PERFORMANCE

Term	Remarks	Staff Member's Signature
I	Completed	Ali ng! 22/10/19
II	Completed	AK 07/02/2022

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who has worked for the year ________ in the Computer Laboratory.

07/9/2020

Teacher In-Charge

Head of Department

Date : ____________

Examiner

Practical no 1.

Limits and continuity.

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x})+2x\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\;\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\cancel{4}^{2}\sqrt{a}}{2\sqrt{3}\sqrt{a}}=\frac{2}{3\sqrt{3}}$$

2. $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$\lim_{y\to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0} \frac{y}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\frac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a})}$

$\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$\frac{1}{\sqrt{a}(2\sqrt{a})} = \frac{1}{2a}$

$$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$$

by substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h + \frac{\pi}{6}) - \sqrt{3} \sin(h - \frac{\pi}{6})}{\pi - 6(h + \frac{\pi}{6})}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/6 - \sin h \, \pi/6 - \sqrt{3} \sin h \cos \pi/6 + \cos h - \sin \pi/6}{-6h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \sqrt{3}h - \sin h - \sin \frac{3/h}{2} - \cos \frac{\sqrt{3}}{2}h}{-6h}$$

$$= \lim_{h \to 0} \frac{-\sin 4h/2}{-6h}$$

$$= \frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3}$$

(4)

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

$$=\lim_{x\to\infty}\left(\sqrt{x^2+5}-\sqrt{x^2-3}\times\frac{\sqrt{x^2+3}\sqrt{x^2+1}}{\sqrt{x^2+3}++\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2+1}}{\sqrt{x^2+6}+\sqrt{x^2+1}}\right.$$

$$=\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5})+\sqrt{x^2+1})}\right]$$

$$\cancel{=\lim_{x\to\infty}\frac{(x^2+5-x^2+3)}{(\sqrt{\ }}}$$

$$=\lim_{x\to\infty}\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2+3})}$$

$$=\lim_{x\to\infty}\frac{8}{4}\frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2+3}}$$

$$=4\lim\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/4^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}$$

$$=4$$

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$f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 \leq x \leq \pi/2$ $\Big\}$ at $x = \pi/2$

$= \frac{\cos x}{\pi - 2x}$ for $\pi/2 < x < \pi$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

$\therefore f$ at $x = \pi/2$ define

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} + \frac{\cos x}{\pi - 2x}$

by sub method.

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2[h + \pi/2)}$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{-2h}$ $\because$ using $\cos(A+B)$

$= \sin A \cos B - \sin A \sin B$

$$\lim_{h\to 0} \cos h \cdot 0 \frac{-\sin h}{-2h}$$

$$\lim_{h\to 0} \frac{\cancel{-}\sin h}{\cancel{-}2h} = \frac{1}{2}\lim_{h\to 0}\frac{\sin h}{h}$$

$$= \frac{1}{2}$$

b. $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} -\frac{\sin 2x}{\sqrt{1-\cos 2x}}$ using $\sin 2x = 2\sin\cdot\cos x$

$$\lim_{x\to\pi/2} \frac{2\sin x\cdot\cos x}{\sqrt{2}\sin x}$$

$$\lim_{x\to\pi/2} -\frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}}\lim_{x\to\pi/2} -\cos x$$

LHL $\neq$ RHL

$\therefore$ f is not continuous at $x = \pi/2$

$$f(x) = \frac{x^2-9}{x-3} \quad 0 < x < 3$$

$$= x+3 \quad 3 \le x \le 6)$$

$$= \frac{x^2-9}{x+3} \quad 6 \le x < 9$$

at $x=3$ and $x=6$

at $x=3$

$$f(3x) = \frac{x^2-9}{x-3} = 0$$

f at $x=3$ def

$$\lim_{x\to 3} f(x) = \lim_{x \to 3^+} x+3$$

$f(3) = x+3 = 3+3 = 6$

f is bijctue at $x=3$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

~~$\therefore$ LHL = RHL~~

f is contnuum at $x=3$

for $x=6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}\,3}{\cancel{9}} = 3$$

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iii $\lim_{x \to 6} \frac{x^2 - 9}{x+3}$

$\lim_{x \to 6} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x \to 6} (x-3) = 6-3 = 3$

$\lim_{x \to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL ≠ RHL

function is not continuous $x = 0$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = 4$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

$2\lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = 4$

$2(2)^2 = 4$

$k = 8$

$$f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$$
$$= 4 \quad x = 0 \Bigg\} \text{ at } x = 0$$

$$\Rightarrow f(x) = (\sec^2 x)^{\cot^2 x}$$

Using

$\sec^2 - \tan^2 - \sec^2 x - 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

and

$\cot^2 x = \frac{1}{\tan^2 x}$

$$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$$

we know that

$$\lim_{x \to 0} (1 + px)^{1/px} = e$$

$= e$

$4 \neq e$

$$f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \pi/3$$
$$= 4 \quad x \in \pi/3 \Bigg\} \text{ at } x = \pi/3$$

