

# Project #1: Climate Change

Math 4334: Mathematical Modeling

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**Climate Change.** In this project, we will continue exploring data on global temperature and CO<sub>2</sub> levels, and make predictions about future outcomes under various scenarios. You will turn in a 4-8 page report with five sections, the first of which is an introduction requiring outside reading, and the last of which is a conclusion. In addition, there are two optional “bonus” sections that ambitious / curious students may complete for extra credit.

**Section I: Introduction.** Do some research on the basic method by which CO<sub>2</sub> warms the atmosphere. To get you started, try Googling “how does the greenhouse effect work” and click on “Images.” Once you believe you understand the basic physical mechanism, write a summary (in 2-4 paragraphs) as an introduction to your report. Address each of the following in your introduction:

- (a) What is a “greenhouse gas”? Why are they called that?
- (b) When was it first realized that CO<sub>2</sub> was a greenhouse gas?
- (c) When was it first postulated that fossil fuel use *\*might\** someday warm the climate?
- (d) When was it first realized that we were on track to *\*actually\** significantly warm the climate?

**Section II: Data and a Linear Model.** Here we will analyze data on the mean global temperature for the years 1850-2020. In the field of climate science, the convention is to express temperature as the “Global Mean Temperature Anomaly,” which is the *difference* in degrees Celsius between the global mean temperature now from its value in pre-industrial times. More specifically, we compute the average temperature over the years 1850-1900, and subtract that number from our temperature data. For fitting purposes, we will subtract 1900 from the year (but remember to add this number back in whenever you plot something). In addition, we know that pre-industrial CO<sub>2</sub> levels were about 280 ppm, so we will subtract 280 from our CO<sub>2</sub> values to obtain “excess CO<sub>2</sub>” (ECO<sub>2</sub>). Finally, some data may be given on both a monthly and yearly basis. For simplicity, we will only use *yearly averages* in this assignment.

- (a) Create a plot with the ECO<sub>2</sub> levels vs time on the left, and GMTA levels vs time on the right. Note that the data on GMTA is much noisier over time than are the data on ECO<sub>2</sub> levels, because GMTA is affected by several important factors besides ECO<sub>2</sub> (such as El Nino cycles).
- (b) Next, eliminate the time variable by plotting the relationship **between GMTA vs ECO<sub>2</sub>**. The data is now in the same format as other exercises we have done in Unit #01. Does there appear to be a strong correlation, that could be fit by a simple, linear model?

- (c) For any years that look significantly different than their neighbors, determine whether unusual circumstances may have applied. If you find such combinations, you may remove those years from your data. Justify each removal.
- (d) Obtain a linear fit between ECO2 levels and the GMTA. State the fitted model and its coefficients. Discuss the value of  $R^2$ , and analyze the residual as we did in unit 1b. Does this seem like a reasonable model?
- (e) Climate scientists strongly advocate keeping the GMTA under 1.5 degrees Celsius to avoid “the worst effects of warming.” Based on your model, what atmospheric CO2 concentration would correspond to that GMTA?
- (f) At the current rate of CO2 emissions – 2.5 ppm/year, how many years remain before we reach the CO2 levels you calculated in part (e)?

In your report, include a single large plot with the following characteristics: the top left quadrant contains a small plot of ECO2 vs time, and the label **(a)**. The top right quadrant contains a small plot of GMTA vs time, and the label **(b)**. The bottom half contains a large plot of the GMTA vs. ECO2, together with a line of best fit, under the label **(c)**. This figure should have a caption describing each of the three panels. The description for panel (c) should include the equation for the line of best fit, and the value of  $R^2$ . This plot should be around 6.5 inches wide (i.e., the full distance between left and right margins). Make it as visually appealing as you can.

**Section III: the “Business as Usual” scenario.** Given advances in medical technology, it is likely that some of you (perhaps many) will live to see the year 2100. Certainly most of your children will live to see this year. Therefore, in this section we will turn our attention from the past to the future, exploring what could potentially happen over the next 80 years.

