Mini-Project #2

Math 4334: Mathematical Modeling
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Background. An important component of the world's climate goals will be the electrification of automobiles, and a well-known hesitation that people express when thinking about electric cars is "range anxiety." The distance an automobile can travel on a single charge depends on the speed at which it is traveling through a variety of mechanisms. The data set `ev-range.xlsx` describes the range vs speed curve for the first popular EV – the original Tesla Roadster. In this assignment we will search for a model that agrees with, and helps us understand, this data.

Note: This data set has essentially no random noise. With the right model, I can obtain an extremely good fit for it, in which the relative magnitude of the residual is less than 1%. However, the residual for this model exhibits a jagged 'sawtooth'-like structure, with around 3 teeth. Do *not* attempt to get rid of this sawtooth-shaped residual (My working hypothesis is that this is an artifact of the way the data was prepared prior to being released).

Warm-up: A basic polynomial fit. First, let's attempt to naiively fit this data to a polynomial:

$$R_1(v) \approx a_0 + a_1 v + a_2 v^2 + a_3 v^3 + \dots \tag{1}$$

Experiment with polynomials of different orders (say, up to 5th order). What do you observe as you increase the order of the polynomial? How accurate does your model get? Do you think it is reliable?

A transform from efficiency to consumption. The range of a car obviously depends on battery size. Since the Tesla Roadster has a battery pack with a capacity of 55 kwh, its range rating could be stated "miles per 55 kwh." And if we then divide this number by 55, we obtain numbers with units of "miles per kwh" – a measure of *energy efficiency*, where higher is better.[1] On the other hand, most other countries rate vehicles on the inverse measure of "kwh per mile" – a measure of *energy consumption*, where lower is better.[2] Let's compute and plot the consumption per mile of our vehicle at various speeds using

$$C_i = \frac{55}{R_i} \tag{2}$$

[1] A gallon of gasoline contains 33.7 kwh of thermal energy, of which a maximum of about 10 kwh gets converted to mechanical energy. So whereas a gasoline-powered car is considered fairly efficient if it achieves more than about 40 miles per *gallon*, an electric car is considered fairly efficient if it achieves more than about 4 miles per *kilowatt-hour*. Note that the automotive industry's metric of "MPGe" is defined such that 1 MPkwh = 33.7 x MPGe; i.e. it does not account for the engine's conversion efficiency of only around 30%. It is therefore highly misleading from the perspective of carbon emissions. (Power plants can attain efficiencies of almost 50%, which is why an electric car typically generates fewer emissions than a gasoline-powered car even if it is charged from a grid running on fossil fuels. Of course, as the grid becomes powered by more renewables, the emissions per mile for electric cars falls).

[2] There are lots of reasons for this, but the simplest is that we rarely choose our destination based on available energy, whereas we frequently purchase energy based on chosen destination.

Plotting this quantity, we see that our vehicle consumes about 250 wh / mile at highway speeds, which is pretty good.[3] Repeat the exercise of finding a reasonable polynomial fit to this data, this time of the form

$$C(v) \approx b_0 + b_1 v + b_2 v^2 + b_3 v^3 + \dots$$

(again, staying below 5th order). Is this representation of the data any easier to fit to a polynomial? Why or why not?

A further transform: from energy to power. An interesting feature of the "kwh per mile" measure is that it has units of force. Moreover, when multiplied by the speed (in "miles per hour"), it becomes simply "kilowatts," which is a measure of *power* (i.e. energy per hour). This quantity tells us how fast the car is consuming energy at each speed. Let's compute power for our vehicle using the equation

$$P_i = v_i C_i = \frac{55 v_i}{R_i} \tag{3}$$

One last time, repeat the exercise of finding a reasonable polynomial fit, this time of the form

$$P(v) \approx c_0 + c_1 v + c_2 v^2 + c_3 v^3 + \dots$$

Is this curve any easier to fit to a polynomial? Why or why not?

Comparing Range models. At this point you have three models: one for $R(v)$, one for $C(v)$, and one for $P(v)$. Using the relationships described in Equations (2) and (3), convert the polynomial models for $C(v)$ and $P(v)$ into *rational function* models for $R(v)$ of the form

$$R_2(v) = \frac{55}{C(v)}$$
$$R_3(v) = \frac{55v}{P(v)}$$

so that you now have three different models for range. Plot the original range data, along with all three range models. In addition, extrapolate all three models to obtain predictions for the range when traveling 150mph. Which model do you think is most trustworthy for extrapolation? Why?

Going Further. Until this point our approach has been purely empirical. In the next unit we will learn *why* the model has the form that it does, but here I invite you to try and anticipate some of those results on your own. In your model for power above, you should have found a good fit with a finite number of terms. It turns out that each of those terms is associated with a specific type of energy drain on a moving car, including baseline electronics usage, rolling resistance, powertrain friction, and wind resistance.

(a) Do some research to learn which coefficients in your model connect to which mechanisms described in the introduction. Report what you find.

(b) Create two plots showing (a) the power consumption of each mechanism, and (b) the *relative* power consumption of each mechanism (which is just each mechanism's power use divided by the total power use).

(c) What are the dominant sources of power loss (a) at "city" speeds of around 30mph, and (b) at "highway" speeds of around 70mph? If you were a car manufacturer, what approaches might you take to improve the range of your car?

[3] In contrast, the new electric GMC Hummer requires around 650 wh / mile at such speeds, demonstrating that "electric" doesn't magically mean "efficient."

(d) Despite all of the advancements that have been made in automobile design, the most efficient form of mechanized land-based transportation, by far, is the first one humans invented: the train. Discuss the advantages trains have over cars and trucks, using what you learned in part (c).