$= \cancel{x} - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$ when $h = 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h}{\pi - 3(\pi/3 + h}$$

using
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \frac{\tan\frac{\pi}{3} + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$$

$$\lim_{h \to 0} \frac{\frac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\frac{(\sqrt{3} - 3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{-4 \tan h}{-3h(1 - \sqrt{3} \tan h}$$

$$\lim_{h\to 0} \frac{4 - \sin h}{3h(1-\sqrt{3}\sin h)}$$

$$\frac{4}{3} \cdot \lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\sin h)}$$

$$\frac{\sin h}{h} = 1$$

$$= \frac{4}{3} \cdot \frac{1}{(1-\sqrt{3}(0))}$$

$$\frac{4}{3}\left[\frac{1}{1}\right] = \frac{4}{3}$$

$$f(x) = \frac{1-\cos 3x}{x \sin x} \quad \begin{matrix} x = 0 \\ x = 0 \end{matrix} \Big\} \text{ at } x = 0$$

$$f(x) = 1 - \frac{\cos 3x}{x \sin x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x \sin x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 \frac{3x}{2} \times x^2}{x^2}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$= 2 \times \frac{9}{4} = \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \quad 9 = f(0)$$

f is not continuous at $x = 0$ [illegible]

$f(x) = \frac{1-\cos x}{x \sin x}$ $x \neq 0$

$\frac{9}{2}$ $x=0$

now $\lim_{x \to 0} f(x) = f(0)$

f is removable discontinuity at $x=0$

II $f(x) = (e^{3x}) \sin x$; $x \neq 0$ } at $x=0$

$= \frac{\pi}{0}$ $x=0$

$\Rightarrow \lim_{x \to 0} \frac{(e^{3x}-1)\sin(\pi x/180)}{x^2}$

$= \lim_{x \to 0} \frac{e^{3x}-1}{x} \lim_{x \to 0} \frac{\sin(\pi|x/180)}{x}$

$= 3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin(\pi|x/180)}{x}$

$= 3 \log e \; \pi/60 = \pi/60 = f(0)$

f is continuous at $x=0$

$f(x) = e^{x^2} - \cos x \qquad x = 0$

is continuous at $x = 0$

given

f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$

$\log e + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2 \lim_{x \to 0} \left[\sin x/2\right]^2$

$1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$

9) $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x = \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2}\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$ 3/12/19

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

Practical 2

Q1] Show that the following function def from $\mathbb{R}$ are differentiable. derivative.

1. $\cot x$)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\cot(x+h) - \cot(x)}{h}$$

$$\lim_{h \to 0} \frac{\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos(x)}{\sin(x)}}{h}$$

$$\lim_{h \to 0} \frac{\sin \cos(x+h) - \cos x \sin(x+h)}{h - \sin(- \sin(x+h)}$$ by using $\sin(A+B)$ formula.

$$\lim_{h \to 0} \frac{\sin(x-(x+h)}{h \sin(x+h) \cdot \sin x}$$

$$\lim_{h \to 0} \frac{\sin(x-(x+h)}{h \sin(x+h) \cdot \sin x}$$

$$\lim_{h \to 0} \frac{\sin h}{h} \quad \lim \frac{1}{\sin(x+h) \cdot \sin x}$$

$$-1 \cdot \frac{1}{\sin x \cdot \sin x} = -\operatorname{cosec}^2 x$$

2. cosec x

we know that $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$\lim_{h\to 0} \frac{\operatorname{cosec}(x+h)-\operatorname{cosec}(x)}{h}$$

$$\lim_{h\to 0} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin(x)}}{h}$$

$$\lim_{h\to 0} \frac{\sin x - \sin(x+h)}{h - \sin x \cdot \sin(x+h)}$$ by using formula $\sin a - \sin B$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{x+x+h}{2}\right) \cdot \sin\left(\frac{x-x+h}{2}\right)}{h \cdot \sin x \cdot \sin(x+h)}$$

$$\lim_{h\to 0} 2\cos\left(\cancel{2x}\left(\frac{2x+h}{2}\right)\right) \lim_{h\to 0} \sin\left[\frac{-h}{2}\right] \cdot \frac{1}{h - \sin x \cdot \sin(x+h)}$$

$$\lim_{h\to 0} -\sin\frac{h}{2} \times \frac{1}{2} \times 2\cos\left[\frac{2x+h}{2}\right]$$

$$= -\frac{1}{2} \times 2\cos\frac{(2a+0)}{2} = -\frac{\cos a}{\sin^2 a}$$

$$= -\cot a \operatorname{cosec} a$$

sec x

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)(\cos a \cos x)}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(h+a)}{h \times \cos a \cos(a+h)}$$

$$= \frac{-1}{2} \; \frac{-2 \sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= \frac{1}{2} \times 2 \; \frac{\sin a}{\cos a + \cos a}$$

$= \tan a \sec a.$

Q2 If $f(x) = 4x+1$, $x \le 2$
$= x^2+5$ $x > 0$; at $x=2$ then
find the diff or not

→ $Df(2^-) = \lim\limits_{x \to 2^-} \dfrac{f(x) - f(2)}{x-2}$

$= \lim\limits_{x \to 2^-} \dfrac{4x+1-(4 \times 2+1)}{x-2}$

$= \lim\limits_{x \to 4} \dfrac{4x+1-9}{x-2}$

$= \lim\limits_{x \to 2} \dfrac{4(x-2)}{(x-2)} = 4$

$Df(2^-) = 4$

RHD =

$Df(2^+) = \lim\limits_{x \to 2^+} \dfrac{x^2+5-9}{x-2}$

~~$= \lim\limits_{x \to 2}$~~ $\dfrac{x^2-4}{x-2}$

$= \lim\limits_{x \to 2^+} \dfrac{x^2+2x-2x^2}{x-2}$

$= 2+2 = 4$

$Df(2^+) = 4$

RHD = LHD

Q3. If $f(x) = 4x+7 ; x < 3$

$= x^2 + 3x + 1 , x \geq 3$ at $x = 3$ then

find f is diff or not.