- (a) The rate of emissions has been continually increasing, from about 1.0 ppm/year in 1965, to about 2.5 ppm/year in 2020. Assuming a steady acceleration over time, what rate  $r$  of acceleration is this?
- (b) Assume eCO2 at the beginning of 2020 was 134 ppm, emissions in 2020 were 2.5 ppm, emissions in 2021 were  $2.5 * (1 + r)$  ppm, emissions in 2022 were  $2.5 * (1 + r)^2$  ppm, and so on. Compute cumulative eCO2 levels for the years 2020-2100.
- (c) Use the linear model you developed in part II to convert these projected eCO2 levels into projected GMTA levels. What is the projected GMTA by the year 2100? (This outcome is known as the “Business as Usual” or “BAU” scenario.)
- (d) Note that GMTA is the average over the whole globe – which is 70% ocean. Water absorbs heat much more readily than land, and doesn’t warm as readily. Consequently, the expected temperature increase over land is *about twice* the global average<sup>1</sup>. Also note that the GMTA is given in degrees Celsius, which although an international standard, is not intuitively familiar to US college students. So, under BAU, what is the expected average warming over land, in degrees Fahrenheit, by the year 2100?<sup>2</sup>

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<sup>1</sup>See, for instance, the first image in this article: <https://climate.nasa.gov/news/2865/a-degree-of-concern-why-global-temperatures-matter/>

<sup>2</sup>Remember that when converting temperature \*differences\* from one system to another, you do not add or subtract a constant value.

- (e) With at least two classmates, spend at least 15 minutes brainstorming what societal consequences can you think of if summer high temperatures increase to these levels amount? In Texas? In the United States? Across the globe? Conclude this section with a discussion of your thoughts.

Your plot for this section should include a single panel containing three data sets: (a) past actual GMTA levels as blue squares, (b) past fitted GMTA levels (based on actual ECO2 levels fed to your model from Section II) as a solid green line, and (c) future predicted GMTA levels (based on the described emissions scenario fed to your model from Section II) as a solid red line.

**Section IV: Alternative scenarios.** Because CO2 is a very stable molecule, emitted carbon remains in the atmosphere a very long time, and the resulting temperature increases are essentially permanent<sup>3</sup>. Consequently, reaching the 1.5 degree goal (or any other goal) requires humanity to completely eliminate fossil fuels before reaching the CO2 level you calculated above. Practically, this can only be done gradually, so consider a scheme in which the nations of the world reduce their emissions by a percentage  $p$  every year.

- (a) Construct a plot containing each of the following:
- (i) actual historical GMTA levels from 1950-2020 (black squares)
  - (ii) predicted historical GMTA levels from 1950-2020 (black curve)
  - (iii) projected GMTA from 2020 until 2100, assuming the following annual changes in emissions:
    - 1.6% increases (red curve, BAU)
    - constant emissions (orange)
    - 1.5% annual decreases (yellow)
    - 3.0% annual decreases (green)
    - 4.5% annual decreases (blue)
    - 6.0% annual decreases (purple)
- (b) Under a policy such as the ones considered here, ECO2 levels  $\Delta C(t; p)$  should eventually saturate at a value  $\lim_{t \rightarrow \infty} \Delta C(t; p) = \Delta C^\infty(p)$  that depends on the reduction percentage  $r$ . These values, in turn, can be converted into “permanent” future GMTA levels  $\Delta T^\infty(p)$  using the model from part II. Write a function that determines this value, and plot  $\Delta T^\infty(p)$  versus  $p$ , for values of  $p$  between 0.01 and 0.1.
- (c) From your graph, approximately what rate of reduction is currently needed to meet the 1.5 degree target?

The graphs described in parts (a) and (b) should appear side-by-side within a single figure, with added labels (a) and (b), and a shared caption that describes each of them in turn.

**Section V: Conclusions.** Conclude your report with a short discussion (1-2 paragraphs) of what you learned over the course of this assignment. As you have seen, the basic science and data underlying climate change are not complicated. So if what is happening now was so easy to predict, why do you think we waited so long to do anything about it? What does this failure say about our society, and our capability for collective action? What do you think the role of scientists should be when the “technical” work is straightforward but the “social” work is difficult? This is very open-ended, and has no right or wrong answer.

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<sup>3</sup>Geological processes gradually remove it over tens of thousands of years