle's performance, and it even became a strong predictor range anxiety in a 4th order polynomial model. This model's robustness was evident—especially in comparison to other models for predicting range and energy consumption—and its success can be attributed to the monotonically increasing nature of power in relation to speed. $P(v)$, which is represented by **Equation (3)**. The equation effectively reveals the core challenge of range anxiety by having interpretable coefficients, which offer realistic and practical insights into the vehicle's capabilities, especially at higher speeds.

The contributing factors to range anxiety were dissected, highlighting the roles of mechanical variables such as internal system energy consumption, rolling resistance, aerodynamic drag, air turbulence, and vehicle stability. While rolling resistance, air turbulence, and vehicle stability contribute heavily to power consumption across all speeds of the Tesla Roadster, rolling resistance plays the strongest role in energy consumption at "city" speeds around 30 mph, whereas air turbulence and vehicle stability play the strongest role in energy consumption at "highway" speeds around 70 mph.

To mitigate range anxiety and enhance EV efficiency, it is important for manufacturers to prioritize the optimization of these variables. Range anxiety is critical for both slower and faster speeds, but it's especially more critical to consider at higher speeds where people may be using EVs for road trips. Durability for these road trips is essential, so refining the vehicle's physical design to reduce wind turbulence and advancing stability control systems will play pivotal roles in achieving this goal. These improvements are instrumental in bolstering the appeal and reliability of EVs for long-distance travel, thereby addressing a key concern among potential EV consumers.

Thinking through a broader perspective on the future of EVs, it is paramount for EVs to defeat this range anxiety problem that they experience. As the world grapples with the challenges of climate change and sustainable transportation, the insights gained from this study are vital. Improving EV efficiency and range is essential for the transportation norms of the world to align more closely with the larger goal of transitioning towards a more sustainable and environmentally friendly transportation ecosystem. EVs are far more energy-efficient than standard gas cars and trucks, which individually products on average 4.6 metric tons of CO2 emissions per year [9]. If we do not make advances in EVs and transitioning towards them soon, we may contribute in failing to decrease our annual CO2 emissions every year, which may lead to an exponentially increasing global mean temperature anomaly (GMTA) that exceeds 5 degrees Celsius by 2100 [10].

8: References

[1] "Electric Car Range and 5 Reasons Why Your Range Anxiety Is Unwarranted." EVBox, blog.evbox.com/eliminate-range-anxiety

[2] "Tesla Roadster (First Generation)." Wikipedia, Wikimedia Foundation, 30 Nov. 2023, en.wikipedia.org/wiki/Tesla_Roadster_(first_generation)

[3] "Roadster Efficiency and Range." Tesla, www.tesla.com/blog/roadster-efficiency-and-range.

[4] "Your Guide to Calculating the Energy Consumption of an Electric Vehicle." Energy Matters, 17 Oct. 2023, www.energymatters.com.au/renewable-news/your-guide-to-calculating-the-energy-consumption-of-an-electric-vehicle/: :text=Electric%20vehicle%20energy%20consumption %20calculation&text=If%20you%20drive%20around%2030,kilowatt%2Dhours%20per %20100%20km.

[5] Modern Electric Hybrid Electric and Fuel Cell Vehicles (3rd Edn.) Mehrdad Ehsani, Yimin Gao, Stefano Longo, Kambiz M. Ebrahimi CRC Press, Boca Raton, FL, USA, 2018

[6] "The Power of the Wind: Cube of Wind Speed." Danish Wind Industry Association, xn–drmstrre-64ad.dk/wp-content/wind/miller/windpower%20web/en/tour/wres/enrspeed.htm.

[7] "Steel Wheels or Rubber Tires?" Steel Wheels or Rubber Tires? | Steel Interstate Coalition, steelinterstate.org/topics/steel-wheels-or-rubber-tires.

[8] Zhiyong Yao, Nan Zhang, Xinzhong Chen, Cheng Zhang, He Xia & Xiaozhen Li (2020) The effect of moving train on the aerodynamic performances of train-bridge system with a crosswind, Engineering Applications of Computational Fluid Mechanics, 14:1, 222-235, DOI: 10.1080/19942060.2019.1704886

[9] Greenhouse Gas Emissions from a Typical Passenger Vehicle, www.epa.gov/greenvehicles/greenhouse-gas-emissions-typical-passenger-vehicle. Accessed 9 Dec. 2023.

[10] Iriye, Reece. "An Outlook on Projected Global Temperatures in Relation to Excess Carbon Emissions." 2023, Sep 2023, MATH 4334: SMU

9: Appendix (ChatGPT Usage)

I used ChatGPT for all of my plots and refining my calculations in my code. Additionally, throughout the paper when I found the verbiage I was using to not make too much sense, I asked ChatGPT every now and then to reword sentences to make them more refined. ChatGPT also was a nice research tool that helped provide some sources for me in understanding the role of the coefficients in the $P(v)$ equation.