RHD = $\lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$

$= \lim_{x \to 3} \frac{x^2 + 3x + 1 - (3^2 + 3 + 3 + 1)}{x - 3}$

$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$

$= \lim_{x \to 3} \frac{x^2 + 6x - 3x - 18}{x - 3}$

$= \lim_{x \to 3^+} \frac{x(x-(6) - 3(x - 6)}{x - 3}$

$= \lim_{x \to 3^+} \frac{(x + 6(x-3)}{(x - 3)} = 3 + 6 = 9$

$0 f(3^+) = 3$

LHD = $\cancel{0f(3^-)}$

$= \lim_{x \to 3^-} f(x) - f(3)$

$= \lim_{x \to 3} \frac{4x - 12}{x - 3}$

$= \lim_{x \to 3} \frac{4(x - 3)}{(x - 3)}$

$0 f(3^+) = 4$

RHD = LHD

f is not diff at $x = 3$

~~off~~

~~$Of'(2^+) = 8$~~

Q4.

RHD :

$$Of(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} - \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2} + \frac{(3x + 2)(x - 2)}{(x - 2)}$$

$$= 3 + 2 + 2 = 8$$

$$of(2^+) = 8$$

$$Of(2)^- = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2} \cdot \frac{8(x - 2)}{x - 2)}$$

$$of(2)^- = 8 = 8$$

LHD = RHD

f is diff at $x = 2$

19/2/19

Prac 3

Topic : Application of Derivative

Find the intervals in which function is increasing or decreasing.

$f(x) = x^3 - 5x - 11$

$f(x) = x^2 - 4x$

$f(x) = 2x^3 - 20x + 4$

$f(x) = x^3 - 27x + 5$

$f(x) = 69 - 24x - 9x^2 + 2x^3$

Find the intervals in which function is concave upwards.

$y = 3x^2 - 2x^3$

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$y = x^3 - 27x + 5$

$y = 69 - 24x - 9x^2 + 2x^3$

$y = 2x^3 + x^2 - 20x + 4$

Solution.

$fx = x^3 - 5x - 11$

$\therefore f(x) = 3x^2 - 5$

$\therefore f$ is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

+   −   +

$-\sqrt{5/3}$ $\sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$ $\quad x \in (\sqrt{5/3}, \sqrt{5}$

$-\sqrt{5/3} \quad \sqrt{5/3}$

2. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, -2)$

3. $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2 - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore (x+2)(3x-5) > 0$

$+ \quad - \quad +$ $\quad x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

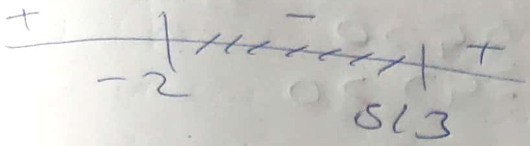
$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$



$x \in (-2, 5/3)$

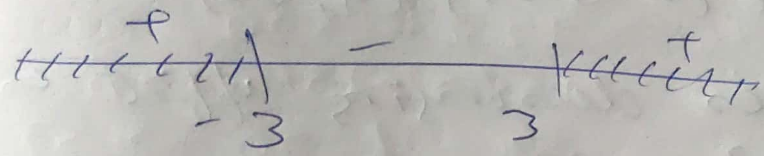
$f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

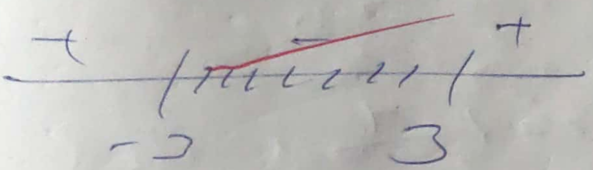
$\therefore (x-3)(x+3) > 0$



$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$



$\therefore x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

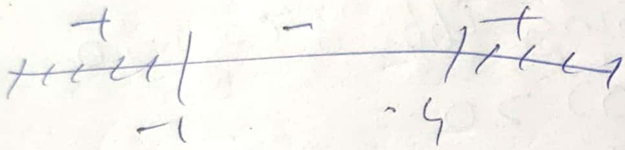
$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$(x-4)(x+1) > 0$



$\therefore x \in (-\infty, -1) \cup (4, \infty)$

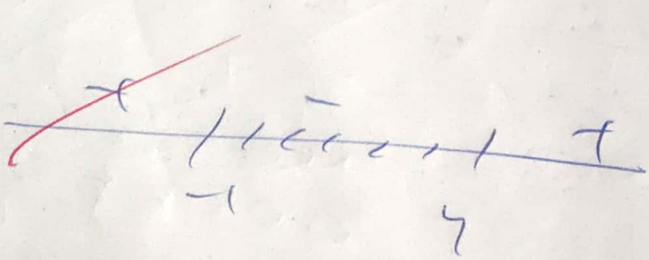
and f is decreasing iff f is ca

$6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x(x-4) + 1(x-4) < 0$

$(x-4)(x+1) < 0$



$x \in (-1, 4)$

$y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$2(3 - 6x)$

$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$x \in (1/2 ; \infty)$

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x - 2) - 1(x - 2) > 0$

$\therefore (x - 2)(x - 1) > 0$

$+ \quad - \quad +$

$1 \quad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

3. $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$x > 0$

$x \in (0, \infty)$

4. $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 - x > 3/2$

$\therefore x \in (3/2, \infty)$

5. $y = 2x^3 + x^2 - 20x + 4$

$\Rightarrow f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff f''

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x > -1/6$

$\therefore f''(x) > 0$

then is ~~AEO~~ interval must

A1 19/12/19

Prac-4

Topic: Application of derivative and newton's method.

