

Steps for PCA Analysis:

Suppose data set is like:

x	y
2	1
3	5
4	5
5	6
6	7
7	8

1. Find the zero mean:

Mean of $x = \mu_x$, Mean of $y = \mu_y$

$\mu_x = 4.5$ and $\mu_y = 5.33$

Then,

x	y	$x - \mu_x$	$y - \mu_y$
2	1	-2.5	-4.333333
3	5	-1.5	-0.333333
4	5	-0.5	-0.333333
5	6	0.5	0.666667
6	7	1.5	1.666667
7	8	2.5	2.666667

Now A=

-2.5	-4.333333
-1.5	-0.333333
-0.5	-0.333333
0.5	0.666667
1.5	1.666667
2.5	2.666667

2. Find the covariance matrix of A.

$$\text{cov}A = \frac{1}{m} (A * A^T)$$

Here m is number of rows

for this example, it is,

$$\text{cov}A = \begin{bmatrix} 3.5 & 4.2 \\ 4.2 & 5.9 \end{bmatrix}$$

3. Now we have to find the PC scores (Eigen value) & PC loadings (Eigen vector). Here are the steps with equations

Equation for Eigen vector of the matrix $covA$ is

$$covA.\bar{X} = \lambda.\bar{X}$$

Here,

$\bar{X} = PC \text{ loadings (eigen vector)}$

$\lambda = PC \text{ scores (eigen value)}$

Now go with the steps to find Eigen vectors of a matrix

- i. Calculate: $A - \lambda I$

$$\text{Here } \begin{bmatrix} 3.5 - \lambda & 4.2 \\ 4.2 & 5.9 - \lambda \end{bmatrix}$$

- ii. Now $\det(A - \lambda I) = 0$ ————— (eq1)

$$\det \begin{bmatrix} 3.5 - \lambda & 4.2 \\ 4.2 & 5.9 - \lambda \end{bmatrix} = 0$$

- iii. From eq1 we will get the value of λ_1 & λ_2

$$\text{Here } \lambda_1 = 0.32 \text{ \& } \lambda_2 = 9.05$$

- iv. Now $[covA - \lambda I][\bar{X}] = \bar{0}$ ————— (eq2)

As $\lambda_2 > \lambda_1$, we will take λ_2 as the best PC score

So for λ_2 the eq2 is

$$\begin{bmatrix} -5.5 & 4.2 \\ 4.2 & -3.15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

From this we will get two equations to solve x_1 and x_2 and get the Eigen vector \bar{X}

From above we get

$$-5.5 x_1 + 4.2 x_2 = 0 \text{ ————— (eq3)}$$

$$4.2 x_1 - 3.15 x_2 = 0 \text{ ————— (eq4)}$$

Solving eq3 & eq4 we get $x_1 = 0.604$ & $x_2 = -0.797$

- v. You will find out the (PC1)Eigen vector \bar{X} for λ_2

$$\text{Here } \bar{X} = \begin{bmatrix} 0.604 \\ -0.797 \end{bmatrix}$$

4. Calculate the projected value as $PC1$

$$PC1 = A.(\bar{X})^T$$

5. Now plotting the data:

Here in graph, red line is PC1, green line is PC2 and blue dots are our data. We have observed that PC1 is the best one.

The equation to plot for PC1 is

$$0.604x_1 - 0.797x_2 = 0$$

[All calculations is approximate]

Wow this is awesome!

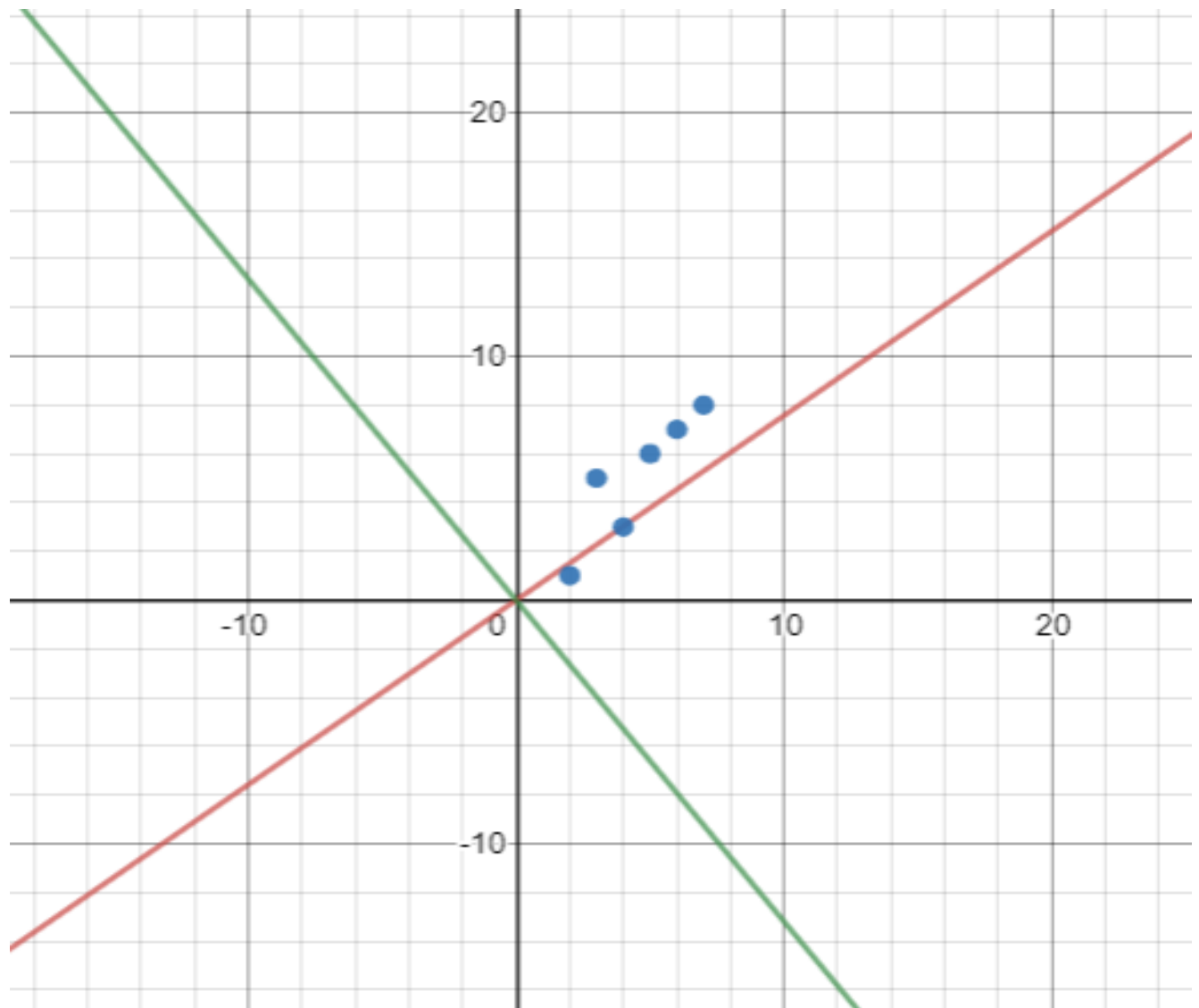


Figure 1: PCA Analysis graph.