

Steps for PCA Analysis:

Suppose data set is like:

x	y
2	1
3	5
4	5
5	6
6	7
7	8

1. Find the zero mean:

$$\text{Mean of } x = \mu_x, \text{Mean of } y = \mu_y$$

$$\mu_x = 4.5 \text{ and } \mu_y = 5.33$$

Then,

x	y	$x - \mu_x$	$y - \mu_y$
2	1	-2.5	-4.333333
3	5	-1.5	-0.333333
4	5	-0.5	-0.333333
5	6	0.5	0.666667
6	7	1.5	1.666667
7	8	2.5	2.666667

Now A=

-2.5	-4.333333
-1.5	-0.333333
-0.5	-0.333333
0.5	0.666667
1.5	1.666667
2.5	2.666667

2. Find the covariance matrix of A.

$$covA = \frac{1}{m} (A * A^T)$$

Here m is number of rows

for this example, it is,

$$covA = \begin{bmatrix} 3.5 & 4.2 \\ 4.2 & 5.9 \end{bmatrix}$$

3. Now we have to find the PC scores (Eigen value) & PC loadings (Eigen vector). Here are the steps with equations

Equation for Eigen vector of the matrix $covA$ is

$$covA \cdot \bar{X} = \lambda \cdot \bar{X}$$

Here,

\bar{X} = PC loadings (eigen vector)

λ = PC scores (eigen value)

Now go with the steps to find Eigen vectors of a matrix

i. Calculate: $A - \lambda I$

$$\text{Here } \begin{bmatrix} 3.5 - \lambda & 4.2 \\ 4.2 & 5.9 - \lambda \end{bmatrix}$$

ii. Now $\det(A - \lambda I) = 0$ ----- (eq1)

$$\det \begin{bmatrix} 3.5 - \lambda & 4.2 \\ 4.2 & 5.9 - \lambda \end{bmatrix} = 0$$

iii. From eq1 we will get the value of λ_1 & λ_2

Here $\lambda_1 = 0.32$ & $\lambda_2 = 9.05$

iv. Now $[covA - \lambda I] \bar{X} = \bar{0}$ ----- (eq2)

As $\lambda_2 > \lambda_1$, we will take λ_2 as the best PC score

So for λ_2 the eq2 is

$$\begin{bmatrix} -5.5 & 4.2 \\ 4.2 & -3.15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

From this we will get two equations to solve x_1 and x_2 and get the Eigen vector \bar{X}

From above we get

$$-5.5 x_1 + 4.2 x_2 = 0 \quad \text{--- (eq3)}$$

$$4.2 x_1 - 3.15 x_2 = 0 \quad \text{--- (eq4)}$$

Solving eq3 & eq4 we get $x_1 = 0.604$ & $x_2 = -0.797$

v. You will find out the (PC1)Eigen vector \bar{X} for λ_2

$$\text{Here } \bar{X} = \begin{bmatrix} 0.604 \\ -0.797 \end{bmatrix}$$

4. Calculate the projected value as $PC1$

$$PC1 = A \cdot (\bar{X})^T$$

5. Now plotting the data:

Here in graph, red line is **PC1**, green line is **PC2** and blue dots are our **data**. We have observed that PC1 is the best one.

The equation to plot for PC1 is

$$0.604x_1 - 0.797x_2 = 0$$

[All calculations is approximate]

Wow this is awesome!

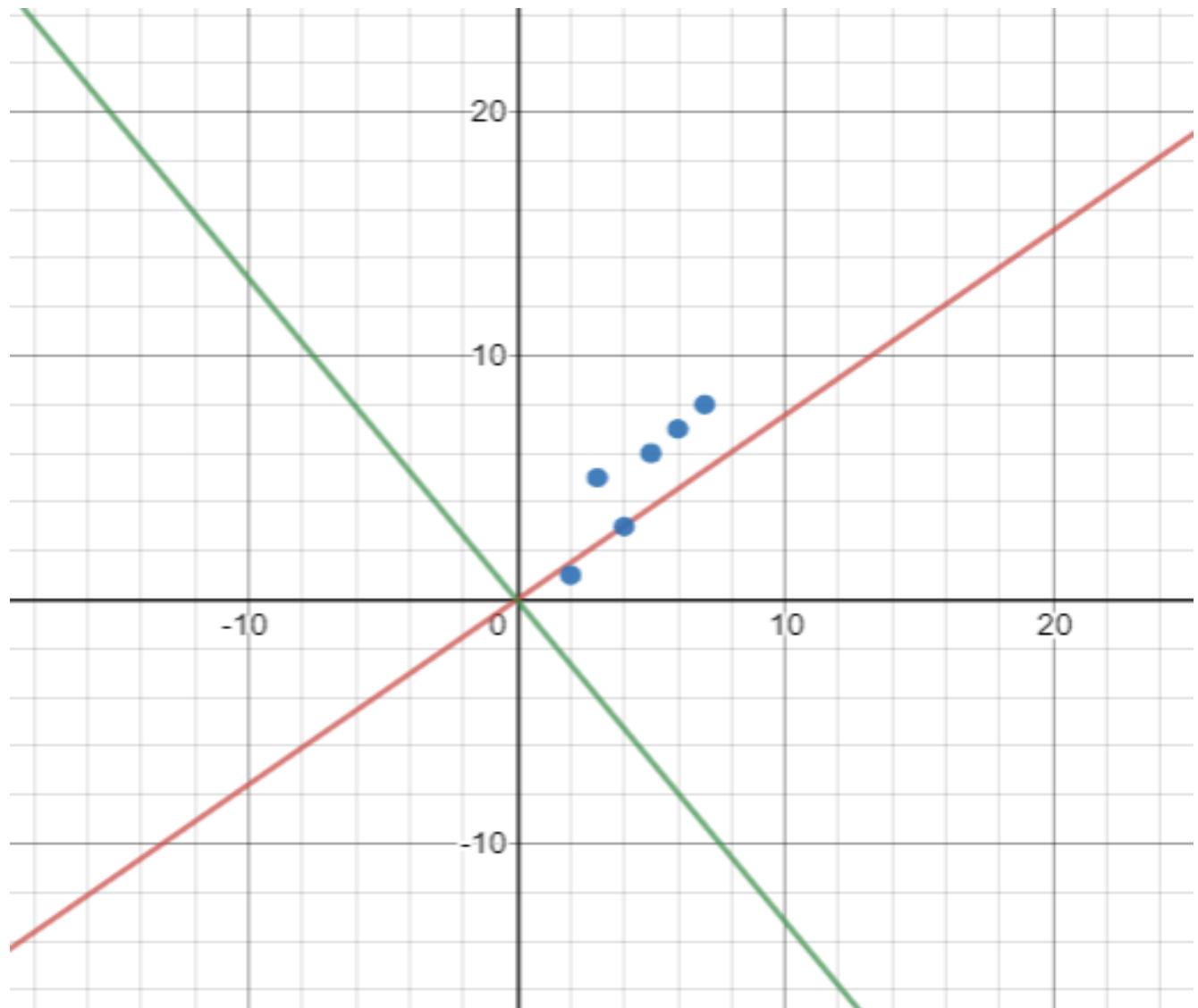


Figure 1: PCA Analysis graph.