Find max and min values of following functions.

1. $f(x) = x^2 + \frac{16}{x^2}$

2. $f(x) = 3 - 5x^3 + 3x^5$

3. $f(x) = x^3 - 3x^2 + 1$ in $[\frac{1}{2}, 4]$

4. $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Find the root of following eq by newtons method (Take 4 iteration only) correct upto 4 decimal.

1. $f(x) = x^3 - 3x^2 - 55x + 9.5$

2. $f(x) = x^3 - 4x - 9$ in $[2, 3]$

3. $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2)$

Q1. $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has min value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has min value at $x = -2$ $\therefore$ Function reaches min value at $x = 2$, and $x = -2$

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2. $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider ; $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore 15x^2 = 15x^4$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0 \therefore$ f has min value at $x = 1$

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0 \therefore$ f has max value at $x = -1$

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has max value 5 at $x = -1$ and has the min value 1 at $x = 1$

3. $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x - 2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$, or $x = 2$

$\therefore f''(x) = 6(0) - 6$

$= \cancel{-6 < 0}$

$= -6 < 0$ $\therefore$ f has max value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has min value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has max value 1 at $x = 0$ and f has min value -3 at $x = 2$

4. $f(x) = 2x^2 - 3x^2 - 12x + 1$

$\therefore f(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$\therefore x^2 - x - x = 0$

$\therefore x^2 \cancel{+ x} - 2x - 2 = 0$

$\therefore x(\cancel{x+1}) - 2(x - 1) = 0$

$(x - 2)(x + 1) = 0$

$x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has min value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

Scanned with CamScanner

$\therefore f''(-1) = 12(-1) - 6$
$= -12 - 6$
$= -18 < 0$

$\therefore$ f has max value
at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$
$= -2 - 3 + 12 + 1$
$= 8$

$\therefore$ f has max value
8 at $x = -1$ and
f has max value
-19 at $x = 2$

Q2. $f(x) = x^3 - 9x^2 - 55x + 9.5$

$x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

by newtons Ex method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5 / 55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 55(0.1727) + 9.5$
$= 0.0051 - 0.0895 - 9.4985 + 9.5$
$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= 0.0895 - 1.0362 - 55$

020

$= -55.9467$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.022 - 55$

$= -55.9393$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.00011/55.9393$

$= 0.1712$

$\therefore$ The root of the equation is

0.1712

$f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ by newtons method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 0.5961 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= (19.7158 - 10.806 - 9 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4$

$= 17.8943$

$x_4 = 2.7015 + 0.0901 / 17.8943 = 2.7015 + 0.0050$

$= 2.7065$

3. $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 + 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approx by Newtons method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = \cancel{x} - f(x_2) / f'(x_2)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4267 - 15.77 + 17$

$= 66755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2)/f'(x_2)$
$= 1.6592 + 0.0204/7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.28417 - 5.9824 - 10$
$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$
$= 1.618 + \frac{0.0004}{7.6977} = \underline{\underline{1.6618}}$

Practical 5.

Topic: Integration.

Q1. Solve the following:

1. $\int \frac{dx}{\sqrt{x^2+2x-3}}$

2. $\int (4e^{3x}+1)\,dx$

3. $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

4. $\int \frac{x^2+3x+4}{\sqrt{x}}\,dx$

5. $\int t^7 \sin(2t^4)\,dt$

6. $\int \sqrt{x}\,(x^2-1)\,dx$

7. $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

8. $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

9. $\int e^{\cos^2 x} \sin 2x\,dx$

10. $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

$\int \frac{1}{x^2+2x-3} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$\#\ a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx$

Substituting put $x = t$

$dx = \frac{1}{t} \times dt$ where $t = 1$ $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt.$

$\therefore \ln \cancel{(|t+\sqrt{t^2-4}|)}$

$t = x+1$

$\therefore \ln (|x+1+\sqrt{(x+1)^2-4}|)$

$\therefore \ln (|x+1+\sqrt{x^2+2x-3}) + C$

$\int (4e^{3x}+1)\,dx$

(2) $= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$ # $\int e^{ax}\,dx = \frac{1}{a} e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

(3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

(4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}} dx$

split the dinominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2} dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

(8) $\int \sqrt{5}\,(x^2 - 1) dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2}\, x^2 - \sqrt{x}^{1/2} dx$

$= \int x^{5/2} - x^{1/2} dx$

$$= \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = 2x^3$$

$$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2}{3}$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$$

$$\cancel{\int \frac{\cos x}{3\sqrt{\sin(x)^2}}\,dx}$$

$$\cancel{2\int \frac{\cos x}{\sin x^{2/3}}\,dx}$$

$\sqrt{x}\ (x^2+1)\,dx$

$= \int \sqrt{x}\ x^2 - \sqrt{x}\ dx$

$= \int x^{5/2} - x^{4/2}\,dx$

$= \int x^{5/2}dx - \int x^{1/2}dx$

$= I_1 = \dfrac{x^{5/2+1}}{5/2+1} = \dfrac{x^{7/2}}{7/2} = \dfrac{2x^{7/2}}{7}$

$= \dfrac{2x^3\sqrt{x}}{7}$

$= I_2 = \dfrac{x^{1/2+1}}{1/2+1} = \dfrac{x^{3/2}}{3/2} = \dfrac{2x^{3/2}}{3/2}$

$= \dfrac{2\sqrt{x}^3}{3}$

$= \dfrac{2x^3\sqrt{x}}{7} + \dfrac{2\sqrt{x}^3}{3} + c$

$\int \dfrac{\cos x}{3\sqrt{\sin x x^2}}\,dx$

$= \int \dfrac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin(x)$

$$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)} \, dt$$

$$= \frac{1}{t^{2/3}} \, dt$$

$$\int \frac{1}{t^{2/3}} \, dt = \frac{-1}{(2/3 - 1)t^{2/3-1}}$$

$$= \frac{-1}{-1/3 \, t^{2/3-1}} = \frac{1}{1/3 \, t^{-1/3}} = \frac{t^{1/3}}{1/3}$$

$$= \underline{3(t^{1/3})}$$

$$= 3\sqrt[3]{t}$$

Return sub $t = \sin(x)$

(5)

$\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4}\,dx$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)}{8}\,du$

sub t^4 with $\frac{u}{2}$

$= \int \frac{u/2 \times \sin(u)}{8}\,du$

$= \int \frac{u \times \sin(u)}{2} / 8\,du$

$= \int \frac{u \times \sin(u)}{16}\,du$

$= \frac{1}{16} \int u \times \sin(u)\,du$

$= \frac{1}{16} \int (u \times (\cos)(u) + \int -\cos(u)\,du$

$= \frac{1}{16} \times (u \times (\cos(u)) + \sin(u)$

$= 1/16 \times (2t \times (-\cos(2t+4)) + \sin(2t+4)$

$= \frac{-t^4 \times \cos(2t+4)}{8} + \frac{\sin(2t+4)}{16} + C$

IX $\int \frac{x^3+3x+4}{\sqrt{x}} dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}} dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

$$\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$$

put $x^3-3x^2+1 = dt$

$$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dt$$

$$= \int \frac{1}{x^3+3x^2+1} \times 1/3\,dt$$

$$= \int \frac{1}{3(x^3-3x^2+1)}\,dt = \int \frac{1}{3t}\,dt$$

$$= 1/3 \int 1/t\,dt \qquad \int 1/x\,dx = \ln|x|$$

$$= 1/3 \times \ln(t) + c$$

$$= 1/3 \times \ln(x^3-3x^2+1) + c$$

AK
02/01/2020

dx

Prac-6.

Topic: Application of integration and numerical integration.

Q1. Find the length of the following curve.

1. $x = t\sin t$, $y = 1-\cos t$ $[t \in (0, 2\pi]]$
for t belongs to $[0, 2\pi]$

$\rightarrow$ $$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = t - \sin t$

$\frac{dx}{dt} = t - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = t - \sin t$

$\frac{dy}{dt} = \sin t$

$$L = \int_0^{2\pi} \sqrt{(t - \cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt \qquad \sin^2\frac{t}{2} = \frac{1-\cos t}{2}$$

Q58.

$$= \int_0^{2\pi} 2\sin\frac{t}{2}\, dt$$

$$= \left[-4\cos\left(\frac{t}{2}\right)\right]_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$= 4 + 4$

$= 8$

2. $y = \sqrt{4 - x^2} \qquad x \in [-2, 2]$

$$2 = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4 - x^2}}\, dx$$

$$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$$

$= 2\pi$

820

3) $y = x^{3/2}$ in $[0, 4]$

$\rightarrow f'(x) = \frac{3}{2} x^{1/2}$

$(f'(x))^2 = \frac{9}{4} x$

$$l = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_0^x \sqrt{1 + 9/4\, x}\, dx$$

put $u = 1 + \frac{9}{4} x$, $du = \frac{9}{4}\, dx$

$$l = \int^{1 + 9/4 x} \frac{4}{9} \sqrt{u}\, du = \left[\frac{4}{9} \cdot \frac{2}{3} (u^{3/2})\right]^{1 + \frac{9}{4} x}$$

$$= \frac{8}{27} \left[\left(1 + \frac{9x}{4}\right) - 1\right]$$

$x = 3\sin t \; , \; y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t$$

$$\frac{dy}{dt} = -3\sin t$$

$$L = \int_{0}^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_{0}^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_{0}^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_{0}^{2\pi} 3\,\cancel{dt}$$

$$= 3\int_{0}^{2\pi} 1\, dt$$

$$= 3\,[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$L = 6\pi$ units.

5. $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = (1, 2)$

$\rightarrow \frac{dy}{dx} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int \frac{y^4 + 1}{2y^2}\, dy$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\cancel{\frac{y^3}{3}} - \frac{y^{-2}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$ units.

Q2.

1. $\int_{0}^{2} e^{x^2} dx$ with $n=4$

$\int_{0}^{2} e^{x^2} dx = 16.452$

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

by Simpsons rule

$\int_{0}^{2} dx = \frac{1/2}{3} (y_0 + + 4y_1 + 2y_2 + 4y_3 + y_4)$

$= \frac{1/2}{3} (e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5+2)^2})$

$= 17.353$

2. $\int_{0}^{4} x^2 dx$ $n=4$

$\Delta x = \frac{4-0}{4} = 1$

$\int_{a}^{b} f(x) dx = \frac{\Delta x}{3} [y_0 + 4y_2 + 4y_3 + y_4]$

$= \frac{64}{3} = 21.33$

030

Q3. $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ n=6

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	π/8	2π/18	3π/18	4π/18	5π/18
y	0	0.416	0.58	0.70	0.80	0.875

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{\Delta x}{3} (y_0 + 4(y_1 + y_2 + y_3) + 2(y_2 + y_4) + y_6)$

$= \frac{\pi/18}{3} (0 + 4(0.4167 + 0.707 + 0.875) +$

$= 2(0.54 + 0.801) + 0.930$

$= 0.687$

AK
09/01/2020

prac - 7.

Topic: Solve the following differential eq (Differential eq)

$x \frac{dy}{dx} + y = e^x$

div dividing by x

$\frac{dy}{dx} + y/x = \frac{e^x}{x}$

by comparing with

$\frac{dy}{dx} + p(x)y = Q(x)$

If : $e^{\int p(x)dx}$

$e^{\int 1/x \, dx}$ $\quad \int \log x \, dx = x$

3)

Q2 $e^{x}\frac{dy}{dx} + 2e^{x}y = 1$

$\frac{dy}{dx} + \frac{2e^{x}y}{e^{x}}$ y $\frac{1}{e^{x}}$

$\frac{dy}{dx} + 2y = \frac{1}{e^{x}}$

$\frac{dy}{dx} + 2y = e^{-x}$

$p(x) = 2 \quad Q(x) = e^{-x}$

$\int p(x)\,dx$

$If = e^{\int 2dx}$

$= e^{2x}$

$y\,(I.f) = \int Q(x)\,(I.f)\,dx\,dx$

$y \cdot e^{2x} = \int e^{-x} + 2x\,dx + c$

$= \int e^{x}\,dx + c$

$y \cdot e^{2x} = e^{x} + c$

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

$$P(x) = 2(x) \qquad Q(x) = \frac{\cos x}{x^2}$$

$$If = e^{\int P(x)\,dx}$$

$$= e^{\int 2 \cdot 1/x\,dx}$$

$$e^{2 \ln x}$$

$$= \ln x^2$$

$$If = x^2$$

$$y(If) = \int Q(x)(If)\,dx + c$$

$$= \int \frac{\cos x}{\cancel{x^2}} - x^2\,dx + c$$

$$= \int \cos x + c$$

$$x^2 y = \sin x + c$$

4. $x\dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$

$\dfrac{dy}{dx} + \dfrac{3y}{x} = \dfrac{\sin x}{x^2}$ ($\div$ by x on both sides)

$p(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int p(x)\,dx}$

$= e^{\int 3/x\,dx}$

$= e^{3/x}\ x$

$= e^{\ln x^3}$

$If = x^3$

$y(If) = \int Q(x)(IF)\,dx + c$

$= \int \dfrac{\sin x}{x^3}\, x^3\,dx + c$

$= \int \sin x\,dx + c$

$x^3 y = -\cos x + c$

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$p(x) = 2$ $\quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$$\begin{aligned} If &= e^{\int p(x)\,dx} \\ &= e^{\int 2\,dx} \\ &= e^{2x} \end{aligned}$$

$$\begin{aligned} y\,(If) &= \int Q(x)\,(If)\,dx + c \\ &= \int 2x^{-x} \times e^{2x}\,dx + c \\ &= \int 2x\,dx + c \\ &= \int 2x\,dx + c \end{aligned}$$

$$ye^{2x} = x^2 + c$$

6. $\sec^2 x \cdot \tan y \, dx + \sec^2 y + \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dx$

$$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$$

$$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dx}{\tan y}$$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan y \cdot \tan y| = c$

$\tan x \cdot \tan y = c^x$

7. $\frac{dy}{dx} = \sin^2 (x - y + 1)$

put $x - y + 1 = v$

differening on both the sides

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$y=0$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x+c$$

8. $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+4y+6}$$

put $2x+3y = v$

$$2 + \frac{3dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2 \qquad \frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$\int \frac{v+1}{v} \, dx + \int \frac{1}{v+1} \, dv = 3x$$

$$v + \log|x| = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log(2x+3y+1) + c$$

AB
15/01/2020

Topic: Euler method.

$\frac{dy}{dx} = y + e^{x} - 2$ $\quad y(0) = 2 \quad h = 0.5$ find $y(2)$

$\frac{dy}{dx} = 1 + y^{2}$ $\quad y(0) = 0, \quad h = 0.2$ find $y(1)$

3. $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0) = 1, \; h = 0.2$ find $y(1)$

4. $\frac{dy}{dx} = 3x^{2} + 1$ $\quad y(1) = 2$, find $y(2)$ for $h = 0.5 \quad h = 0.25$

5. $\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y(1) = 1$ find $y(1.2)$ with $h = 0.2$

1. $\frac{dy}{dx} = y + e^{x} - 2$

$f(x,y) = y + e^{x} - 2$, $y_0 = 2$, $x_0 = 0$

$h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.48	3.57
2	1	3.75	4.29	5.361

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.36	7.8.4	9.28305
4	2	9.2		

$\therefore$ by Euler's formula.

$y(2) = 9.2837$

$\frac{dy}{dx} = 1+y^2$

$f(x,y) = 1+y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

using Euler's iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.104	0.48
2	0.4	0.4	1.1665	0.6
3	0.6	0.6	1.4113	0.9
4	0.8	0.9	1.853	1.2
5	1	1.29		

$\therefore y(1) = 1.2942$

4. $\frac{dy}{dx} = 3x^2+1$, $y(1) = 2$ find $y(x)$; $h = 0.5$

$y_0 = 2$ $x_0 = 1$ $h = 0.25$

by Euler formula.

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.8
2	2	7.8		
3	2.5			

$y(2) = 7.87$

when $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	5.6	4.42
1	1.25	3	7.7	6.35
2	1.5	4.4	10.1	8.90
3	1.75	6.3		
4	2	8.9		

$y(2) = 6.3594$

$\frac{dy}{dx} = \sqrt{xy} + 2, \quad y(1)=1, \quad h=0.2$

$x_0 = 1 \quad y_0 = 1, \quad h = 0.2$

by Euler rule

$y_{n+1} = y_n + h\,f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6	1.38	1.6
2	1.4	1.8	1.62	1.8
3	1.6	2.2		2.2

$y(1.2) = 1.6$

Practical no - 9.

Aim: Limits and partial order derivative

Q1. Evaluate the following

1. $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

2. $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2+y^2-z^2}{x^3-x^2y^2}$

3. $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

Q2. Find f_x, f_y for the following.

1. $f(x,y) = xye^{x^2+y^2}$

2. $f(x,y) = e^x \cos y$

3. $f(x,y) = x^3y^2 - 3x^2 + y^3 + 1$

III. Using def find values of f_x; f_y, at $(0,0)$ for $f(x,y) = \dfrac{2x}{1+y^2}$

IV. Find all second order partial of f also verify whether $f_{xy} = f_{yx}$

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2. $f(x,y) = \frac{y^2 - xy}{x^2}$

3. $f(x,y) = x^3 + 3x^2y^2 + \log(x^2+1)$

4. $f(x,y) = \sin(xy) + e^{x+y}$

5. Find the linearization of $f(x,y)$ at given point

1. $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$
2. $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$
3. $f(x,y) = \log x + \log y$ at $(1,1)$

1. $\lim_{(x,y)\to(-4,1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

at $(-4, -1)$ denominator $\neq 0$

$\therefore$ by applying limit.

$= \frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1) + 5}$

$= \frac{-64 + 3 + 1 - 1}{4 + 5}$

$= \frac{-61}{9}$

ii. $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

at $(2,0)$; denominator $\neq 0$

by applying the limit.

$= \frac{(0+1)((2)^2 + 0 - 4(2))}{2+0}$

$= \frac{1(4 + 0 - 8)}{2}$

$= \frac{-4}{2}$

$= -2$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$$

At (1,1,1) denom = 0

$$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2-z^2}{x^3-x^2y^2}$$

$$= \lim_{(x,y,z)\to(x,1,1)} \frac{-(x+yz)}{x^2}$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

on applying limit

$$= \frac{1+1(1)}{(1)^2}$$

$$= 2$$

Q2

1. $f(x,y) = xy\, e^{x^2+y^2}$

$\therefore f_x = \frac{\partial}{\partial x} (f(x,y))$

$= \frac{\partial}{\partial x} (xy e^{x^2+y^2})$

$= y e^{x^2+y^2}(2x)$

$\therefore f(x) = 2xy e^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y} \cdot (f(x,y))$

$= x e^{x^2+y^2}(2y)$

$\therefore f(y) = 2yx e^{x^2+y^2}$

ii. $f(x,y) = e^x \cos y$

$f(x) = \frac{\partial}{\partial x} f(x;y))$

$= \frac{\partial}{\partial} (e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = -e^x \sin y$

3. f(
f

$f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f(x) = 3x^2y^2 - 6xy$

$fy = \frac{\partial}{\partial y}(f(x:y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore fy = 2x^3y - 3x^2 + 3y^2$

3.

1. $f(x,y) = \frac{2x}{1+y^2}$

$f(x) = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{\cancel{1+y^2\frac{\partial}{\partial x}(2x)} - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2(1+y)^2}{\cancel{(1+y^2)}(1+y)^2}$

$= \frac{2}{1+y^2}$

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Q4

$$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= x^4\left(\frac{\partial}{\partial x}(-x^2 - y - 2xy^2 + 2x^2 y)\right) -$$

$$(x^2 - y - 2xy^2 + 2x^2y)\frac{\partial}{\partial x}(x^4)$$

$$= \frac{\partial}{\partial y} \frac{(2y - x)}{x^2}$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial xy} \frac{(-x^2y - 2xy^2 + 2x^2y^2)}{x^4}$$

$$= \frac{\partial}{\partial x} \frac{(2y - x)}{x^2}$$

$$= \frac{x^2 \frac{\partial}{\partial x}(2y - x) - 2y - x) \frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$= f_{xy} = f_{yx}$$

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III $f(x,y) = \sin(xy) + e^{x+y}$

$fx = y\cos(xy) + e^{x+y}$
$= y\cos(xy) + e^{x+y}$

$fxx = \frac{\partial}{\partial x}\left(y\cos(xy) + e^{x+y}\right)$

$fyy = \frac{\partial}{\partial y}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy)(x) + e^{x+y}$ .. ①

$= -x^2\sin(xy) + e^{x+y}$ — ②

$fxy = \frac{\partial}{\partial y}\left(y\cos(xy) + e^{x+y}\right)$

$= -y^2\sin(xy) + \cos(xy) + (e^{x+y})$

$fyx = \frac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(\cos xy) + e^{x+y}$

Scanned with CamScanner

$f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad = \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_b(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y+1)$

$= \sqrt{2} + \frac{1}{2}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$fx = 0 - 1 + y\cos x$. fy at $\left(\frac{\pi}{2}, 0\right) = \sin\frac{\pi}{2} = 1$

$l(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$

$= 1 - \cancel{\frac{\pi}{2}} - x + \frac{\pi}{2} + y$

$= 1 - x + y$

iii) $f(x,y) = \log x + \log y$

$f(1,1) = \log(1) + \log(1) = 0$

$f(x) = \frac{1}{x} + 0 \qquad fy = 0 + \frac{1}{y}$

fy at $(1,1) = 1 \qquad fy$ at $(1,1) = 1$

$\therefore$ $l(x,y) = f(a,b) + fx(1,1)(x-a) + fy(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= (x-1) + (y-1)$

$= x + y - 2$

Practical no 10

Topic: Derivatives, vectors, max, min
Tangents.

$f(x,y) = x + 2y - 3 \quad a = (1,-1) \quad u = 3i - i$

$|u| = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

$f(a+hu) = f(1,1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$fa = f(1,-1) = (1) + 2(1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f\left(1 + \frac{3}{\sqrt{10}}\right), \left(1 - \frac{h}{\sqrt{10}}\right)$

$1 + \frac{3}{10} \quad -2 - \frac{2\sqrt{h}}{10} \quad -3$

$D_u f(a) = \lim_{h \to 0} f(a+hu) - f(a)$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

2. $f(x) = y^2 - 4x + 1 \quad a(3,4) \quad u = i + 5j$

$u = i + 5j$ is not a unit vector

$|u| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left[\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right]$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a + hu) = f(3,4) + h\left[\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right]$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{3h}{\sqrt{26}} + 5$

$= \frac{25h}{26} + \frac{36}{\sqrt{26}}$

$2x+3y$ $a=(1,2)$; $u=3i+4j$

$u=3i+4j$

$|u|=\sqrt{(3)^2+(4)^2}=\sqrt{25}=5$

unit vector along u is $\frac{u}{|u|}=\frac{1}{5}(3,4)$

$=\left(\frac{3}{5},\frac{4}{5}\right)$

$f(a)=f(1,2)=2(1)+3(2)=8$

$f(a+hu)=f\left((1,2)+h\left(\frac{3}{5},\frac{4}{5}\right)\right)$

$=f\left(\left(1+\frac{3h}{5}\right),2+\frac{4h}{5}\right)$

$\therefore\ 2+\frac{6h}{5}+6+\frac{12h}{5}$

$=\frac{18h}{5}+8$

$=\frac{18h+8-8}{h}$

$=\frac{18}{5}$

$(4,5)$

Q3:

1. $x^2\cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1,0) \therefore x_0 = 1 \quad y_0 = 0$

eq of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 \cdot (-\sin 0) + e^0 1$

$= 0 + 1 =$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

eq of normal

$= ax + by + c = 0$

$= bx + ay + c = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

$x^2 + y^2 - 2x + 3y + 2 = 0$. at $(2, -2)$

$f(x) = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$fy = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2) \therefore x_0 = 2, y_0 = -2$

$f(x)(x_0, y_0) = 2(2) - 2 = 2$

$fy(x_0, y_0) = 2(-2) + 3 = -1$

Eq - of normal at $(-2, 5, -2)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$= \frac{x - 1}{-2} = \frac{y + 1}{5} = \frac{z - 2}{-2}$$

Q5

1. $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 2y - 3x - y$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \; \ldots \; (1)$

$f_y = 0$

$2y - 3x - y = 0$

$2y - 3x = 4$

multiply eq 1 with 2

$4x - 2 = -4$

$2y - 3x = 4$

$x = 0$

$20 - y = -2$

$= 40 - 12$

$y = 4$

$f(x,y) = 2x^3 + 3x^2 y - y^2$

$f(x) = 8x^3 + 6xy$

$fy = 3x^2 - 2y$

$fx = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y^2 = 0$

$4x^2 + 3y = 0 \ldots (1)$

$fy = 0$

$\therefore 3x^2 - 2y = 0$

$- 12x + 9y = 0$

$\cancel{12x^2} - 8y = 0$

$y = 0$

Sub value of y is eq^n (1)

$4x^2 + 8(0) = 0$

$2 = fxy = 24x^2 + 6x6$

$t = fyy = 0 - \cancel{2 = -2}$

$S = fxy = \cancel{6x - 0} = 6x = 6(0) = 0$

$f(x, y)$ at $(0,0)$

$2(0)4 + 3(0)^2 - (0) - (0)$

$= 0 + 0 - 0$

$= 0$

iii

$f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad 2x + 2 = 0$

$x = \frac{-2}{2} \quad = x = -1$

$f_y = 0$

$-2y + 8 = 0$

$y = \frac{-8}{-2} \quad 4 \quad = y = 4$

$\therefore$ critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$= (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 32 - 70 = \underline{-33}$